Indian Statistical Institute, Delhi Centre

Measure Theoretic Probability

Fall 2010

Quiz # 3

Date: October 12, 2010 (Tuesday) **Total Points:** 10

Note:

- **Please write your name.**
- **There are 2 problems carrying 5 points each. Answer all of them.**
- **Please write your answer for each of the problems in the space provided and show all your work.**
- **This is a CLOSE NOTE and CLOSE BOOK examination.**
- **You have 20 minutes to complete the quiz.**

Name: ______________________

1. Give an example of a sequence of Riemann integrable functions $f_n : [0, 1] \to \mathbb{R}$ such that, $|f_n| \leq 1$ for all $n \geq 1$, $f_n \to f$ everywhere, but $f : [0, 1] \to \mathbb{R}$ is **not** Riemann integrable.

2. Let $(\mathbf{X} \times \mathbf{Y}, \mathcal{F} \otimes \mathcal{G}, \mu \otimes \nu)$ be a σ-finite product space. Suppose $C \in \mathcal{F} \otimes \mathcal{G}$ such that $\mu \otimes \nu\,(C) = 0$. Show that $\nu\,(C^x) = 0$ almost surely for all x with respect to the measure μ and $\mu\,(C_y) = 0$ almost surely for all y with respect to the measure ν.