## Game Theory - Assignment 4

Due date: October 28, 2022.

1. The stage game is shown in Table 1.

|  | $H$ | L |
| :---: | :---: | :---: |
| H | $(3,1)$ | $(0,0)$ |
| L | $(1,2)$ | $(5,3)$ |

Table 1: Stage game
Consider the infinite repetition of the game in Table 1 with discounted criterion to evaluate payoffs. Find a subgame perfect equilibrium of this game such that
(a) the equilibrium payoff of Players approach $(4,2)$ as $\delta \rightarrow 1$.
(b) the equilibrium payoff of Players approach $(3,2)$ as $\delta \rightarrow 1$.
2. If we repeat prisoner's dilemma game for two periods, how many strategies does each player have in this repeated game?
3. Consider the stage game $G$ shown in Table 2.

|  | $a$ | $b$ | $c$ |
| :---: | :---: | :---: | :---: |
| $A$ | $(4,4)$ | $(-1,5)$ | $(2,2)$ |
| $B$ | $(5,-1)$ | $(1,1)$ | $(2,2)$ |
| $C$ | $(2,2)$ | $(2,2)$ | $(3.5,3.5)$ |

Table 2: Stage game
(a) Find the worst Nash equilibrium (pure action) for each player in $G$ and the corresponding payoffs.
(b) Consider $G^{2}$ : the finitely repeated game, where $G$ is repeated for two periods.
(i) Is there a subgame perfect equilibrium of $G^{2}$ where $(A, a)$ is played in the first period? Explain your answer.
(ii) Is there a Nash equilibrium of $G^{2}$ where $(A, a)$ is played in the first period? Explain your answer.
(c) Consider the infinitely repeated game $G^{\infty}$. Describe a carrot and stick strategy profile where punishment is carried out for one period and $(A, a)$ is played in normal state. Show that it is a subgame perfect equilibrium strategy profile and find the corresponding discount factor.
4. Suppose instead of discounting criterion for evaluating payoffs, we evaluate payoff of Player $i$ from a stream of payoffs $\left\{v_{i}^{t}\right\}_{1}^{\infty}$ as

$$
\lim _{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^{T} v_{i}^{t}
$$

|  | $C$ | $D$ |
| :---: | :---: | :---: |
| C | $(2,2)$ | $(0,3)$ |
| D | $(3,0)$ | $(1,1)$ |

Table 3: Prisoner's dilemma
Verify if the grim-trigger strategy continues to be the Nash and subgame perfect equilibrium of the Prisoner's Dilemma game of Table 3 using this criterion for evaluating payoffs.

