Game Theory - Assignment 2

Due date: August 26, 2025

1. Consider a two player symmetric game where each player has three possible strategies $\{a, b, c\}$. The best response map of each player looks the same, and is shown in Figure **??**.

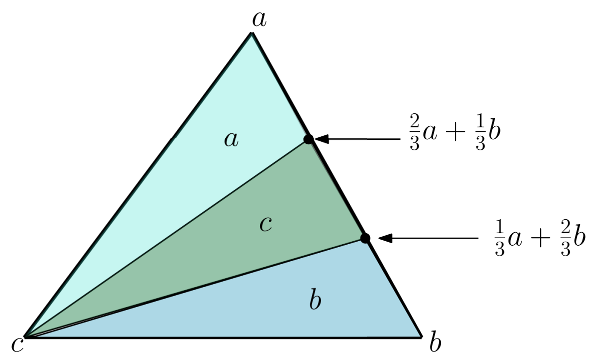

Figure 1: Best response map

- Find all pure strategy Nash equilibria of this game.
- Which strategies are never best responses?
- Find all mixed strategy Nash equilibria of this game.

2. Consider the two-player game in Table **??**, where utilities of the players are given by the real numbers a, b, c, d, e, f, g, h. Suppose there is a unique Nash equilibrium in this game in completely mixed strategies (i.e., each player uses all of its strategies with positive probability in this mixed strategy). Derive necessary conditions on a, b, c, d, e, f, g, h so that there is a unique Nash equilibrium in this game. Also, derive an expression for this Nash equilibrium. Illustrate this Nash equilibrium by considering matching pennies and some other games (by considering specific values of a, b, c, d, e, f, g, h) where a unique Nash equilibrium in mixed strategies exist.

	a	b
A	a, b	e, f
B	c, d	g, h

Table 1: A two-player game

3. Consider the following voluntary contribution game. There is a society with n citizens. A public good is developed if at least one citizen volunteers to put effort (which is binary, i.e., you put one unit of effort or zero unit of effort). Effort costs one unit to those who contribute. Public good, if created, gives a utility of $U > 1$ to everyone (including those who do not contribute). If noone volunteers, then the public good is not created and everyone gets zero utility. If player i does not volunteer and somone else volunteers, then player i can enjoy the public good with utility U without incurring cost.

   - Formulate this as a strategic form game.
   - Is there a weakly dominant strategy for each player in this game?
   - What are the (pure strategy) Nash equilibria in this game?
   - Consider the mixed extension of this game. Is there a mixed strategy Nash equilibrium where each player contributes with probability σ and does not contribute with probability $(1 - \sigma)$? Notice that the probability of contribution is same σ for all players. If yes, find σ such that the resulting mixed strategy is a Nash equilibrium. What happens to this Nash equilibrium as $n \to \infty$?

4. A Nash equilibrium s^* in a finite strategic form game $\Gamma = (N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$ is a **strict Nash equilibrium** if for every $i \in N$, for every $s_i \in S_i \setminus \{s_i^*\}$,

$$u_i(s_i^*, s_{-i}^*) > u_i(s_i, s_{-i}^*).$$

   Prove that if the process of iterative elimination of strictly dominated strategies results in a unique strategy profile s^*, then s^* is a strict Nash equilibrium.