Solutions to Assignment 11
Stat 155: Game Theory

Question 1

This follow from the definition of safety values and Nash Equilibrium. Indeed,

$$x^T A y^* \geq \min_y x^T A y \text{ for all } x$$

$$\text{taking max on both sides, } \max_x x^T A y^* \geq \max_x \min_y x^T A y$$

$$\text{by definition of NE, } x^{*T} A y^* \geq v_I \text{ by definition of safety value}$$

The calculation for the other expression is exactly the same.

Question 2

a) The safety value for PI is 0 as his payoff is always 0 when PII plays ColumnI.

The safety value for PII is $\max_y \min_i \sum_j B_{ij} y_j$. Noticing that the first and third columns of B are same, it is enough, while calculating the safety value, to work with the smaller payoff matrix,

$$\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$$

Then, safety value $= \max_{y_1} \min\{2 - 2y_1, y_1\} = \frac{2}{3}$.

b) From PII's safety value, $(0, 0)$ and $(2, 0)$ can not be NE. Of the 3 positions left, it is easy to observe that the $(0, 1)$ in the $(2, 1)^{th}$ position of the payoff matrix is the only possible pure NE.

c) To find a mixed NE, we first observe that there are no dominated actions in the payoff matrix. Next we assign mixed strategies $(p, 1-p)$ and $(q, r, 1-q-r)$

to actions of PI and PII respectively. By equalizing payoffs, we get $2 - p = 2p, 2r = 1 - r$, which means $p = \frac{2}{3}, r = \frac{1}{3}$. So there are many mixed NE, parametrized by q, and given by the pair of mixed strategies $(\frac{2}{3}, \frac{1}{3}), (q, \frac{1}{3}, \frac{2}{3} - q)$.