Game Theory - Assignment 6

Due date: **November 03, 2025**.

1. There is a circular table of diameter 100 unit and there are coins of diameter 1 unit. Consider the following game between two players. Players move in turn and place the coin in an empty place on the table (they cannot stack coins). The game ends when a player cannot place a coin on the table. The player who places the last possible coin on the table gets utility 1 and the other player gets 0. Call the player who moves first Player 1 and the other player Player 2.

   - Consider the following strategy of Player 1. She places the first coin in the center of the table and at every subsequent move, places a coin radially opposite from the other player's previous move (i.e., draws a diameter of the table through the center of other player's last coin and places her coin on this diameter at the same distance as the other player but on the other side of the center of the table). Argue that this is a *valid* strategy for Player 1, i.e., Player 1 will always be able to place her coin like this.
   - Describe a subgame perfect equilibrium of this game.

2. Find a subgame perfect equilibrium of the game in Figure 1. In this game, in the vertex where Nature moves, the probabilities (for actions T and B) are shown on the edges.

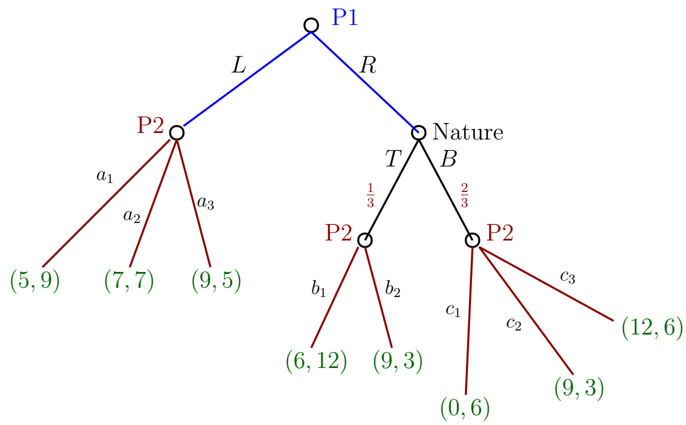

Figure 1: Extensive form game

3. Consider the extensive form game in Figure 2.

   (a) Find all pure strategy Nash equilibria and subgame perfect Nash equilibria of this game.

   (b) Suppose Player 2 can observe the move of Player 1 with probability $p \in (0, 1)$, i.e., if Player 1 plays L, Player 2 will observe L with probability p and R with probability $(1-p)$ and if Player 1 plays R, Player 2 will observe R with probability p and L with probability $(1-p)$. So, Player 2 observes L or R but does not know if it is the correct one.

      i. Describe this as an extensive form game of incomplete information - a game tree representation with information sets is sufficient.

      ii. Find a perfect Bayesian equilibrium of this game for $p = \frac{1}{2}$.

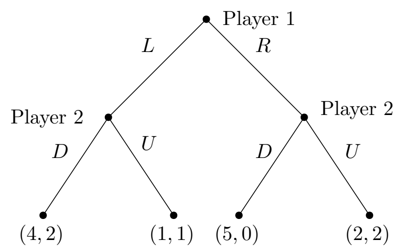

Figure 2: Extensive form game

4. King Solomon is faced with two women, Elizabeth and Mary, who both claim to be the mother of an infant. King Solomon does not know the true mother. If the true mother gets the child, then she gets a utility of 100. On the other hand, the woman who is not the true mother only gets a utility of 50 if given the child. By not getting the child, both women get zero utility.

   King Solomon sets up the following game.

Step 1. He will ask Elizabeth whether the child is hers. If she answers negatively, the child will be given to Mary. If she answers affirmatively, the king will continue to the next step.

Step 2. He will ask Mary whether the child is hers. If she answers negatively, the child will be given to Elizabeth. If she answers affirmatively, the king will ask Mary to pay 75 and Elizabeth to pay 10, and give the child to Mary. Utility from money is linear, i.e., paying p gives a utility of $-p$.

Since King Solomon does not know the true mother, there are two extensive form games possible - denote them as Γ_M (where Mary is the true mother) and Γ_E (where Elizabeth is the true mother).

(a) Describe Γ_M and Γ_E.

(b) Argue that there is a unique subgame perfect equilibrium of each of the games where the true mother gets the infant.

5. A firm goes to a bank for loan. The firm is one of two types: (a) a honest (H) type or (b) a cheat (C) type. The probability that the firm is of type C is $p = \frac{2}{3}$ and of type H is $1 - p$. Bank does not know the type of the firm. The bank can either *approve* or *reject* the loan request of the firm. If the loan request is approved, then the firm can either *default* the loan or *repay* the loan.

If the bank rejects the loan request, then both the bank and the firm receive a payoff of 10 each. If the bank approves the loan request and the firm repays the loan, then the bank receives a payoff of 40 and the firm receives a payoff of 60. If the bank approves the loan request and the firm defaults, then the bank has a **loss** of 100 (i.e., payoff is -100). On the other hand, if the firm defaults, his payoff is zero if he is of type H and 150 if he is of type C.

- Describe this as an extensive form game of incomplete information (a graphical representation describing all information sets is good enough).
- Describe a perfect Bayesian equilibrium of this game.