# Indian Statistical Institute, Delhi Centre

## Linear Models and GLM

## Spring 2008

## Answers to the Quiz # 1

**Date: February 8, 2008 (Friday)** **Total Points:** $2 \times 5 = 10$

1. Consider the following models:

   (a) $y_i = \alpha + \beta x_i^2 + \varepsilon_i^2$, $1 \le i \le n$.

   (b) $y_i^2 = \theta + \varepsilon_i$, $1 \le i \le n$.

   (c) $\log \hat{p_{i,j}} = \mu + \alpha_i + \beta_j + \gamma_{i,j} + \varepsilon_{i,j}$, $1 \le i, j \le N$.

   (d) $y_{i,j} = \mu + \alpha_i + \beta_j + \alpha_i \beta_j + \varepsilon_{i,j}$, $1 \le i, j \le N$.

   In all of above the errors are i.i.d. Normal $(0, 1)$ random variables.

   State which of the above can be described as a *Linear Model* and which can not be.

   **Ans.** **(a)** `A Linear Model` **(b)** `A Linear Model`
   **(c)** `A Linear MOdel` **(d)** `Not a Linear Model`

2. Fill in the following table:

| **Symmetric Matrix** | **Is it p.d./p.s.d./n.d./n.s.d./indefinite ?** |
|---|---|
| $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ | `Indefinite` |
| $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ | `Indefinite` |
| $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ | `p.s.d.` |
| $\begin{pmatrix} 1 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 1 \end{pmatrix}$ | `p.d.` |

3. Suppose $A_{n\times n}$ be a real matrix and consider its *Singular Value Decomposition* given by:

$$A = P\begin{pmatrix} \Delta & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} Q\,,$$

where $P$ and $Q$ are two orthogonal matrices and $\Delta$ is a diagonal matrix with strictly positive diagonal entries.

Indicate if the following statements are **True** or **False**:

(a) $\Delta$ has all the non-zero eigen-values of $A$. `False`

(b) If $A = A^T$ then $\Delta$ has all the non-zero eigen-values of $A$. `False`

(c) $PQ = QP$. `False`

(d) $\exists$ an orthogonal matrix $B_{n\times n}$ such that $P = BQ$. `True`

4. Consider the following model:

$$y_i = \mu_i + \varepsilon_i,\ \ 1 \le i \le n\,,$$

where $\{\varepsilon_i\}_{i\ge 1}$ are i.i.d. with Normal $(0, 1)$ random variables.

Indicate if the following statements are **True** or **False**:

(a) This is not a *Linear Model*. `False`

(b) For each $1 \le i \le n$ the parameter $\mu_i$ is *estimable*. `True`

(c) $R_0^2 > 0$. `False`

(d) We can test $H_0 : \mu_1 = \mu_2 = \cdots = \mu_n$. `True`

5. Consider the following the linear model:

$$y_{1,j} = \mu + \nu + \varepsilon_{1,j} \text{ for } 1 \le j \le n_1$$

$$y_{2,j} = \mu + \varepsilon_{2,j} \text{ for } 1 \le j \le n_2\,,$$

where $\mu, \nu \in \mathbb{R}$ are unknown parameters, and $\left\{\varepsilon_{i,j} \,\middle|\, 1 \le i \le 2, 1 \le j \le n_i\right\}$ are i.i.d. Normal $\left(0, \sigma^2\right)$, where $\sigma^2$ is not know.

Indicate if the following statements are **True** or **False**:

(a) The MLE of $\mu^2$ is $\left(\frac{1}{n_2}\sum_{j=1}^{n_2} y_{2,j}\right)^2$. `True`

(b) The MLE of $\nu$ is not unique. `False`

(c) The MLE of $\sigma^2$ exists no matter what $n_1$ and $n_2$ are. `False`

(d) Under a hypothesis $H_0 : \mu = 0$ a MLE of $\sigma^2$ exists no matter what $n_1$ and $n_2$ are. `True`