Corrected Proof of Lemma 2 in

"Implementation with Contingent Contracts"

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There is a mistake in the proof of Lemma 2 (the statement of which is correct) in Deb and Mishra (2014) - the partition created in Lemma 2 is not f-ordered partitioned as claimed. We are very grateful to Kiho Yoon who brought this error to our attention.

Below is the corrected proof of Lemma 2.

LEMMA 2: Suppose the type space is finite and f is an acyclic scf. Then, the type space can be f-ordered partitioned.

Corrected Proof: We define a new relation $\rhd^f$ as follows: for every v_i, v_i', we say $v_i \rhd^f v_i'$ if there is a finite sequence $\{v_i^1, \ldots, v_i^k\}$ such that $v_i \equiv v_i^1 \succeq^f v_i^2 \succeq^f \ldots \succeq^f v_i^k \equiv v_i'$, with at least one of the above relations strict ($\succ^f$). Now, we do the proof in several steps.

STEP 1. First, we show that $\rhd^f$ is acyclic. Consider a sequence $v_i^1, \ldots, v_i^k$ such that $v_i^1 \rhd^f \ldots \rhd^f v_i^k$. Assume for contradiction $v_i^k \rhd^f v_i^1$. So, we get $v_i^1 \rhd^f \ldots \rhd^f v_i^k \rhd^f v_i^{k+1} \equiv v_i^1$. Between any consecutive types v_i^j and v_i^{j+1} in this sequence, we have $v_i^j \succeq^f \ldots \succeq^f v_i^{j+1}$, with strict relation holding for at least one. Hence, we get a finite sequence $v_i^1 \succeq^f \ldots \succeq^f v_i^{k+1} \equiv v_i^1$ with strict relation holding for at least one. Acyclicity of f implies that $v_i^1 \not\succeq^f v_i^1$, contradicting the reflexivity of $\succeq^f$.

STEP 2. Next, choose any nonempty subset $V_i' \subseteq V_i$, and let $M(V_i') \subseteq V_i'$ be the set of maximal elements of V_i' corresponding to the relation $\rhd^f$. Formally

$$M(V_i') := \{v_i \,|\, v_i \in V_i' \text{ and there is no } v_i' \in V_i' \text{ such that } v_i' \rhd^f v_i\}.$$

Since $\rhd^f$ is acyclic, $M(V_i')$ is non-empty.

STEP 2(A). $v_i' \not\succ^f v_i$ for every $v_i, v_i' \in M(V_i')$. To see this, if $v_i' \succ^f v_i$, then $v_i' \rhd^f v_i$, contradicting $v_i \in M(V_i')$.

STEP 2(B). $v_i' \not\succeq^f v_i$ for every $v_i \in M(V_i')$ and every $v_i' \in (V_i' \setminus M(V_i'))$. Assume for contradiction $v_i' \succeq^f v_i$. Since $v_i' \notin M(V_i')$, there exists a v_i'' such that $v_i'' \rhd^f v_i'$. Hence, there is a

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finite sequence, $v_i'' \succeq^f \ldots \succeq^f v_i' \succeq^f v_i$, with strict relation holding at least once. As a result, we have $v_i'' \rhd^f v_i$, contradicting the fact that $v_i \in M(V_i')$.

Step 3. Now, we recursively define a partition. We set $V_i^1 = M(V_i)$. Having defined, $(V_i^1, \ldots, V_i^{k-1})$, we define $R^k := V_i \setminus (V_i^1 \cup \ldots \cup V_i^{k-1})$. If $R^k = \emptyset$, we stop, else, we set $V_i^k := M(R^k)$. Suppose $(V_i^1, \ldots, V_i^K)$ is the partition created. We show that $(V_i^1, \ldots, V_i^K)$ satisfies Property P1 and P2. To do that, pick $v_i, v_i' \in V_i^j$ for some j. Since $V_i^j = M(R^j)$, by Step 2(A), we have $v_i' \not\succ^f v_i$. So, Property P1 is satisfied.

Similarly, pick $v_i \in V_i^j$ and $v_i' \in (V_i^{j+1} \cup \ldots \cup V_i^K)$. Since $V_i^j \equiv M(R^j)$ and $(V_i^{j+1} \cup \ldots \cup V_i^K) \equiv R^j \setminus V_i^j$, by Step 2(B), we get $v_i' \not\succeq^f v_i$. So, Property P2 is satisfied. ■

References

Deb, R. and D. Mishra (2014): "Implementation with Contingent Contracts," *Econometrica*, 82, 2371–2393.