UNIVERSITY OF CALIFORNIA, BERKELEY

DEPARTMENT OF STATISTICS

STAT 134: Concepts of Probability

Spring 2014

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Practice Midterm Examination

Date Given: March 10, 2014 **Duration: 80 minutes** **Total Points: 60**

Note: There are five problems with a total of 60 points. Show all your works.

1. Suppose there are 10 drawers, each containing **two** coins. Drawers 1 and 2 contain only **gold** coins, while in drawers $3, 4$ and 5 one is a **gold** coin and the other one is a **silver** coin, and rest of the drawers contain only **silver** coins. Suppose you pick a drawer D at random, and then a coin at random from it. Given that you choose a **gold** coin find the conditional probability that the other coin is also **gold**. Find the conditional distribution of the random drawer D. [6 + 6]

2. An urn contains 3 tickets labeled $1, 2$ and 3. The tickets 1 and 3 are green and the ticket 2 is red. Two tickets are drawn at random *without replacement* from the urn. Let X be the number of green tickets in the sample and Y be the total of the two numbers selected.

   (a) Write the joint distribution of X and Y as form of a table. Are X and Y independent? [4 + 1]

   (b) Name the distribution of X and specify the parameter values. [1]

   (c) Calculate the expected value and variance of Y. [3 + 3]

3. Suppose $X_1, X_2, \cdots, X_n$ are independent and identically distributed random variables. Each X_i takes only two values namely ± 1 with equal probabilities. Let $S_n := X_1 + X_2 + \cdots + X_n$.

   (a) Find the distribution of S_n. [5]

   (b) Suppose $n = 2m$ then find $\lim_{m \to \infty} \sqrt{m}\, \mathbf{P}\left(S_{2m} = 0\right)$. [3]

   (c) If $n = 100$ then find approximate numerical value for $\mathbf{P}\left(|S_n| < 10\right)$. [4]

4. Roll a standard six sided fair die till a 6 appears. Let X be the total number of rolls and Y be the number of times 1 has appeared.

   (a) What is the distribution of X? [1]

   (b) Find the conditional distribution of Y given $X = x$. [5]

(c) Find $\mathbf{E}[Y]$ and $\mathbf{Var}(Y)$. [3 + 3]

5. There are 10 empty boxes numbered $1, 2, \ldots, 10$ placed sequentially on a circular table. We perform 100 independent trials. At each trial, a box is selected at random and one ball is added in the two neighboring boxes of the selected box. Let X_k be the number of balls in the k^{th} box at the end of 100 trials.

   (a) Is the sequence of random variables $(X_1, X_2, \cdots, X_{10})$ *exchangeable*? Explain your answer. [4]

   (b) Are they independent? Explain your answer. [4]

   (c) Find $\mathrm{E}[X_k]$ for $1 \leq k \leq 10$. [4]