UNIVERSITY OF CALIFORNIA, BERKELEY

DEPARTMENT OF STATISTICS

STAT-155: Game Theory

Fall 2013

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Assignment # 12

Date Given: December 02, 2013 (Monday) **Total Points: 20**
Date Due: December 09, 2013 (Monday)

1. Recall the definition of a *symmetric* many player general sum game:

   Definition: A n-player game $((X_1, X_2, \cdots, X_n), (U_1, U_2, \cdots, U_n))$ is said to be *symmetric* if

   (i) $X_1 = X_2 = \cdots = X_n =: X$ (say); and

   (ii) for any permutation π of the numbers $\{1, 2, \cdots, n\}$ we have

   $$U_{\pi(i)}\left(x_{\pi(1)}, x_{\pi(2)}, \cdots, x_{\pi(n)}\right) = U_i\left(x_1, x_2, \cdots, x_n\right)$$

   for every $x_1, x_2, \cdots, x_n \in X$.

   Show that

   (a) For a two-person zero-sum game with payoff matrix A, symmetric means A is *skew symmetric*.

   (b) For a two-person general-sum game with payoff matrices (A, B), symmetric means $A = B^T$.

   (c) Suppose for every $i, j \in \{1, 2, \cdots, n\}$ we have $U_i(x_1, x_2, \cdots, x_n) = U_j(x_1, x_2, \cdots, x_n)$ for all $x_1, x_2, \cdots, x_n \in X$. Is the game necessarily symmetric?

2. **Definition:** A graph $G := (V, E)$ is called a *complete bipartite graph* if $V = V_1 \cup V_2$ with $V_1 \cap V_2 = \emptyset$, $V_1 \neq \emptyset$ and $V_2 \neq \emptyset$ and $E = \left\{(v_1, v_2) \,\middle|\, v_1 \in V_1, v_2 \in V_2\right\}$.

   Suppose G is a finite complete bipartite graph and consider the *game of coloring* on G, that is, a many player game where the players are the vertices and moves are coloring your own vertex with utilities given by

   $$U_v(C) := \begin{cases} \sum_{u \in V} \mathbf{1}(C(u) = C(v)) & \text{if } C \text{ is proper coloring;} \\ 0 & \text{otherwise.} \end{cases}$$

   Show that G has a unique pure Nash equilibrium and find it. From here or otherwise find the *chromatic number* of G.