Connectivity Properties and Structure of Random Graphs Obtained by Vertex Percolation on Erdős-Rényi Binomial Random Graph

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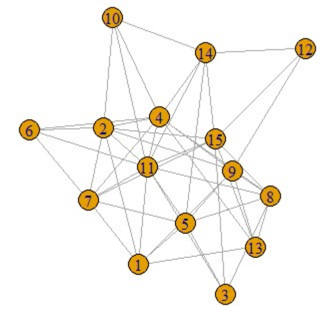
Guide: Antar Bandyopadhyay

June 18, 2020



Introduction

The Problem

There is a large state with several cities with a road between every pair of cities. Suddenly, the state decides to block a few randomly chosen roads for maintenance. Also, some of the cities randomly decide not to allow travel through roads that have one end in that city, independently of the other cities. Now, a traveller standing in one of the remaining cities wants to know the chances that he can travel to all of the remaining cities without having to break any rule.

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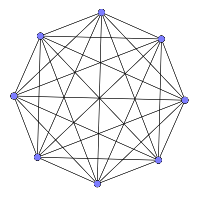

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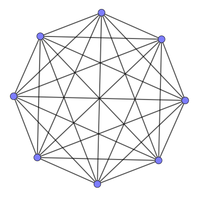

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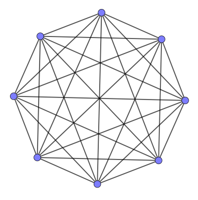

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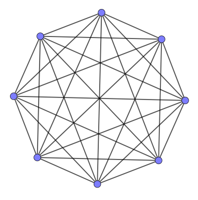

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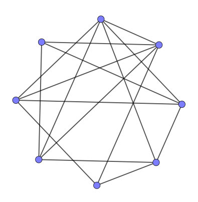

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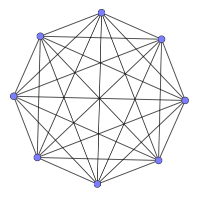

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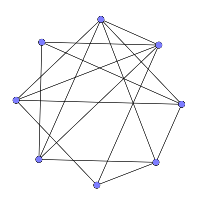

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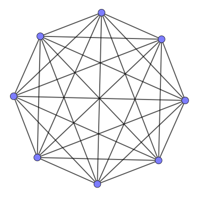
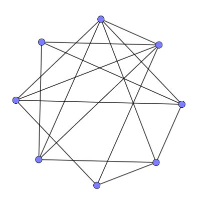
Complete Graph, K_n → Edge Deletion → Vertex Percolation





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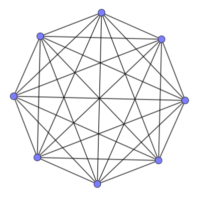

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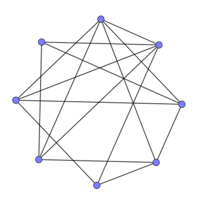

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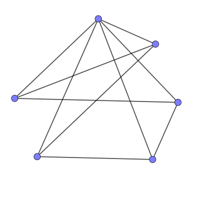
Vertex Percolation



Mathematical Formulation of The Problem

Let, the following be defined,

$$\mathcal{V}_n := V(K_n) = [n]$$
$$\mathcal{E}_n := E(K_n) = \{\{u,v\} | u \neq v \ ; \ u,v \in [n]\}$$

Let

$$(X_v)_{v \in \mathcal{V}_n} \sim \text{i.i.d. Bernoulli}\,(q_n)$$
$$(Y_e)_{e \in \mathcal{E}_n} \sim \text{i.i.d. Bernoulli}\,(p_n)$$
$$X_v \ \&\ Y_e \text{ are independent } \forall\, v \in \mathcal{V}_n \ \&\ e \in \mathcal{E}_n$$

Let G be random graph such that

$$V(G) := \{v | v \in \mathcal{V}_n \ \&\ X_v = 1\}$$
$$E(G) := \{e := \{u,v\} | e \in \mathcal{E}_n \ \&\ X_v = 1, X_u = 1, Y_e = 1\}$$

Such a G is our graph of interest. We will denote such a graph as

$$G \sim \mathcal{G}(n, p_n, q_n)$$

Relation with Erdős-Rényi Binomial Random Graph

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If vertex percolation is done first, the graph obtained is an Erdős-Rényi Binomial Random Graph on the remaining set of vertices i.e., $G|(X_v)_{v \in \mathcal{V}_n} \sim \mathcal{G}\left(|V(G)|, p_n\right)$

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$\mathbb{E}\left[|V(G)|\right] = nq_n$

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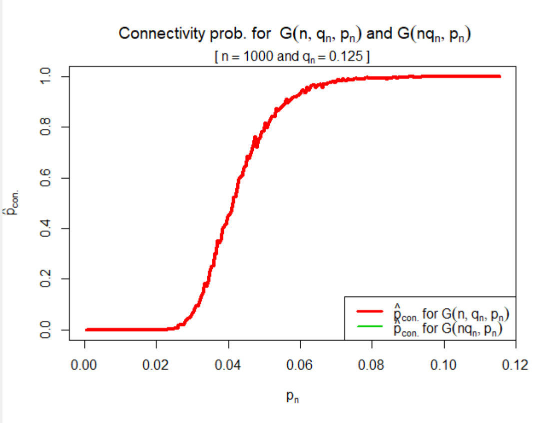

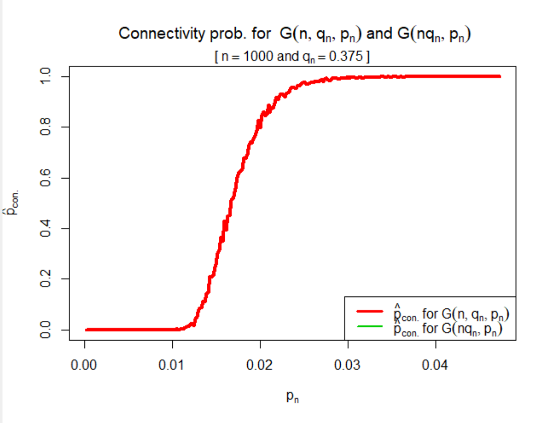

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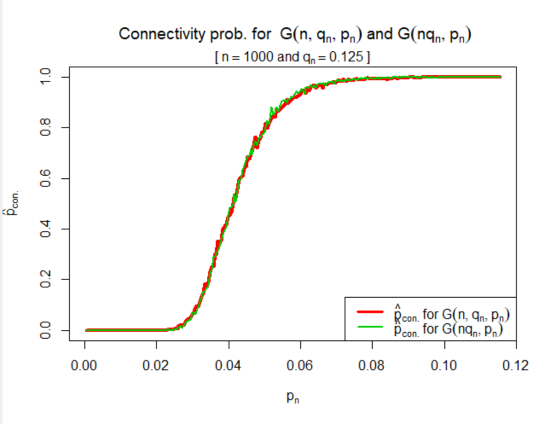

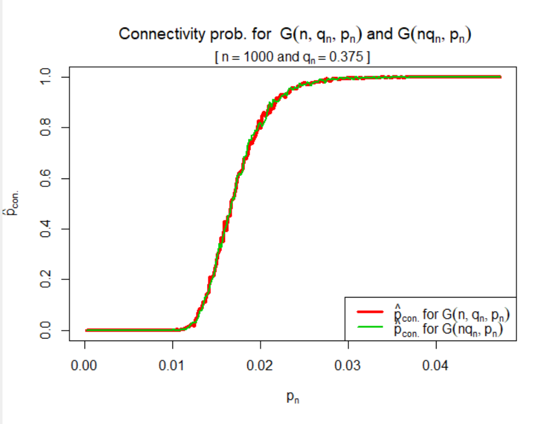

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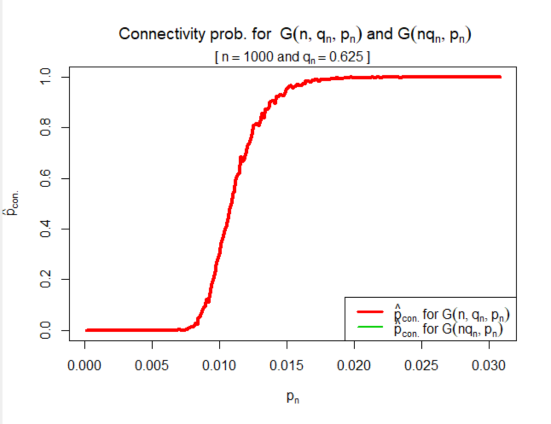

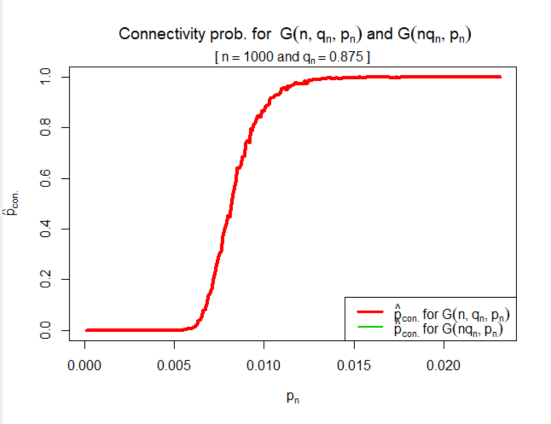

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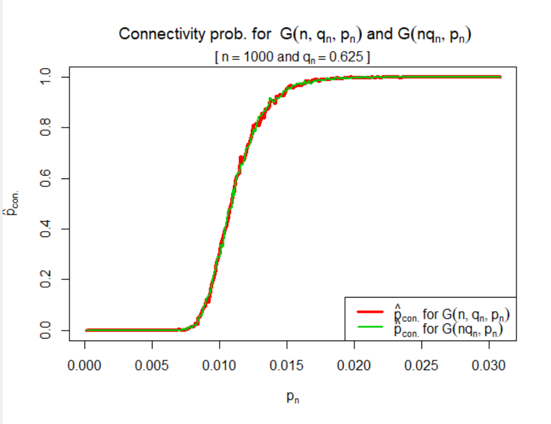

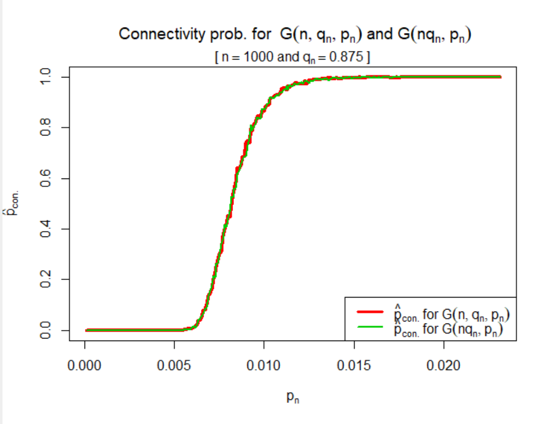

Connectivity in Erdős-Rényi Binomial Random Graph

Theorem (Connectivity Threshold)

Let $G \sim \mathcal{G}(n, p_n)$. Then,

$$\lim_{n\to\infty} \mathbb{P}_{\mathcal{G}(n,p_n)}(G \text{ is connected}) = \begin{cases} 0 & ; \dfrac{p_n}{\log(n)/n} \to 0 \\ 1 & ; \dfrac{p_n}{\log(n)/n} \to \infty \end{cases}$$

Theorem (Critical Window)

Fix $t \in \mathbb{R}$ and $\lambda_n = \log n + t$;

$$\lim_{n\to\infty} \mathbb{P}_{\mathcal{G}(n, \frac{\lambda_n}{n})}(G \text{ is connected}) = \exp\{-\exp\{-t\}\}$$

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We have already seen the relation between our graph of interest and Erdős-Rényi Binomial Random Graph, We proceed to check the relation between the estimated probability of connectivity in $\mathcal{G}(n, p_n, q_n)$ (denoted as $\hat{p}_{con.}$) and edge probability, p_n.

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- Checking the relation between the estimated parameters and n.

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- Checking the relation between the estimated parameters and n.
- Compare those relations with our guess.

In Erdős-Rényi Binomial Random Graph, we also have an another version of the theorem related to Critical window,

Theorem

Fix $p \in (0,1)$,*then* $\mathbb{P}_{\mathcal{G}(n,p)}(G \text{ is connected}) = \exp\{-\exp\{-n(p - \log n/n)\}\}(1 + o(1))$

Our guess is that, in case of connectivity it should behave like Erdős-Rényi Binomial Random Graph but with number of vertices as its expected number of vertices, nq_n.
Hence, we have done various simulations, to

- Plot $\hat{p}_{con.}$ vs p for wide range of q_n.
- Fit the functional form, $\hat{p}_{con.} \approx \exp\{-\exp\{-\delta_n(p - c_n)\}\}$
- Estimate δ_n and c_n

Our observations from these simulations are,

- The rapid increase in connectivity probability, are mostly around $\log(nq_n)/nq_n$ for several q_n values.
- Fit of the functional form, $\hat{p}_{con.} \approx \exp\{-\exp\{-\delta_n(p - c_n)\}\}$ produces a very high value of multiple correlation.

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Simulation 1[Change in $\hat{\delta}_n$ with q_n]

- Fix a 'large enough' value of n.
- Fix a wide range of value for q_n distributed over the region $(0, 1)$.
- For each value of q_n, plot $\hat{p}_{con.}$ vs $p^* := p - \hat{c}_n$ on same plot.

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Simulation 2[Change in $\hat{\delta}_n$ with n]

- Fix a constant value of q_n.
- Fix a set of value for large enough n.[In our case $\{1000, 1001, \cdots, 1500\}$]
- For each value of n, plot $\hat{p}_{con.}$ vs p^* on same plot.

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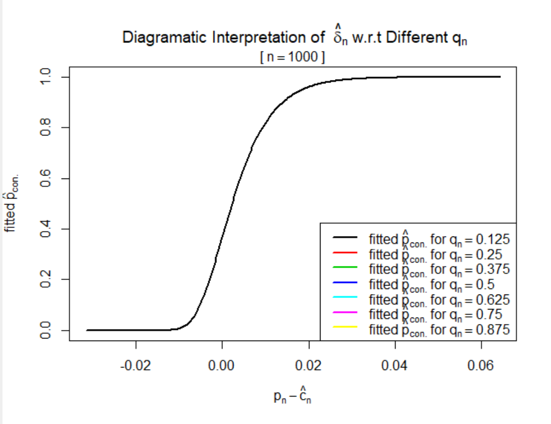
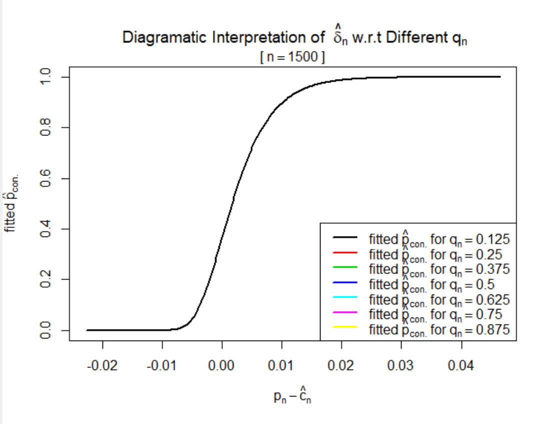


Figure: Change in $\hat{\delta}_n$ w.r.t. q_n



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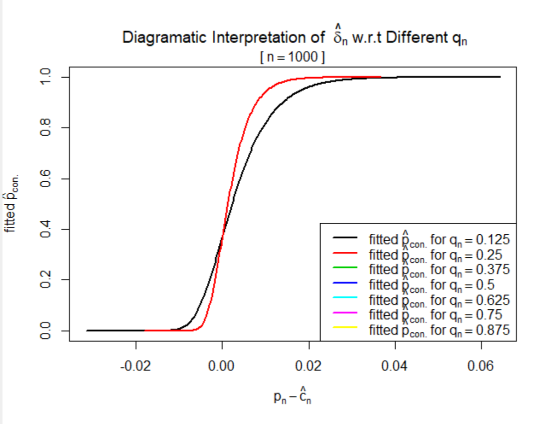

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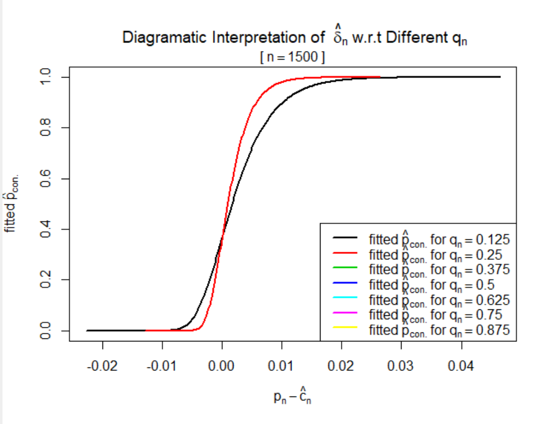

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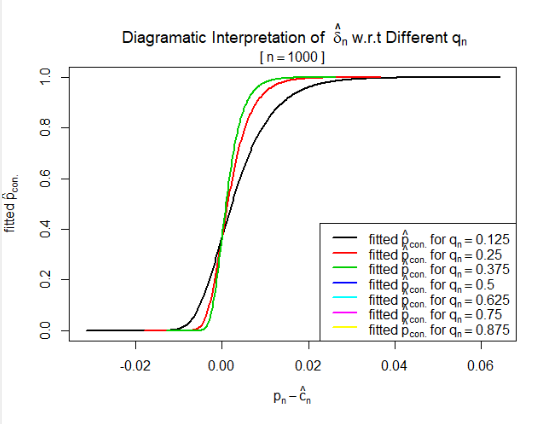

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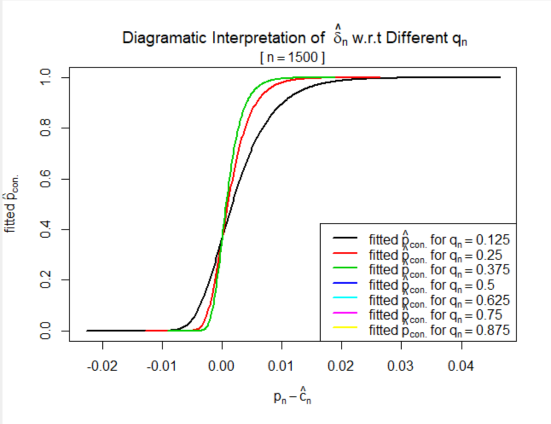

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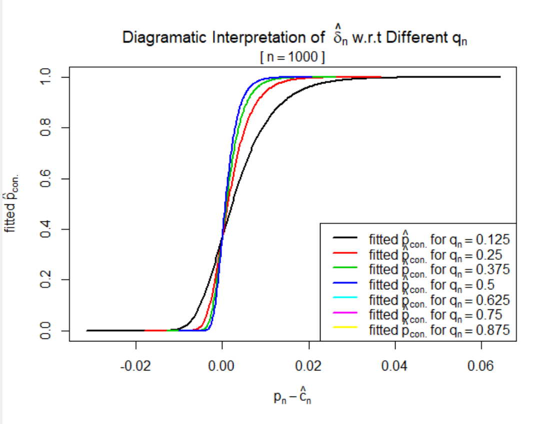

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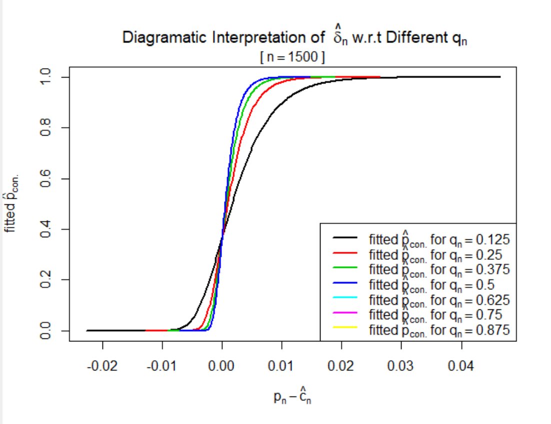

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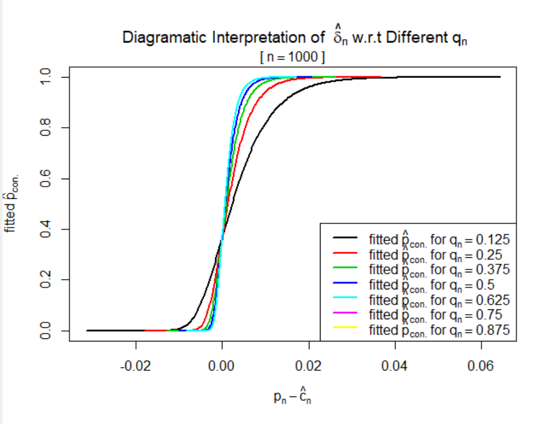
CHANGE IN $\hat{\delta}_n$ WITH q_n



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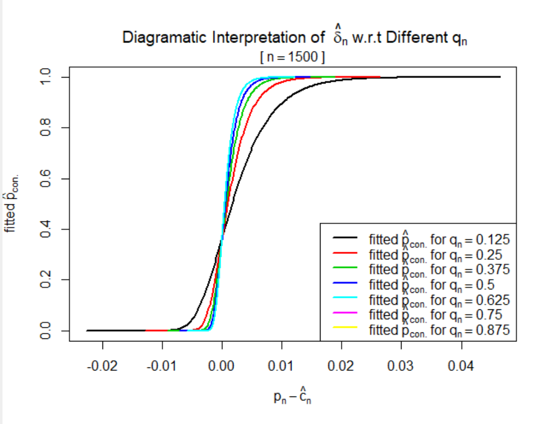

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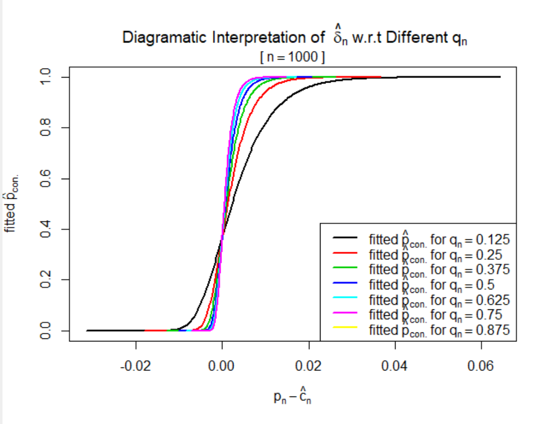

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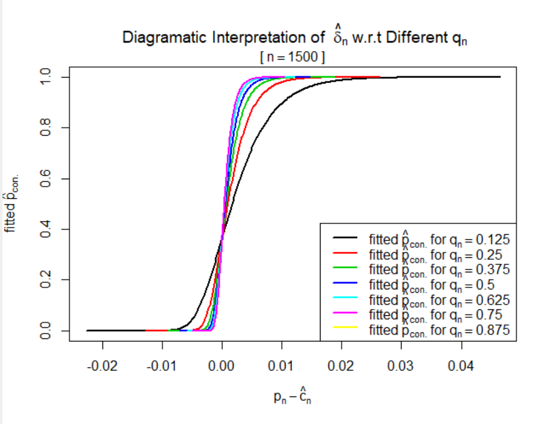

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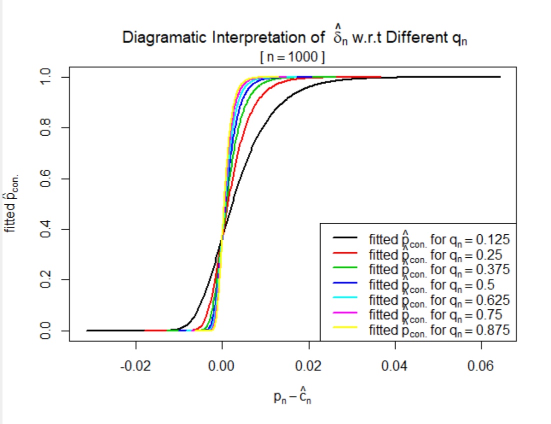

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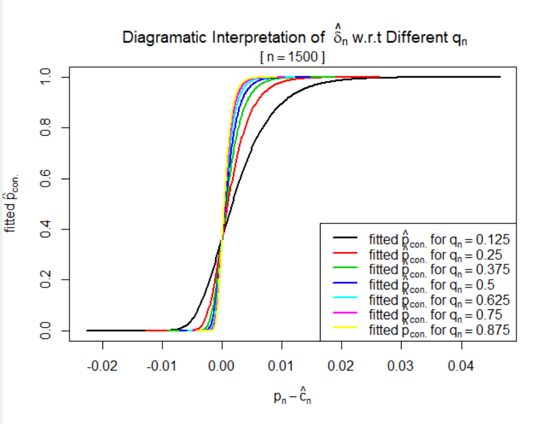

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CHANGE IN $\hat{\delta}_n$ WITH n

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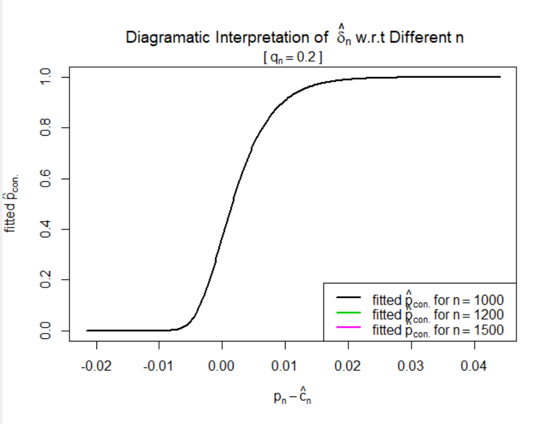
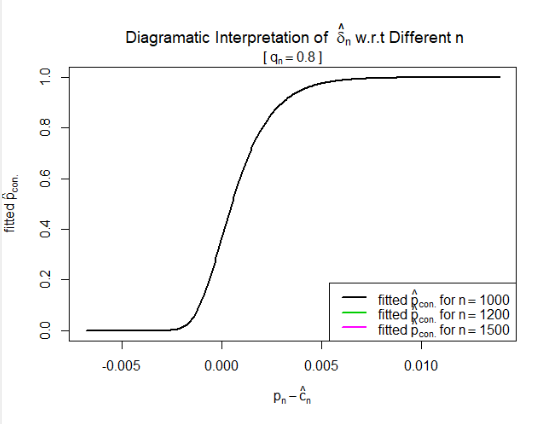


Figure: Change in $\hat{\delta}_n$ w.r.t. n



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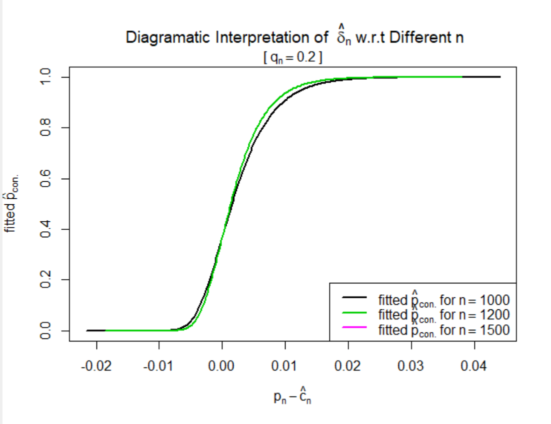

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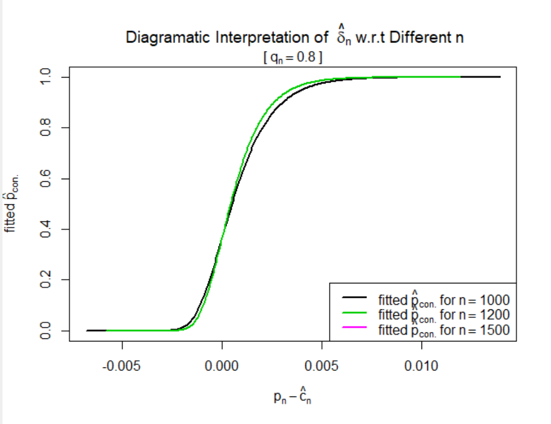

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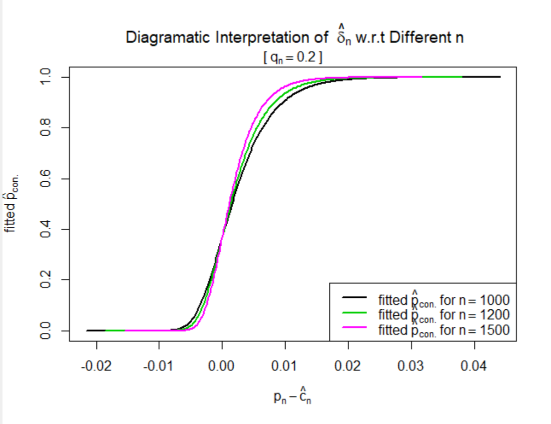

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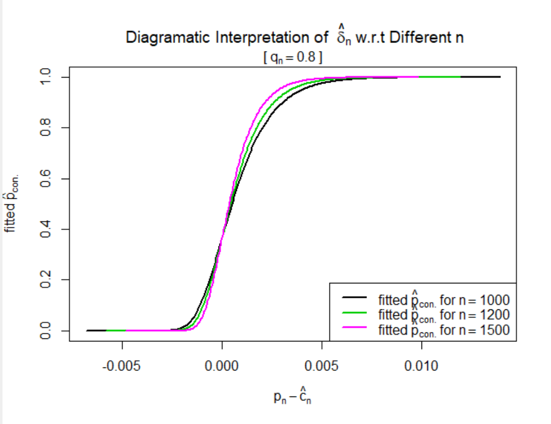

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Estimating the Parameters as Function of n

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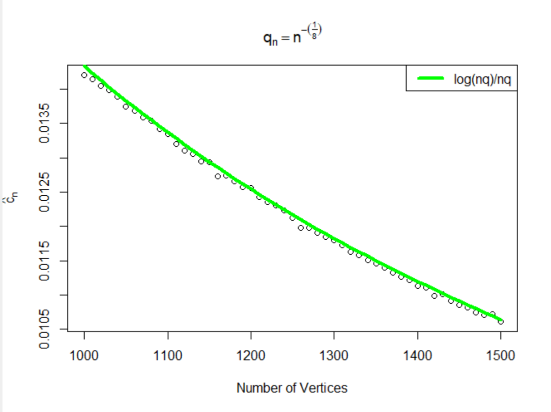
Simulation 3

- Fix an $n - q_n$ relationship.
- For $n \in \{1000, 1001, \cdots, 1500\}$,
  - Simulate values for $\hat{p}_{con.}(p)$ for different values of p.
  - Fit $\hat{p}_{con.}(p) \approx \exp\{-\exp\{-\delta_n(p - c_n)\}\}$ and estimate $\hat{c}_n$ and $\hat{\delta}_n$
- Plot n *vs* $\hat{c}_n$ and n *vs* $\log nq_n/nq_n$ in same plot and plot n *vs* $\hat{\delta}_n$ and n *vs* nq_n in another same plot.
- Calculate $Cor(\{\hat{c}_n\}, \{\log nq_n/nq_n\})$ and $Cor(\{\hat{\delta}_n\}, \{nq_n\})$.

Estimating the Parameters as Function of n $[q_n = 1/n^{\frac{1}{8}}]$

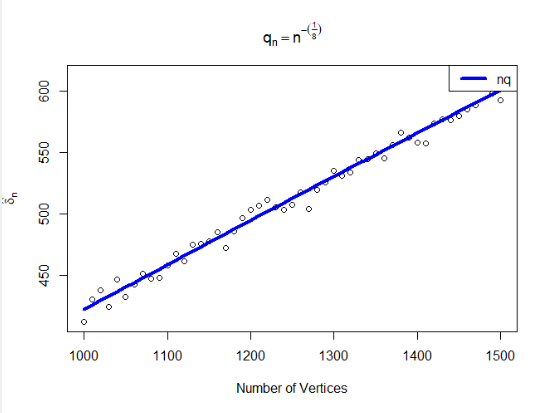
ESTIMATING THE PARAMETERS AS FUNCTION OF n $[q_n = 1/n^{\frac{1}{8}}]$

Figure: Plot of n *vs* $\hat{c}_n$ and n *vs* $\log nq_n/nq_n$



$$Cor\left(\hat{c}_n, \log nq_n/nq_n\right) = 0.9998316$$

Figure: Plot of n *vs* $\hat{\delta}_n$ and n *vs* nq_n



$$Cor\left(\hat{\delta}_n, nq_n\right) = 0.9936192$$

Estimating the Parameters as Function of n [$q_n = 0.3$]

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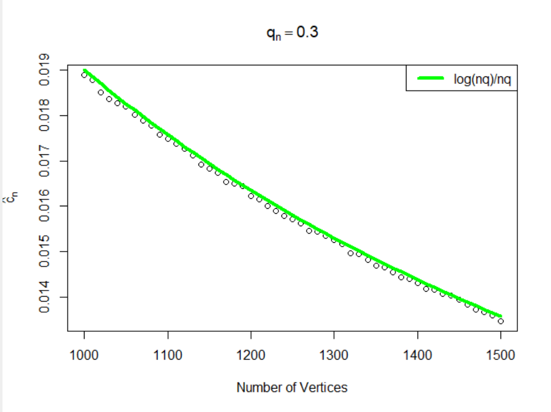
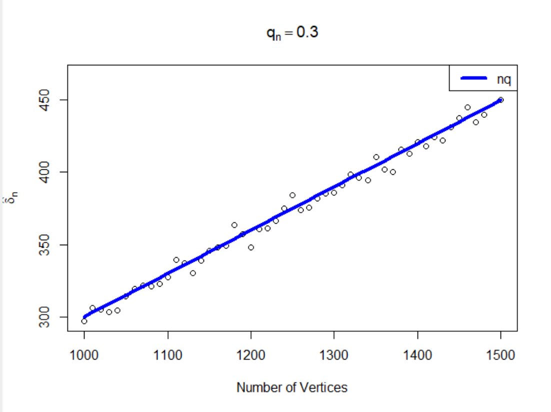
Figure: Plot of n *vs* $\hat{c}_n$ and n *vs* $\log nq_n/nq_n$



$$Cor\left(\hat{c}_n, \log nq_n/nq_n\right) = 0.9996224$$

Figure: Plot of n *vs* $\hat{\delta}_n$ and n *vs* nq_n



$$Cor\left(\hat{\delta}_n, nq_n\right) = 0.9905715$$

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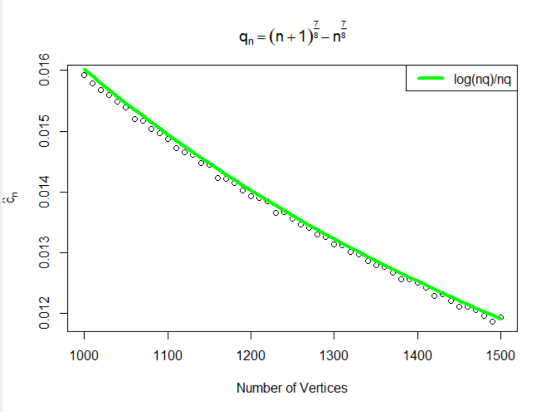
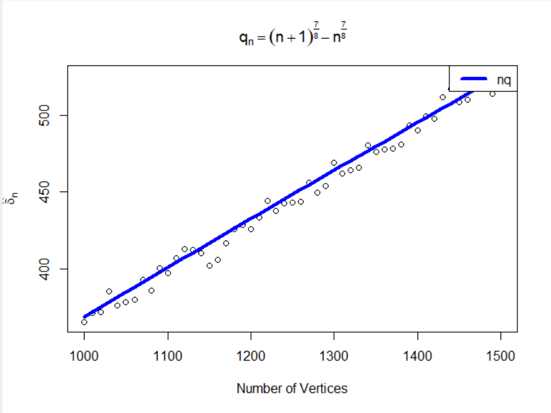


$$Cor\left(\hat{c}_n, \log nq_n / nq_n\right) = 0.9998839$$

Figure: Plot of n *vs* $\hat{\delta}_n$ and n *vs* nq_n



$$Cor\left(\hat{\delta}_n, nq_n\right) = 0.9968299$$

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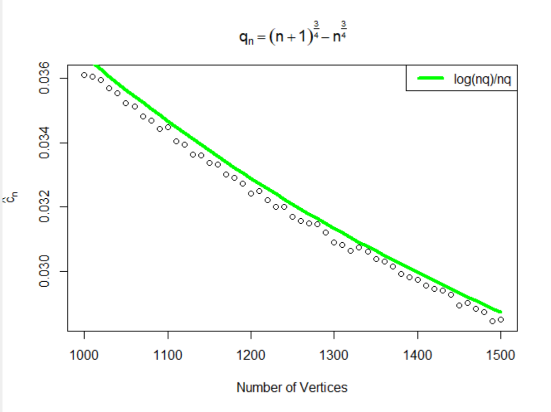
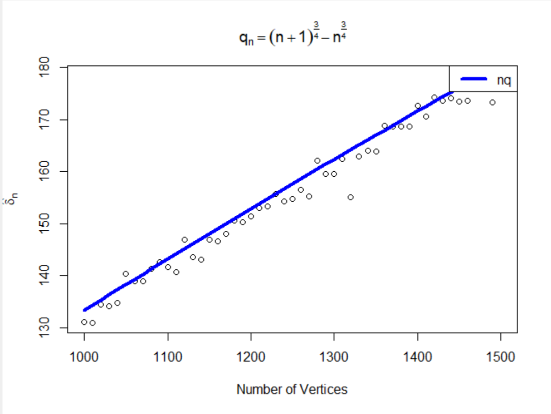
Figure: Plot of n vs $\hat{c}_n$ and n vs $\log nq_n/nq_n$



$Cor\left(\hat{c}_n, \log nq_n/nq_n\right) = 0.9996602$

Figure: Plot of n vs $\hat{\delta}_n$ and n vs nq_n



$Cor\left(\hat{\delta}_n, nq_n\right) = 0.9945886$

Conclusion

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Conjecture (Connectivity 'Threshold' after Percolation)

Let $G \sim \mathcal{G}(n, p_n, q_n)$ such that, $nq_n \to \infty$ as $n \to \infty$,

$$\lim_{n\to\infty} \mathbb{P}\left(G \text{ is connected}\right) = \begin{cases} 0 & ; \dfrac{p_n}{\log nq_n / nq_n} \to 0 \\ 1 & ; \dfrac{p_n}{\log nq_n / nq_n} \to \infty \end{cases}$$

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Conjecture ('Critical Window' for $\mathcal{G}(n, q_n, p_n)$)

Fix $t \in \mathbb{R}$ and $\lambda_n = \log nq_n + t$. Provided that $nq_n \to \infty$ as $n \to \infty$,

$$\lim_{n\to\infty} \mathbb{P}_{\mathcal{G}\left(n, \frac{\lambda_n}{nq_n}, q_n\right)}\left(G \text{ is connected}\right) = \exp\left\{-\exp\left\{-t\right\}\right\}$$

Thank You