Game Theory - Assignment 1

Due date: **August 07, 2025**.

Please use your own thinking. Do not use AI softwares for solutions.

1. There are two firms. Each firm i decides the quantity q_i it wants to produce. Given the quantities (q_1, q_2) a price equal to $(a - q_1 - q_2)$ is determined at which the products (of both the firms) are sold in the market. The cost of producing q_i units of the product for firm i is cq_i (note c is same for both the firms).

   - Model this as a strategic form game.
   - What are the strictly dominated strategies of the firms.
   - Do two rounds of IESDS and show the strategies that survive.
   - Do infinite rounds of IESDS and show the strategies that survive.

2. Three indivisible objects (houses) need to be assigned to three agents. Each agent needs to be assigned a unique house. Each agent has a strict preference ordering over the set of objects.

   The agents play an *allocation game* to allocate objects. First, agent 1 goes and selects an object from the three objects. Second, agent 2 goes and selects an object from the remaining two objects. Finally, agent 3 gets the remaining object.

   Write down the strategic form game by clearly specifying the strategies of the players.

3. An indivisible good is sold to 3 buyers. If any buyer i gets $q_i \in \{0, 1\}$ quantity of the goods makes a payment of p_i, her payoff is

$$q_i v_i - p_i.$$

   Payment p_i can be positive, negative or zero (some buyers may be *paid* or compensated).

   The seller asks each buyer to place a bid. If (b_1, b_2, b_3) are the bids of the buyers then the highest bidder wins (with ties broken in favor of highest indexed bidder[1]). If bidder

[1] For instance, if buyer 1 and 2 are joint winners, buyer 1 wins the object.

i wins, she pays $\max_{j \neq i} b_j$. Out of this payment, the seller returns

$$\frac{1}{3} \min_{j \neq i} b_j$$

to highest and second highest bidder and

$$\frac{1}{3} \max_{j \neq i} b_j$$

to the lowest bidder.

Show that bidding their own value is a weakly dominant strategy for each bidder.