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Deterministic mechanisms and the revelation principle

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Abstract

This note shows: (1) the classical revelation principle does not hold for deterministic mechanisms, (2) with one agent, a revelation principle in terms of payoffs holds, and (3) with more than one agent the result fails and direct mechanisms may by suboptimal.

Keywords: Adverse selection; Optimal mechanisms; Revelation principle

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1. Introduction

In most applications of mechanism design economic analysis restricts attention to deterministic mechanisms. This note shows that an ad hoc restriction to deterministic mechanisms leads to a failure of the classical revelation principle (e.g. Gibbard, 1973; Green and Laffont, 1977; Dasgupta et al., 1979 and Myerson, 1979). However, in mechanism design problems with only one agent a revelation principle can be formulated in terms of payoffs: Any payoff implementable by a deterministic mechanism can at least be matched with a deterministic, direct mechanism that induces truthful revelation.[1] I show in an example that with more than one agent this result fails. As a consequence, the structure of optimal deterministic mechanisms differs from the structure of optimal mechanisms in general. Moreover, in settings with multiple agents the justification of the use of direct mechanisms when restricting attention to deterministic ones is unclear, because they may be suboptimal.

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[1] The result therefore differs from Bester and Strausz (2001), where optimal direct mechanisms may require the agent to lie with a strictly positive probability.

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2. The model

Consider a contracting problem between a principal, the mechanism designer, and n agents: The principal has no private information. Each agent $i = 1, \dots, n$, however, is privately informed about his type $t_i \in T_i$. For simplicity, I take T_i to be finite. The agents' types $t = (t_1, \dots, t_i, \dots, t_n) \in T = T_1 \dots \times T_i \dots \times T_n$ are drawn from some objective distribution $p(t)$. After agent i observes his type t_i, his conditional probability about the other agents' types is $p(t_{-i}|t_i)$. The principal's problem consists of selecting deterministically an allocation $x \in X$. This allocation together with the agents' types determines the players' von Neumann–Morgenstern utilities. I denote the principal's payoff by $V(x,t)$ and the agents' payoffs by $U_i(x,t)$, $i = 1, \dots, n$.

A *mechanism* or contract, (M,x), specifies for each agent i a message space M_i and an *allocation function* x: $M \to X$, that commits the principal to implement the allocation $x(m_1, \dots, m_n)$ when the agents send the combined message $(m_1, \dots, m_n) \in M \equiv \times_i M_i$. To avoid measure theoretical problems, I take M_i to be finite and denote by $\mathcal{M}_i$ the set of probability distributions over M_i. A mechanism (M,x) yields a game between the agents: After agent i has learned his type t_i, he chooses a message; his *reporting strategy* $\mu_i(t_i)$: $T_i \to \mathcal{M}_i$ selects the message m with probability $\mu_i(m|t_i)$. A mechanism is called a *direct* mechanism if $M_i = T_i$ for all i. With a direct mechanism the message space coincides with the agents' type space such that one may interpret the message game as a game in which each agent simply announces his type.

Since the agents' types are private information, the principal's choice of x can depend on t only through the agents' messages $m = (m_1, \dots, m_i, \dots, m_n) \in M = M_1 \dots \times M_i \dots \times M_n$. Note that the allocation function x is deterministic, since it maps the message $m \in M$ directly into a deterministic allocation $x \in X$. In this note the principal is restricted to such deterministic allocation functions.

3. Failure of the revelation principle

Suppose there is one agent who is privately informed about his type; he is equally likely to be either of type t_1 or of type t_2. Moreover, there exist three possible allocations A, B, and C, i.e. $X = \{A,B,C\}$.

Given the agent is of type t_i, the principal's payoff, V, and the agent's payoff, U_1, are:

$$V(A,t_1) = 1 \quad V(B,t_1) = 0 \quad V(C,t_1) = 0$$

$$V(A,t_2) = 0 \quad V(B,t_2) = 1 \quad V(C,t_2) = 0$$

$$U_1(A,t_1) = 1 \quad U_1(B,t_1) = 0 \quad U_1(C,t_1) = 0$$

$$U_1(A,t_2) = 0 \quad U_1(B,t_2) = 1 \quad U_1(C,t_2) = 1$$

Consider the following mechanism (M,x) with $M = \{m_1,m_2,m_3\}$ and $x(m_1) = A$, $x(m_2) = B$ and $x(m_3) = C$. Note that the mechanism is indirect, since it requires the agent to send some message m_i instead of announcing a certain type. Moreover, the mechanism is deterministic, as a certain message leads to an allocation deterministically.

It is straightforward to see that the mechanism (M,x) leads to a game with an equilibrium in which type t_1 sends the message m_1, i.e. uses strategy $\mu_1(t_1) = (1,0,0)$, and type t_2 sends the message m_2 and m_3 with equal probability, i.e. uses reporting strategy $\mu_1(t_2) = (0,1/2,1/2)$. Hence, an equilibrium outcome is that, if the agent is of type t_1, the allocation A is implemented and, if the agent is of type t_2, allocation B and C are equally likely to be implemented. This equilibrium yields payoffs $U_{11} = 1$ for type 1, $U_{12} = 1/2*1 + 1/2*1 = 1$ for type 2, and $V = 1/2*1 + 1/2*(1/2*1 + 1/2*0) = 3/4$ for the principal.

The principal is unable to reach this outcome with a direct, non-stochastic mechanism. The restriction to a direct mechanism requires that the agent can choose from two messages only. The requirement therefore that the mechanism is deterministic implies that with a direct mechanism the principal can induce at most two different allocations.

The reason behind the failure of the revelation principle is straightforward. In the considered equilibrium the agent mixes between two messages. This causes the allocation for type t_2 to become stochastic. Yet, if the principal is unable to use a random implementation strategy, he cannot replicate the randomness.

4. One agent

The previous example illustrates the failure of the classical revelation principle. Yet, in the spirit of Bester and Strausz (2001) one may ask if this failure is economically relevant when parties are only interested in the final payoffs they achieve rather than the underlying allocations. In this section, I show that if the mechanism designer is concerned with only one agent, this is indeed the case and a revelation principle can be reformulated in terms of payoffs:

Proposition 1. *Suppose $n = 1$ and consider an equilibrium of a game induced by a deterministic indirect mechanism (M,x). Then there exists a deterministic direct mechanism, (T,x'), that induces an equilibrium with truthful revelation with associated payoffs that weakly Pareto dominate the equilibrium payoffs of the considered equilibrium under the indirect mechanism.*

Proof. Let $n = 1$ and consider a mechanism (M,x) with a deterministic allocation function $x:M \to X$ and equilibrium reporting strategies $(\mu_1(t_1), \ldots, \mu_1(|T_1|))$. Define for each type t_i a set of allocations, $X_i \subset X$, that under reporting strategy $\mu_1(t_i)$ are reached with positive probability, i.e. $X_i = \{x(m)|\mu_1(m|t_i) > 0\}$. Note that X_i is necessarily non-empty and, since M is finite, also finite. Since $\mu_1(t_i)$ is an equilibrium strategy, for all $x \in X_i$ and $x' \in X$ it holds $U_1(x,t_i) \geq U_1(x',t_i)$. Moreover, for all $x,x' \in X_i$ it holds $U_1(x,t_i) = U_1(x',t_i)$. That is, in equilibrium type t_i must be indifferent over all allocations $x \in X_i$ that are reached with positive probability under the reporting strategy of type t_i. Now define the set P_i as the set of allocations that, given type t_i, yields the principal the highest utility among those allocations that are reached with positive probability under reporting strategy of type t_i, i.e. $P_i = \{x \in X_i|V(x,t_i) = \max_{y \in X_i} V(y,t_i)\}$. Since X_i is finite the set P_i is non-empty. Now construct a direct mechanism (T,x') with $x'(t_i) \in P_i$. Since $P_i \subset X_i$ the allocation function x' leads to a subset of allocations under the indirect mechanism x. Hence, under the direct mechanism (T,x') it is an optimal strategy for type t_i to announce his type truthfully. This equilibrium yields type t_i the same payoff as the equilibrium under the indirect mechanism (M,x) and yields the principal weakly more (The principal receives strictly more if $P_i \neq X_i$ for some t_i). Q.E.D.

The idea behind Proposition 1 is straightforward and depends on two observations: First, if an indirect mechanism leads to an equilibrium that induces for every type a deterministic allocation $x \in X$, then the logic of the classical revelation principle holds: The mechanism designer can use a direct mechanism that, for a certain type of agent, selects the allocation that the indirect mechanism would have implemented for this type. In this case, the restriction to deterministic mechanisms is irrelevant, because given a certain type of agent the equilibrium outcome itself is deterministic.

Second, as shown in the previous example, the revelation principle fails to hold, if, for some type of agent, the equilibrium outcome under the indirect mechanism yields a stochastic allocation. If there is only one agent this can only be attributed to a mixing behavior of the agent himself. But this requires that the agent is indifferent between these messages and ultimately all the allocations that are induced by the messages he chooses. Hence, from the equilibrium behavior of the agent, the principal can construct a deterministic direct mechanism by selecting for each type that mixes over messages the message which yields the principal the most.

In the example of the previous section, type t_2 is mixing over message m_2 and m_3. Of these two messages the principal prefers m_2 since it leads to the more preferred allocation B. Applying the idea behind Proposition 1 to the example, yields therefore the direct mechanism (T,x') with $x'(t_1) = A$ and $x'(t_2) = B$. The direct mechanism (T,x') induces a game in which truthful revelation is an equilibrium. The associated equilibrium payoffs of this equilibrium are $U_{11} = 1$ for type 1, $U_{12} = 1$ for type 2, and $V = 1/2*1 + 1/2*1 = 1$ for the principal. Note that in comparison with the considered equilibrium under the indirect mechanism, the agent's utility is unchanged and the principal has gained.

In contrast to the result in Bester and Strausz (2001). the reformulated revelation principle is closer to the classical one, since it shows that there is no loss of generality when the principal focuses on equilibria in which the types reveal themselves truthfully. Therefore, even though one may view the inability of a mechanism designer to use stochastic mechanisms as a form of imperfect commitment, this type of imperfect commitment is less dramatic than imperfect commitment in general. As shown in Bester and Strausz (2001), optimal direct mechanisms in general may require lying.

5. Multiple agents

Suppose there exists a second agent who has no private information, but whose utility depends on the implemented allocation as follows:

$$U_2(A,t_1) = 0 \quad U_2(B,t_1) = -1 \quad U_2(C,t_1) = 1.$$
$$U_2(A,t_2) = 0 \quad U_2(B,t_2) = -1 \quad U_2(C,t_2) = 1$$

Note that the agent's utility is independent of the type of agent 1. The indirect mechanism (M,x) of the example yields the agent an expected payoff of 0. It is straightforward to see that if the principal uses a deterministic direct mechanism rather than the indirect mechanism (M,x) at least one of the players is worse off: To guarantee type t_1 his payoff of 1, the direct mechanism must exhibit $x(t_1) = A$, but for any $x(t_2) \in \{A,B,C\}$ the principal or agent 2 loses. Hence, Proposition 1 fails to hold.

In a contracting-like setting in which parties must first approve to participate in the mechanism, the failure of Proposition 1 may have the important consequence that direct mechanisms are suboptimal. For example, if agent 2 in the extended example has a reservation utility of $\bar{U}_2 = 0$, he will veto any mechanism that leads to a negative equilibrium payoff. In this case any direct mechanism is suboptimal in comparison to the indirect mechanism (M,x).

6. Conclusion

This note shows that the classical revelation principle does not hold for deterministic mechanisms. If the mechanism designer deals with one agent only, a revelation principle in terms of payoffs may be established. In settings with more than one agent such a result cannot be obtained, implying that direct mechanisms may turn out to be suboptimal.

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