Labour Policy and Multinational Firms: the "Race to the Bottom" Revisited

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Abstract

This paper revisits the phenomenon of "race to the bottom" in labour markets in a model of strategic interaction with one monoposonist multinational producer and two countries providing labour inputs. The firm has to employ labour from both countries for its production and it has a constant elasticity of substitution production function. The government in each country seeks to maximize the country's labour income. The countries simultaneously decide on their labour policies which in turn decides the effective wages the firm has to pay. Following this, the firm chooses its labour input in each country. The wages are bounded above and below, where the lower bound stands for the minimum wage prevailing in a country and the upper bound is the maximum wage acceptable to the firm. We show that there is no equilibrium with "race to the bottom" (i.e. at least one country setting the minimum wage). On the contrary, depending on the substitutability of the labour inputs of the two countries, it is possible to have equilibrium where "race to the top" (i.e. both countries setting the maximum wage) takes place.

Keywords: constant elasticity of substitution; race to the bottom; race to the top

JEL Classification: F63, J42, O24

1 Introduction

The phenomenon of competitive dilution of labour standards—"race to the bottom"—by the governments of developing countries for attracting multinational corporate economic activities to one's country, especially for direct investment, is one of the most crucial current issues in the political economy of such countries. In this paper we revisit the phenomenon of "race to the bottom" in a simple game-theoretic framework. We explore whether drastic strategic undercutting of labour's bargaining power as reflected in the expected wage labourers can get is an *inevitable* outcome of strategic competition between policy-makers of different countries. We model the strategic behavior of two countries as a two-player game with simultaneous moves where the action of each country is to choose a wage for its labourers (or a labour policy that might generate such a price as the expected wage). The payoffs to the players are determined by the production decision of a multinational monopsonist availing the labour inputs provided by these countries which are substitutes but not *perfect* substitutes. Using the properties of supermodular games (see, e.g., Milgrom and Roberts, 1990), we show that this game has a unique Nash equilibrium in pure strategies. Exploring the properties of this equilibrium, we find that "race to the bottom" never emerges as an equilibrium outcome while the complete opposite—"race to the top"—is possible for a range of relevant parameters of our model.

While anecdotal descriptions of "race to the bottom" in labour standards or wages are quite common (see, e.g., EPW (2014)), concrete identification of the phenomenon often proved elusive (see, e.g., Singh and Zammit, 2004; Potrafke 2013). Recently Davies and Vadlamannati (2013) and Olney (2013) have provided empirical evidence in favour of this.

There has been consideration of whether the phenomenon of "race to the bottom" is inevitable and some channels through which this can be endogenously counteracted have been identified. Of course, generating externality of increased demand through increased wages is one well-known channel. In the context of tax competition, Baldwin and Krugman (2004) analyze how agglomeration effect can counteract "race to the bottom". Dorsch et al. (2011) see how pressure of getting re-elected in a democracy may induce a government to adopt ways to woo foreign investments other than lowering labour standards. Our research is along this line: that of identifying factors which can endogenously counteract this phenomenon.

Our work is motivated by the following simple observations. Race to the bottom in labour markets is akin to Bertrand competition: competitive undercutting of prices to increase demand for one's product. However, the possibility of relaxing such competition in presence of product differentiation is well-known (from onwards Shaked and Sutton, 1982). A multinational firm (MNC) often organizes production in more than one countries for intermediate products to create its final product and it sells the product also all over the world. This implies that for such an MNC, productive inputs obtainable in different countries may not be *perfect* substitutes although near-perfect substitution is still possible. As an example one might think of a car manufacturer obtaining ore and processed metal from one country and having the assembly line in another country and in the first country it may obtain labour experinced in mining activities and in the second the labour skilled in works related to a modern automobile industry. We explore the implication of such production processes involving imperfectly substitutable labour inputs.

To model the imperfect substitutability of the inputs in the MNC's production process, we adopt the usual approach of taking its production function to be of the CES type. Then we analyze the policy-setting game played by the two countries (as outlined above and described in detail in the next section). We look at the properties of the pure strategy Nash equilibria as parameters in the model—especially the elasticity of substitution—change. However, our focus is on ascertaining whether race to the bottom is feasible (and we find the answer to be negative) and on identifying situations for which the completely opposite phenomenon, "race to the top", rather, is possible. While analyses of MNC's decisions of input choices are profuse (see, e.g., Sly and Soderbery, 2014, whose work is close to the theme of this paper and the survey by Antras and Yeaple, 2014), our finding of the possibility of "race to the top" even when inputs are substitutes seems novel.

The model is described in the following section. Section 3 gives the propositions describing the properties of the equilibria. Section 4 provides a brief discussion of the significance of the results and include some concluding remarks. All the proofs are collected in the Appendix.

2 The model

There are two countries a, b. A multinational firm, a monopsonist in the labour markets in these countries, has to employ labour from both countries to carry out production, but the labour inputs from these two countries are imperfect substitutes. Let x_a, x_b denote the labour employed from countries a, b. The firm has a constant elasticity of substitution production function (Arrow et al., 1961) given by

$$F(x_a, x_b) = \left[\alpha x_a^{-\rho} + (1-\alpha)x_b^{-\rho}\right]^{-1/\rho} \tag{1}$$

where $0 < \alpha < 1$ and $\rho \in (-1, 0) \cup (0, \infty)$, i.e., $\rho > -1$ and $\rho \neq 0$. The function F also stands for the firm's profit (we can assume that the product is the numeraire which is sold in the rest of the world).

Let w_a, w_b be the wages paid by the firm in countries a, b. Notice that it is not necessary that the government or the decision-maker in each country has to *actually* administer a fixed wage. Think of a more realistic scenario that the labourers in each country and the management of the firm get into a bilateral conflict over the wages to be paid and where the probability of a party's winning depends on the labour policy taken by the respective government. Then w_i can be thought of as the expected price to be paid/received to the labourers of country i in the face of this possible conflict. However, in what follows, we shall adopt the simple convention as if the government in each country administers its wage and offers that wage to the firm.

Assume that the firm has a fixed amount of capital $K > 0$ that it uses to pay for the labour inputs. The budget constraint of the firm is given by

$$w_a x_a + w_b x_b = K \tag{2}$$

For any $w_a, w_b > 0$, the firm's constrained profit maximization problem has a unique solution. Let the solution be (x_a^*, x_b^*). Let $\pi(w_a, w_b, K) = F(x_a^*, x_b^*)$ be the maximized value of the profit of the firm.

The total labour population in country $i \in \{a, b\}$ is denoted by $\overline{x}_i$. It is assumed that $\overline{x}_a, \overline{x}_b$ are sufficiently large positive numbers so that for the firm's problem, the labour constraint is never binding. The labourers in country i can either work for the monopsonist firm or get some reservation payoff. For $i \in \{a, b\}$, let $\psi_i^M(w_a, w_b, K) = w_i x_i^*$ be the labour income accruing from the firm in country i. Labour that is not employed in the firm gets the reservation payoff (for example, by working in a traditional sector) and earns wage $\underline{w}_i > 0$ in country i. Hence for $i \in \{a, b\}$, the income for the labour that is not employed by the firm is $\psi_i^T(w_a, w_b, K) = \underline{w}_i(\overline{x}_i - x_i^*)$. Consequently the total labour income in country i is

$$\psi_i(w_a, w_b, K) = \psi_i^M(w_a, w_b, K) + \psi_i^T(w_a, w_b, K) = w_i x_i^* - \underline{w}_i x_i^* + \underline{w}_i \overline{x}_i \tag{3}$$

For any (w_a, w_b), the payoff of the decision-maker in each country is a weighted sum of the firm's profit and its labour income. Specifically, the payoffs of countries a, b are given by

$$\phi_a(w_a, w_b) = \lambda_a \pi(w_a, w_b, K) + (1 - \lambda_a)\psi_a(w_a, w_b, K)$$

$$\phi_b(w_a, w_b) = \lambda_b \pi(w_a, w_b, K) + (1 - \lambda_b)\psi_b(w_a, w_b, K) \tag{4}$$

where $\lambda_a, \lambda_b \in [0, 1]$ and $\lambda_a + \lambda_b < 1$ (which ensures that the firm gets some non-negative profit).

The interpretation is that a country's decision-maker, a priori, may have two kinds of incentives. It can get a share of the firm's profit which may be thought of a pecuniary gain of it or bribe paid to it by the firm. However, the decision-maker may also have some incentive for increasing the labourers' income (perhaps so that it does not get too unpopular). However, for the remainder of this analysis we shall assume that $\lambda_i = 0$ for both i. Later, in the concluding section we make a remark on the implication of having $\lambda_i \neq 0$.

The strategic interaction between countries a, b is modeled as a simultaneous-move game G where two countries simultaneously set wages (or, as we remarked above, equivalently, set policies resulting in effective wages) w_a, w_b. The payoff of country $i \in \{a, b\}$ is its total labour income given by ψ_i in (3). We consider $w_i \geq \underline{w}_i$. We also assume there exists a $\overline{w} > \underline{w}_i$ such that the wages $w_a, w_b \leq \overline{w}$. Hence $w_i \in [\underline{w}_i, \overline{w}]$. We look for *Nash Equilibrium in pure strategies* (called simply NE) for the game G.

Remark 1: We outline a few notable features of our set-up. First, we focus on the labour policy and extract away from the other general equilibrium features of international trade: e.g., in our model the firm presumably sells its output in a third country. Next, we endow the firm with maximum market power. Also, we allow substitutability of inputs for the firm apart from the single-point of *perfect* substitutability. And finally, with $\lambda_i = 0$, our model is equivalent to a variant of Bertrand duopoly with differentiated products.

Lemma 1 *The following hold for $i, j \in \{a, b\}$ and $i \neq j$.*

(i) *x_i^* is decreasing in w_i.*

(ii) *x_i^* is increasing in w_j if $\rho \in (-1, 0)$ and decreasing in w_j if $\rho \in (0, \infty)$.*

(iii) *π is decreasing in w_i.*

(iv) *ψ_i^M is decreasing in w_i if $\rho \in (-1, 0)$ and increasing in w_i if $\rho \in (0, \infty)$.*

Proof See the Appendix. ■

We explore the properties of NE of G. In particular, we are interested in whether the equilibria show strategic undercutting or otherwise. Therefore, we introduce the following definitions.

Definitions An NE of G has

(i) *race to the bottom* property if $w_i = \underline{w}_i$ for some $i \in \{a, b\}$;

(ii) *complete race to the bottom* property if $w_i = \underline{w}_i$ for both $i \in \{a, b\}$;

(iii) *race to the top* property if $w_i = \overline{w}$ for some $i \in \{a, b\}$;

(iv) *complete race to the top* property if $w_i = \overline{w}$ for both $i \in \{a, b\}$.

3 The results: equilibria and their properties

Proposition 1 characterizes best responses of the players in G and shows that G has a unique NE. It also identifies some initial properties of the NE.

Proposition 1

(I) *The best responses of countries in the game G have the following properties.*

(i) *If $\rho \in (-1, 0)$, then for any $w_j \in [\underline{w}_j, \overline{w}]$, country i has a unique best response $B_i(w_j)$. The best response function B_i is non-decreasing in w_j and $B_i(w_j) > \underline{w}_i$ for any $w_j \in [\underline{w}_j, \overline{w}]$.*

(ii) *If $\rho \in (0, \infty)$, then for any $w_j \in [\underline{w}, \overline{w}]$, country i has a unique best response $\overline{w}$.*

(II) *The game G has a unique NE. The NE has the following properties.*

(i) *If $\rho \in (-1, 0)$, then at the NE, $w_i > \underline{w}_i$ for $i \in \{a, b\}$, i.e., the NE does not have the race to the bottom property.*

(ii) *If $\rho \in (0, \infty)$, then the NE has the complete race to the top property.*

(iii) *The NE value of w_i is increasing in $\underline{w}_i$ for $i \in \{a, b\}$.*

Proof See the Appendix. ■

Since the case where $\rho \in (0, \infty)$ is immediately clear, next we get on to the case where $\rho \in (-1, 0)$ and identify the property of the equilibrium as the parameters affecting demands vary.

Proposition 2 *Consider the game G. Let $\rho \in (-1,0)$ and $\delta \equiv -\rho \in (0,1)$. Let $\tau_a \equiv \alpha$, $\tau_b \equiv 1-\alpha$ and for $i,j \in \{a,b\}$, $i \neq j$, define*

$$\widetilde{\delta}_i \equiv \tau_j^{1/(1-\delta)}\delta/[(1-\delta)\tau_i^{1/(1-\delta)} + \tau_j^{1/(1-\delta)}] \in (0,\delta) \tag{5}$$

(i) (**not race to the top**) *If $\underline{w}_i < \widetilde{\delta}_i\overline{w}$ for $i \in \{a,b\}$, then at the NE, $w_i < \overline{w}$ for both i.*

(ii) (**partial race to the top**) *If $\underline{w}_i < \widetilde{\delta}_i\overline{w}$ and $\underline{w}_j \geq \delta\overline{w}$ for $i,j \in \{a,b\}$, $i \neq j$, then at the NE, $w_i < \overline{w}$ and $w_j = \overline{w}$.*

(iii) (**complete race to the top**) *If $\underline{w}_i \geq \widetilde{\delta}_i\overline{w}$ for both $i \in \{a,b\}$, then the NE has complete race to the top property.*

(iv) *If $\widetilde{\delta}_i\overline{w} \leq \underline{w}_i < \delta\overline{w}$ and $\underline{w}_j < \widetilde{\delta}_j\overline{w}$ then the NE has either has not race to the top or partial race to the top.*

Proof See the Appendix. ■

Now let $\underline{w}_a = \underline{w}_b = \underline{w}$: i.e., the two countries are symmetric. Then we explore what we can say additionally. First we get the following corollary of Proposition 2 above:

Corollary 1 *Let $\underline{w}_a = \underline{w}_b = \underline{w}$: i.e., the two countries are symmetric. Let $\overline{w}/\underline{w} \equiv \theta >$ 1. Then, if $\delta \leq 2/(1+\theta)$, then the NE has complete race to the top property.*

Proof See the Appendix. ■

Notice that the most interesting aspect of our results so far is the possibility of having an equilibrium with "race to the top"—opposite to "race to the bottom"—even when $\rho \in (-1,0)$: i.e., even when the labour inputs can be said to be substitutes in production. Therefore, we explore, in the case of symmetric countries, whether we can provide any stricter bound for ρ for which the equilibrium possesses this property. We obtain:

Proposition 3 *Let $\overline{w}/\underline{w} \equiv \theta > 1$ and $m = min\{\alpha/(1-\alpha),(1-\alpha)/\alpha\}$. If $\delta > 2/(1+\theta)$ and $m \leq (1-\delta)/(\delta\theta-1)$ [or equivalently $2/(1+\theta) < \delta \leq (m+1)/(m\theta+1)$], then the NE has race to the top property.*

Proof See the Appendix. ■

Therefore, to summarize, the unique equilibrium of G does not have "race to the bottom" property. When inputs are complementary in production, the equilibrium has "complete race to the top" property. However, even when the labour inputs can be said to be substitutes in production (but when they are not perfect substitutes) it is possible to have equilibria showing "race to the top", the opposite feature of "race to the bottom". We provide characterization of such equilibria in terms of the parameters affecting demand.

Given the nature of the conditions in Proposition 2, the central proposition characterizing the equilibrium in our work, the economics behind the proposition seems as follows. At an equilibrium $(\overline{w}, \overline{w})$ if a country reduces wage, then indeed the demand for its labour goes up. However, if the substitution of labour in its favour, driven by the parameters controlling the firm's demand for labour is low enough, then the total labour income resulting from such a unilateral lowering of wage, may, however, go down owing to the lowering of the wage. Moreover, as the reservation wage in the country goes up, the volume of incremental labour income of the labour units shifting from the reservation sector to the monopsonist would also be low enough. Therefore, as $\underline{w}_i$ increases or as the degree of plausible substitution of labour goes down, the propensity of "race to the top" being an equilibrium phenomenon goes up.

4 Discussion and conclusion

Notice that our set-up is not meant to provide a comprehensive model of MNCs' competition and input choice decisions. We focussed on *one* aspect of an MNC's organization of production and looked into the possibility of this feature generating a counteracting effect to the "race to the bottom". We find that indeed this feature of imperfect substitutability can act as a counteracting factor to the "race to the bottom".

In fact, in our set-up we have rather deliberately left out from our model some other additional factors, like any demand externality for producers from increased wage of labourers or any agglomeration effects etc, which can act as additional countervailing factors to "race to the bottom".

Note however, that our specification of payoff for the countries—consisting of labour income only—is crucial for our result. It is easy to see that if λ_i, the weight put by the decision-maker of country i to the profit of the firm, is large enough, then it would be optimal for the decision-maker of country i to push the effective wages down. In fact, when both these incentives are present for the decision-maker, what emerges for the labourers in a dynamic extension of this model is a matter for future research.

Appendix

Proof of Lemma 1 (i)-(ii) Solving the firm's problem, optimal labour inputs for the firm are given by

$$x_a^* = \frac{K}{w_a + [(1-\alpha)w_a/\alpha w_b]^{1/(1+\rho)} w_b}, x_b^* = \frac{K}{w_b + [\alpha w_b/(1-\alpha)w_a]^{1/(1+\rho)} w_a} \tag{6}$$

Using the expressions above, standard reasoning proves (i)-(ii).

(iii) Recall that $\pi(w_a, w_b, K)$ is the maximized value of the profit of the firm, i.e., $\pi(w_a, w_b, K) = F(x_a^*, x_b^*)$. Invoking Roy's identity (see, e.g., Kreps, 1990, p.57) we have

$$\partial\pi(w_a, w_b, K)/\partial w_i = -x_i^*[\partial\pi(w_a, w_b, K)/\partial K] \text{ for } i \in \{a, b\} \tag{7}$$

By (6), both x_a^*, x_b^* are increasing in K. Since F is increasing in both x_a, x_b, we conclude that $\pi(w_a, w_b, K)$ is increasing in K. As $x_i^* > 0$, from (7) it follows that $\pi(w_a, w_b, K)$ is decreasing in w_i for $i \in \{a, b\}$.

(iii) The labour income in the monopoly sector for countries a, b, are given by

$$\psi_a^M(w_a, w_b, K) = w_a x_a^* = \frac{K}{1 + [(1-\alpha)/\alpha]^{1/(1+\rho)}(w_b/w_a)^{\rho/(1+\rho)}}$$

$$\psi_b^M(w_a, w_b, K) = w_b x_b^* = \frac{K}{1 + [\alpha/(1-\alpha)]^{1/(1+\rho)}(w_a/w_b)^{\rho/(1+\rho)}} \tag{8}$$

If $\rho \in (-1, 0)$, we have $\rho/(1+\rho) < 0$ and hence $w_a^{\rho/(1+\rho)}$ is decreasing in w_a. Consequently the denominator of ψ_a^M is increasing in w_a and hence ψ_a^M is decreasing in w_a for any $w_b \geq 0$. By the same reasoning, ψ_b^M is decreasing in w_b for any $w_a \geq 0$.

If $\rho \in (0, \infty)$, we have $\rho/(1+\rho) > 0$ and hence $w_a^{\rho/(1+\rho)}$ is increasing in w_a. Consequently the denominator of ψ_a^M is decreasing in w_a and hence ψ_a^M is increasing in w_a for any $w_b \geq 0$. By the same reasoning, ψ_b^M is increasing in w_b for any $w_a \geq 0$. ■

Proof of Proposition 1

Throughout let $i, j \in \{a, b\}$ and $i \neq j$. First we prove parts (I)(ii) and (II)(ii), then parts (I)(i) and (II)(i).

(I)(ii), (II)(ii): Let $\rho \in (0, \infty)$. Then ψ_i^M is increasing in w_i (Lemma 1(iv)). As x_i^* is decreasing in w_i (Lemma 1(i)), so is ψ_i^T. Then from (3), it follows that ψ_i is increasing in w_i for any w_j, so the unique best response of country i to any w_j is to choose $w_i = \overline{w}$. This proves (I)(ii), implying that G has a unique NE $(\overline{w}, \overline{w})$, proving (II)(ii).

(I)(i), (II)(i): Let $\rho \in (-1, 0)$ and $\delta \equiv -\rho \in (0, 1)$. Denote $\tau_a \equiv \alpha$ and $\tau_b \equiv 1 - \alpha$. For this case ψ_i^M is decreasing in w_i (Lemma 1(iii)(a)). As x_i^* is decreasing in w_i (Lemma 1(i)), so is ψ_i^T. Define

$$g_i(w_i) := \tau_j^{1/(1-\delta)}(\delta w_i - \underline{w}_i) w_i^{\delta/(1-\delta)} \text{ and } h_i(w_j) := (1-\delta)\tau_i^{1/(1-\delta)} \underline{w}_i w_j^{\delta/(1-\delta)} \tag{9}$$

Observe that $g_i(w_i)$ is increasing in w_i, $h_i(w_j)$ is increasing in w_j and $\lim_{w_i\to\infty} g_i(w_i) = \lim_{w_j\to\infty} h_i(w_j) = \infty$. Note from (3) that $\partial\psi_i/\partial w_i \gtreqless 0 \Leftrightarrow g_i(w_i) \lesseqgtr h_i(w_j)$.

To prove (I)(i), we consider the following two cases.

Case 1 If $\underline{w}_i \geq \delta\overline{w}$, then for any w_j, we have $g_i(w_i) \leq 0 < h_i(w_j)$ and hence $\partial\psi_i/\partial w_i > 0$ for all $w_i \in [\underline{w}_i, \overline{w}]$. So the unique best response of country i to any w_j is to choose $w_i = \overline{w}$.

Case 2 If $\underline{w}_i < \delta\overline{w}$, then for $w_i \in [\underline{w}_i, \underline{w}_i/\delta]$, we have $g_i(w_i) \leq 0 < h_i(w_j)$ and hence ψ_i is increasing in w_i in this interval. So for any w_j, best response of country i is to choose $w_i \in [\underline{w}_i/\delta, \overline{w}]$. As $g_i(\underline{w}_a/\delta) = 0 < h_i(w_j) < \lim_{w_i\to\infty} g_i(w_i) = \infty$, by the monotonicity of g_i, $\exists$ a unique $w_i = b_i(w_j) \in (\underline{w}/\delta, \infty)$ such that $g_i(w_i) \lesseqgtr h_i(w_j) \Leftrightarrow \partial\psi_i/\partial w_i \gtreqless 0 \Leftrightarrow w_i \lesseqgtr b_i(w_j)$. Therefore the unique best response of country i to any $w_j \in [\underline{w}_j, \overline{w}]$, is $B_i(w_j) = \min\{b_i(w_j), \overline{w}\}$. As h_i is increasing in w_j, it follows that $b_i(w_j)$ is increasing and $B_i(w_j)$ is non-decreasing in w_j. This completes the proof of (I)(i).

To prove (II)(i), first we show that G has a unique NE for $\rho \in (-1, 0)$. Note from the proof of (I)(i) that for $i \in \{a, b\}$, $\exists\ 0 < \varepsilon_i < \overline{w} - \underline{w}_i$ such that $B_i(w_j) \in [\underline{w}_i + \varepsilon_i, \overline{w}]$ for any[1] w_j. Also observe that the constant term $\underline{w}_i\overline{x}_i$ in (3) does not play any role in determining NE outcomes of G. Consider the two-person “transformed” game H in which countries a, b choose w_a, w_b, where the strategy set of i is $[\underline{w}_i + \varepsilon_i, \overline{w}]$ and its payoff is

$$\Phi_i(w_a, w_b) = \log[(w_i - \underline{w})x_i^*(w_a, w_b)] = \log(w_i - \underline{w}_i) + \log(x_i^*(w_a, w_b)) \qquad (10)$$

The log function is well defined for the game H. Note that the set of NE of G coincides with the set of NE of H.

Observation 1 *The log labour demand* $\log(x_i^*(w_a, w_b))$ *of any country* $i \in \{a, b\}$ *has increasing differences in* (w_a, w_b), *i.e., the following hold for* $w_a' > w_a$, $w_b' > w_b$.

$$[\log(x_i^*(w_a', w_b')) - \log(x_i^*(w_a, w_b'))] - [\log(x_i^*(w_a', w_b)) - \log(x_i^*(w_a, w_b))] > 0$$

Consequently the game H is a supermodular game.

Proof We prove the increasing difference result for $i = a$ (the proof is similar for $i = b$). Let $t_\alpha(w) := \alpha^{1/(1+\rho)} w^{\rho/(1+\rho)}$. Using (6) and simplifying, we have

$$[\log(x_a^*(w_a', w_b')) - \log(x_a^*(w_a, w_b'))] - [\log(x_a^*(w_a', w_b)) - \log(x_a^*(w_a, w_b))]$$
$$= \log \frac{[t_\alpha(w_a') + t_{1-\alpha}(w_b)][t_\alpha(w_a) + t_{1-\alpha}(w_b')]}{[t_\alpha(w_a') + t_{1-\alpha}(w_b')][(t_\alpha(w_a) + t_{1-\alpha}(w_b)]} > 0$$

[1]For example, take $\varepsilon_i = (\underline{w}_i + \overline{w})/2$ in Case 1 and $\varepsilon_i = \delta\overline{w}$ in Case 2 of (I)(i).

Using the increasing difference result and the conclusions of Milgrom and Roberts (1990) [see p. 1271, the paragraph before eqn. (5)], it follows that the game H is supermodular. ■

Observation 2 *For $i \in \{a,b\}$, let $y_i = \log(w_i)$. The payoff of i in the game H has the following property: $\partial^2\Phi_i/\partial y_a \partial y_b + \partial^2\Phi_i/(\partial y_a)^2 < 0$. Consequently H has a unique NE.*

Proof We prove the inequality above for $i = a$ (similar reasoning applies for $i = b$). Denote $\widetilde{\alpha} \equiv [(1-\alpha)/\alpha]^{1/(1+\rho)}$. Using (6) in (10) and simplifying:

$$\Phi_a = \log[\exp(y_a) - \underline{w}_a] + \log K - y_a + y_b/(1+\rho) - \log[\nu(y_a, y_b)]$$

where $\nu(y_a, y_b) := \exp[y_b/(1+\rho)] + \widetilde{\alpha}\exp[y_a/(1+\rho)]$. For $i, j \in \{a,b\}$, let $\nu_i = \partial\nu/\partial y_i$ and $\nu_{ij} = \partial^2\nu/\partial y_i \partial y_j$. Note that $\nu_{ab} = 0$ and $\nu\nu_{aa} - (\nu_a)^2 - \nu_a\nu_b = 0$. Hence $\partial^2 \log[\nu]/\partial y_a \partial y_b + \partial^2 \log[\nu]/(\partial y_a)^2 = [\nu\nu_{aa} - (\nu_a)^2 - \nu_a\nu_b]/\nu^2 = 0$ implying that $\partial^2\Phi_a/\partial y_a \partial y_b + \partial\Phi_a/(\partial y_a)^2 = -\underline{w}_a w_a/(w_a - \underline{w}_a)^2 < 0$.

Since H is a supermodular game, the inequalities above imply that H has a unique NE [see Milgrom and Roberts, 1990 (eqn. (6), p.1271)]. ■

Since H has a unique NE, so does G. Having shown that G has a unique NE, observe from part (I)(i) that for any country i, the unique best response $B_i(w_j)$ to any w_j has either $B_i(w_j) = \overline{w} > \underline{w}_i$, or $B_i(w_j) > \underline{w}_i/\delta > \underline{w}_i$. So the unique NE of G does not have $w_i = \underline{w}_i$ for any i. This completes the proof of part (II)(i).

(III) This is immediate from the facts that for any w_j, best response of i is increasing in $\underline{w}_i$ and that G possesses a unique NE. ■

Proof of Proposition 2

Note from the proof of Proposition 1 that if $\underline{w}_i \geq \delta\overline{w}$ for $i \in \{a,b\}$, then the unique NE is $(w_a = \overline{w}, w_b = \overline{w})$. To characterize NE for other cases, let $\underline{w}_i < \delta\overline{w}$ for some i. Then the best response function of i is $B_i(w_j) = \min\{b_i(w_j), \overline{w}\}$. By the monotonicity of g_i, we have $b_i(\overline{w}) \gtreqless \overline{w} \Leftrightarrow g_i(b_i(\overline{w})) = h_i(\overline{w}) \gtreqless g_i(\overline{w})$. Using the expressions of g_i and h_i from (9), we conclude that $\exists\ \widetilde{\delta}_i \in (0, \delta)$ (given by (5)) such that

$$b_i(\overline{w}) \gtreqless \overline{w} \Leftrightarrow \underline{w}_i \gtreqless \widetilde{\delta}_i\overline{w} \tag{11}$$

(i) If $\underline{w}_i < \widetilde{\delta}_i\overline{w}$, then $b_i(\overline{w}) < \overline{w}$ and hence $b_i(w_j) < \overline{w}$ for all $w_j \in [\underline{w}_j, \overline{w}]$. So $B_i(w_j) = b_i(w_j)$ for all $w_j \in [\underline{w}_j, \overline{w}]$. Therefore if $\underline{w}_i < \widetilde{\delta}_i\overline{w}$ for $i \in \{a,b\}$, then there is no NE where $w_i = \overline{w}$. Therefore, the unique NE has $w_i < \overline{w}$ for both i.

(ii) Let $\underline{w}_i < \widetilde{\delta}_i\overline{w}$ and $\underline{w}_j \geq \delta\overline{w}$. Then $B_j(w_i) = \overline{w}$ for all $w_i \in [\underline{w}_i, \overline{w}]$. So, the NE has $w_j = \overline{w}$ and $w_i = B_i(\overline{w})$. As $\underline{w}_i < \widetilde{\delta}_i\overline{w}$, we have $b_i(\overline{w}) < \overline{w}$ and hence $B_i(\overline{w}) = b_i(\overline{w}) < \overline{w}$. So the unique NE has $w_i < \overline{w}$ and $w_j = \overline{w}$.

(iii) By inequality (11) above, in this case we have $b_i(\overline{w}) \geq \overline{w}$ for both i and hence $B_i(\overline{w}) = \overline{w}$. So the unique NE has $(w_a = w_b = \overline{w})$.

(iv) The proof is exactly similar to the three cases above. ■

Proof of Corollary 1 With $\underline{w}_a = \underline{w}_b = \underline{w}$, the condition $\widetilde{\delta}_i\overline{w} \leq \underline{w}$ is equivalent to: $\underline{w} \geq \delta\overline{w}/[1 + (1-\delta)(\tau_i/\tau_j)^{1/(1-\delta)}]$ for $i, j \in \{a, b\}, i \neq j$. As $\min\{\tau_a/\tau_b, \tau_b/\tau_a\} \leq 1$, $\widetilde{\delta}_i\overline{w} \leq \underline{w}$ implies $\underline{w} \geq \delta\overline{w}/[1 + (1-\delta)]$ which can be simplified as $\delta \leq 2/(1+\theta)$. ■

Proof of Proposition 3

The following lemma will be useful for this proof.

Lemma A1 *Let $\rho \in (-1, 0)$ and $\delta \equiv -\rho \in (0, 1)$. Denote $\overline{w}/\underline{w} \equiv \theta > 1$ and let $\delta \in (1/\theta, 1)$. Denote $\widetilde{\alpha} \equiv \beta/\alpha$, $\widetilde{\beta} \equiv \alpha/\beta$ and for $t > 0$,*

$$\ell^{t,\delta}(w) := t^{1/\delta}[(\delta w - \underline{w})/(1-\delta)\underline{w}]^{(1-\delta)/\delta}w, r^{\delta}(w) := \underline{w}/\delta + (1-\delta)^2\underline{w}^2/\delta(\delta w - \underline{w}) \quad (12)$$

(i) *$\ell^{t,\delta}(w)$ is increasing and $r^{\delta}(w)$ is decreasing in w.*

(ii) *If G has an NE where $w_i < \overline{w}$ for $i \in \{a, b\}$, then $\ell^{\widetilde{\alpha},\delta}(w_a) = r^{\delta}(w_a)$ and $\ell^{\widetilde{\beta},\delta}(w_b) = r^{\delta}(w_b)$.*

(iii) *Let $m = min\{\widetilde{\alpha}, \widetilde{\beta}\}$. If $r^{\delta}(\overline{w}) > \ell^{m,\delta}(\overline{w})$, then G cannot have an NE where $w_i < \overline{w}$ for $i \in \{a, b\}$.*

Proof Part (i) is immediate. For (ii), let $\delta \in (1/\theta, 1)$. If G has an NE where $w_i < \overline{w}$ for $i \in \{a, b\}$, then $g^{\beta,\delta}(w_a) = h^{\alpha,\delta}(w_b)$ and $g^{\alpha,\delta}(w_b) = h^{\beta,\delta}(w_a)$. We obtain $w_b = \ell^{\widetilde{\alpha},\delta}(w_a)$ from the first and $w_a = \ell^{\widetilde{\beta},\delta}(w_b)$ from the second equation. The system of equations together imply $(\delta w_a - \underline{w})(\delta w_b - \underline{w}) = (1-\delta)^2\underline{w}^2$ which implies that $w_b = r^{\delta}(w_a)$ and $w_a = r^{\delta}(w_b)$. This proves (ii). Since $w_a, w_b \leq \overline{w}$, part (iii) is immediate from (i) and (ii).

The final part of the proposition follows immediately from the expressions of the best-response functions. ■

Proof of the proposition We have already shown that if $\delta \leq 1/\theta$, then the NE has the race to the top property. So let $\delta > 1/\theta$.

We shall use Lemma A1 for this proof. Note that $r^{\delta}(\overline{w})$ is decreasing in δ. Denoting $\delta\theta - 1 \equiv \tau > 0$, we have

$$\partial\ell^{m,\delta}(\overline{w})/\partial\delta = [m\tau^{1-\delta}]^{1/\delta}[(\theta-1)\delta + \tau\log((1-\delta)/m\tau)]/(1-\delta)^{(1-\delta)/\delta}\delta^2\tau$$

Observe that if $(1-\delta)/m\tau \geq 1$, i.e., $m \leq (1-\delta)/(\delta\theta+1)$, then $\ell^{m,\delta}(\overline{w})$ is increasing in δ. Noting that $1/\theta < 2/(1+\theta) < 1$ (since $\theta > 1$), let $\delta \in (2/(1+\theta), 1)$. For this case $(1-\delta)/(\delta\theta+1) < 1$ and $\ell^{m,\delta}(\overline{w})$ is increasing in δ for any $m \leq (1-\delta)/(\delta\theta+1)$, or equivalently $\delta \leq (m+1)/(m\theta+1)$. Since $\theta > 1$, for any $m \in (0,1)$, we have $2/(1+\theta) < (m+1)/(m\theta+1) < 1$. Let $\delta \in (2/(1+\theta), (m+1)/(m\theta+1)]$. Note that $r^{(m+1)/(m\theta+1)}(\overline{w}) = [1+m(\theta-1)]\overline{w} > \overline{w} = \ell^{(m+1)/(m\theta+1),\delta}(\overline{w})$. As $\ell^{m,\delta}(\overline{w})$ is increasing and $r^{\delta}(\overline{w})$ is decreasing in δ, it follows that for this case $r^{\delta}(\overline{w}) > \ell^{m,\delta}(\overline{w})$ and by Lemma A1, there is no NE where $w_i < \overline{w}$ for $i \in \{a,b\}$. Therefore, the unique NE has race to the top property. ■

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