Theory of Games - Problem Set 4

November 2008

1. Consider a two person game where player 1 believes with probability $\frac{1}{2}$ that he is playing the game

	B	S
B	$(2,1)$	$(0,0)$
D	$(0,0)$	$(1,2)$

and that with probability $\frac{1}{2}$ that he is playing the game

	B	S
B	$(1,2)$	$(0,0)$
S	$(0,0)$	$(2,1)$

Player 2 knows which game is being played.

(a) Model this as a game of incomplete information; i.e. write down action sets, type sets etc.

(b) Show that player 1 playing B and player 2 playing B in the top game and playing S in the bottom game is a Bayes Nash equilibrium.

2. Consider the following game of incomplete information. There are 2 players and each has an action set $\{C, N\}$. The type of player 1 is denoted by c and that of player 2 by d. The payoffs are given by

	C	N
C	$(1-c, 1-d)$	$(1-c, 1)$
N	$(1, 1-d)$	$(0,0)$

Assume that c and d are random variables distributed independently and uniformly on $[0,2]$. (In this game each player has to decide whether or not to contribute to a "common pool". The cost of contributing are c and d to the two players. Each player would like the other to contribute rather than contributing himself).

(a) Prove that there exists a unique Bayes-Nash equilibria of the game.

(b) Suppose that c and d are distributed i.i.d and uniformly on the interval $[\frac{1}{2}, \frac{5}{4}]$. Show that there are two asymmetric equilibria of the game.

3. Two players 1 and 2 compete for a single object worth v_i to player $i = 1, 2$. The winner of the game is the player who remains "aggressive" longer where the cost of being aggressive is 1 per unit of time. An action x_i of player i is a non-negative real number and signifies that i will remain aggressive till x_i. The object is won by the player who remains aggressive longer but both players must pay the costs of remaining aggressive, i.e.

$$\pi_i(x_i, x_j, v_i) = \begin{cases} v_i - x_j & \text{if } x_j < x_i \\ -x_i & \text{if } x_j > x_i \\ \frac{v_i}{2} - x_i & \text{if } x_j = x_i \end{cases}$$

Assume that a player's valuation is observed only by the player. Assume also that v_i and v_j are two independent random variables distributed uniformly on $[0, 1]$. Compute a symmetric Bayes-Nash equilibrium. Is this equilibrium efficient for every possible realization of v_i and $_j$?

4. Consider a first-price auction with two bidders whose valuations x_1 and x_2 are random variables distributed independently according to the distribution functions F_1 and F_2 over the supports $[0, \omega_1]$ and $[0, \omega_2]$ respectively. Consider equilibrium bidding functions β_1, β_2 which are increasing and differentiable. Denote $\phi_i \equiv \beta_i^{-1}$, for $i = 1, 2$. Prove

(a) $\beta_1(\omega_1) = \beta_2(\omega_2)$.

(b) The following differential equation is satisfied

$$\phi_j'(b) = \frac{F_j(\phi_j(b))}{f_j(\phi_j(b))} \frac{1}{(\phi_i(b) - b)}$$

where f_j is the density associated with F_j.

5. Player A takes player B to court in a dispute. Player A knows whether he will win the case but player B does not. Player B believes that A will win with probability $\frac{1}{3}$. If A wins, he gets 3 while B gets -4; if he loses he gets -1 and B gets 0. Before going to court, A offers an out-of-court settlement of m where either $m = 1$ or $m = 2$. If B accepts m, then A gets m and B gets $-m$. Compute all separating and pooling Perfect Bayes-Nash equilibria in this game.

6. (Beer-Quiche game). Consider the following game with incomplete information. First Nature chooses whether player 1 is a "Strong" (S) type (probability 0.9) or a "Weak" (W) type (probability 0.1). Player 1 learns her type (but player 2 does not) and decides whether to have a "beer" or a "quiche" for breakfast. Player 2 sees the breakfast and has to decide whether to "fight" or "not fight". For the S type of player 1, having beer adds 1 to payoff; for the W type, having quiche adds 1 to payoff. For both types of player 1 not being fought adds 2 to payoff. For player 2 fighting the W type yields 1 in payoff and not fighting the W type yields zero. Fighting the S type of player 1 yields player 2 a payoff of zero and not fighting the S type yields a payoff of 1. Compute all Perfect Bayesian equilibria in this game.