# Game Theory - Assignment 1

Due date: **August 14, 2023**.

1. Consider the following game with two players $\{b, s\}$: Player $b$ is the buyer of *time* from Player $s$ who has 1 unit of time to sell. Strategies of the players are as follows:

   - $b$ chooses a price $p_b \in \{3, 4\}$.
   - $s$ chooses a price $p_s \in \{3, 4\}$.

   Depending on the prices $(p_b, p_s)$ chosen by the players, an amount of time $q(p_b, p_s)$ is given by Player $s$ to Player $b$:

   - $q(p_b, p_s) = 0$ if $p_b < p_s$.
   - $q(p_b, p_s) = 1$ if $p_s = 3 \leq p_b$ at a per unit price $p_s = 3$.
   - $q(p_b, p_s) = \frac{2}{3}$ if $p_s = 4 = p_b$ at a per unit price $p_b = 4$.

   Utilities of players are as follows. Player $s$ incurs a cost $c \in [0, 4]$ by giving one unit of time and Player $b$ gets a value $v \in [3, 5]$ per unit time of Player $s$. The per unit price paid by $b$ to $s$ is the price chosen by the seller ($p_s$). Utilities are

   $$u_b(p_b, p_s; v) = q(p_b, p_s)(v - p_s)$$
   $$u_s(p_b, p_s; c) = q(p_b, p_s)(p_s - c),$$

   where $u_b$ is the utility of buyer and $u_s$ is utility of seller. Answer the following.

   - Show that for every $v \neq 4$, Player $b$ has a weakly dominant strategy in this game. Specify the weakly dominant strategy. What happens at $v = 4$?
   - Show that for $c \geq 3$ and $c \leq 1$, Player $s$ has a weakly dominant strategy in this game.
   - What happens if $c \in (1, 3)$ for Player $s$?

2. Three indivisible objects (houses) need to be assigned to three agents. Each agent needs to be assigned a unique house. Each agent has a strict preference ordering over the set of objects.

The agents play an *allocation game* to allocate objects. First, agent 1 goes and selects an object from the three objects. Second, agent 2 goes and selects an object from the remaining two objects. Finally, agent 3 gets the remaining object.

Write down the strategic form game by clearly specifying the strategies of the players.

3. An indivisible good is sold to 3 buyers. If any buyer $i$ gets $q_i \in \{0, 1\}$ quantity of the goods makes a payment of $p_i$, her payoff is

$$q_i v_i - p_i.$$

Payment $p_i$ can be positive, negative or zero (some buyers may be *paid* or compensated).

The seller asks each buyer to place a bid. If $(b_1, b_2, b_3)$ are the bids of the buyers then the highest bidder wins (with ties broken in favor of highest indexed bidder[1]). If bidder $i$ wins, she pays $\max_{j \neq i} b_j$. Out of this payment, the seller returns

$$\frac{1}{3} \min_{j \neq i} b_j$$

to highest and second highest bidder and

$$\frac{1}{3} \max_{j \neq i} b_j$$

to the lowest bidder.

Show that bidding their own value is a weakly dominant strategy for each bidder.

[1]For instance, if buyer 1 and 2 are joint winners, buyer 1 wins the object.