Solutions to Assignment 3
Stat 155: Game Theory

Question 1

a)

$$12 = (01100)_2$$
$$21 = (10101)_2$$
$$Nimsum = (11001)_2 = 25$$

$$15 = (1111)_2$$
$$10 = (1010)_2$$
$$5 = (0101)_2$$
$$Nimsum = (0000)_2 = 0$$

Question 2

This game is a game of Staircase Nim under disguise. To see this, consider each north-west to south-east diagonal as a step in the staircase, with a certain non-negative number of chips on each step. Verify that, at each step, a move in the chessboard is exactly the same as a move in Staircase Nim where the step towards the right is the lower step.

So, to paraphrase, we have a Staircase Nim game with 14 steps + Step 0, where the north-east corner is Step 0. Lets denote the state of the game, i.e. the position as $(x_1, x_2, \ldots, x_{14})$ where x_i is the number of chips in the ith lowest step.

a) As discussed in supplementary section and also in 2.6.6 in Ferguson's book, the P-positions are exactly those positions where $x_1 \oplus x_3 \oplus \cdots \oplus x_{13} = 0$, i.e. the Nimsum of the number of chips in the odd steps is 0. All other positions are N. Notice that this does not depend on the number of chips in even positions.

b) The starting position written in the above notation is $(2, 3, 4, 5, 6, 7, 8, 7, 6, 5, 4, 3, 2, 1)$, the string of number of chips in odd positions is $(2, 4, 6, 8, 6, 4, 2)$ and the desired Nimsum is $8 \neq 0$. So it is a N-position and the first player has a winning strategy.