Homework # 1

Statistics 134, Bandyopadhyay, Spring 2014

1.1.3 a) If the tickets are drawn with replacement, then, as in Example 1, there are n^2 equally likely outcomes. There is just one pair in which the first number is 1 and the second number is 2, so $P(\text{first ticket is 1 and second ticket is 2}) = 1/n^2$.

b) The event (the numbers on the two tickets are consecutive integers) consists of $n-1$ outcomes: $(1,2)$, $(2,3)$, $\ldots$ $(n-1,n)$. So its probability is $(n-1)/n^2$.

c) Same as Problem 3 of Example 3. Answer: $(1-1/n)/2$

d) If the draws are made without replacement, then there are only $n^2 - n$ equally likely possible outcomes, since we have to exclude the outcomes $(1,1)$, $(2,2)$, $\ldots$, (n,n). So replace the denominators in a) through c) by $n(n-1)$.

1.1.6 a) $52 \times 52 = 2704$

b) $(52 \times 4)/(52 \times 52) = 1/13$

c) Same as b)

d) $(4 \times 4)/(52 \times 52) = 1/169$

e) $P(\text{at least one ace}) = P(\text{first card ace}) + P(\text{second card ace}) - P(\text{both cards aces})$ $= \frac{1}{13} + \frac{1}{13} - \frac{1}{169} = \frac{25}{169}$.

1.1.8 a-d) As in Example 1.1.3, the outcome space consists of n^2 equally likely pairs of numbers, each number between 1 and n. The event "the maximum of the two numbers is less than or equal to x" is represented by the set of pairs having both entries less than or equal to x. There are x^2 possible pairs of this type, so for x = 0 to n, $P(\text{maximum } \le x) = x^2/n^2$; for $x = 1$ to n,

$$\begin{aligned} P(x) &= P(\text{maximum is exactly } x) \\ &= P(\text{maximum } \le x) - P(\text{maximum } \le x-1) \\ &= x^2/n^2 \;-\; (x-1)^2/n^2 \\ &= (2x-1)/n^2. \end{aligned}$$

e) Use the form $P(1) = 1^2/n^2$, $P(2) = 2^2/n^2 - 1^2/n^2$, $P(3) = 3^2/n^2 - 2^2/n^2$, etc. to see that the sum telescopes to give $\sum_{x=1}^{n} P(x) = n^2/n^2 = 1$. So the formula for $P(x)$ results in a probability distribution.

Remark. It follows that $\sum_{x=1}^{n}(2x-1) = n^2$. In other words, the sum of the first n odd numbers is n^2, a fact which you can check in other ways.

1.3.4 a) Yes: $\{0,1\}$

b) Yes: $\{1\}$

c) No. This is a subset of the event $\{1\}$ but it is not identical to $\{1\}$ because the event (first toss tails, second toss heads) also is a subset of $\{1\}$.

d) Yes: $\{1, 2\}$

1.3.8 a) $A \cup B =$ "male or undeclared".
$P(A \cup B) = P(A) + P(B) - P(AB) = 0.6 + 0.4 - 0.2 = 0.8$

b) $A^c =$ "female"
$P(A^c) = 1 - P(A) = 0.4$

c) $B^c =$ "declared major"
$P(B^c) = 0.6$

d) $A^cB =$ "female and undeclared"
$P(A^cB) = P(B) - P(AB) = 0.4 - 0.2 = 0.2$

e) $A \cup B^c =$ "male or declared".
$P(A \cup B^c) = P[(A^cB)^c] = 1 - 0.2 = 0.8$

f) $A^cB^c =$ "female and declared"
$P(A^cB^c) = P[(A \cup B)^c] = 1 - 0.8 = 0.2$