Indian Statistical Institute, Delhi Centre

Advanced Analysis

Spring 2011

Quiz # 1

Date: _____   **Total Points:** 10

Note:

- Please write your name.

- There are 5 true/false statements each with 2 points. Answer all of them. Write brief reasons supporting your answers in the space provided.

- This is a **CLOSE NOTE** and **CLOSE BOOK** examination.

- You have <u>20 minutes</u> to complete the quiz.

Name: _____

1. A metric space is always Hausdorff. _____.

2. A compact topological space is necessarily Hausdorff. _____.

3. Let f be a uniformly continuous function on $\mathbb{R}$ then $f \in C_c(\mathbb{R})$. _____.

4. Suppose $(\mathbf{X}, \tau)$ is a *locally compact* Hausdorff space. Fix $a, b, c \in X$ which are distinct. Then there is a continuous function $f : \mathbf{X} \to [0, 1]$ with compact support such that $f(a) = 0$, $f(b) = 1/2$ and $f(c) = 1$. _____.

5. Consider $\mathbb{R}^2$ as a metric space with metric $\rho(\mathbf{x}, \mathbf{y}) := |x_1 - y_1| + |x_2 - y_2|$. Let $\mathcal{B}$ be the σ-algebra generated by the open sets and let $\mathbf{P}$ be a probability measure defined on it. Then $\mathbf{P}$ is *regular*. _____.