Indian Statistical Institute, Delhi Centre

Measure Theoretic Probability

Fall 2010

Quiz # 1

Date: August 5, 2010 (Thursday) **Total Points:** 10

Note:

- **Please write your name.**
- **There are 2 problems carrying 5 points each. Answer all of them.**
- **Please write your answer for each of the problems in the space provided and show all your work.**
- **This is a CLOSE NOTE and CLOSE BOOK examination.**
- **You have 20 minutes to complete the quiz.**

Name: ______________________

1. Let $\mathcal{F} := \left\{ B \in \mathcal{B}_{\mathbb{R}} \,\middle|\, B = -B \right\}$ show that $\mathcal{F}$ is a σ-algebra on $\mathbb{R}$.

2. Let $(\Omega, \mathcal{F})$ be a measurable space. A *set function* $\mathbf{P} : \mathcal{F} \longrightarrow [0,1]$ with $\mathbf{P}(\emptyset) = 0$ and $\mathbf{P}(\Omega) = 1$ is said to be a *finitely additive probability* if for any finite sequence $A_1, A_2, \ldots, A_n \in \mathcal{F}$ which are pairwise disjoint $\mathbf{P}(\cup_{i=1}^n A_i) = \sum_{i=1}^n \mathbf{P}(A_i)$. Show that a finitely additive probability $\mathbf{P}$ on $(\Omega, \mathcal{F})$ is a probability, if and only if, it is *continuous at* $\emptyset$ *from above*, that is, if and only if, for any decreasing sequence $A_n \downarrow \emptyset$ such that $A_n \in \mathcal{F}$ we have $\mathbf{P}(A_n) \downarrow 0$.