Social Choice and Political Economy

February 29, 2008

Time: 2 hours

Attempt all questions

1. Define the following terms:

(i) Independence of Irrelevanant alternatives (for social welfare functions).

(ii) An oligarchic social welfare function.

(iii) The Pareto Indifference axiom (for social welfare functionals).

(iv) The k^{th} positional dictatorship.

[20]

2. Assess each of the claims below. Either prove them or provide an appropriate counterexample. The set of alternatives is A and the set of individuals is N. We let $\mathcal{R}$ denote the set of all orderings over A. Counterexamples can be given for specific values of A and N.

(i) Consider the following social welfare aggregator F which maps every preference profile $R \in \mathcal{R}^N$ to a social binary relation Q_R. For all alternatives a and b, aQ_Rb (a is socially at least as good as b) if the number of individuals i who rank a as a best alternative according to R_i is at least as great as the number of individuals j who rank b as a best alternative according to R_j. (This is the plurality rule). Then F satisfies Independence of Irrelevant alternatives.

(ii) The F described in (i) violates the Weak Pareto axiom.

(iii) The symmetric component, I of an ordering R must be transitive.

(iv) The generalized utilitarian social welfare functional satisfies the Ordinal Measurability non Comparablity axiom.

(v) The maxmin social welfare functional satisfies the Binary Independependence of Irrelevant Alternatives axiom.

[20 marks]

3. Let the set of alternatives $A = [0, 1] \times [0, 1]$. Each individual i has a most preferred point (or peak) $\alpha_i \in A$. Her preference ordering R_i over A is then defined as follows: for all $\beta, \gamma \in A$,

$$\beta R_i \gamma \text{ if } d(\beta, \alpha_i) \leq d(\gamma, \alpha_i)$$

where $d(.)$ is the Euclidean distance function. In other words, β is preferred to γ if the former is closer to α_i than the latter. Each individual i's preference ordering is therefore characterized completely by a point α_i, her peak.

Assume that there is a continuum of individuals with peaks distributed uniformly over A. We construct a social binary relation Q as follows: for all $\beta, \gamma \in A$, $\beta Q \gamma$ if the "measure" of the set of individuals who prefer β to γ is greater than the "measure" of the set of individuals who prefer γ to β. (The "measure" of a subset B of A is simply the area of B). In other words, β is socially preferred to γ if the number (appropriately defined) of individuals who prefer the former to the latter is greater than the number who prefer the reverse.

(i) In a diagram, identify the set of individuals who prefer the alternative $(0.30, 0.50)$ to the alternative $(0.90, 0.50)$.

(ii) Is the alternative $(0.2, 0.2)$ socially preferred to the alternative $(0.90, 0.90)$?

(iii) Is there an alternative β^* such that $\beta^* Q \gamma$ for all $\gamma \in A$?

[2]+[2]+[6]