Solutions to Practice Midterm
Stat 155: Game Theory

Question 1

a) This game is same as a game of Nim if you think of each diagonal leading from bottom-left to top-right as a Nim pile. The size of the pile depends on the number of moves the bishop in that diagonal can make. Then this is a 14-pile Nim with the size of piles being $(0, 1, 2, \ldots 6, 6, \ldots, 2, 1, 0)$. The Nim-Sum of the given starting position is 0 which implies this is a P-position, hence Player II is going to win the game.

b) For a game of Nim, the Sprague-Grundy function is same as the Nim-Sum, so the SG function for the given starting position is 0.

c) Now the game becomes a 15-pile Nim with starting position being $(0, 1, 2, \ldots 6, 7, 6, \ldots, 2, 1, 0)$. Now the SG function is the Nim-Sum for this new starting position, which is 7. Thus, it is a N-position and Player I wins the game.

Question 2

The payoff matrix of this Two-Person Zero-Sum Game is as follows,

0	-1	-2
1	0	-1
2	1	0

a) As the $(3, 3)$ position is the unique saddle point, the optimal strategies for each player is to call 3 and the value of the game is 0.

b) If we play with the strategy set $(1, 2, \ldots, 10)$ instead then also it can be shown that the unique saddle point will be $(10, 10)$ which means that the optimal strategy for each player is to call 10 and the value of the game is 0, the payoff under the optimal strategies.

To see this, notice that as the payoff for playing strategy (i, j) is $i - j$, the row minimum (i.e. the minimum payoff when i is fixed) is achieved only when $j = 10$ and the column maximum (i.e. the maximum payoff when j is fixed) is achieved only when $i = 10$, which means (i, j) can not be a saddle point if $j \neq 10$ (it can not be row minimum) or $i \neq 10$ (it can not be column maximum). Thus $(10, 10)$ is the unique saddle point.