Game Theory - Problem Set 2

September 13, 2008

1. Find the set of rationalizable strategies of each player in the games below. In the last game $\alpha \in (0, 1)$.

	A	B	C	D
A	$0, 7$	$2, 5$	$7, 0$	$0, 1$
B	$5, 2$	$3, 3$	$5, 2$	$0, 1$
C	$7, 0$	$2, 5$	$0, 7$	$0, 1$
D	$0, 0$	$0, -2$	$0, 0$	$10, -1$

	A	B	C	D
A	$2, 9$	$4, 7$	$9, 2$	$2, 3$
B	$7, 4$	$5, 5$	$7, 4$	$2, 3$
C	$9, 2$	$4, 7$	$2, 9$	$2, 3$
D	$2, 2$	$2, 0$	$2, 2$	$12, 1$

	H	T
H	$1, -1$	$-1, 1$
B	$-1, 1$	$1, -1$
C	$\alpha, 0$	$\alpha, 0$

2. Consider the two-person game $\Gamma = (\{1, 2\}, S_1, S_2, \pi_1, \pi_2)$ where $\pi_i(s_i, s_j) = s_i(30 - 3s_1 - 3_2)$. Determine the set of rationalizable strategies for each player.

3. Consider an n firm quantity setting game where the cost function for firm i is given by $C_i(x_i) = c_i.x_i$ where $c_i \geq 0$. The inverse demand function is given $P(X) = a - bX$ where $a, b > 0$ and $X = x_1 + ... + x_n$. The payoff function for firm i is therefore $\pi_i(x_1, .., x_n) = (a - b(\sum_j x_j)x_i - c_i.x_i$.

(i) Compute a (pure strategy) Nash equilibrium for the game. Compute also the equilibrium price.
(ii) What happens to the equilibrium quantity choice of firm j if there is an increase in firm i's cost; i.e in c_i? What happens to equilibrium price?

4. Consider a firm where there is an employer and a worker. The employer provides capital K and the worker, labour L to produnce output $Y = \sqrt{KL}$ which they share equally. The two parties determine their investment levels (i.e. the employer's K and the worker's L) simultaneously. The per-unit costs of providing capital and labour are $r \geq \frac{1}{4}$ and $c \geq \frac{1}{4}$ respectively. The worker cannot provide more than $\bar{L} > 0$ units of labour. The payoffs to the employer and worker are therefore $\frac{1}{2}\sqrt{KL} - rK$ and $\frac{1}{2}\sqrt{KL} - cL$ respectively. Find all rationalizable strategies for the two players.

5. Find all pure and mixed strategy Nash equilibria in the games below.
(i)

	L	M	R
U	$1,-2$	$-2,1$	$0,0$
M	$-2,1$	$1,-2$	$0,0$
D	$0,0$	$0,0$	$1,1$

(ii)

	L	M	R
U	$1,3$	$0,0$	$2,-1$
M	$0,0$	$4,2$	$0,-2$
D	$0,1$	$0,1$	$0,0$

6. Find all pure and mixed strategy Nash equilibria in the three person game below where players choose rows, columns and matrices.

$$\begin{pmatrix} 1,1,1 & 0,0,0 \\ 0,0,0 & 0,0,0 \end{pmatrix} \qquad \begin{pmatrix} 0,0,0 & 0,0,0 \\ 0,0,0 & 2,2,2 \end{pmatrix}$$

7. Prove that in a 2×2 game (2 players each of whom has 2 strategies) cannoy have exactly 1 pure strategy and 1 completely mixed strategy Nash equilibrium.

8. Assume that the inverse demand function facing an industry producing a homogenous good is given by $P(X) = \sqrt{(\frac{1}{X} - 1)}$. Consider the duopoly quantity setting game where each player's strategy set is $[0, \frac{1}{2}]$. Prove that the payoff function of each player is concave with respect to her strategy. Compute the best reply function of each player and the Nash equilibrium outcome.

9. Players 1 and 2 choose an element of the set $\{1, ..., K\}$. If the players choose the same number, then player 2 pays 1 to player 1; otherwise no payment is made. Find all pure and mixed strategy Nash equilibria of this game.

10. A group of n students go to a restaurant. Each person will simultaneously choose his own meal but the total bill will be shared amongst all the students. If a student chooses a meal of price p and contributes x towards paying the bill, then his payoff is $\sqrt{p} - x$. Compute all pure strategy Nash equilibria of this game. Is the equilibrium unique? Symmetric? Discuss the limiting cases of $n = 1$ and $n \to \infty$.

11. Consider the class of symmetric two person games described below.

	H	T
H	a,a	$0,0$
B	$0,0$	b,b

Derive the set of all (pure and mixed) Nash equilibria for all games where $a \neq b$, $a \neq 0$ and $b \neq 0$. Which of these games admit asymmetric equilibria? (We say s^* is a symmetric equilibrium if $s_1^* = s_2^*$).

12. Each of $n \geq 2$, $i = 1, ..., n$ can make contributions $s_i \in [0, w]$ ($w > 0$) to the production of some public good. Their payoff functions are given by

$\pi_1(s_1, .., s_n) = n \min\{s_1, .., s_n\} - s_i$. Find all pure strategy Nash equilibria in the game.

13. Consider the two person game below.

	A	B	C
A	$0, 0$	$5, 4$	$4, 5$
B	$4, 5$	$0, 0$	$5, 4$
C	$5, 4$	$4, 5$	$0, 0$

Show that the probability distribution over $\{A, B, C\}^2$ which assigns 0 to the diagonal elements and $\frac{1}{6}$ to all off-diagonal elements is a correlated equilibrium.