Indian Statistical Institute, Delhi Centre

Advanced Analysis

Spring 2011

Quiz # 2

Date: April 21, 2011 **Total Points:** 10

Note:

- **Please write your name.**
- **There are 5 true/false statements each with 2 points. Answer all of them. Write brief reasons supporting your answers in the space provided.**
- **This is a CLOSE NOTE and CLOSE BOOK examination.**
- **You have 30 minutes to complete the quiz.**

Name: ______________________

1. Suppose $(X_n, \mathcal{F}_n)_{n \geq 0}$ is a (forward) martingale. Let $Y_{-n} := X_n$ for $n \geq 0$. Then $(Y_m, \mathcal{F}_m)_{m \leq 0}$ is a *reverse* martingale. ______________.

2. A predictable martingale always converges. ______________.

3. There is a probability $\mathbf{P}$ on $\left([0,1], \mathcal{B}_{[0,1]}\right)$ which is neither *absolutely continuous singular* with respect to the *Lebesgue measure* λ. ______________.

4. If $\mathbf{P}$ and $\mathbf{Q}$ are two probabilities on $(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$ which are absolutely continuous with respect to the Lebesgue measure λ. Then so is the product probability $\mathbf{P} \otimes \mathbf{Q}$ with respect to $\lambda \otimes \lambda$. ______________.

5. If $(X_n, \mathcal{F}_n)$ is a non-negative martingale with limit X_∞. Then $\mathbf{E}[X_n] \longrightarrow \mathbf{E}[X_\infty]$. ______________.