Homework # 4 *Statistics 134, Bandyopadhyay, Spring 2014*

2.2.4 Let X be the number of patients helped by the treatment. Then $E(X) = 100$, $SD(X) = 8.16$ and
$P(X > 120) = P(X \geq 120.5) \approx 1 - \Phi(2.51) = .006$.

2.2.6 The number of opposing voters in the sample has the binomial (200, .45) distribution. This gives $\mu = 90$ and $\sigma = \sqrt{200 \times .45 \times .55} = 7.035$. Use the normal approximation:

a) The required chance is approximately

$$\Phi\left(\frac{90.5 - 90}{7.035}\right) - \Phi\left(\frac{89.5 - 90}{7.035}\right) = .5283 - .4717 = .0567 \text{ (about 6\%)}$$

b) Now the required chance is approximately

$$1 - \Phi\left(\frac{100.5 - 90}{7.035}\right) = 1 - \Phi(1.49) = 1 - .9319 = 0.0681 \text{ (about 7\%)}$$

2.2.10 The number of heads that Student A gets has the binomial $(200, 0.5)$ distribution, which is approximately normal with $\mu = 100$ and $\sigma = \sqrt{200 \times 0.5 \times 0.5} = 7.07$. The chance that this student gets exactly 100 heads is approximately

$$P(100 \text{ heads }) = \Phi\left(\frac{(100.5) - 100}{7.07}\right) - \Phi\left(\frac{(99.5) - 100}{7.07}\right) \approx .5282 - .4718 = .0564$$

The chance that Student A gets something other than exactly 100 heads is $1 - 0.0564 = 0.9436$. The probability that none of the 30 students gets exactly 100 heads is the probability that all of the students get something other than exactly 100 heads, which is approximately $0.9436^{30} = 0.1752$.

2.4.6 a) The number of black balls seen in a series of 100 draws with replacement has binomial $(1000, 2/1000)$ distribution, which is approximately Poisson with $\mu = 2$. So

$$P(\text{fewer than 2 black balls}) \approx e^{-\mu}\frac{\mu^0}{0!} + e^{-\mu}\frac{\mu^1}{1!} = e^{-\mu}(1 + \mu) = .406006.$$

$$P(\text{exactly 2 black balls}) \approx e^{-\mu}\frac{\mu^2}{2!} = .270671.$$

Calculate the probability of more than 2 black balls by subtraction, conclude that getting fewer than 2 black balls is most likely.

b) $P(\text{both series get the same number of black balls}) = \sum_{k=0}^{\infty} P(\text{both series get k black balls})$

$$\approx \sum_{k=0}^{\infty} e^{-\mu}\frac{\mu^k}{k!}e^{-\mu}\frac{\mu^k}{k!} \text{ by Poisson approx and independence}$$

$$= e^{-4}\sum_{k=0}^{\infty} \frac{4^k}{(k!)^2} \approx 0.207002$$

Only a few terms in the sum are noticeable. After that they are essentially 0.

2.4.10 Distribution of the number of successes is

$$\text{binomial } (n, 1/N) \approx \text{ Poisson } (n/N) \approx \text{ Poisson } (5/3).$$

$$P(\text{at least two}) \ = 1 \ - \ P(0) \ - \ P(1) \ \approx 1 - e^{-5/3}(1+5/3) = 1 - e^{-5/3}\cdot\frac{8}{3} \approx 0.49633 \approx 0.5$$