Indian Statistical Institute, Delhi Centre

Linear Models and GLM

Spring 2008

Quiz # 2

Date: April 4, 2008 (Friday) **Total Points:** $2 \times 5 = 10$

Note:

- **Please write your name.**
- **There are 5 problems carrying 2 points each. Answer all of them.**
- **For each problem credit will be given only if you have answered all the parts correctly. There will be NO partial credit.**
- **Write the answers in the space provided. You can use the back of this page for rough calculations.**
- **This is a CLOSE NOTE and CLOSE BOOK examination.**
- **You have 20 minutes to complete the quiz.**

Name: ______________________________

1. Consider the following model:

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{C}\gamma + \varepsilon\,,$$

where co-ordinates of ε are i.i.d. Normal $(0, 1)$.

Indicate if the following statements are **True** or **False**:

(a) If $\mathbf{C}^T\mathbf{C}$ invertible and $\mathbf{P}_{\mathbf{X}}C = \mathbf{0}$ then γ is estimable. ____________________

(b) If γ is estimable with LSE $\hat{\gamma}$ then $\hat{\gamma}$ has a multivariate normal distribution which is singular. ____________________

(c) Suppose γ is estimable, put $\mathbf{Z} = \mathbf{Y} - \mathbf{C}\hat{\gamma}$ then $\mathbf{E}[\mathbf{Z}] = \mathbf{X}\beta$. ____________________

(d) Two-way classification model with no interaction and one observation per cell can be written as a special case of the above model. ____________________

2. Consider the following model:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk} \;\; 1 \le k \le K,\; 1 \le j \le J,\; 1 \le i \le I\,,$$

where ε_{ijk}'s are i.i.d. Normal $\left(0, \sigma^2\right)$.

Fill in the blanks:

(a) It is called the ____________________-way classification model with no interaction.

(b) The degrees of freedom for the *residual sum of square* is ____________________.

(c) The *maximum likelihood estimate* of σ^2 is given by ____________________.

(d) The linear parametric functions $\alpha_i - \alpha_{i'}$ for $1 \le i \ne i' \le I$ are ____________________.

3. Consider the following model:

$$y_{ij} = \mu + \alpha_i + \gamma_{ij} + \varepsilon_{ij} \;\; 1 \le j \le k_i,\; 1 \le i \le I\,,$$

where ε_{ij}'s are i.i.d. Normal $(0, 1)$.

Indicate if the following statements are **True** or **False**:

(a) $\alpha_1 - \alpha_2$ is estimable. ____________________

(b) If we fix an i then the observations indexed by j form an *one-way classification model.* ____________________

(c) Suppose $k_1 = 10$ then we can do multiple comparison using *Tukey's Honest Significant Difference* to test for $\gamma_{11} - \gamma_{12} = 0$ and $\gamma_{12} - \gamma_{13} = 0$. ____________________

(d) For the multiple comparison in (c) above if we use *Bonferroni's method* then we should do the one-degrees of freedom testing at a level 0.025 to achieve an experimental error rate of 5%. ____________________

4. Indicate if the following statements are **True** or **False**:

   (a) A *log-linear model* is a linear model. ____________________

   (b) A two-way classification data represented as a $I \times J$ table can be modeled by a *log-linear model*. ____________________

   (c) The estimates obtained in *logistic regression* are MLEs under appropriate model. ____________________

   (d) The following *log-linear model* is a *saturated model* ____________________

$$\log m_{ijk} = u_0 + u_{1(i)} + u_{2(j)} + u_{3(k)} + u_{12(ij)} + u_{23(jk)} + u_{31(ki)} \, .$$

5. Fill in the blanks:

   (a) For a linear model the residuals are always ______________________________ of the LSEs.

   (b) One-way classification model is a ______________________________ of two-way classification model.

   (c) The degrees of freedom for the residual sum of square from a *four-way classification model* with no interaction and one observation per cell is ______________________________ where each classification has K categories.

   (d) *Tukey's one degrees of freedom test* is a test of ______________________________.

Good Luck