Indian Statistical Institute, Delhi Centre

Linear Models and GLM

Spring 2008

Quiz # 1

Date: February 8, 2008 (Friday) **Total Points:** $2 \times 5 = 10$

Note:

- **Please write your name.**
- **There are 5 problems carrying 2 points each. Answer all of them.**
- **For each problem credit will be given only if you have answered all the parts correctly. There will be NO partial credit.**
- **Write the answers in the space provided. You can use the back of this page for rough calculations.**
- **You have 15 minutes to complete the quiz.**

Name: ____________________________

1. Consider the following models:

   (a) $y_i = \alpha + \beta x_i^2 + \varepsilon_i^2$, $1 \leq i \leq n$.
   (b) $y_i^2 = \theta + \varepsilon_i$, $1 \leq i \leq n$.
   (c) $\log \hat{p_{i,j}} = \mu + \alpha_i + \beta_j + \gamma_{i,j} + \varepsilon_{i,j}$, $1 \leq i, j \leq N$.
   (d) $y_{i,j} = \mu + \alpha_i + \beta_j + \alpha_i \beta_j + \varepsilon_{i,j}$, $1 \leq i, j \leq N$.

   In all of above the errors are i.i.d. Normal$\,(0, 1)$ random variables.

   State which of the above can be described as a *Linear Model* and which can not be.

   Ans. (a) ____________________ **(b)** ____________________
   (c) ____________________ **(d)** ____________________

2. Fill in the following table:

Symmetric Matrix	**Is it p.d./p.s.d./n.d./n.s.d./indefinite ?**
$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$	
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	
$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	
$\begin{pmatrix} 1 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 1 \end{pmatrix}$	

3. Suppose $A_{n\times n}$ be a real matrix and consider its *Singular Value Decomposition* given by:

$$A = P \begin{pmatrix} \Delta & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} Q\,,$$

   where P and Q are two orthogonal matrices and Δ is a diagonal matrix with strictly positive diagonal entries.

   Indicate if the following statements are **True** or **False**:

   (a) Δ has all the non-zero eigen-values of A. ____________________
   (b) If $A = A^T$ then Δ has all the non-zero eigen-values of A. ____________________
   (c) $PQ = QP$. ____________________
   (d) $\exists$ an orthogonal matrix $B_{n\times n}$ such that $P = BQ$. ____________________

4. Consider the following model:

$$y_i = \mu_i + \varepsilon_i, \ \ 1 \leq i \leq n\,,$$

where $\{\varepsilon_i\}_{i \geq 1}$ are i.i.d. with Normal $(0,1)$ random variables.
Indicate if the following statements are **True** or **False**:

(a) This is not a *Linear Model.* ____________________

(b) For each $1 \leq i \leq n$ the parameter μ_i is *estimable.* ____________________

(c) $R_0^2 > 0$. ____________________

(d) We can test $H_0 : \mu_1 = \mu_2 = \cdots = \mu_n$. ____________________

5. Consider the following the linear model:

$$y_{1,j} = \mu + \nu + \varepsilon_{1,j} \text{ for } 1 \leq j \leq n_1$$

$$y_{2,j} = \mu + \varepsilon_{2,j} \text{ for } 1 \leq j \leq n_2\,,$$

where $\mu, \nu \in \mathbb{R}$ are unknown parameters, and $\left\{\varepsilon_{i,j} \,\middle|\, 1 \leq i \leq 2, 1 \leq j \leq n_i\right\}$ are i.i.d. Normal $\left(0, \sigma^2\right)$, where σ^2 is not know.
Indicate if the following statements are **True** or **False**:

(a) The MLE of μ^2 is $\left(\frac{1}{n_2}\sum_{j=1}^{n_2} y_{2,j}\right)^2$. ____________________

(b) The MLE of ν is not unique. ____________________

(c) The MLE of σ^2 exists no matter what n_1 and n_2 are. ____________________

(d) Under a hypothesis $H_0 : \mu = 0$ a MLE of σ^2 exists no matter what n_1 and n_2 are. ____________________

Good Luck