Infectious Diseases, Human Capital and Economic Growth

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Abstract

Cross-country evidence shows that developed countries have almost eradicated infectious diseases, grow at a higher rate and have high educational attainment; developing countries grow at a moderate rate with low disease prevalence and an intermediate level of educational attainment; and the least developed countries are stuck in a poverty trap with highest disease prevalence and lowest educational attainment. To capture these empirical facts we enrich the endogenous growth model (Lucas 1988) with SIS epidemiological model incorporating explicitly the dynamics of disease transmission. Not only does the disease prevalence have an adverse effect on economic activity and human capital accumulation, but individuals can affect the disease transmission through health expenditures. We show there are multiple balanced growth paths mirroring the empirical evidence. The prevalence of infectious diseases, which is endogenous determined itself, is the key factor in deciding whether countries are in a poverty trap or undergoing a growth path. Compared with a decentralized economy where individuals fail to fully internalize positive externality associated with controlling diseases, we further show that with a public health policy, countries are more likely to take off and grow at a higher rate. We characterize the optimal Pigouvian health subsidy and show that in countries with a higher incidence of diseases it can be lower than in those with a lower incidence. In addition, we examine when an economy may take off in response to a demographic transition or enter into a Malthusian scenario.

JEL Classification: E19, I10, D90, O11.

Key Words: Infectious Diseases; Endogenous Growth; Epidemiology; Poverty Trap; Public Health Policy; Human Capital

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1 Introduction

Diseases and income are inextricably tied together. Empirical evidence shows that incidence of infectious diseases is typically high in poorer countries, and low in richer countries. The evidence also shows that human capital, measured by years of schooling, displays a similar pattern. This paper addresses the issue of how incidence of infectious diseases, human capital, income and growth are determined simultaneously. Understanding the endogenous determination of the three in a dynamic general equilibrium will enable us to understand the important but often contentious relationship between the burden of disease and growth.[1] Most of the macroeconomic literature either does cross country regressions or uses some variant of the Solow model without fully accounting for the effect of diseases on incentives for human and physical capital accumulation. Using quasi-experiments, the microeconomic empirical literature tends to find larger effects of diseases on human capital (Bleakley (2007, 2010)). However, how this may aggregate in the macroeconomy is not yet fully understood. Therefore, in this paper we model the evolution of infectious diseases, human capital and income in a dynamic general equilibrium framework.

This paper builds a Lucas (1988) endogenous growth model where individuals allocate time between working and accumulating human capital. The difference is that in our environment, individuals are exposed to the risk of being infected by an infectious disease. To model the transmission of infectious diseases, we incorporate insights from the mathematical biology literature on epidemiology of infectious diseases. These epidemiology models lend themselves to integration into dynamic economic models as they capture disease transmission via dynamical systems. There are many epidemiology models describing different disease transmission mechanisms. Models with different epidemiological structure, however, may bring little economic insight with added burden of complexity. In light of this, we adopt the canonical and simplest SIS epidemiological model.[2] Moreover, individuals can intervene in the transmission of diseases through preventive health expenditure. This is im-

[1] There is an empirical literature, trying to resolve the endogeneity issue and estimate effect of diseases on the economy. A debate still remains on its quantitative significance. Some papers find the effect of diseases to be large (Bloom et al (2009), Gallup and Sachs (2001)), while others find the effect is modest (Ashraf et al (2009), Weil (2008, 2010)) or there might even be an adverse effect due to the dilution effect of a larger population and increase in dependency ratio (Acemoglu and Johnson (2007), Young (2005)).

[2] These include influenza, malaria, dengue, the so-called neglected diseases which are endemic in sub-Saharan Africa (soil transmitted helminths, including: roundworms such as Ascaris Lumbricoides which causes ascariasis; whipworm which causes trichuriasis; hookworms which, depending on the species, cause necatoriasis and ancylostomiasis; schistosomiasis; lymphatic filariasis; trachoma; river blindness (onchocerciasis) kala-azar black fever (and other clinical forms of leishmaniasis); Chagas disease (American trypanosomiasis); leprosy; African sleeping sickness (human African trypanosomiasis); Guinea-worm (dracunculiasis); Buruli ulcer), STDs (including syphilis and gonorrhea), cholera, diarrohea, tuberculosis, etc. Some of these are vector borne diseases which from a modeling point of view entail would require the modeling of the evolution of the vector which we abstract from in the analysis.

portant, as when the incidence of diseases, income and human capital evolve the incentives for controlling the disease also change.

The economic and epidemiological structure of the model is as follows. An individual is born healthy or susceptible to the disease (i.e., in state S). He/she can become infective (i.e., in state I) – infected with the disease and capable of transmitting it to others. If the person is infected, then he/she is incapacitated and cannot work or acquire human capital; if the person is not infected, then he/she can choose the amount of time for both working and education. As a result of the latter, aggregate human capital increases, and so does labor productivity. The infected, with some probability, recovers and transits to the state S. In order to capture the fact that individuals can control the disease transmission, the contact rate, the key parameter in epidemiology model, is endogenized. Not only individuals can reduce the likelihood of being infected by investing in health capital, but higher physical capital and stress of higher economic activity could impair the immune system or lead to more social activity, such that individuals could easily get infected. Therefore, the contact rate is assumed to be a function of the ratio of health and physical capital. This specification is also necessary to ensure the economy to grow at a balanced path. We use the framework of a large representative household, commonly used for embedding a labor search structure into a dynamic general equilibrium model. This brings the model closer to the canonical endegenous growth model as we can avoid from keeping track of the cross-sectional distribution of the health status and various economic variables.

We use two different specifications to study the impact of household choices on the evolution of the disease. The first is a decentralized decision-making framework where the household does not internalize the externality of infectious disease transmission. This is like a "disease-taking" behavior where the household takes the fraction of infected population as given and does not take into account the effect of private health expenditure on the evolution of the infected in the population (Gersovitz and Hammer (2004)). We then look at the optimal health policy where a social planner internalizes the externality.

We show there are multiple balanced growth paths (BGP), mirroring the cross-country empirical evidence. First, infectious diseases are eradicated, and countries grow at a fast rate with high educational attainment. This is the standard endogenous growth model with human capital accumulation. Second, countries grow at a moderate rate with low disease prevalence and an intermediate level of educational attainment. Third, countries are stuck in a poverty trap with highest disease prevalence and lowest educational attainment. The intuition is that when infectious diseases are endemic and the severity of disease prevalence, which is itself endogenously determined, decides whether countries are in a poverty trap or grow at a moderate rate. This is because the time allocation for human capital accumulation – the engine of economic growth – depends on the effective labor force or the fraction of

healthy individuals in the economy, which is in turn determined by the disease prevalence and thus the ratio of health and physical capital. When the return to human capital is extremely low due to the prevalence of the disease and the country may be in a poverty trap.

Unlike the decentralized economy where individuals fail to take into account the positive externality of their preventive behavior on the others, a social planner fully internalizes this. As a result, growth rate tends to be higher or it is more likely for countries to escape from poverty trap. Thus the effect of infectious diseases is especially severe in poorer countries with a tighter budget constraint and lacking an effective public health policy. In the paper, we examine the role of a particular public health policy, that is subsidizing private health expenditure. This is a Pigouvian subsidy that would cause households to make the socially optimal health expenditures. This subsidy is financed through lump-sum taxes. We show that the subsidy is proportional and increasing in the externality. The subsidy also depends on whether the economy is in a poverty trap or not, being smaller in the former case. The reason is that when the socially optimal equilibrium is a poverty trap, the returns to health expenditures is small as there is no human capital accumulation. Thus, poor economies in a poverty trap may be perversely having low optimal subsidies.

To study the effect of exogenous change in life expectancy (due to medical innovations etc.), we examine the effect of decrease in death rate. One issue is whether a decrease in death rate can give rise to a Maltusian scenario, that is the consumption levels of households decrease due to dilution effects. We show this can happen only for economies which are in the poverty trap, for others a decrease in mortality unambiguously increases welfare. This is shown analytically and the possibility of a Malthusian scenario is illustrated in a calibrated exercise. In this example we show that while there can be a Malthusian scenario, the welfare of households, however, increases as the effect of increased longevity (which implies a lower discount rate) dominates the drop in consumption.

There are a few papers examining economic epidemiological models from various different angles. Geoffard and Philipson (1997), Kremer (1996) and Philipson (2000) develop microeconomic models at rational choices of individuals in face of the diseases and how it may affect the spread of the diseases. Delfino and Simmons (2000) combines an epidemiological model and Solow growth model with a fixed savings rate and exogenous labor supply. In Goenka and Liu (2012), we endogenize savings and the labor-leisure choice, but there is only a one-way effect: diseases affect the economy but their transmission is largely biological. The paper shows that the non-linearities in disease dynamics can be a source for cycles and chaos. However, to address endogeneity of both diseases and economic activities, we need to simultaneously model both capital accumulation and the epidemiological structure of the diseases. Goenka, Liu and Nguyen (2013) is a first step in this direction within a framework of a neo-classical growth model. In the current paper, we extend the analysis to

an endogenous growth model. In this paper we incorporate the *SIS* disease dynamics into an endogenous growth model with human capital accumulation costly skill acquisition or schooling, fully endogenizing both disease transmission and choice of physical, human and health capital.

There are two kinds of effects of diseases: disease related mortality and morbidity (illness). Much of the macroeconomic literature in explaining diseases and poverty, concentrate on premature death and its consequences on education and fertility (e.g. Chakrabory et al. (2010), Kalemli-Ozcan, Romer and Weil (2000), Lagerlof (2003), and Soares (2005)). However, there are many endemic diseases that have low mortality but the burden of diseases is estimated to be large. Our paper demonstrates diseases have adverse effect on human capital accumulation and thus economic growth even without the disease related mortality. The focus of morbidity rather than mortality is also there in the microeconomic literature looking at the effect of control of infectious diseases on schooling and income (see Bleakley (2007, 2010), Cutler et al (2010) and Lucas (2010). In this paper, we also abstract from the endogenous fertility choice that has been the focus of some of the literature so as to focus on endogenous disease transmission on income, health and human capital.[3] Furthermore, this literature has been built extensively on the OLG model and does not use epidemiological modeling. In contrast, we carry on the tradition of endogenous growth model with continuous time and infinite lived agents.

The paper is organized as follows. Section 2 provides empirical facts and Secion 3 presents the economic epidemiological model in a decentralized economy. Section 4 examines multiple balanced growth paths. Section 5 investigates a centralized economy and provides policy implications, and Section 6 studies effects of rising life expectancy. Section 7 concludes.

2 Empirical Evidence

In this section, we present cross-country evidence on the relationship among diseases, human capital and growth. We use a cluster analysis to group countries based on their characteristics: infectious disease prevalence, educational levels and economic growth. The reason we do this rather than reduced form regression as the literature is two-fold: economic growth, human capital and disease prevalence are simultaneously determined, causing an endogeneity problem; and there is an asymmetric effect of disease control, causing a non-linearity

[3]The evidence of diseases on fertility is somewhat mixed. Some papers find evidence of the standard Beckerian channel of decrease in disease incidence increases demand for quality rather than quantity of children, and hence, fall in fertility (e..g. Bleakley and Lange (2009) who study hookworm eradication in southern USA. Others find no evidence of changes of disease incidence on fertility (e.g. Fortson (2009), and Kalemli-Ozcan and Turan (2011) who both study effect of HIV/AIDS in Africa.)

problem. Both these issues impose a challenge for reduced-form regression and can be a reason for the sensitivity of the estimate for the impact of disease control on the economy (see e.g. the differing results of Acemoglu and Johnson (2007) and Bloom, Canning and Fink (2009)).

How should one measure the burden of infectious diseases? The mortality rate is often used, both for reasons of humanity and easy data accessibility. However, morbidity caused by infectious diseases is at least as important as mortality (see Bleakley (2007, 2010) for impact of diseases with morbidity but low mortality). Diseases with a low mortality rate but a high morbidity rate have have effect in terms of both the direct cost of treating, and indirect of cost of being disabled from the disease. As a result, World Health Organization (WHO) provides a summary measure - disability adjusted life year rates (DALY) - to give a better indication of the burden of diseases from both mortality and morbidity. It is calculated as the ratio of sum of the years of life lost due to premature mortality (YLL) and the years lost due to disability (YLD) in the population.[4] Table 1 presents DALY, YLL and YLD at aggregate country level. The number indicates for each individual the proportion of time lost due to infectious diseases. For instance, in 2002 for a representative individual living in the developing countries, on average 1.7 percent of his time is lost due to infectious diseases, among which 1.2 percent is due to premature death and 0.5 percent is due to disability. As this paper focuses more on disability caused by infectious diseases, ideally we should be using YLD as the measure for the burden of infectious diseases. However, since YLD is not available at the country level, we use DALY in the following cluster analysis. From Table 1 we can see countries bearing the heavier burden of infectious diseases - higher in DALY – are higher in both YLL and YLD. Thus, for the cluster analysis, our results should be robust to any of the above measurements.

For educational attainment at the country level, we use quality adjusted average schooling years from Barro and Lee (1986). The rest of data used for the cluster analysis is from the World Bank database. One problem with all these variables is availability of comparable data: educational levels are available from 1965 to 1990 at 5 years intervals, while DALY is only available starting from 2002. In order to have more accurate country classifications, we incorporate more information including mortality rate caused by infectious diseases in year 1965, 1990 and 2002. Thus, the variables used in the cluster analysis are: average schooling years in year 1965 and 1990, GDP per capita in year 1965 and 1990, GDP growth rate from year 1965 to 1990, life expectancy in year 1965 and 1990, mortality rate in 1965, 1990 and

[4]YLL basically corresponds to the number of deaths caused by infectious diseases multiplied by the standard life expectancy at the age at which death occurs. To estimate YLD, the number of incident cases in a certain period is multiplied by the average duration of the diseases and a weight factor that reflects the severity of the disease on a scale from 0 (perfect health) to 1 (death). For more details, please refer to WHO website: http://www.who.int.

Table 1. Burden of Infectious Diseases

	DALY	YLL	YLD
Developed Countries	0.006	0.004	0.002
Developing Countries	0.017	0.012	0.005
Least Developed Countries	0.128	0.112	0.016

Source: The Global Burden of Disease (GBD) for year 2002, downloaded from WHO website. All countries and regions are divided into three groups: developed countries including France, Japan, US etc., low mortality developing countries (called developing countries here) including Brazil, Mexico, Thailand, etc. and high mortality developing countries (called least developed countries here) including Kenya, Mali, Zimbabwe etc. For a complete list of countries and regions, please refer to WHO website. DALY measures the difference between a current situation and an ideal situation where everyone lives up to the age of the standard life expectancy and in perfect health. Based on life tables, the standard life expectancy at birth is set at 80 years for men and 82.5 for women. DALY combines in one measure the time lost due to premature mortality (YLL) and the time lived with disability (YLD). YLL basically corresponds to the number of deaths caused by infectious diseases multiplied by the standard life expectancy at the age at which death occurs. To estimate YLD, the number of incident cases in a certain period is multiplied by the average duration of the diseases and a weight factor that reflects the severity of the disease on a scale from 0 (perfect health) to 1 (death).

2002, and DALY in 2002, including both the ratio of years lost caused by infectious diseases and total population, and the ratio of years lost caused by infectious diseases and all causes. There are in total 67 countries for which we have the data. For a complete list of countries used in the cluster analysis, refer to the Table Appendix.

Through the cluster analysis, we classify all the countries into three groups. Based on the statistics, shown in Table 2, it is clear that these groups represent developed countries, developing countries and the least developed countries, respectively. The average growth rate for developed countries is around 2.85 percent annually, for developing countries it is around 1.86 percent. In contrast, the least developed countries are stuck in the poverty trap with negative average growth rate from 1965 to 1990. In terms of the spread of infectious diseases, the least developed countries bear the heaviest burden of infectious diseases. On average, for each individual 26 percent of their time is lost due to either premature death or disability caused by infectious diseases. As a comparison, an individual in developing countries loses 2.5 percent of his time due to infectious diseases and this number is almost zero for developed countries. The life expectancy at birth in developed countries is significantly higher than the one in developing countries, which again is significantly higher than the one in the least developed countries for both the year 1965 and 1990. For the educational attainment, developed countries have the highest educational levels with 6.96 average schooling years in 1965 and 8.76 in 1990, while the least developed countries have the lowest educational levels with 1.15 in 1965 and 2.42 in 1990. Therefore, these statistics from the cluster analysis

indicate that there is a close link between disease prevalence, human capital and economic growth.

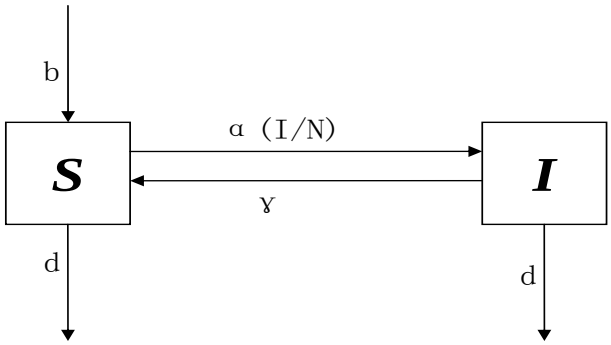
More evidence is shown by cross-country joint distribution of GDP per capita and DALY ratio in Figure 2. In 1965 (left panel), countries with heaviest burden of infectious diseases are countries with lowest GDP per capita. This relationship remains true in 1990 (right panel) although on average GDP per capita rises. Thus, there is a positive relationship between disease control and economic development, and the causality is likely to run both ways. Moreover, Figure 3 provides the cross-country joint distribution of average schooling years and DALY ratio caused by infectious diseases. It mirrors the cross-country joint distribution of GDP per capita and DALY ratio in Figure 2. In 1965 (left panel), countries with high DALY ratio are associated with low average schooling years. The same cross-country relationship holds true in 1990 (right panel) with on average rising educational attainment, particularly for countries with low disease prevalence. Thus, it suggests that there is a positive relationship between disease control and educational attainment.

This cross-country evidence is consistent with some of the micro empirical studies in the literature. For instance, Bleakley (2007) evaluates the economic consequence of the successful eradication of hookworm disease from the American South, and finds that areas with higher level of hookworm infection prior to the intervention experienced greater increase in school enrolment, attendance and literacy. Miguel and Kremer (2004) evaluate a Kenyan project with deworming drugs targeting intestinal helminths, and find that the program substantially reduce school absenteeism. The evidence on eradication or control of malaria also indicates positive effects on schooling, health capital and subsequent income (Bleakley (2010), Lucas (2010). Cutler et al (2010) find weak effects of malaria eradication in India.These micro empirical studies focus on diseases where the burden is predominantly in the childhood. There is a concern that if there is child labor then part of the effect of decline in morbidity increase child labor supply. Our model is an infinitely agent framework (as we want to abstract from mortality effects of diseases) and agents can accumulate human capital in any period. This is consistent with the evidence as increase in human capital will subsequently increase income, but it also takes a more general view of human capital accumulation through non-schooling acquisition of skills.

3 The Model

In this section, we build a theoretical model to capture the cross-country pattern of diseases prevalence, educational attainment and economic growth. First, we introduce the canonical SIS epidemiology model. Then, we integrate it into an endogenous growth model with human capital accumulation.

Figure 1. The Transfer Diagram For the SIS Epidemiology Model



Note: In a SIS epidemiology model, the total population is divided into two groups: the susceptible denoted as 'S' and the infected denoted as 'I'. The birth rate is b and newborns are born healthy and susceptible. All individuals irrespective of health status die at the rate d. The susceptible get infected at the rate $\alpha\frac{I}{N}$ and the infected recover at the rate γ. For more details, refer to Hethcote (2005).

3.1 SIS epidemiology model

Epidemiology is the study of the distribution and determinants of disease prevalence. The mathematical modeling is done by dividing the population into different health states (e.g. susceptible, infectious), and the movements between these states is given by a dynamical system. [5] Depending on the characteristics of the disease, there are many different epidemiology models and some of the details bring little economic insight with the added burden of complexity. As we want to highlight the role of morbidity associated with recurring diseases, we choose the canonical SIS model.[6]

Figure 1 describes the transfer diagram for the SIS model. The total population is divided into two groups: the susceptible (healthy and susceptible to the disease) and the infective (infected and capable of transmitting the disease), whose size change with time. Let N_t be the total population size, S_t be the number of susceptibles and I_t be the number of infected. Individuals are born at the rate b, healthy and susceptible to the disease. We assume homogeneous mixing so that the likelihood of any individual contracting the disease is the same. There is only horizontal incidence of the disease i.e. from peers. Let α_t be the average number of adequate contacts of a person to catch the disease per unit time or the contact rate. Then, the number of new cases per unit of time is $\alpha(I/N)S$,[7] depending on the *fraction*

[5]This is very similar to labor search model, in which all individuals are divided into two groups: employed and unemployed. The flow between groups is described by a law of motion.

[6]For more details on epidemiology models, see Hethcote (1994, 2005). *SIS* epidemiological model is applicable to infectious diseases which are absent of immunity or which mutate rapidly so that people will be susceptible to the newly mutated strains of the disease even if they have immunity to the old ones. For instance, *SIS* is the epidemiology model for recurring diseases, but not for HIV/AIDS where the individual never recovers from it, or disease which confer disease related immunity such as measles. Many of the diseases which cause morbidity and are not amenable to effective vaccinations are of the *SIS* type.

[7]The time subscript, t is omitted if there is no confusion.

of infected people. This contact structure is the standard incidence or frequency dependant model, commonly used in the epidemiology literature. The basic idea is that the pattern of human interaction is relatively stable and what is important is the fraction of infected people rather than the total number. If the population increases, the pattern of interaction will be invariant.[8] The contact rate α is the key parameter and reflects two different aspects of disease transmission: the biological infectivity of the disease and the pattern of social interaction. Changes in either will change α. The recovery of individuals is governed by the parameter γ and the total number of individuals who recover from the disease at each time period is γI. Upon recovery, individuals do not have any disease conferred immunity, and move back to the class of susceptible individuals. Each individual faces the exogenous death rate d, irrespective of health status.[9]

The SIS epidemiology model is given by the following system of differential equations (Hethcote, 2005):

$$\begin{aligned}
&\dot{S} = bN - dS - \alpha(I/N)S + \gamma I \\
&\dot{I} = \alpha(I/N)S - \gamma I - dI \\
&N = S + I \ \forall t;\ S, I \geq 0 \ \forall t;\ S_0, I_0 > 0 \text{ given.}
\end{aligned}$$

The first equation says that the change in the number of the susceptibles equals to the inflow of newborns, bN, and the recovered, γI, minus the outflow due to both being infected, $\alpha(I/N)S$, and death, dS. Similarly, the second equation shows that the change in the number of the infected is the difference between the inflow of newly infected, $\alpha(I/N)S$, and the outflow of the those recovered, γI and due to death, dI. The total population grows at the rate $b - d$. Let $s = S/N$ and $i = I/N$ be the fractions of individuals in the susceptible and infected class, respectively. We can rewrite the above equations as:

$$\begin{aligned}
\dot{s} &= b - ds - \alpha is + \gamma i - s(b - d) \\
\dot{i} &= \alpha is - \gamma i - di - i(b - d).
\end{aligned}$$

[8]Naively, it might seem plausible that the population density and hence the contact rate would increase with population size, but the daily contact patterns of people are often similar in large and small communities, cities and regions. For human diseases the contact rate seems to be only very weakly dependent on the population size. The other commonly used model, i.e., new cases equal to αIS, is used typically for herd animals. For more discussion about the form of the incidence, see Hethcote (2005).

[9]Introducing disease-related mortality rate will make the discount factor endogenous, since population growth is affected by the composition of the healthy and infected individuals, which eventually are endogenous variables. This will become clear in the following subsection. Nevertheless, we provide comparative statics of varying death rate or life expectancy.

Since $i = 1 - s$, one of these equations is redundant, and we can simplify them to:

$$\dot{s} = (b + \gamma)(1 - s) - \alpha(1 - s)s \tag{1}$$

with the total population growing at the rate $b-d$. We maintain the assumption that $b-d \geq 0$, that is, the net population growth is always non-negative. Otherwise, the population will become extinct.

The SIS epidemiology model admits multiple steady states, which mirrors the multiple balance growth paths we have later. One steady state is the disease-free steady state ($s^* = 1$), and the other is the disease-endemic steady state ($s^* = \frac{b+\gamma}{\alpha}$). We notice that the former exists for all parameter values, while the latter exists only when $\frac{b+\gamma}{\alpha} < 1$. [10]

3.2 Economic epidemiology model

The economic model follows from the Lucas (1988) endogenous growth model with human capital accumulation, in which we incorporate the dynamics of disease transmission. To avoid from keeping track of the cross-sectional distribution of the healthy and infected individuals and their respective economic variables, and stay close to the canonical endogenous growth model, we adopt the "large household" framework, which is commonly used to embed a labor search structure into a general equilibrium model.

The households. We assume the economy is populated with many identical households, which are taken as the representative decision-making agents. The size of population in each household grows over time at the rate of $b - d$. In each household, an individual is either healthy or infected by the diseases. Each household is assumed to be sufficiently large so that the proportion of the household in each disease status is identical to the corresponding population proportion. Thus, within a household, the proportion of health individuals is s and the proportion of infected individuals is i. Following Gersovitz and Hammer (2004), each household understands and anticipates how the epidemic will evolve and is fully forward-looking with regard to its possible future states as well as its present situation. The only difference from the SIS epidemiology model above is that the household is assumed to be small relative to the population as a whole. Thus, a household believes that the proportion of itself in any disease status does not affect the proportion of the population as a whole in that status. In particular, the household takes as given the proportion of the population that is infected, denoted as Π, and thinks the probability for the healthy individuals to contract disease is $\alpha\Pi$, rather than αi. As a result, the disease transmission dynamics perceived by

[10] When both steady state co-exist, that is $\frac{b+\gamma}{\alpha} < 1$, the disease-free steady state is unstable.

the households is now given as follows:

$$\dot{s} = (b+\gamma)(1-s) - \alpha \mathbf{\Pi} s.$$

This captures the idea that the household is small relative to the population and is like competitive "disease taking." It captures the idea that the household does not take into account the externality on disease transmission. This distinguishes the competitive model from the social planner's problem where this externality is taken into account. The two different formulations also help distinguish between private health (where the externality is ignored) and public health (where it is internalized).

There is a two-way interaction between the economy and the disease. On the one hand, diseases have direct adverse effects on the economy by reducing the labor force participation. Being infected with a disease affects the productivity of an individual. How much productivity is affected varies across diseases. The recent comprehensive estimates of disability weights used to compute DALYs is one possible measure of affect on productivity (see Salomon, et al (2012), Murray, et al (2012)). For some specific diseases there are estimates in the economic literature on loss of income from which effect on productivity is imputed (e.g. Baldwin and Weisbrod (1974) study effect of five parasitic diseases on banana plantation workers in St. Lucia; Fox, et al (2004) study loss of income to tea pickers in Kenya). The burden of diseases varies considerably, and the estimates in these studies are annualized. Our model is a however, an aggregated continuous time model making it difficult to use these estimates. Thus, we make the simplifying assumption that an infected individual is incapacitated by the disease or that the productivity falls to zero. That is, the infected are unable to work or accumulate human capital.[11] With disease incidence there is thus variation in the extensive margin. There could also be variation in the intensive margin. Goenka and Liu (2012) endogenize the labor-leisure choice with SIS disease dynamics and show that the dynamics are invariant under standard assumptions. Thus, assume that the labor is supplied inelastically. Thus, for each household labor supply L is given by the proportion of the healthy individuals, and its dynamics inherits the dynamics of s:

$$\dot{L} \quad = \quad (b+\gamma)(1-L) - \alpha \Pi L. \tag{2}$$

We do not take the disease dynamics as biologically given: the household can intervene in the disease transmission through private health expenditures. We model this by endogenizing the contact rate α - the key parameter in the epidemiology model.[12] We assume both health

[11] Assuming that the productivity falls to an intermediate level but not to zero will not affect the qualitative results.

[12] The recovery rate γ can also be endogenized, but we abstract from this for simplicity: see Goenka, Liu and Nguyen (2013) where both are endogenized.

capital H and physical capital K can affect the contact rate: The higher the health capital or the lower the physical capital, the lower the contact rate. This incorporates the fact that investing in private health expenditure can strength the immune system and prevent risk of infection, and thus reduce the likelihood of getting infected. However, higher physical capital can be detrimental to the immune system and lead to increased stress which increases the likelihood of infections. [13] Higher physical capital can also proxy greater specialization in economic activity which leads to greater social interaction leading to higher probability of getting infected. Specifically, the contact rate is assumed to be a function of the ratio of health and physical capital, denoted as $q = \frac{H}{K}$. This is also required for guaranteeing a balanced growth path.

Assumption 1. *The contact rate $\alpha(q)$ is a C^2 function:*

1. *$\alpha' < 0$, $\alpha'' > 0$ and $\lim_{q\to 0} \alpha' \to -\infty$, $\lim_{q\to\infty} \alpha' \to 0$;*

2. *Let $\overline{\alpha}$ and $\underline{\alpha}$ be the upper and lower bound, respectively, and we assume*

$$\frac{b+\gamma}{\underline{\alpha}} < 1.$$

The Inada condition is not necessary for the analysis but in its absence there can be another equilibrium where the disease is prevalent but there are no positive health expenditures. The evidence does suggest that at low levels of income individuals make sub-optimal choices where there is insufficient or no expenditures on preventive health expenditures but there is on curative health expenditures (Banerjee and Duflo (2011)). In order to simplify the analysis we abstract from these two issues in the current paper. [14] Eradication of endemic diseases is difficult and smallpox is the first, and so far only infectious disease, to have been eradicated. It was largely due to a long-run coordinated vaccination program involving WHO and national assumptions. In the absence of sustained public efforts, diseases that were previously controlled can re-emerge as in the case of leprosy in India (Gokhale (2013)). Thus, we make the second assumption an endemic disease cannot be eradicated by private health expenditures alone, and the disease free steady state is unstable (see below). This also reflects the externalities associated with the transmission of infectious diseases.

[13] More pollution can increase the incidence of diseases by contaminating the ecosystem; viruses causing the infection may mutate and become resistant to existing interventions (such as resistance to antibiotics in MRSA or resistance to DDT in malarial bacteria); expansion of economic activity may change the natural nidality of diseases (Pavlovsky (1966)) or cause diseases to cross over to humans (as seems to be the case with Avian Flu).

[14] See Goenka, Liu and Nguyen (2013) for analysis of the corner solution with no health expenditure in the absence of this Inada condition. This paper also considers public expenditures for curative purposes, that is endogenizes γ. It does not however distinguish between curative and preventive health expenditures.

Health and physical capital follow the standard law of motion with the common depreciation rate δ. Having a common depreciation rate is inessential and it the assumption is made for the sake of simplicity:

$$\dot{K} = I_K - \delta K - (b-d)K \tag{3}$$
$$\dot{H} = I_H - \delta H - (b-d)H. \tag{4}$$

The average human capital e affects labor productivity. The effective labor supply for each household is eL, of which u is used for production and $1-u$ is used for accumulating human capital. Thus, the law of motion for human capital is given as:

$$\dot{e} = \psi eL(1-u), \tag{5}$$

where ψ is the effectiveness of human capital accumulation. The linearity in the above equation, i.e. non diminishing returns on human capital accumulation, suggests human capital is the engine of economic growth. Unlike the standard endogenous growth model (Lucas, 1988), here it depends on the effective time spent in accumulation human capital $L(1-u)$, which is affected by the severity of disease prevalence.

Households take the interest rate R and wage W as given, rent out physical capital K and provide effective labor supply eLu. The income is either consumed C, invested inphysical capital I_K or health capital I_H. Thus, the budget constraint is:

$$C + I_K + I_H = RK + WeuL. \tag{6}$$

We further assume there is full insurance within each household and all individuals have the same consumption irrespective of their health status. This is indeed the case if the household welfare aggregator is concave. The representative household's preferences is given as:

$$\int_0^\infty e^{-\rho t} u(C) N dt = \int_0^\infty e^{-(\rho - b + d)t} u(C) dt, \tag{7}$$

where ρ is the discount factor with $\rho > b - d$, and the initial size of household is assumed to be one. For analytical convenience, we assume the felicity function to take the following form: $u(C) = \log(C)$.[15]

The firms There are many perfectly competitive firms, which maximize profit by choosing physical capital and effective labor as inputs. We assume the Cobb-Douglas production

[15]The adoption of the usual CES utility function affects the quantitative results of the paper, but not the qualitative results. The results of CES utility function is upon request from authors. For simplicity of exposition, we use log utility.

function $Y = AK^{\beta}(eLu)^{1-\beta}$, where A is the total factor productivity and $\beta \in (0,1)$ is the capital share. Thus, we have:

$$R = \beta A K^{\beta-1}(eLu)^{1-\beta} \tag{8}$$
$$W = (1-\beta)AK^{\beta}(eLu)^{-\beta}. \tag{9}$$

Definition 1. *A competitive equilibrium is a feasible allocation* $\{C, K, H, I_K, I_H, L, u, e\}$ *and a price system* $\{R, W\}$ *such that, given prices:*

- *Households maximize equation (7) by choosing consumption* C*, health expenditure* I_H*, physical capital investment* I_K *and time allocation* u*, subject to the constraints equation (2) - (6), and* $0 \leq u \leq 1$*,* $0 \leq L \leq 1$*,* $I_H \geq 0$*, with* $e_0, K_0, H_0,$ *and* L_0 *given;*
- *Firms maximize profits, given by equation (8) and (9);*
- *Markets clear:*
  - *Capital market, labor market and goods market clear;*
  - *Since each household is representative of the population, we have in equilibrium*

$$\Pi = 1 - L. \tag{10}$$

The last conditon (10) says that while each household takes the disease dynamics as given, in equilibrium, the disease dynamics reflect the situation in the representative household.

4 Competitive Equilibria

In this section, we analyze the competitive equilibrium balanced growth paths (BGPs). Along these paths, consumption, all types of capital and investment, and total output grow at a constant rate - either positive or zero, except for labor (L^*) and time allocation (u^*), which are constant since both are bounded between 0 and 1. Moreover, both L^* and u^* are strictly positive: the first is true by the disease dynamics as $\dot{L}(0) = b+\gamma > 0$, and the second from Inada conditions.

The current value Hamiltonian for household's optimization problem is given as:

$$\begin{aligned} H = {} & \log(C) + \lambda_1[RK + WeuL - C - I_H - \delta K - (b-d)K] + \lambda_2[I_H - \delta H - (b-d)H] + \\ & + \lambda_3 \psi eL(1-u) + \lambda_4[(b+\gamma)(1-L) - \alpha\left(\frac{H}{K}\right)\Pi L] + \theta_1(1-u) + \theta_2(1-L) + \theta_3 I_H. \end{aligned}$$

We substitute out physical capital investment I_K. $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are costate variables or shadow value of increments to physical capital, health capital, human capital and labor supply, respectively. θ_1, θ_2 and θ_3 are the Lagrange multiplies for the inequality constraints.[16]

The three control variables, consumption C, health expenditure I_H, and time allocation u, are chosen optimally and the first-order conditions are given as:

$$\frac{1}{C} = \lambda_1 \tag{11}$$

$$\lambda_1 = \lambda_2 + \theta_3, \quad \theta_3 \geq 0, \quad I_H \geq 0, \quad \theta_3 I_H = 0 \tag{12}$$

$$\lambda_1 WeL = \lambda_3 \psi eL + \theta_1, \quad \theta_1 \geq 0, \quad 1 - u \geq 0, \quad \theta_1(1-u) = 0. \tag{13}$$

On the margin, goods must be equally valuable in their use as consumption, physical capital investment and health expenditure, described by equation (11) and (12). Labor time must be equally valuable in either production or human capital accumulation by equation (13).

The rates of change of shadow value for the four state variables satisfy the following conditions:

$$\dot{\lambda}_1 = (\rho - b + d)\lambda_1 - \lambda_1(R - (\delta + b - d)) - \lambda_4 \alpha' \left(\frac{H}{K}\right) \frac{H}{K^2} \Pi L \tag{14}$$

$$\dot{\lambda}_2 = (\rho - b + d)\lambda_2 + \lambda_2(\delta + b - d) + \lambda_4 \alpha' \left(\frac{H}{K}\right) \frac{1}{K} \Pi L \tag{15}$$

$$\dot{\lambda}_3 = (\rho - b + d)\lambda_3 - \lambda_3 \psi L(1-u) - \lambda_1 WuL \tag{16}$$

$$\dot{\lambda}_4 = (\rho - b + d)\lambda_4 - \lambda_3 \psi e(1-u) + \lambda_4 (b + \gamma + \alpha \left(\frac{H}{K}\right) \Pi) - \lambda_1 Weu + \theta_2$$
$$\theta_2 \geq 0, \quad 1 - L \geq 0, \quad \theta_2(1-L) = 0. \tag{17}$$

Then, the competitive equilibrium is described by equation (2) - (6), (8) - (10) and (11) - (17). In addition, the following TVCs: $\lim_{t\to\infty} e^{-(\rho-b+d)t}\lambda_1 K = 0$, $\lim_{t\to\infty} e^{-(\rho-b+d)t}\lambda_2 H = 0$, $\lim_{t\to\infty} e^{-(\rho-b+d)t}\lambda_3 e = 0$, and $\lim_{t\to\infty} e^{-(\rho-b+d)t}\lambda_4 L = 0$ have to be satisfied in the equilibrium.

The following assumption is made to ensure that controlling the disease is relevant for

[16] It has been recognized in the literature that SIS dynamics are not concave which can make the Hamiltonian non-concave, and difficult to check whether the maximized Hamiltonian is concave or not. Thus, the usual Mangasarian and Arrow sufficiency conditions cannot be used here. Goenka, Liu and Nguyen (2013) investigate this issue in detail. They show that if the growth rate of capital is bounded from below, $\dot{K}/K \geq -\kappa, \kappa > 0$, then there is a solution to the maximization problem. It relies on showing that the feasible set is relatively compact in $L^1(e^{-(\rho-b+d)t})$. They then show that the first order conditions to the maximization problems are indeed optimal. Thus, we work with the first order conditions in this paper. See also d'Albis et al (2008). There is an additional problem due to the decentralized decision problem. For existence in the Lucas (1988) model also see D'Albis and Le Van (2006)). These methods can be adapted to the model, but is beyond the scope of the current paper.

the growth of a country.

Assumption 2. *We assume the model parameters satisfy:*

$$\frac{b+\gamma}{\overline{\alpha}} < \frac{\rho - b + d}{\psi} < \frac{b+\gamma}{\underline{\alpha}}.$$

If this is violated, for countries inflicted by infectious diseases, when $\frac{\rho-b+d}{\psi} \leq \frac{b+\gamma}{\overline{\alpha}}$, all of them have positive economic growth rate, and when $\frac{\rho-b+d}{\psi} \geq \frac{b+\gamma}{\underline{\alpha}}$, all of them are stuck in the poverty trap, regardless of whether they control the diseases or not which makes the analysis uninteresting. This point will become clearer in the following analysis.

There are two situations when $\dot{L} = 0$. The first is $L^* = 1$, that is infectious diseases are eradicated, all individuals are healthy and working. This the disease-free case. The other is $L^* = \dfrac{b+\gamma}{\alpha(q^*)}$, with $q \equiv \dfrac{H}{K}$, which is strictly less than one by assumption 1. This is the disease-endemic case, where infectious diseases are prevalent, a fraction of individuals are infected and unable to work. These two cases directly mirror the two steady states in the pure SIS epidemiology model. The difference between the two is that here households can affect diseases transmission through health expenditure, and in the other the steady states are determined entirely by biology.

Since for the disease-endemic case, labor supply is a function of ratio of health and physical capital, for easy exposition, we define the function

$$L(q) = \frac{b+\gamma}{\alpha(q)}.$$

Clearly it is increasing in q. Moreover, we define $\hat{q}$ such that

$$L(\hat{q}) = \frac{\rho - b + d}{\psi}.$$

Proposition 1. *(Existence)*

1. *There exists a disease-free BGP. We have $L^* = 1$, $u^* = \frac{\rho-b+d}{\psi}$, and the growth rate*

$$g = \psi - (\rho - b + d);$$

2. *There exists a disease-endemic case. The ratio of health and physical capital q^* is determined by equation*

$$G(q) = max\{G_L(q), G_R(q)\} = 0,$$

where

$$G_L(q) = -\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q) - \alpha(q) - (\rho - b + d), \quad \textit{and}$$
$$G_R(q) = -\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q)\frac{\psi L(q)}{\rho - b + d} - \alpha(q) - (\rho - b + d).$$

Then, we have $L^* = L(q^*)$*, and*

(a) If $L^* \leq \frac{\rho-b+d}{\psi}$ *(or* $q^* \leq \hat{q}$*), there exists a disease-endemic poverty trap, with* $u^* = 1$*;*

(b) If $L^* > \frac{\rho-b+d}{\psi}$ *(or* $q^* > \hat{q}$*), there exists a disease-endemic BGP, with* $u^* = \frac{\rho-b+d}{\psi L^*}$*, and the growth rate*

$$g = \psi L^* - (\rho - b + d).$$

Proof. See the Appendix. □

In the disease-free case, infectious diseases are completely eradicated, and thus health expenditure for controlling diseases is zero. The maximization problem degenerates to the standard Lucas (1988) model. Whether there is positive economic growth or not depends on whether any time is spent on human capital accumulation. Hence, it depends on the relative magnitude of marginal value of time use in either human capital accumulation or production. In the extreme case when all the time is spent for production and growth rate is zero, marginal value of spending du unit of time in human capital accumulation is $\lambda_3\psi edu$, and marginal cost is the value associated with loss in production, $\lambda_1(1-\beta)AK^\beta e^{1-\beta}du$. By equation (16) and $\dot{\lambda}_3 = 0$ as growth rate is zero, we have $\lambda_3(\rho - b + d) = \lambda_1(1-\beta)AK^\beta e^{-\beta}$. Therefore, time is allocated in human capital accumulation or there is a positive economic growth rate if and only if:

$$\frac{\lambda_3\psi edu}{\lambda_1(1-\beta)AK^\beta e^{1-\beta}du} = \frac{\psi}{\rho - b + d} > 1.$$

From Assumption 1 and 2, i.e. $\frac{\rho-b+d}{\psi} < \frac{b+\gamma}{\underline{\alpha}} < 1$, the above inequality always holds, and there exists a disease-free BGP with growth. It implies that when the effectiveness of human capital accumulation is large enough, larger than the effective discount rate, the country undergoes positive growth path, and the growth rate is determined by effectiveness of human capital accumulation and effective discount rate.

In the disease-endemic case, infectious diseases are prevalent and labor force participation rate $L(q) = \frac{b+\gamma}{\alpha(q)} < 1$. Due to the Inada condition in Assumption 1, health expenditure is

strictly positive. So $\lambda_1 = \lambda_2$ and

$$\lambda_1 \beta A K^{\beta-1}(euL)^{1-\beta} + \lambda_4 \alpha'(q)\frac{H}{K^2}(1-L)L = -\lambda_4 \alpha'(q)\frac{1}{K}(1-L)L, \tag{18}$$

that is, marginal value of physical capital equals to marginal value of health capital. The equation can be rewritten as:

$$-\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q) = \frac{\lambda_1}{\lambda_4}(1-\beta)AK^{\beta}(euL)^{-\beta}eu. \tag{19}$$

Consumption, physical, health and human capital grow at the same rate $g = \psi L(1-u)$, $\frac{\dot{\lambda}_1}{\lambda_1} = \frac{\dot{\lambda}_3}{\lambda_3} = -g$ and $\frac{\dot{\lambda}_4}{\lambda_4} = 0$. Through some manipulations, equation (16) is given as:

$$\lambda_3 \psi L(1-u) + \lambda_1(1-\beta)AK^{\beta}(euL)^{-\beta}uL = \lambda_3(\rho - b + d + g), \tag{20}$$

which says marginal value of human capital contributing to both human capital accumulation and production equals to its marginal cost. Similarly, equation (17) becomes:

$$\lambda_1(1-\beta)AK^{\beta}(euL)^{-\beta}eu - \lambda_4(b+\gamma+\alpha(q)(1-L)) + \lambda_3\psi e(1-u) = \lambda_4(\rho - b + d), \tag{21}$$

the R.H.S. of which is marginal cost of labor supply and the L.H.S is marginal value of labor supply, consisting of its contribution to production, evolution of labor force participation and human capital accumulation. Divide both sides of the above equation by λ_4, substitute into equations (18) and (20), and we have:

$$\begin{aligned} &-\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q) - \alpha(q) - \frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q)\frac{\psi L(q)(1-u)}{\rho - b + d} \\ &= \rho - b + d, \end{aligned} \tag{22}$$

which is a function of both q and u. Hence, equation (22) along with equation (13) determine q^* and u^*.

There are two scenarios. One is the poverty trap, $u^* = 1$ and $g = 0$. Equation (22) simplifies to $G_L(q) = 0$, suggesting q^* is chosen such that marginal cost of labor is equal to its marginal value, consisting of the first two terms in the L.H.S. of equation (22). Because there is no economic growth and hence no human capital accumulation, the third term disappears. This case exists only if $\psi L^* \leq \rho - b + d$ or $q^* \leq \hat{q}$. The other scenario is positive economic growth path, $u^* = \frac{\rho - b + d}{\psi L^*}$ and $g = \psi L^* - (\rho - b + d)$. q^* is determined by equation $G_R(q) = 0$, derived by substituting u^* into equation (22). This case exists only if $\psi L^* > \rho - b + d$ or $q^* > \hat{q}$. Moreover, $G_L(q) > G_R(q)$ if $q < \hat{q}$, $G_L(q) < G_R(q)$ if $q > \hat{q}$, and $G_L(q) = G_R(q)$ if $q = \hat{q}$. Combining the two scenarios, q^* is determined by the upper contour of functions G_L

and G_R. That is, it is determined by function $G(q) = \max\{G_L(q), G_R(q)\} = 0$.

The intuition for positive economic growth when diseases are endemic, is very similar to the disease-free case, that is, whether there is positive growth path or not depends on the relative marginal value of time use in human capital accumulation or production. The difference is that here marginal value of time use depends on the proportion of healthy individuals in a household, which in turn depends on the ratio of health and physical capital. In the extreme case, when all the time is spent for production only and growth rate is zero, marginal value of spending du unit of time in human capital accumulation is $\lambda_3\psi eLdu$, and marginal cost is the value associated with loss in production, $\lambda_1(1-\beta)AK^\beta(eL)^{1-\beta}du$. By equation (16) and $\dot{\lambda}_3 = 0$, we have $\lambda_3(\rho - b + d) = \lambda_1(1-\beta)AK^\beta(eL)^{-\beta}L$. Therefore, time is allocated for human capital accumulation or there is positive economic growth rate if and only if:

$$\frac{\lambda_3\psi eLdu}{\lambda_1(1-\beta)AK^\beta(eL)^{1-\beta}du} = \frac{\psi L}{\rho - b + d} > 1.$$

It implies that when the effectiveness of human capital accumulation, now proportional to the labor force participation rate, is larger than the effective discount rate, the country undergoes positive growth path.

Compared with countries where diseases are eradicated, the prevalence of infectious diseases directly affects the labor force participation rate, which reduces the effectiveness of human capital accumulation. As a result more time is allocated for production rather than human capital accumulation, and there is slower growth. In the extreme case, all the time is allocated for production and there is a poverty trap. The following lemma details the resource allocation for each type of countries.

Proposition 2. *The resources are allocated as follows:*

1. *For countries in a disease-free BGP, we have*

$$\frac{I_K}{Y} = \beta\left(1 - \frac{\rho - b + d}{\psi + b - d + \delta}\right), \quad \frac{I_H}{Y} = 0, \quad \textit{and} \quad \frac{C}{Y} = 1 - \beta\left(1 - \frac{\rho - b + d}{\psi + b - d + \delta}\right);$$

2. *For countries in a disease-endemic BGP, we have*

$$\frac{I_K}{Y} = \beta\left(1 - \frac{\rho - b + d}{\psi L^* + b - d + \delta}\right)\cdot\frac{1}{1+q}, \quad \frac{I_H}{Y} = \beta\left(1 - \frac{\rho - b + d}{\psi L^* + b - d + \delta}\right)\cdot\frac{q}{1+q},$$

$$\textit{and} \quad \frac{C}{Y} = 1 - \beta\left(1 - \frac{\rho - b + d}{\psi L^* + b - d + \delta}\right);$$

3. For countries in a disease-endemic poverty trap, we have

$$\frac{I_K}{Y} = \beta\left(1 - \frac{\rho - b + d}{\rho + \delta}\right) \cdot \frac{1}{1+q}, \quad \frac{I_H}{Y} = \beta\left(1 - \frac{\rho - b + d}{\rho + \delta}\right) \cdot \frac{q}{1+q},$$

$$\textit{and} \quad \frac{C}{Y} = 1 - \beta\left(1 - \frac{\rho - b + d}{\rho + \delta}\right).$$

Proof. See the Appendix. □

Since $\psi + b - d + \delta > \psi L^* + b - d + \delta > \rho + \delta$, from the above Lemma, we see that countries in a disease-free BGP in fact have the highest saving rate and countries in a disease-endemic poverty trap have the lowest saving rate. For the countries without infectious diseases, all the savings are invested in physical capital as infectious diseases are eradicated and there is no need to spend resources on controlling them. In contrast, for the countries afflicted by infectious diseases, $\frac{1}{1+q}$ fraction is invested in physical capital and the rest is invested in health capital for controlling diseases.

So far, we have shown the existence of the disease-endemic case, or the existence of q^*. We introduce the following assumption to guarantee the uniqueness of a disease-endemic case, or the uniqueness of q^*.

Assumption 3. *We assume $\alpha''(q)$ is big enough such that*

$$\alpha''(q) > -\alpha'(q)\ \max\{\frac{\beta}{(1-\beta)(1-L(q))(1+q)} + \frac{L(q)}{1-L(q)} \cdot \frac{\alpha'(q)}{\alpha(q)} + \frac{1}{1+q},$$
$$\frac{\beta}{(1-\beta)(1-L(q))(1+q)} \cdot \frac{\rho - b + d}{\psi L(q)} + \frac{L(q)}{1-L(q)} \cdot \frac{\alpha'(q)}{\alpha(q)} + \frac{1}{1+q} - \frac{\alpha'(q)}{\alpha(q)}\}.$$

Lemma 1. *Under Assumption 3, the function $G(q) = \max\{G_L(q), G_R(q)\}$ is monotonically decreasing in q.*

Proof. See the Appendix. □

Since function $G(q)$ describes net marginal value of labor force participation rate, Assumption 3 and Lemma 1 imply that the net marginal value is diminishing as L increases or q increases. We define

$$\hat{G} = G(\hat{q}) = -\frac{1-\beta}{\beta}\alpha'(\hat{q})(1 - L(\hat{q}))(1 + \hat{q}) - \alpha(\hat{q}) - (\rho - b + d).$$

Proposition 3. *(Uniqueness)*

When infectious diseases are endemic, there is a unique q^* determined by equation $G(q) = 0$, and

1. *If $\hat{G} \leq 0$, it is a disease-endemic poverty trap;*

2. *If $\hat{G} > 0$, it is a diseases-endemic BGP.*

Proof. See the Appendix. □

This suggests that in the disease-endemic case, whether there is positive or zero economic growth depends on the magnitude of $\hat{G}$, which in turn depends on all the economic parameters (discount rate, capital share etc.), demographic parameters (birth and death rate) and disease transmission mechanism (epidemiological functions). The following lemma gives the conditions, under which countries are more likely to be in a growth path.

Figure 4 describes the two scenarios under the disease-endemic case. In both panels, the functions $G_L(q)$ and $G_R(q)$ are monotonically decreasing in q, and intersect at the point $\hat{q}$. The function $G(q)$ is given by the upper contour of both functions. The upper panel gives the disease-endemic BGP with $\hat{G} > 0$ and $q^* > \hat{q}$, and the bottom panel gives the disease-endemic poverty trap with $\hat{G} < 0$ and $q^* < \hat{q}$.

Proposition 4. *When infectious diseases are endemic, countries are more likely to undergo a positive economic growth path, under the following conditions:*

1. *Capital share, β, is smaller;*

2. *Households are more patient, i.e. ρ is smaller;*

3. *Death rate, d, is lower or life expectancy increases;*

4. *Effectiveness of human capital accumulation, ψ, is higher.*

Proof. See the Appendix. □

The results are very intuitive. When labor becomes more important in production, that is, capital share is smaller, households care more about labor force participation rate and spend more on health expenditure. When households becomes more patient due to either lower discount factor or higher life expectancy, they are more willing to postpone consumption and invest more in health capital. As result of this, labor force participation rate increases and hence countries are more likely to be in a growth path. When effectiveness of human capital accumulation is higher, it is more profitable to spend time in investing human capital rather than production, and the probability of taking off increases.

The effects of changing the birth rate and recovery rate is ambiguous. On the one hand, due to the assumption that all newborns are healthy, higher birth rate is beneficial for controlling diseases (and so does a higher recovery rate). On the other hand, when diseases

are not severe and the fraction of the infected is low, there is less chance for the healthy individuals to catch diseases, which lowers the incentive for diseases control and thus reduces the health expenditure. Thus, either an epidemiological transition (increased recuperation rates due to medical advancements) and a demographic transition (change in net birth rate $b-d$) can have an ambiguous effect on the control of diseases as how the incentive to control depends on the prevalence of the disease, and this leads to an ambiguous effect on income and growth. This is consistent with the mixed evidence that exists in the empirical literature.

To sum up, as the result of the introduction of SIS epidemiological model, there are multiple competitive equilibria, in which infectious diseases are either be eradicated or are endemic. In the disease-free case, countries grow at a fast rate,[17] while in the disease-endemic case, countries either grow at a slow rate or are in a poverty trap, depending on the investment in human capital accumulation - the engine of economic growth. The reason that prevalence of infectious diseases affects human capital investment is that the ratio of marginal value of time use in human capital accumulation and time use in production is proportional to labor force participation rate or the fraction of healthy individuals. Therefore, countries with lower disease prevalence or larger fraction of healthy individuals are more likely to invest in human capital, and hence be in a economic growth path. The intuition is that as the incidence of disease prevalence goes down, households expect a larger proportion to be healthy which increases the rate of return on human capital accumulation. This has the natural effect of increasing its accumulation. An implication of this is that projections of the economic burden of disease which largely focus on lost productivity and cost of treatment are going to underestimate the cost as they do not account for the changed incentives for human capital accumulation and thus not account for the change in the growth rate.

5 Optimal Paths and Policy Implications

In this section, we examine the social planner's problem. This can be interpreted as the outcome that will obtain when the optimal public health policy is implemented. There are potentially three externalities in the model: the externality associated with human capital accumulation (Lucas (1988)) because of the effect of human capital accumulation in the production function $AK^{\beta}(euL)^{(1-\beta)}e^{\eta}$, and the externality associated with disease transmission, and a health capital externality. The first is well understood in the literature. The second has been recognized but studied primarily at the micro level (Geoffard and Philipson (1996)) and the dynamic general equilibrium effects are not that well understood. Similarly, the health

[17]However, we know from the disease dynamics in section 3.1 disease-free equilibrium is not stable if $b+\gamma<\alpha$. Since $b+\gamma<\alpha(q)$ for all q by the assumption, economic growth with disease eradicated is not a stable BGP. This explains why in developed countries, even though diseases are eradicated people are still concerned about the possible outbreak of infectious diseases.

capital externality is that the contact rate is a function of both ratio of private health and physical capital and ratio of public health and physical capital, that is, $\alpha(q^{\upsilon_1}(q^{pub})^{\upsilon_2})$. Since households fail to take into account the ratio of public health and physical capital q^{pub}, there can be under investment in health expenditure. In order to concentrate on the disease transmission externality we shut down the human capital externality as $\eta = 0$, and abstract from the health capital externality. Looking at the difference from the competitive equilibrium, will give a sense of the impact of the externality associated with infectious disease transmission. As the two solutions differ only when a disease is endemic, we concentrate on this case.

5.1 Centralized economy

In our model, the centralized economy differs from the decentralized one in that social planner takes into account that the intervention can effectively control the proportion of the infected in total population. Recall that in the decentralized economy household takes the proportion of the infected in total population as fixed, shown in equation (2). The social planner's maximization problem is essentially similar to the one we considered above with the only difference being in the law of motion for labor force participation, which is now given as:

$$\dot{L} \quad = \quad (b+\gamma)(1-L) - \alpha(1-L)L.$$

In the following analysis, the superscript c is used in denoting variables in the centralized economy.

Proposition 5. *In a centralized economy,*

1. *There exists a unique disease-free BGP with growth rate $g^c = \psi - (\rho - b + d)$;*
2. *There exists a unique disease-endemic case. The ratio of health and physical capital $q^{*,c}$ is determined by equation*

$$G(q) + b + \gamma = max\{G_L(q), G_R(q)\} + b + \gamma = 0.$$

Then, we have $L^{*,c} = L(q^{*,c})$, and

   (a) *If $L^{*,c} \leq \frac{\rho-b+d}{\psi}$ or $q^{*,c} \leq \hat{q}$ or $G(\hat{q}) + b + \gamma \leq 0$, it is a poverty trap;*
   (b) *If $L^{*,c} > \frac{\rho-b+d}{\psi}$ or $q^{*,c} > \hat{q}$ or $G(\hat{q}) + b + \gamma > 0$, it is a BGP with $u^{*,c} = \frac{\rho-b+d}{\psi L^{*,c}}$, and $g^c = \psi L^{*,c} - (\rho - b + d)$.*

Proof. The proof is the similar to the proof of Proposition 1, and hence ignored here. □

We do not present the proposition for the resource allocation in the centralized economy here, since it is the same as the one in the decentralized economy, shown in Proposition 2. The only difference is that L^* is replace with $L^{*,c}$ in the disease-endemic BGP.

Similar to the decentralized case, there always exists a disease-free balance growth path. Since social planner and the households only differ in how they view the impact of their behavior on the disease transmission, there is no difference between the optimal growth path and competitive equilibrium path when diseases are eradicated.

There also exists a disease-endemic case. The ratio of health and physical capital is optimally chosen according to:

$$\lambda_1(1-\beta)AK^{\beta}(euL)^{-\beta}eu - \lambda_4(b+\gamma+\alpha(q)(1-L)-\alpha(q)L) + \lambda_3\psi e(1-u) = \lambda_4(\rho-b+d).$$

The R.H.S. of the above equation is marginal cost of labor supply and the L.H.S. is marginal value of labor supply, consisting of its contribution to production, the evolution of labor force participation and human capital accumulation. Compared with equation (21) since in the decentralized case individual households fail to take into account the positive externality of controlling infectious diseases, for the same amount of health and physical capital ratio q, net marginal value of labor is always higher in the centralized than the decentralized, which differ exactly by the amount $\lambda_4\alpha(q)L$ or $\lambda_4(b+\gamma)$. Thus, in the centralized economy, the optimal ratio $q^{*,c}$ is determined by the equation $G(q)+b+\gamma=0$.

Under Assumption 3, the optimal ratio q^* is always higher in centralized economy, and it is more likely that countries can escape the poverty trap or grow at a faster rate. To be more specific, Figure 5 describes three scenarios for the comparison between the decentralized and centralized economies, which depends on the parameters or the magnitude of $\hat{G}$. In all the panels, the solid line is the funtion $G(q)$, determining the ratio q^*, and the dash line is the function $G(q)+b+\gamma$, determining the ratio $q^{*,c}$. The critical ratio of health and physical capital $\hat{q}$ for the positive growth is the same in the centralized and decentralized cases.

There are three scenarios. First, in the upper panel, since $\hat{G} > 0$, there is positive economic growth path in both the decentralized and centralized economies, and we have

$$q^{*,c} > q^* > \hat{q}, \quad L^{*,c} > L^* > \frac{\rho-b+d}{\psi} \quad \text{and} \quad g^c > g > 0.$$

In this case, centralized economy is better in controlling infectious diseases, and hence, grows at a faster rate. From Lemma 2, the saving rates in both economies are given as $\beta(1-\frac{\rho-b+d}{\psi L+b-d+\delta})$, $\frac{q}{1+q}$ fraction of which is invested in health expenditure. Notice these are increasing functions of labor force participation rate. Thus, the centralized economy has a higher saving rate and investment rate for health expenditure.

Second, in the bottom left panel, since $-(b+\gamma) < \hat{G} \leq 0$, the centralized economy is in

a positive growth path while the decentralized economy is stuck in the poverty trap, and

$$q^{*,c} > \hat{q} \geq q^*, \quad L^{*,c} > \frac{\rho - b + d}{\psi} \geq L^* \quad \text{and} \quad g^c > g = 0.$$

In this case, because individuals fail to take into account the positive externality in disease control, the economy is stuck in the poverty trap, which otherwise would have taken off in a centralized economy. From Lemma 2, the saving rate in the centralized economy is given as $\beta(1 - \frac{\rho-b+d}{\psi L^{*,c}+b-d+\delta})$, while in the decentralized economy, it is given as $\beta(1 - \frac{\rho-b+d}{\rho+\delta})$. Since $\psi L^{*,c} + b - d + \delta > \rho + \delta$, the saving rate is higher in the centralized economy, and hence more resources are allocated for controlling infectious diseases.

Third, in the bottom right panel, since $\hat{G} \leq -(b + \gamma)$, both the centralized and decentralized economies are in the poverty trap, and

$$\hat{q} \geq q^{*,c} > q^*, \quad \frac{\rho - b + d}{\psi} \geq L^{*,c} > L^* \quad \text{and} \quad g^c = g = 0.$$

From Lemma 2, the saving rates in both economies are given as $\beta(1 - \frac{\rho-b+d}{\rho+\delta})$, of which $\frac{q}{1+q}$ fraction is invested for controlling infectious diseases. Thus, both economies share the same saving rate, of which centralized economy spends more in health expenditure than the decentralized ones. The prevalence of infectious diseases is less severe in the centralized economy. However, the effectiveness of human capital accumulation is still not large enough for justifying its time allocation, and hence there is no economic growth.

In this case, the welfare comparison between two economies is ambiguous. Since human capital is indeterminate, we assume it is given by its initial level $e_0 = 1$. The output and consumption in both economies are given as:

$$\begin{aligned} Y^{*,j} &= A^{\frac{1}{1-\beta}} \left(\frac{\rho + \delta}{\beta}(1 + q^{*,j}) \right)^{-\frac{\beta}{1-\beta}} L(q^{*,j}), \\ C^{*,j} &= \left(1 - \beta \frac{\delta + b - d}{\rho + \delta} \right) Y^{*,j}. \end{aligned}$$

depending on $q^{*,j}$, where $j = c$ for the centralized economy and $j = nil.$ for the decentralized economy. Even though labor force participation rate is higher in the centralized economy, which increases the production, the investment in physical capital is less compared with the decentralized ones, which leads to lower production. Thus, the overall effect on the total production and consumption is ambiguous. The quantitative analysis, shown in the next section, suggests that output, consumption and hence welfare are in fact higher in the centralized economy.

5.2 Optimal health subsidies

Compared with the decentralized economy, the centralized economy, which successfully takes into account the positive externatlity of controlling infectious diseases, either has a higher growth rate, or is more likely to take off, or has a higher consumption level even in a poverty trap. This provides a justification for introducing effective public health policy. One of the issue with infectious diseases is that households do not account for the effect of their actions on the transmission of the disease. There is evidence (e.g. Banerjee and Duflo (2011) who discuss preventive health care in general and Tarozzi, et al (2009) who focus on use of insectiside-treated bednets for prevention of malaria) that households seem to under-invest in preventive health care. It raises the question what is the nature of the Pigouvian subsidy that will induce households to internalize preventive health expenditures. This is especially important as some of the countries that are most afflicted with infectious diseases have weak public health delivery mechanisms. External aid raises the question whether it is actually delivered for the specific need. There are, of course international health organizations (e.g. WHO) and NGOs (e.g. Carter Foundation that has worked for eradication of Guinea Worm in sub-Saharan Africa, Gates Foundation, and the earlier Rockefeller Foundation that played a key role of eradication of hookworm in southern U.S.A. (Bleakley (2007)). However, a market solution via balanced (self-financing) public health policy may be more sustainable. [18]

We assume for each unit of private health investment, there is a proportional health subsidy τI_H, and the law of motion for health capital now is:

$$\dot{H} = (1+\tau)I_H - \delta H - (b-d)H. \tag{23}$$

The public health expenditure is financed through a lump-sum tax T, and the budget constraint becomes:

$$C + I_K + I_H = RK + WeuL - T. \tag{24}$$

Households maximize equation (7) by choosing consumption C, health expenditure I_H, physical capital investment I_K and time allocation u, subject to the constraints equation (2), (3),(23), (5) and (24). In equilibrium, the period-by-period balance budget (balancedness) implies $T = \tau I_H$. The rest is the same as the competitive equilibrium, defined in Section 3.

We solve the maximization problem and the first order conditions are the similar to equations (11)- (17). The only difference is the equation (12) with positive health expenditure

[18] Here we focus on health subsidies. In fact, any policy distorting marginal benefit of physical capital investment and health expenditure can be equally effective in obtaining the optimal path under the centralized economy, for instance, proportionate capital income tax, educational subsidy, etc.

which is now given as (the others are the same):

$$\lambda_1 = (1+\tau)\lambda_2.$$

It implies:

$$\lambda_1 \beta A K^{\beta-1}(euL)^{1-\beta} + \lambda_4 \alpha'(q)\frac{H}{K^2}(1-L)L + g = (1+\tau)[-\lambda_4\alpha'(q)\frac{1}{K}(1-L)L + g]. \quad (25)$$

Comparing the above equation with equation (18), we see that because there is the additional τ unit health subsidy for each unit of private health expenditure, marginal value of physical capital investment (the L.H.S. of the above equation) equals to $(1+\tau)$ times marginal value of private health expenditure (the R.H.S. of the above equation). In the following analysis, the superscript τ is used in denoting variables in the decentralized economy with the health subsidy.

Proposition 6. *In a decentralized economy with health subsidy,*

1. *There exists a unique disease-free BGP with the growth rate $g^\tau = \psi - (\rho - b + d)$;*
2. *There exists a unqie disease-endemic case. The ratio of health and physical capital $q^{*,\tau}$ is determined by equation*

$$G^\tau(q) = \max\{G_L^\tau(q), G_R^\tau(q)\} = 0,$$

where

$$\begin{aligned} G_L^\tau(q) &= -\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q+\tau) - \alpha(q) - (\rho - b + d), \quad \textit{and} \\ G_R^\tau(q) &= -\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q+\frac{\rho+\delta}{\rho+\delta+(1+\tau)g}\tau)\frac{\psi L(q)}{\rho - b + d} - \alpha(q) - (\rho - b + d). \end{aligned}$$

Then, we have $L^{*,\tau} = L(q^{*,\tau})$, and

   (a) *If $L^{*,\tau} \leq \frac{\rho-b+d}{\psi}$ or $q^{*,\tau} \leq \hat{q}$ or $\hat{G}^\tau = G^\tau(\hat{q}) \leq 0$, it is a poverty trap;*
   (b) *If $L^{*,\tau} > \frac{\rho-b+d}{\psi}$ or $q^{*,\tau} > \hat{q}$ or $\hat{G}^\tau = G^\tau(\hat{q}) > 0$, it is a BGP with $u^{*,\tau} = \frac{\rho-b+d}{\psi L^{*,\tau}}$, and $g^c = \psi L^{*,\tau} - (\rho - b + d)$.*

Proof. The proof is the similar to the proof of Proposition 1, and hence ignored here. □

When infectious diseases are eradicated, there is no health expenditure and thus no need for the health subsidy. The disease-free BGP is the same as those in the decentralized economy shown in Proposition 1 and the centralized economy shown in Proposition 5. When

infectious diseases are endemic, the ratio of health and physical capital $q^{*,\tau}$ is determined by the equation $G^\tau(q) = 0$. Compared with $G(q) = 0$ in Proposition 1, the difference lies in the first term in the net marginal benefit, which is distorted by the relative marginal value of physical capital investment and health expenditure shown in equation (25), due to the subsidy τ.

Lemma 2. *When the health subsidy τ is strictly positive, $G^\tau(q)$ is a monotonically decreasing function with $G^\tau(q) > G(q) \quad \forall q$.*

Proof. See the Appendix. □

The lemma implies that $\hat{G}^\tau > \hat{G}$ and with the health subsidy, countries are more likely to be in the positive economic growth path. Figure 6 describes the three possible cases when comparing the decentralized economy with or without public health policy. It is very similar to Figure 5, where we compare the centralized and decentralized economies. Throughout all the panels, the solid line is the function $G(q)$ and the dash line is the function $G^\tau(q)$. In the upper panel, since $\hat{G}^\tau > \hat{G} > 0$, there are BGPs for both economies and the decentralized economy with public health policy has a higher growth rate with

$$q^{*,\tau} > q^* > \hat{q}, \quad L^{*,\tau} > L^* > \frac{\rho - b + d}{\psi} \quad \text{and} \quad g^\tau > g > 0.$$

In the bottom left panel, since $\hat{G}^\tau > 0 \geq \hat{G}$, it is a poverty trap for the decentralized economy, while with public health policy, there could be a positive growth path. And we have

$$q^{*,\tau} > \hat{q} \geq q^*, \quad L^{*,\tau} > \frac{\rho - b + d}{\psi} \geq L^*, \quad \text{and} \quad g^\tau > g = 0.$$

In the bottom right panel, since $\hat{G} < \hat{G}^\tau \leq 0$, both economies are stuck in a poverty trap with

$$\hat{q} \geq q^{*,\tau} > q^*, \quad \frac{\rho - b + d}{\psi} \geq L^{*,\tau} > L^* \quad \text{and} \quad g^\tau = g = 0.$$

That is, public health policy is still not enough to bring the country out of the poverty trap.

The following proposition gives the optimal subsidy, in the sense that it is chosen such that $q^{*,\tau}$ coincides with the optimal ratio in the centralized economy $q^{*,c}$.

Proposition 7. *Let $q^{*,c}$ be the optimal ratio in the centralized economy.*

1. *When the optimal path is a disease-endemic poverty trap, the optimal health subsidy is given as:*

$$\tau = (1 + q^{*,c}) \frac{b + \gamma}{\alpha(q^{*,c}) - (b + \gamma) + \rho - b + d}.$$

2. *When the optimal path is a disease-endemic BGP with* $g = \psi L^{*,c} - (\rho - b + d)$*, the optimal health subsidy is given as:*

$$\tau = (1 + q^{*,c}) \frac{b + \gamma}{\alpha(q^{*,c}) - (b + \gamma) + \rho - b + d} \cdot \frac{\rho + \delta + g}{\rho + \delta - (1 + q^{*,c}) \frac{b+\gamma}{\alpha(q^{*,c}) - (b+\gamma) + \rho - b + d} g}.$$

Proof. See the Appendix. □

We can see that the optimal subsidy is proportional to the externality $b + \gamma$, and the larger the externality the bigger the subsidy. Moreover, the optimal subsidy is higher when the optimal path is a BGP than poverty trap. The reason is that there is additional distortion resulting from positive growth rate in equation (25).

6 Effects of Rising Life Expectancy

In this section, we examine how a permanent exogenous increase in life expectancy or a decline in death rate d, changes the economic growth paths.[19] The reason we are particularly interested in this is that in the paper we emphasize the interaction between diseases transmission and human capital investment, instead of the interaction between disease transmission and demographics so as to focus on the role of morbidity.The examination of effect of increase in life expectancy gives us a glimpse of how the demographic transition affects the disease control, and hence human capital investment and economic growth.

For countries growing at a rate $g = \psi - (\rho - b + d)$ with the diseases eradicated, an exogenous increase in life expectancy causes them to grow at a even faster rate, because individuals become more patient and more time is allocated for human capital accumulation leading to further economic growth. From Lemma 2, saving rate increases as death rate drops, all of which is allocated for physical capital investment.

When infectious diseases are endemic, there are three scenarios presented in Figure 6. We use superscript d to denote the variables after the drop in death rate. Since

$$\begin{aligned} \frac{\partial G_L(q)}{\partial d} &= -1 < 0 \quad \text{and} \\ \frac{\partial G_R(q)}{\partial d} &= -\frac{1-\beta}{\beta} \alpha'(q)(1 - L(q))(1 + q)\psi L(q)(-\frac{1}{(\rho - b + d)^2}) - 1 < 0, \end{aligned}$$

[19] The dynamical system is too complicated to study the transitional dynamics. So we only focus on comparative statics here. Goenka and Liu (2012) are able to characterize the full dynamics as there is only a one-way interaction which simplifies the dynamics. Goenka, Liu and Nguyen (2013) have a full characterization of local dynamics in the neo-classical version of the model. The analysis here is only about the comparison between the stationary equilibrium before and after the change, and the transitional dynamics are ignored.

the function $G(q)$ shifts up to $G^d(q)$ as the death rate d declines. Moreover, as d drops, the critical value $\hat{q}$ decreases and $\hat{G}$ increases from Lemma 4, that is, $\hat{q} > \hat{q}^d$ and $\hat{G} < \hat{G}^d$.

In the upper panel, for countries initially in the disease-endemic BGP with $g = \psi L(q^*) - (\rho - b + d)$, where q^* is determined by $G(q) = 0$, an exogenous increase in life expectancy causes them to grow at a faster rate, with

$$q^{*,d} > q^* > \hat{q} > \hat{q}^d, \quad L^{*,d} > L^* > \frac{\rho - b + d}{\psi} \quad \text{and} \quad g^d > g > 0.$$

From Lemma 2, we know that saving rate increases as death rate drops, among which a larger fraction is spent on health expenditure.

The bottom panels are for countries initially in the poverty trap, and the effect of change in life expectancy is more complex. In the bottom left panel, if the increase in life expectancy is large enough countries escape from the poverty trap. We have

$$q^{*,d} > \hat{q} \geq q^*, \quad L^{*,d} > \frac{\rho - b + d}{\psi} \geq L^* \quad \text{and} \quad g^d > g = 0.$$

From Lemma 2, initially saving rate is given by $\beta\left(1 - \frac{\rho - b + d}{\rho + \delta}\right)$ and after death rate drops, saving rate becomes $\beta\left(1 - \frac{\rho - b + d}{\psi L^{*,d} + b - d + \delta}\right)$. Since $\psi L^{*,d} + b - d + \delta > \rho + \delta$, saving rate rises, among which a larger fraction is spent on health expenditure. The reason is when life expectancy increases, people become more patient. They tend to spend more in health expenditure and hence labor force participation increases. It's more likely the country will start investing in human capital accumulation when disease prevalence drops. Hence, the country may transit from a poverty trap to an equilibrium with positive economic growth.

In the bottom right panel, after the death rate drops the economy is still in the poverty trap with

$$\hat{q} > \hat{q}^d \geq q^{*,d} > q^*, \quad \frac{\rho - b + d}{\psi} \geq L^{*,d} > L^* \quad \text{and} \quad g^d = g = 0.$$

From Lemma 2, we know that after life expectancy rises, the saving rate increases of which a larger fraction is spent on health expenditure. As a result, the prevalence of infectious diseases becomes less severe. However, the effectiveness of human capital accumulation is still not large enough for justifying its time allocation, and hence there is no economic growth.

In this case, the output initially is given as:

$$Y^* = A^{\frac{1}{1-\beta}} \left(\frac{\rho + \delta}{\beta}(1 + q^*)\right)^{-\frac{\beta}{1-\beta}} L(q^*).$$

And we have

$$\frac{\partial Y^*}{\partial q}|_{q=q^*} = A^{\frac{1}{1-\beta}} \left(\frac{\rho+\delta}{\beta}\right)^{-\frac{\beta}{1-\beta}} (1+q^*)^{-\frac{1}{1-\beta}} L(q^*) \frac{\beta}{1-\beta} \left[-1 + \frac{\rho - b + d + \alpha(q^*)}{\alpha(q^*)} \cdot \frac{\alpha(q^*)}{\alpha(q^*) - (b+\gamma)}\right] > 0.$$

So

$$\frac{\partial Y^*}{\partial d}|_{q=q^*} = \frac{\partial Y^*}{\partial q^*} \cdot \frac{\partial q^*}{\partial d}|_{q=q^*} < 0.$$

It implies that output increases after the drop of death rate. However, the welfare comparison before and after the drop of death rate is ambiguous. The consumption initially is given as:

$$C^* = \left(1 - \beta \frac{\delta + b - d}{\rho + \delta}\right) Y^*.$$

We know as life expectancy increases, more health expenditure is allocated for controlling infectious diseases and labor force participation rate rises. Then, output increases leading to higher consumption level. However, as the result of direct effect of declining death rate, consumption level decreases. The reason is that more people alive diffuse the resource allocation and lower consumption level for each individual. This is so called Malthusian effect. It is not clear in the model which effect dominates.

One thing to note is that the Malthusian effect can take place only when a country is in a poverty trap. For the other situations, a decrease in the death rate or increase in life expectancy unambiguously increases growth through the mechanism of increased incentives for saving due to the decrease in the effective discount rate.

6.1 Numerical analysis

In this subsection, we conduct numerical analysis to examine effects of change in life expectancy, with particular focus on the welfare comparison when countries are in the poverty trap before and after the change. We also incorporate the effects of change in life expectancy in a centralized economy and welfare comparison between the decentralized and centralized economies. Throughout this subsection, we focus on the case when infectious diseases are prevalent.

We calibrate the model on an annual basis. To be consistent with the literature, the parameters we choose are as follows: discount rate $\rho = 0.05$, capital share $\beta = 0.36$, depreciation rate $\delta = 0.05$, birth rate $b = 0.02$, death rate $d = 0.01$ and the scale parameter in the production function $A = 1$. In the developed countries with no prevalence of infectious diseases, growth rate is around 1.5% and time devoted for the education is 18% from Lucas (1988). So the effectiveness of human capital accumulation is $\psi = 0.054$. We assume every year 20% of the infected population recover from the infection, that is, $\gamma = 0.2$. We assume

the functional form of contact rate is $\alpha(q) = (a/q) + b + \gamma$, satisfying Assumption 1 and 2. The parameter a is chosen such that in the developing countries with diseases prevalence, percentage of health expenditure used in controlling diseases among the total GDP is around 2% and years loss due to infectious diseases is 5%.

Figure 8 describes the change of economic growth path for both the decentralized and centralized economies, when life expectancy increases or death rate drops from 2.3% to 0.8%. In the top panel, for both economies as life expectancy rises, people become more patient and invest more in health expenditure: the ratio of health and physical capital rises. The fraction of healthy individuals increases and by assumption the labor supply rises. The interesting observation is that in the centralized economy, when the death rate drops to 1.9% the economy starts taking off with time devoted for human capital accumulation strictly positive, while the decentralized economy is still stuck in the poverty trap, which only takes off when death rates drops further below 1.5%. If we compare the two economies, the ratio of health and physical capital, labor force participation rate and growth rate all tend to be higher in the centralized than the decentralized economy. In the bottom panel, we provide the change in the resource allocation. As people become more patient, saving rate rises. So do the percentage of health expenditure and physical capital investment in the total production.

When death rate is higher, between 2.3% to 1.9%, such that both economies are in the poverty trap, the saving rates are in fact the same. However, in the centralized economy more is allocated towards controlling infectious diseases. In Figure 9, we compare the total output, consumption level and welfare. When the economy is stagnant, the welfare is given as:

$$W = \frac{1}{\rho - b + d} C^*.$$

We normalize the variables in the decentralized economy when $d = 2.3\%$ to 1. For each specific death rate, output, consumption and welfare are higher in the centralized economy. As life expectancy rises, output increases slightly. However, due to Malthusian effect, consumption level declines. But, welfare increases as people now live longer.

7 Conclusion

This paper develops an endogenous growth model with human capital formation where the prevalence of an infectious disease causes ill-health and incapicitates individuals from working as well as accumulating human capital. There is an endogenous choice of health expenditure to prevent infectious diseases. The paper focuses on the effects of morbidity (ill-health) and thus chooses to use an infinitely lived agent framework. We find that the incidence of the

disease can cause poverty traps where there is low capital and income, as well as low human and health expenditure, as well as high growth balanced growth paths with low disease incidence but with higher levels of the other variables. This helps explain the clustering of the data. The model also shows that an exogenous demographic transition could lead to a take-off from poverty trap to a positive growth. The paper also shows that beyond the mortality effects of diseases such as HIV/AIDS and malaria, the so-called "forgotten disease" that are endemic, do not cause significant mortality, and afflict primarily the poor could be an important determinant of poverty traps by affecting the amortization of physical, human and health capital. There are both savings effects and an allocation of savings effects that are ignored by models that treat savings as exogenous. It is worth emphasizing that these exist even when preferences are log-linear.

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A Appendix

Proof of Proposition 1:

Proof. We rewrite the dynamical system of the decentralized economy as follows:

$$C + [\dot{K} + (\delta + b - d)K] + [\dot{H} + (\delta + b - d)H] = AK^{\beta}(euL)^{1-\beta} \tag{A.1}$$

$$\dot{e} = \psi eL(1-u) \tag{A.2}$$

$$\dot{L} = (b+\gamma)(1-L) - \alpha\left(\frac{H}{K}\right)(1-L)L \tag{A.3}$$

$$\frac{1}{C} = \lambda_1 \tag{A.4}$$

$$\lambda_1 = \lambda_2 + \theta_3, \quad \theta_3 \geq 0, \quad I_H \geq 0, \quad \theta_3 I_H = 0 \tag{A.5}$$

$$\lambda_1(1-\beta)AK^{\beta}(euL)^{-\beta}eL = \lambda_3\psi eL + \theta_1, \quad \theta_1 \geq 0, \quad 1-u \geq 0, \quad \theta_1(1-u) = 0 \tag{A.6}$$

$$\dot{\lambda}_1 = (\rho - b + d)\lambda_1 - \lambda_1(\beta AK^{\beta-1}(euL)^{1-\beta} - (\delta + b - d)) - \lambda_4\alpha'\left(\frac{H}{K}\right)\frac{H}{K^2}(1-L)L \tag{A.7}$$

$$\dot{\lambda}_2 = (\rho - b + d)\lambda_2 + \lambda_2(\delta + b - d) + \lambda_4\alpha'\left(\frac{H}{K}\right)\frac{1}{K}(1-L)L \tag{A.8}$$

$$\dot{\lambda}_3 = (\rho - b + d)\lambda_3 - \lambda_3\psi L(1-u) - \lambda_1(1-\beta)AK^{\beta}(euL)^{-\beta}uL \tag{A.9}$$

$$\dot{\lambda}_4 = (\rho - b + d)\lambda_4 - \lambda_3\psi e(1-u) + \lambda_4(b + \gamma + \alpha\left(\frac{H}{K}\right)(1-L))$$
$$-\lambda_1(1-\beta)AK^{\beta}(euL)^{-\beta}eu + \theta_2, \quad \theta_2 \geq 0, \quad 1-L \geq 0, \quad \theta_2(1-L) = 0 \tag{A.10}$$

Disease-free case: In this case, nfectious diseases are eradicated and all individuals are health. In this case, health expenditure for disease control is zero, $I_H = 0$. Otherwise, if $I_H > 0$, we have $\theta_3 = 0$ and $\lambda_1 = \lambda_2$. Combining equations (A.7) and (A.8), we obtain $\lambda_1\beta AK^{\beta-1}(eu)^{1-\beta} = 0$, which contradicts to $\lambda_1 = \frac{1}{C} > 0$.

Differentiating both sides of equation (A.4) and we get $-\frac{\dot{C}}{C} = \frac{\dot{\lambda}_1}{\lambda_1}$. Dividing both sides of equation (A.7) by λ_1, we get $\frac{\dot{\lambda}_1}{\lambda_1} = \rho + \delta - \beta AK^{\beta-1}(eu)^{1-\beta}$. Since u is a constant along BGP, it implies growth rates of human capital and physical capital are the same. Similarly, by dividing both sides of equation (A.1) by K, growth rates of physical capital and consumption are the same. So consumption, physical and human capital all grow at the same rate $g = \psi(1-u)$, given by equation (A.2). Dividing both sides of equation (A.9) by λ_3, we have $\frac{\dot{\lambda}_3}{\lambda_3} = \frac{\dot{\lambda}_1}{\lambda_1} = -g$.

If $u^* = 1$, $g = 0$ and $\dot{\lambda}_1 = \dot{\lambda}_3 = 0$. From equation (A.6), $\theta_1 \geq 0$ and $\lambda_1(1-\beta)AK^{\beta}e^{-\beta} > \lambda_3\psi$. From equation (A.9), $\lambda_1(1-\beta)AK^{\beta}e^{-\beta} = \lambda_3(\rho - b + d)$. So we have $\psi < \rho - b + d$, contradicting to the assumption $\psi > \rho - b + d$. So, u^* is strictly less than one, $\theta_1 = 0$ and $\lambda_1(1-\beta)AK^{\beta}e^{-\beta} = \lambda_3\psi$. Substitute this into equation (A.9), and we get $g = \psi - (\rho - b + d)$ and $u^* = \frac{\rho - b + d}{\psi}$.

Disease-endemic case: In this case, infectious diseases are prevalent and $L(q) = \frac{b+\gamma}{\alpha(q)}$. Since L^* is a constant along BGP, q^* is also a constant, implying physical and health capital grow at the same rate. Due to the Inada condition in Assumption 1, health expenditure is strictly positive. So

in equation (12) $\theta_3 = 0$ and $\lambda_1 = \lambda_2$. Then, we could rewrite equations (A.7) - (A.10) as:

$$\frac{\dot{\lambda}_1}{\lambda_1} = \rho - b + d - [\beta A(\frac{euL}{K})^{1-\beta} - (\delta + b - d)] - \frac{\lambda_4}{\lambda_1}\alpha'(q)\frac{H}{K^2}(1-L)L \tag{A.11}$$

$$\frac{\dot{\lambda}_1}{\lambda_1} = \rho - b + d + (\delta + b - d) + \frac{\lambda_4}{\lambda_1}\alpha'(q)\frac{1}{K}(1-L)L \tag{A.12}$$

$$\frac{\dot{\lambda}_3}{\lambda_3} = \rho - b + d - \psi L(1-u) - \frac{\lambda_1}{\lambda_3}(1-\beta)A(\frac{euL}{K})^{-\beta}uL \tag{A.13}$$

$$\frac{\dot{\lambda}_4}{\lambda_4} = \rho - b + d - \frac{\lambda_3}{\lambda_4}\psi e(1-u) + (b + \gamma + \alpha(q)(1-L)) - \frac{\lambda_1}{\lambda_4}(1-\beta)A(\frac{euL}{K})^{-\beta}eu. \tag{A.14}$$

By some manipulations, we can see that consumption, physical, health and human capital grow at the same rate $g = \psi L(q^*)(1-u^*)$, $\frac{\dot{\lambda}_1}{\lambda_1} = \frac{\dot{\lambda}_3}{\lambda_3} = -g$ and $\frac{\dot{\lambda}_4}{\lambda_4} = 0$. Substitute these into equations (A.11)-(A.13), we have

$$\frac{\lambda_1}{\lambda_3} = \frac{\rho - b + d}{(1-\beta)AK^{\beta}(euL)^{-\beta}uL}, \quad \text{and} \quad \frac{\lambda_4}{\lambda_1} = -\frac{g + \rho + \delta}{\alpha'(q)\frac{1}{K}(1-L)L}.$$

Then substitute these into equation (A.14) and we obtain:

$$-\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q) - \alpha(q) - \frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q)\frac{\psi L(q)(1-u)}{\rho - b + d} = \rho - b + d, \tag{A.15}$$

which is a function of both q and u. Moreover, from equation (A.6), we have:

$$\begin{aligned} \theta_1 &= \lambda_1(1-\beta)A(\frac{euL}{K})^{-\beta}eL - \lambda_3\psi eL \\ &= \lambda_3\frac{e}{u}[(\rho - b + d) - \psi uL] \\ &\geq 0 \end{aligned}$$

Since $\frac{\lambda_1}{\lambda_3} > 0$ and $\lambda_1 > 0$, we have $\lambda_3 > 0$. So equation (A.6) reduces to

$$\rho - b + d - \psi uL \geq 0, \quad u \leq 1, \quad \text{and} \quad (\rho - b + d - \psi uL)(1-u) = 0. \tag{A.16}$$

Therefore, equations (A.15) and (A.16) determines (u^*, q^*).

There are two scenarios. If $u^* = 1$, growth rate $g = 0$. Equation (A.15) simplifies to:

$$G_L(q) = -\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q) - \alpha(q) - (\rho - b + d) = 0.$$

Since $u^* = 1$, we have $\theta_1 \geq 0$, implying $\psi L(q^*) \leq \rho - b + d$. That is, a disease-endemic poverty trap exists if $q^* \leq \hat{q}$.

If $u^* < 1$, we have $\lambda_1(1-\beta)AK^\beta(eL)^{1-\beta}u^{-\beta} = \lambda_3\psi eL$ and $u^* = \frac{\rho-b+d}{\psi L^*}$. q^* is determined by:

$$G_R(q) = -\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q)\frac{\psi L(q)}{\rho-b+d} - \alpha(q) - (\rho-b+d) = 0.$$

Since $u^* < 1$, we have $\psi L(q^*) > \rho - b + d$. That is, a disease-endemic BGP exists if $q^* > \hat{q}$.

If we compare the two functions $G_L(q)$ and $G_R(q)$, we find that $G_L(q) > G_R(q)$ if $q < \hat{q}$, $G_L(q) < G_R(q)$ if $q > \hat{q}$, and $G_L(q) = G_R(q)$ if $q = \hat{q}$. Thus q^* is determined by function

$$G(q) = \max\{G_L(q), G_R(q)\} = 0.$$

Function G is continuous, $\lim_{q\to 0} G = +\infty$ and $\lim_{q\to\infty} G < 0$. By intermediate value theorem, there exists a $q^* > 0$ such that $G(q) = 0$, that is, there exists an endemic-disease case. If $q^* \le \hat{q}$, it is a poverty trap, and if $q^* > \hat{q}$, there is positive growth path.

□

Proof of Lemma 2:

Proof. 1) Countries in the disease-free BGP: Substitute $L^* = 1$ and $\frac{\lambda_1}{\lambda_1} = -g$ into equation (A.7), we have $\beta A(\frac{K}{eu})^{\beta-1} = g + \rho + \delta$. Then

$$\begin{aligned}
\frac{I_K}{Y} &= \frac{\dot{K} + (\delta+b-d)K}{Y} = (\frac{\dot{K}}{K} + \delta + b - d)\frac{1}{A}(\frac{K}{eu})^{1-\beta} = \beta\frac{g+\delta+b-d}{g+\rho+\delta};\\
\frac{C}{Y} &= 1 - \frac{I_K}{Y} = 1 - \beta\frac{g+\delta+b-d}{g+\rho+\delta}.
\end{aligned}$$

Substitute $g = \psi - (\rho - b + d)$, and we obtain the results for countries in the disease-free BGP.

2) Countries in the disease-endemic BGP: Combing equations (A.11) and (A.12), we have $\beta A(\frac{K}{euL})^{\beta-1} = (g+\rho+\delta)(1+q)$. Then

$$\begin{aligned}
\frac{I_K}{Y} &= \frac{\dot{K} + (\delta+b-d)K}{Y} = (\frac{\dot{K}}{K} + \delta + b - d)\frac{1}{A}(\frac{K}{euL})^{1-\beta} = \beta\frac{g+\delta+b-d}{g+\rho+\delta}\cdot\frac{1}{1+q};\\
\frac{I_H}{Y} &= \frac{\dot{H} + (\delta+b-d)H}{Y} = (g+\delta+b-d)\cdot\frac{H}{K}\cdot\frac{K}{Y} = \beta\frac{g+\delta+b-d}{g+\rho+\delta}\cdot\frac{q}{1+q};\\
\frac{C}{Y} &= 1 - \frac{I_K}{Y} - \frac{I_H}{Y} = 1 - \beta\frac{g+\delta+b-d}{g+\rho+\delta}.
\end{aligned}$$

Substitute $g = \psi L^* - (\rho - b + d)$, and we obtain the results for countries in the disease-endemic BGP.

3) Countries in the disease-endemic poverty trap: Similar to countries in the diseases-endemic BGP, we substitute into $g = 0$ and obtain the results for countries in the poverty trap.

□

Proof of Lemma 1:

Proof. Since the functions $G_L(q)$ and $G_R(q)$ are differentiable, we have

$$\begin{aligned}
\frac{\partial G_L(q)}{\partial q} &= -\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q)[\frac{\alpha''(q)}{\alpha'(q)}+\frac{L(q)}{1-L(q)}\cdot\frac{\alpha'(q)}{\alpha(q)}+\frac{1}{1+q}]-\alpha'(q);\\
\frac{\partial G_R(q)}{\partial q} &= -\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))(1+q)\frac{\psi L(q)}{\rho-b+d}[\frac{\alpha''(q)}{\alpha'(q)}+\frac{L(q)}{1-L(q)}\cdot\frac{\alpha'(q)}{\alpha(q)}+\frac{1}{1+q}-\frac{\alpha'(q)}{\alpha(q)}]-\alpha'(q).
\end{aligned}$$

Under Assumption 3, both functions $G_L(q)$ and $G_R(q)$ are monotonically decreasing in q. Moreover, since $G(q)=G_L(q)$ when $q<\hat{q}$, $G(q)=G_R(q)$ when $q>\hat{q}$, and $G(q)=G_L(q)=G_R(q)$ when $q=\hat{q}$, function $G(q)$ is also monotonically decreasing in q.

□

Proof of Proposition 3:

Proof. Under Lemma 1, function $G(q)$ is monotonically decreasing and thus there exists a unique q^* such that $G(q)=0$. When $\hat{G}>0$, that is, $G(\hat{q})>G(q^*)$, it implies $q^*>\hat{q}$, and with Proposition 1 it is a disease-endemic BGP. When $\hat{G}\leq 0$, that is, $G(\hat{q})\leq G(q^*)$, it implies $q^*\leq\hat{q}$, and with Proposition 1 it is a disease-endemic poverty trap. □

Proof of Lemma 4:

Proof. Since $\psi\frac{b+\gamma}{\alpha(\hat{q})}=\rho-b+d$, we have

$$\begin{aligned}
&\frac{\partial\hat{q}}{\partial\rho}=-\frac{\alpha(\hat{q})}{\alpha'(\hat{q})(\rho-b+d)}>0; \quad \frac{\partial\hat{q}}{\partial d}=\frac{-\alpha(\hat{q})}{\alpha'(\hat{q})(\rho-b+d)}>0;\\
&\frac{\partial\hat{q}}{\partial\psi}=\frac{b+\gamma}{\alpha'(\hat{q})(\rho-b+d)}<0.
\end{aligned}$$

Since $\hat{G}=-\frac{1-\beta}{\beta}\alpha'(\hat{q})(1-L(\hat{q}))(1+\hat{q})-\alpha(\hat{q})-(\rho-b+d)$, by Assumption 3, $\frac{\partial\hat{G}}{\partial\hat{q}}<0$. Then, we have

$$\begin{aligned}
&\frac{d\hat{G}}{d\beta}=-(-\frac{1}{\beta^2})\alpha'(\hat{q})(1-L(\hat{q}))(1+\hat{q})<0;\\
&\frac{d\hat{G}}{d\rho}=\frac{\partial\hat{G}}{\partial\rho}+\frac{\partial\hat{G}}{\partial\hat{q}}\cdot\frac{\partial\hat{q}}{\partial\rho}=-1+\frac{\partial\hat{G}}{\partial\hat{q}}\cdot\frac{\partial\hat{q}}{\partial\rho}<0;\\
&\frac{d\hat{G}}{dd}=\frac{\partial\hat{G}}{\partial d}+\frac{\partial\hat{G}}{\partial\hat{q}}\cdot\frac{\partial\hat{q}}{\partial d}=-1+\frac{\partial\hat{G}}{\partial\hat{q}}\cdot\frac{\partial\hat{q}}{\partial d}<0;\\
&\frac{d\hat{G}}{d\psi}=\frac{\partial\hat{G}}{\partial\hat{q}}\cdot\frac{\partial\hat{q}}{\partial\psi}>0.
\end{aligned}$$

□

Proof of Lemma 2:

Proof. We can rewrite $G^\tau(q)$ as follows:

$$\begin{aligned} G_L^\tau(q) &= G_L(q) + \left[-\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))\tau\right], \quad \text{and} \\ G_R^\tau(q) &= G_R(q) + \left[-\frac{1-\beta}{\beta}\alpha'(q)(1-L(q))\frac{\psi L(q)}{\rho-b+d}\frac{\rho+\delta}{\rho+\delta+(1+\tau)g}\tau\right]. \end{aligned}$$

The rest is straightforward from Assumption 3 and Lemma 1. □

Proof of Proposition 7:

Proof. The subside τ is chosen such that $q^{*,\tau}$ determined by equation $G^\tau(q)=0$ is the same as $q^{*,c}$ determined by equation $G(q)+b+\gamma=0$. Let $q^* = q^{*,\tau} = q^{*,c}$.

If $q \le \hat{q}$, we know $G(q^*)+b+\gamma=0$, which implies $-\frac{1-\beta}{\beta}\alpha'(q^*)(1-L(q^*))(1+q^*) - \alpha(q^*) - (\rho-b+d)+b+\gamma=0$. From $G^\tau(q^*)=0$, we have

$$\begin{aligned} \tau &= \frac{\alpha(q^*)+\rho-b+d}{-\frac{1-\beta}{\beta}\alpha'(q^*)(1-L(q^*))} - (1+q^*) \\ &= \frac{\alpha(q^*)+\rho-b+d}{\frac{\alpha(q^*)-(b+\gamma)+\rho-b+d}{1+q^*}} - (1+q^*) \\ &= (1+q^*)\frac{b+\gamma}{\alpha(q^*)-(b+\gamma)+\rho-b+d}. \end{aligned}$$

Similarly, if $q > \hat{q}$, we know $G(q^*)+b+\gamma=0$, which implies $-\frac{1-\beta}{\beta}\alpha'(q^*)(1-L(q^*))(1+q^*)\frac{\psi L^*}{\rho-b+d} - \alpha(q^*) - (\rho-b+d)+b+\gamma=0$. From $G^\tau(q^*)=0$, we have

$$\frac{\rho+\delta}{\rho+\delta+(1+\tau)g}\tau = (1+q^*)\frac{b+\gamma}{\alpha(q^*)-(b+\gamma)+\rho-b+d},$$

and

$$\tau = (1+q^*)\frac{b+\gamma}{\alpha(q^*)-(b+\gamma)+\rho-b+d}\cdot\frac{\rho+\delta+g}{\rho+\delta-(1+q^*)\frac{b+\gamma}{\alpha(q^*)-(b+\gamma)+\rho-b+d}g}.$$

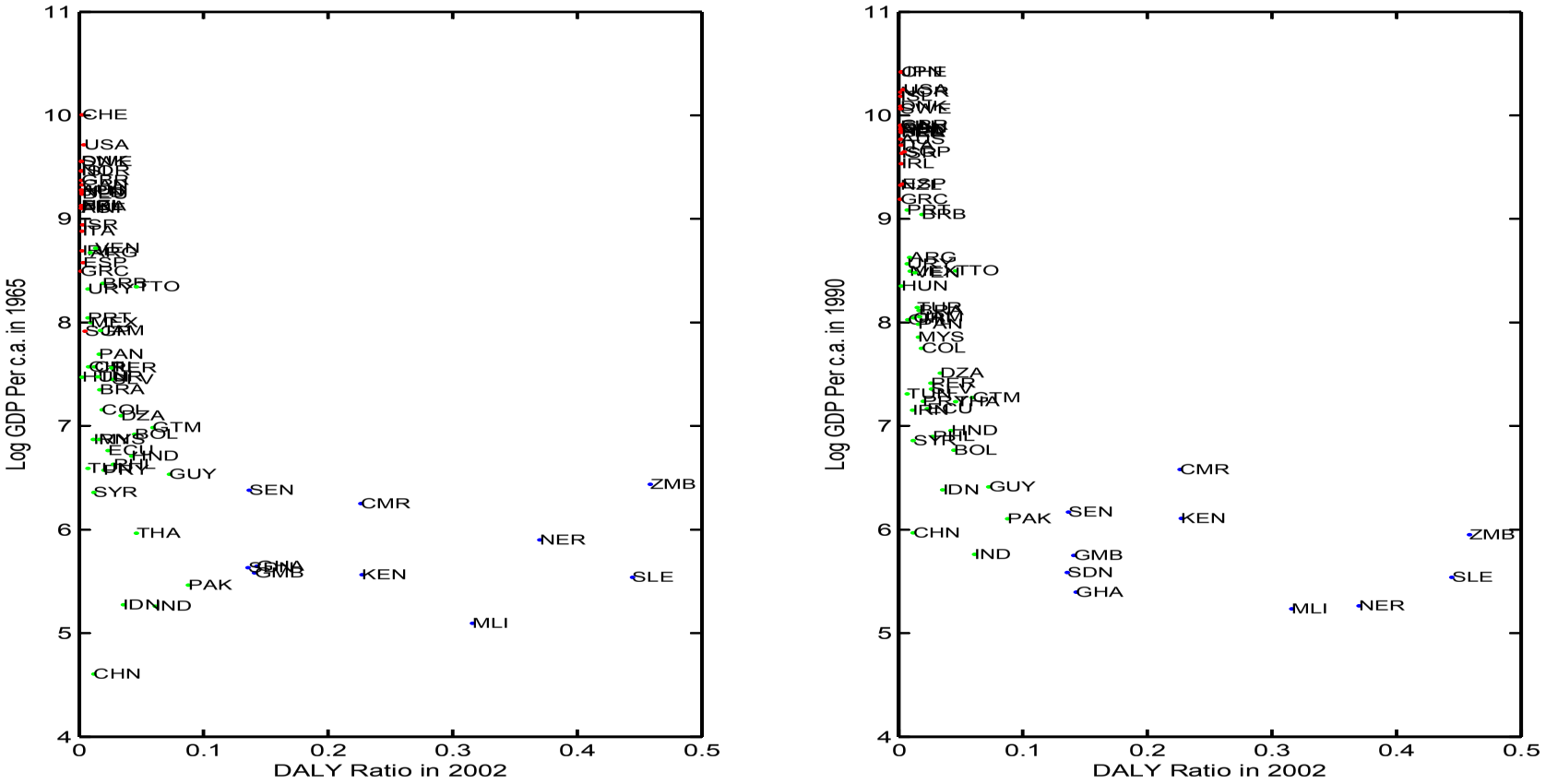
□

Table 2. Results for the Cluster Analysis – Variable Mean for Each Grouped Countries

Panel A: Economic Development			
	Growth Rate(%)	GDP Per Capita (Constant 2000 US$)	
	1965-1990	1965	1990
Developed Countries	2.85	10325	20123
Developing Countries	1.86	1798	2666
Least Developed Countries	−0.21	361	347
Panel B: Burden of Infectious Diseases			
	DALY	Life Expectancy at Birth	
	2002	1965	1990
Developed Countries	0.002	71.2	76.4
Developing Countries	0.025	57.1	67.9
Least Developed Countries	0.260	40.7	50.0
Panel C: Educational Attainment			
	1965	1990	
Developed Countries	6.96	8.76	
Developing Countries	3.28	5.44	
Least Developed Countries	1.15	2.42	

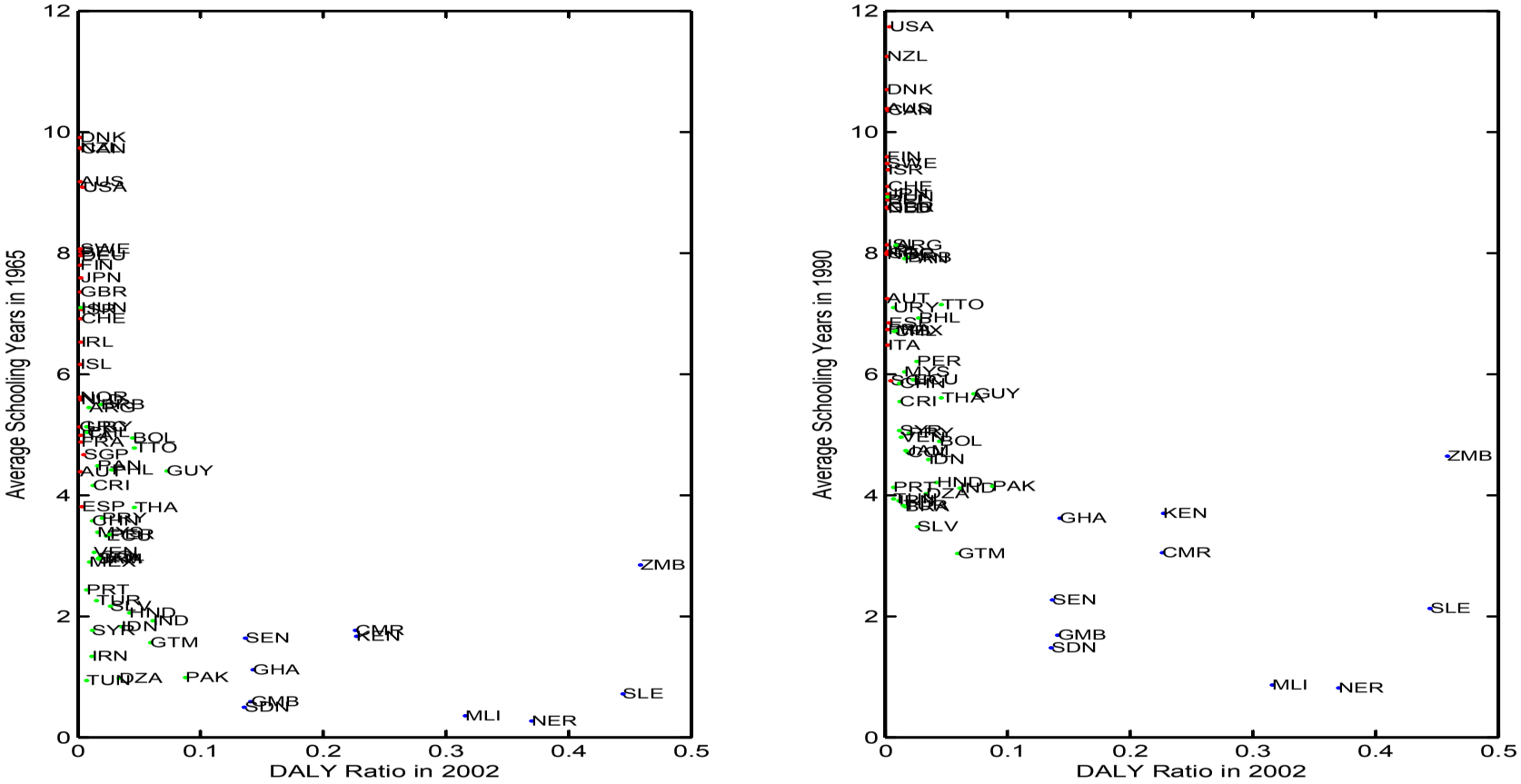
Note: The table describes the variable mean for each grouped countries resulted from the cluster analysis, which we call developed countries, developing countries and least developed countries. For a complete list of countries in each group, refer to the Table Appendix. The variables included in the table are growth rate from 1965 to 1990, GDP Per capita in year 1965 and 1990, the ratio of DALY in total population, life expectancy in year 1965 and 1990, quality adjusted schooling years from Barro and Lee (1986) for year 1965 and 1990.

Figure 2. Cross-country Joint Distribution of DALY and Log GDP Per Capita in the Year 1965 and 1990



Notes: The figure depicts the joint distribution of Log GDP Per capita and DALY ratio for the countries used in the cluster analysis. The Red color denotes the developed countries, the green color denotes the developing countries and the blue color denotes the least developed countries. The left panel is the joint distribution in the year 1965 and the right panel is the one in the year 1990. GDP Per capita is from World Bank database, and DALY ratio is from WHO.

Figure 3. Cross-country Joint Distribution of DALY and Educational Attainment in the Year 1965 and 1990



Notes: The figure depicts the joint distribution of quality adjusted average schooling years and DALY ratio for the countries used in the cluster analysis. The Red color denotes the developed countries, the green color denotes the developing countries and the blue color denotes the least developed countries. The left panel is the joint distribution in the year 1965 and the right panel is the one in the year 1990. Quality adjusted average schooling years are from Barro and Lee (1986), and DALY ratio is from WHO.

Figure 4. Two Scenarios Under the Diseases-endemic Case: Disease-endemic BGP (the Upper Panel) and Disease-endemic Poverty Trap (the Bottom Panel)

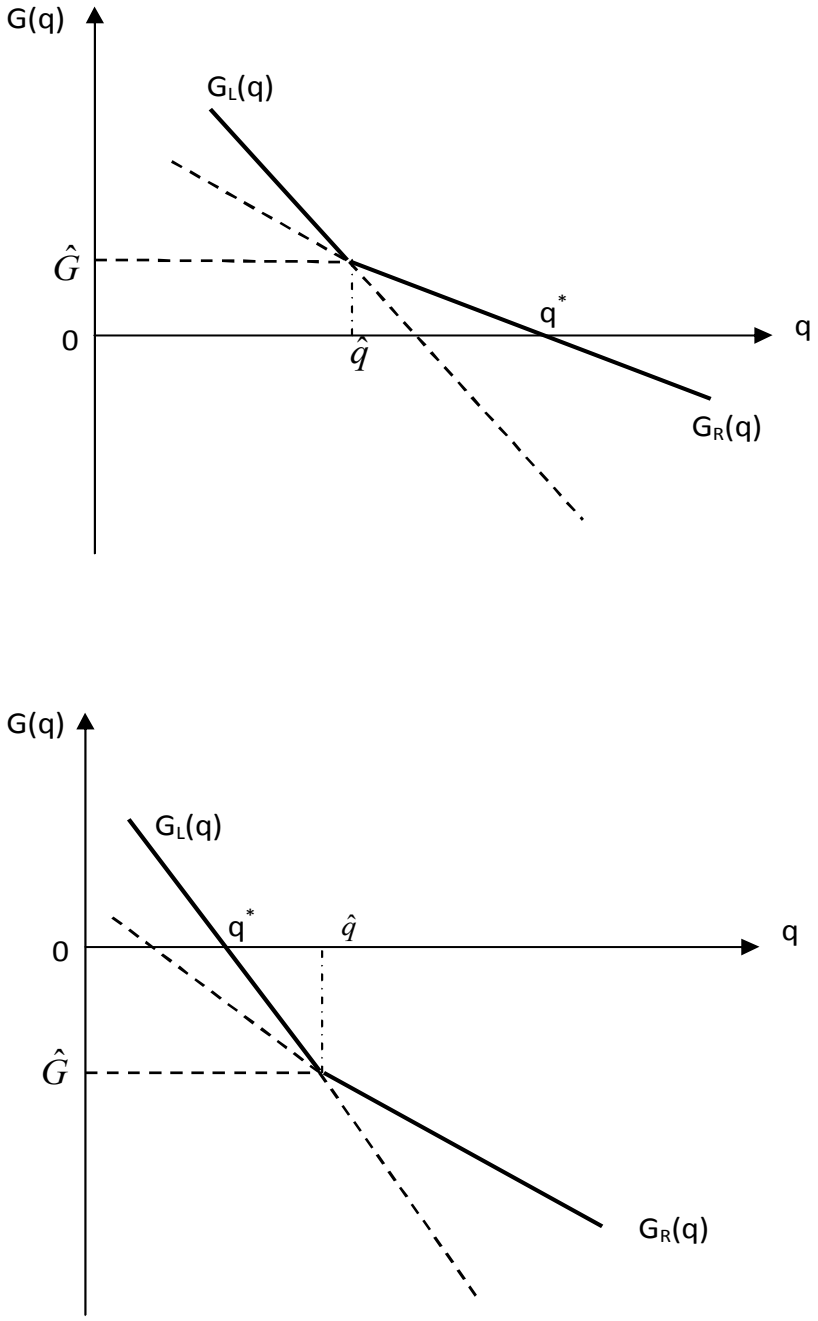


Figure 5. The Comparison Between the Optimal Paths and the Competitive Equilibria

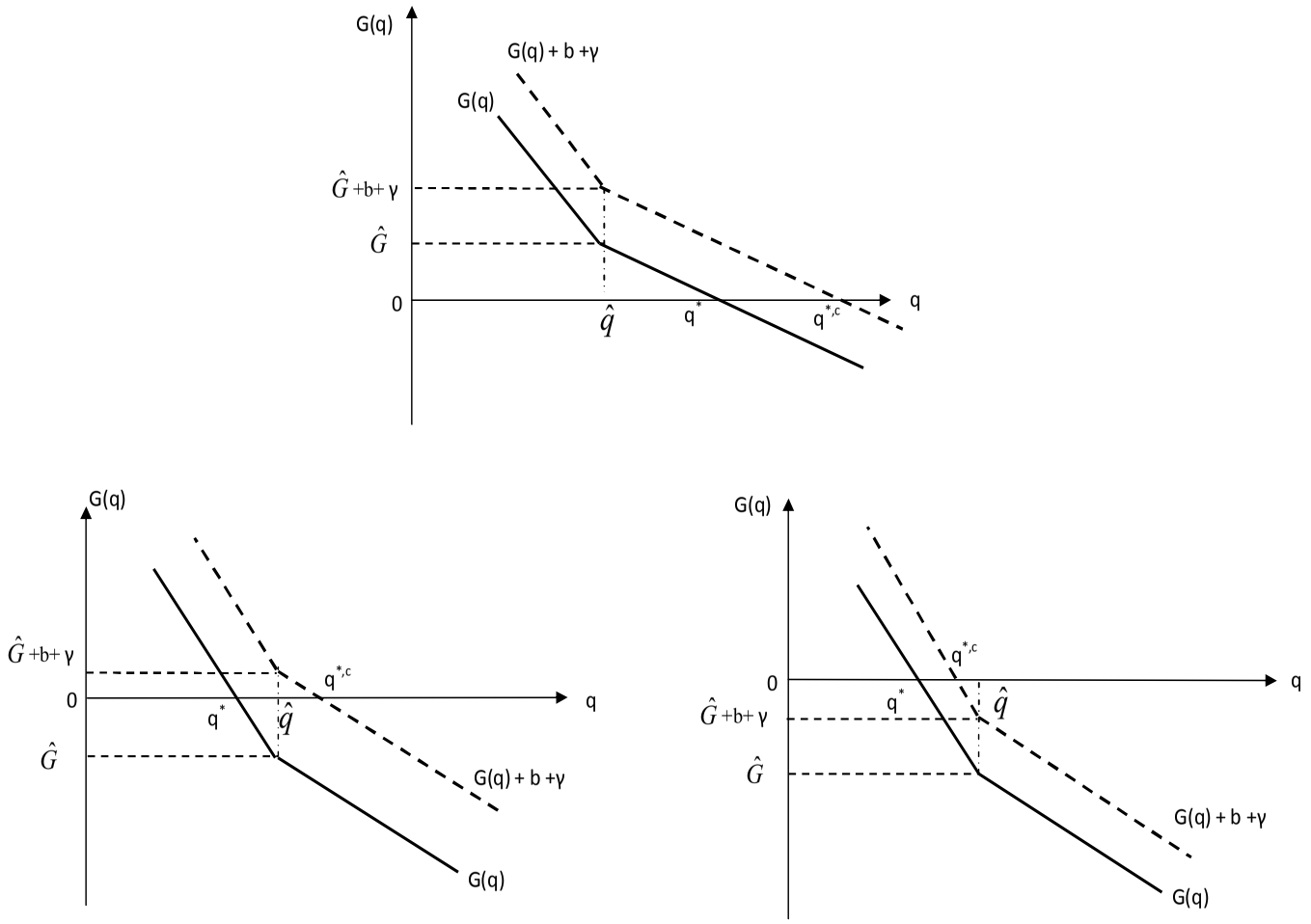


Figure 6. The Comparison Between the Competitive Equilibria in a Decentralized Economy With and Without Public Health Policy

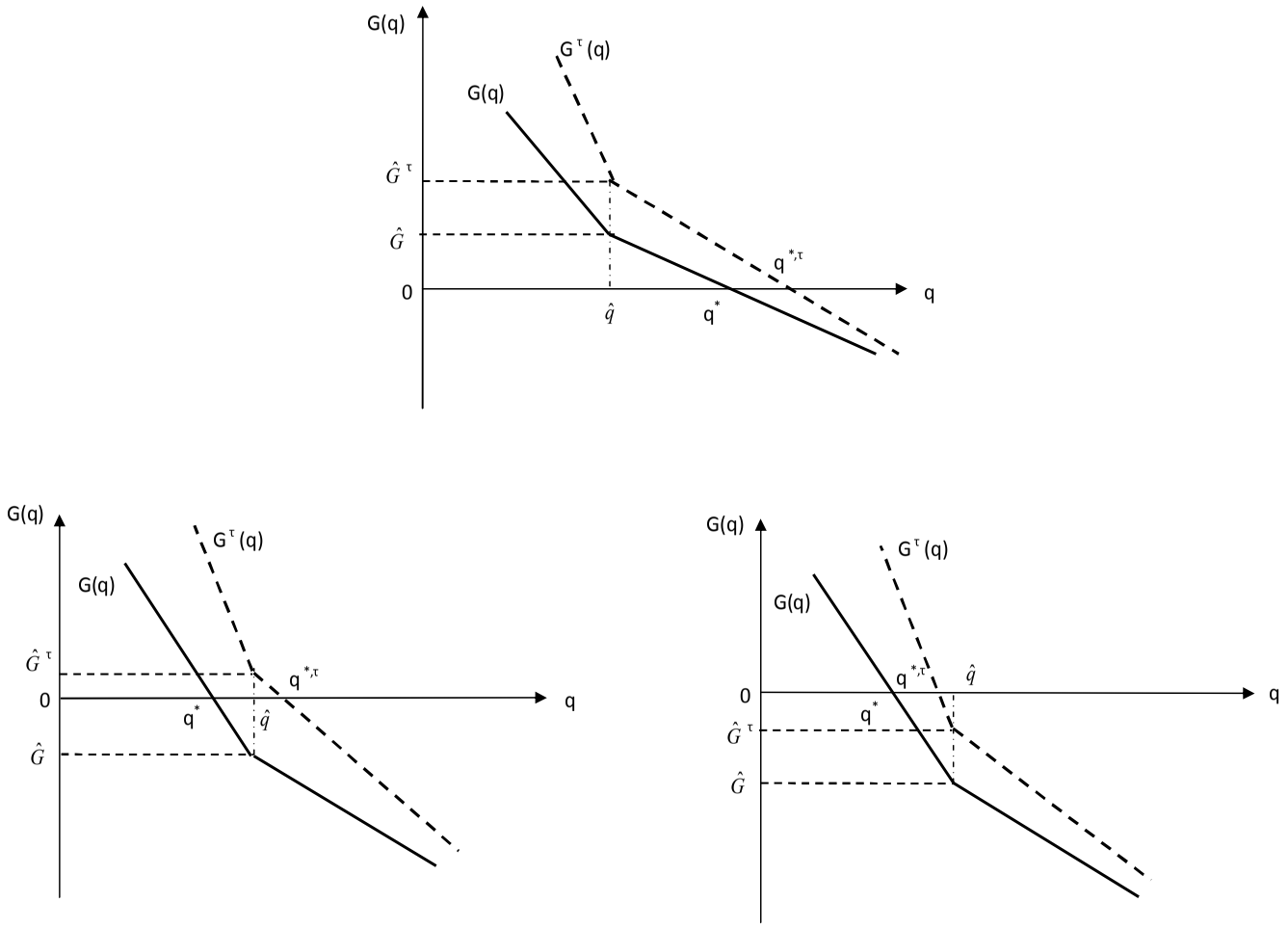


Figure 7. The Comparison Between the Competitive Equilibria in the Decentralized Economy Before and After Life Expectancy Increases

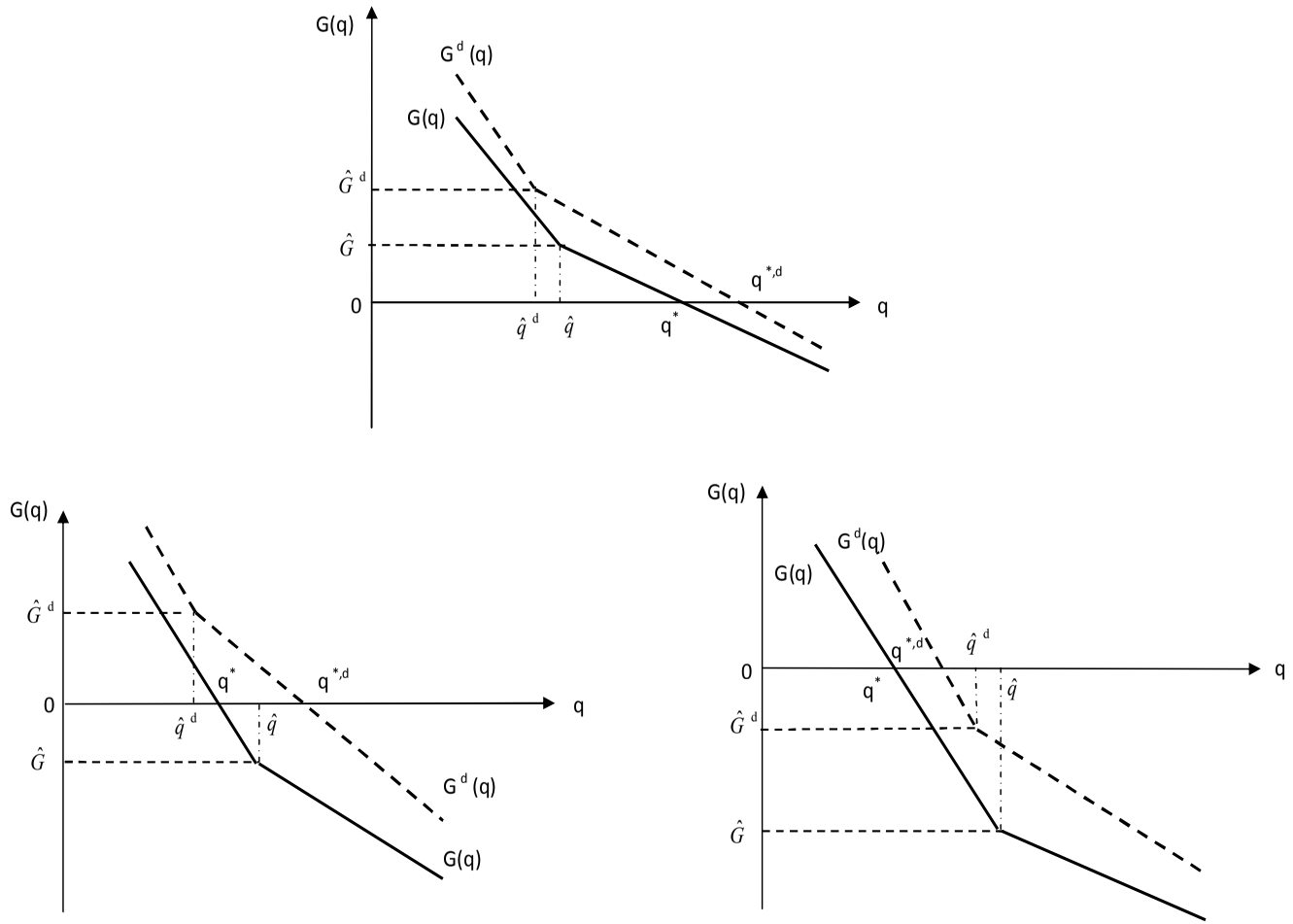


Figure 8. The Numerical Analysis

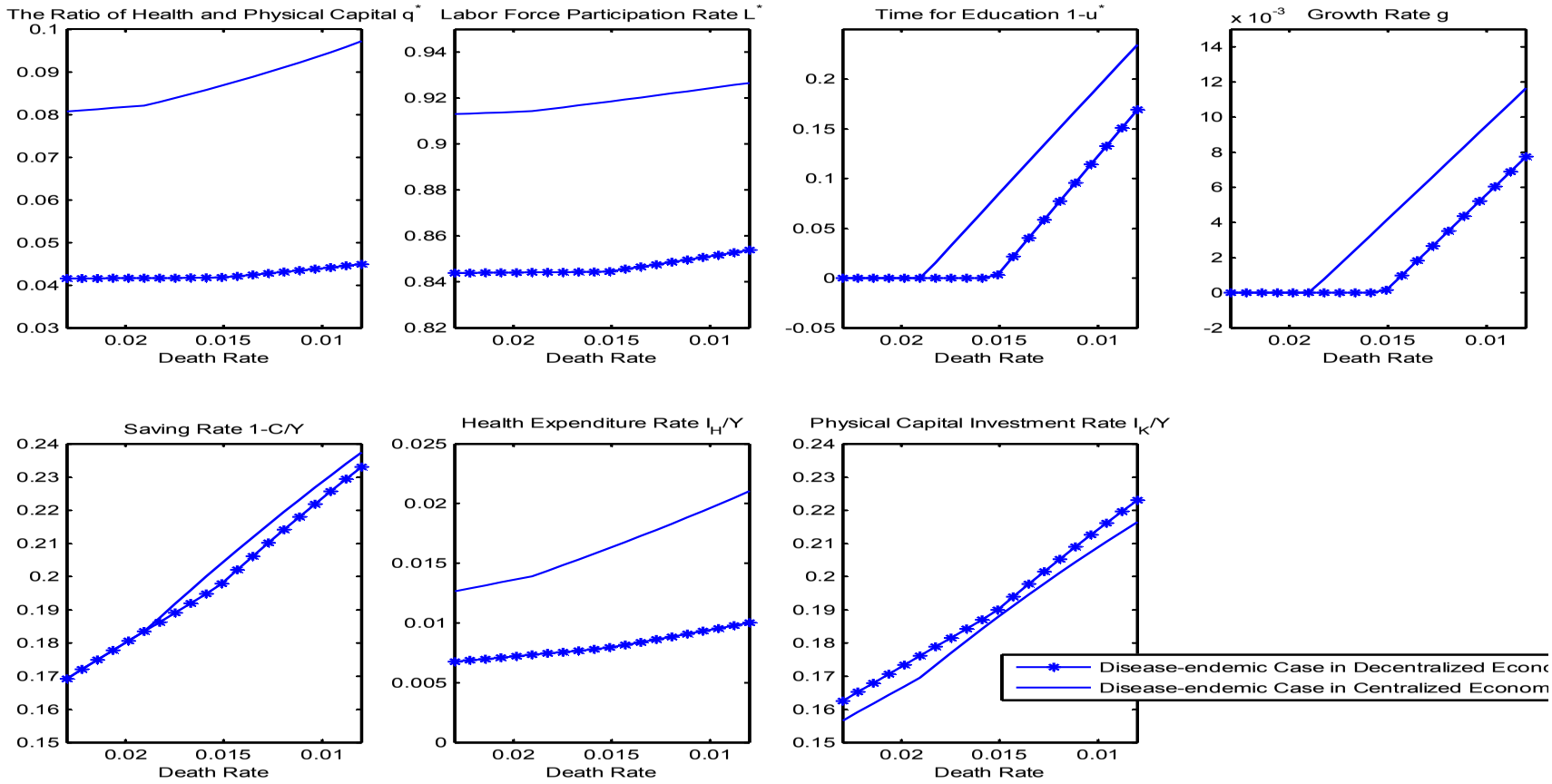


Figure 9. The Numerical Analysis - Welfare Comparison in the Poverty Trap