

# Too Unexpected to Fail: Bail-Out Policy and Systemic Fragility

Arup Daripa  
Birkbeck, University of London

First Draft: September 2014

## Abstract

I present a mechanism that relies on the interaction of coordination and ambiguity and makes precise how a loss of confidence can arise in loan markets, leading to a systemic liquidity crisis. The paper studies a simple global-game coordination model among lenders to a financial intermediary and shows how a market haircut arises in equilibrium. I show how the haircut responds to a variety of parameters. In particular, I show that coordination is non-robust to ambiguity in investor signals and becomes fragile in an environment with ambiguity. This leads to the haircut jumping up suddenly, possibly to 100% when enough lenders are ambiguity-sensitive. Further, I show that the fragility of coordination implies that in such an environment, policy itself becomes a systemic trigger. If the regulator fails to rescue an institution that the market expects to be saved (TUTF), which in turn changes market expectation about policy for other institutions even slightly, an immediate systemic collapse of liquidity ensues. The results explain both the contagious run on liquidity markets at the advent of the recent crisis as well as the liquidity market freeze after the Lehman collapse. The results also clarify the role of liquidity policy and especially the policy of extending central bank guarantees across the financial sector (regulated and shadow).

JEL CLASSIFICATION: G2, C7, E5.

KEYWORDS: Short-term debt, systemic liquidity crises, global games, ambiguity, bail-out policy, liquidity policy

# 1 Introduction

The financial markets experienced multiple rounds of systemic liquidity runs during the last crisis. As Gorton and Metrick (2012) show, with the arrival of bad news from housing markets in 2007, there was a run on the repo market which spread from sub-prime housing assets to non-subprime assets with no direct connection to the housing market. Further, a widespread freeze of liquidity markets followed the failure of Lehman Brothers in September 2008. A run ensued on money-market funds as well as in the asset-backed commercial paper market, even for paper not exposed to subprime mortgages.<sup>1</sup> The interbank market was frozen. To understand the crisis it is important to explain why runs spread to non-subprime assets, and why a systemic liquidity crisis arose immediately after the Lehman failure.

I provide a theory based on the interaction of lender coordination and ambiguity to explain the origins contagion runs in liquidity markets. Further, I show how the interaction of policy with coordination and ambiguity can give rise to a sudden systemic liquidity freeze.

I study a simple global game model of coordination among lenders lending short-term funds to a financial intermediary, and calculate the equilibrium market haircut (the fraction of funds that the intermediary must put up in order to successfully attract enough funds to run an investment project). I show how haircut responds to different parameters and how coordination itself gives rise to an inefficiency. I then show that if lenders are ambiguity averse, the introduction of any (arbitrarily small) degree of ambiguity about the signal received by lenders leads to a complete breakdown of coordination and the market haircut rises to 100%. In other words, coordination is fragile with respect to ambiguity and therefore even the smallest degree of ambiguity gives rise to a liquidity run.

However, if the market expects a financial intermediary to be rescued by policymakers, the intermediary is immune to this problem of collapse-of-coordination. But this immunity depends entirely on policymakers following market expectations about institutions that are too-unexpected-to-fail (TUTF). If policy does not follow market expectations so that the latter adjusts even slightly, a complete collapse of liquidity arises through the interaction of coordination and ambiguity as described above. Thus policy

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<sup>1</sup>See Covitz, Liang and Suarez (2009) for an analysis of the run in the ABCP market.

itself becomes a potential trigger of systemic crisis.

Gorton (2012) defines systemic risk as loss of investor confidence in financial intermediary debt. Here I present a mechanism through which such a loss of confidence arises and make the meaning of loss of confidence precise.

The results also show that avoiding systemic risk implies that the following three things cannot co-exist: (1) short-term debt not covered by guarantees (2) any factor (such as complex financial innovation) that is likely to make investor's signals of return (even slightly) ambiguous in a downturn, and (3) policy making that is market-independent in the sense that the regulator does not rescue an institution that the market expects to be rescued. In other words, the presence of rollover risk and (even slight) ambiguity imply that the extent of bail-out policy is determined by market expectations.

If the market expects an institution to fail under a liquidity run, then at the advent of ambiguity, such an institution would experience a liquidity run. If, on the other hand, the market expects an institution not to fail (TUTF), it does not experience a liquidity run unless policy triggers any lowering of expectations. Thus having a TUTF institution helps with systemic crisis so long as policy conforms to expectations.

Now, does a large institution (the type that is typically labelled TBTF) help with or exacerbate the crisis? If market expectation about which institutions are likely to be rescued is unrelated to size, TBTF does not matter. However, it might reasonably be expected that markets would expect large institutions with complex counterparty obligations to be rescued. In this case, TBTF coincides with TUTF, and, as discussed above, an institution that is TUTF helps with the systemic problem. The same reasoning shows why breaking up a TBTF institution exacerbates the systemic crisis. If broken up, none of the parts might be TUTF, in which case all parts experience liquidity runs.

In other words, the right concept to focus on is TUTF. However, to the extent that TBTF institutions are also TUTF, having a TBTF institution is a good thing from the perspective of systemic liquidity crisis.

Let us describe the modelling of coordination and ambiguity in the model. We adopt a standard global game approach to model coordination among several lenders lending short-term funds to a financial intermediary. Once the project is started, in the next period funds must be rolled over for the project to have a chance of succeeding. If funds

are not rolled over, there is a fire sale, and some return is realized depending on the state of fundamentals. Hence coordination matters: if enough lenders roll-over funds in period 1, the project earns a high expected return in period 2, and a lower return otherwise. Each lender receives a signal of the state of fundamentals with some noise. With a small amount of incomplete information, the game is dominance solvable: there is a unique equilibrium that is attained by iteratively eliminating strictly dominated strategies. However, just as agents are almost, but not entirely sure of the underlying state of fundamentals, if there is even the slightest ambiguity about the signal being biased, and if the agents are ambiguity averse, coordination breaks down completely. Agents lend successfully only when the underlying state of fundamentals is so high that it is a dominant strategy to lend.

The intuition for this result is as follows. We model ambiguity using the maxmin expected utility model of Gilboa and Schmeidler (1989). Suppose the signal bias lies in the interval  $[-b, b]$  (where  $b$  is arbitrarily small). Agents consider the worst case in which there is a bias of  $b$  in the signal. We show since an agent perceives a bias relative to others' signals, when signal noise is small, the agent's optimal action is to calculate what signal threshold  $y^*$  others, who are perceived to receive unbiased signals, arrive at. Then set own signal threshold at  $y^* + b$ . In other words, the agent wants to stay away from the threshold established by others. Note that this agent is not contributing to coordination at all. The agent rolls over own funds only when others have established a coordination threshold  $y^*$ , and then sets own threshold above  $y^*$ . But if everyone (or, as I show, a large enough fraction) behaves this way—each trying to stay ahead of the others—coordination necessarily fails. In this case rollover happens only when it is a dominant strategy for each agent to roll over. In all other cases where coordination matters, no one rolls over, i.e. there is a run on the project and it is liquidated.

Thus starting from a equilibrium with a coordination threshold, if even the slightest degree of ambiguity is introduced and agents are ambiguity averse, a liquidity run follows. Further, policy plays an important role in this process.

Suppose the financial system consists of several financial intermediaries who are expected to be rescued by the regulator (perhaps because investors expect a failure of any of these to cause a fire sale problem). In this case, consider any action by the regulator (such as not rescuing an intermediary that was expected to be rescued) that changes market expectation: financial institutions expected to be rescued are no longer certain

to be saved (i.e. rescue probability  $p$  drops even infinitesimally below 1). Such a move by the regulator then causes the liquidity of the entire system to freeze immediately through the above interaction of coordination and ambiguity as no short term lender wants to renew their loan.

Any small intermediary subject to ambiguity would experience a run immediately as the ambiguity arises. If systemic stability depends on large institutions (who might, for example, cause a fire sale problem if they have a liquidity run), and the market thinks these would be rescued, then crisis management policy itself becomes a source of systemic risk. The only way to avoid this is for policy to become market determined. So markets expecting large intermediaries to be rescued is an equilibrium. Once market expects a bank to be rescued, the regulator must rescue it, or face a systemic liquidity freeze.

The systemic collapse of liquidity does not require all lenders to be affected by ambiguity. I show that if the fraction of lenders affected by ambiguity exceeds the fraction of the project that can be funded by own stable funds, coordination collapses completely if any ambiguity is introduced. If the former fraction is below the latter one, the market haircut rises sharply when ambiguity is introduced, but not to 100%.

The results here imply that having stable funds (long term debt, equity), having access to central bank liquidity provision facilities such as the fed discount window, having a larger deposit base guaranteed by deposit insurance: all help in staving off a rollover run. Extending access to a liquidity provider of last resort to financial intermediaries in the regulated as well as unregulated sectors—a policy advocated by Gorton and Metrick (2010)—is indeed an effective solution for systemic crises. Such guarantees remove the coordination problem and therefore the system is no longer vulnerable to ambiguity. Regulation that requires financial intermediaries to hold more stable funds also reduces systemic vulnerability to ambiguity. On the other hand, ring-fencing of the type proposed by the UK banking commission denies parts of a bank access to deposit funds. Removing access to deposit funds and access to central bank liquidity provision facility reduces a financial intermediary's resilience to a sudden rollover run. Thus while a ring fence protects the core of a bank, reducing access to stable funds to other parts of the bank engaged in short-term borrowing might increase systemic vulnerability.

In their study of the interbank market, Afonso, Kovner and Schoar (2011) find the

following: “...in the days immediately after the Lehman Brothers bankruptcy the market becomes more sensitive to bank-specific characteristics, especially in the amounts lent to borrowers but also in the cost of overnight funds. In particular, large banks with high percentages of non-performing loans (NPLs) showed drastically reduced daily borrowing amounts and borrowed from fewer counterparties in the days after Lehman’s bankruptcy. However, beginning on Tuesday, September 16, 2008, once the AIG bailout was announced, the trend reversed, and spreads for the largest banks fell steeply. We interpret the return to pre-crisis spreads as the effect of the government’s support for systemically important banks, because the same is not true for small banks, which continued to face higher spreads.”

While this paper does not explicitly model the interbank market, it can be readily adapted to that setting. Rochet and Vives (2004) model the interbank market as a coordination game, and derive a unique equilibrium using the global games modelling approach. The results in this paper on the fragility of coordination can be applied to this setting and we can conclude that when banks have some non-performing assets as a result of a downturn which gives rise to signal ambiguity for other participants in the interbank market, coordination would break down and the cost of funds for the borrowing bank would rise dramatically if the lenders perceive that the government might not bail out the bank. However, if other policy measures again convince banks that rescue is likely, coordination would be restored and cost of borrowing would fall. For smaller banks that are not expected to be rescued, cost of funds would rise dramatically at the advent of crisis (and therefore, ambiguity) and stay high.

Our theory relies on the idea of ambiguity arising through counterparty risk. This is precisely the type of theory, in contrast with liquidity-hording theories (e.g. Caballero and Krishnamurthy (2008)), that Afonso et al. (2011) find evidence for. Like many authors and newspaper columnists, they interpret the Lehman failure as an event that changed market expectations. Our theory provides a mechanism to articulate the change in expectations and show its systemic impact.

Our theory can also be used to understand the sudden collapse in any coordination-based market. Consider, for example, the sudden collapse in the auction-rate securities market.<sup>2</sup> While our model does not explicitly capture the institutional details of this market, at the heart of the market is investor coordination, and our theory would sug-

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<sup>2</sup>See Han and Li (2010) for details.

gest that the interaction of such coordination and ambiguity can explain why a sudden failure can arise.

The theory of systemic spread of crisis from breakdown of coordination under ambiguity also fits with events in the Great Depression. In the new preface to Kindleberger (1973) republished in 2012, DeLong and Eichengreen write:

The 1931 crisis began, as Kindleberger observes, in a relatively minor European financial centre, Vienna, but when left untreated leapfrogged first to Berlin and then, with even graver consequences, to London and New York. This is the 20th century's most dramatic reminder of quickly how financial crises can metastasise almost instantaneously. In 1931 they spread through a number of different channels. German banks held deposits in Vienna. Merchant banks in London had extended credits to German banks and firms to help finance the country's foreign trade. In addition to financial links, there were psychological links: as soon as a big bank went down in Vienna, investors, having no way to know for sure, began to fear that similar problems might be lurking in the banking systems of other European countries and the US.

Literature review: Literature on systemic fragility: Brunnermeier and Pedersen (2009), Brunnermeier (2009), Adrian and Shin (2008, 2010), Acharya, Gale and Yorulmazer (2011), Caballero (2010). Ambiguity: Easley and O'Hara (2010), Caballero and Krishnamurthy (2008). Regulation: . To be completed...

## 2 The model

There are three periods, 0, 1 and 2. The intermediary has a project that requires 1 unit of funding initially and in period 1. In period 2 the returns are realized. If in period 1 investment by the intermediary falls below 1, the project must be liquidated. Instead of this risky project, the intermediary can also invest in a riskless asset at the initial period. We normalize the riskfree net return to 0.

There is a unit mass of potential lenders. Lenders are risk neutral. Each lender lends 1 unit of funds initially, and must decide in period 1 whether to rollover the unit to the intermediary or to withdraw (wholly or partially) and invest in a safe asset that has a net return of 0.

The return from the project as well as the liquidation value depends on the underlying economic fundamentals  $\theta$ . If 1 unit is invested in periods 0 and 1, the project returns  $\theta R$  where  $R > 1$ . Here  $\theta$  can be thought of as the probability of success of the project and  $R$  the return if the project succeeds (the return if the project fails is 0 by implication).

The liquidation value depends on market liquidity. We assume that the liquidation value is  $\theta$  in state  $\theta$ . In other words, the state of fundamentals also indicates the state of market liquidity. As  $\theta$  improves, two things happen: the return from the project improves (the project succeeds with higher probability) and the market liquidity improves, which implies that the proceeds from any early liquidation rises.

The intermediary has access to long-term borrowing of  $0 < \psi < 1$  in period 1. Therefore for the project to continue at least  $1 - \psi$  must be rolled over by lenders in period 1.

If the project faces liquidation because of lack of funds, it is bailed-out by the policymaker with probability  $p$ . The policymaker supplies enough funds so that the project continues till maturity and lenders get the same return of  $\theta R$  that they would get if enough funds had been rolled over.

### 2.1 The payoff of lenders and the intermediary

Let us now derive the payoff of lenders, which is a function of the state of fundamentals  $\theta$ . Each lender lending a unit of funds is promised a payoff of  $r > 1$  with limited



liability so long as the project succeeds.

Therefore the expected net return of a lender, denoted by  $\pi_L(L(\theta), \theta)$ , is given by

$$\pi_L(L(\theta), \theta) = \begin{cases} \theta r - 1 & \text{if } \psi + L(\theta) \geq 1 \\ p\theta r + (1 - p)\theta - 1 & \text{otherwise} \end{cases}$$

## 2.2 Dividing the set of fundamentals

If the state of the fundamentals  $\theta$  were common knowledge, we could divide the set of fundamentals as follows.

Let

$$\bar{\theta} = \frac{1}{p\rho + (1 - p)} \quad (1)$$

For any  $\theta > \bar{\theta}$ , the net expected return from rolling over is positive even if the project is liquidated. Therefore rolling over is the dominant strategy in this case.

Next, let  $\underline{\theta}$  be such that  $\theta r = 1$ , which implies

$$\underline{\theta} = \frac{1}{r} \quad (2)$$

For any  $\theta < \underline{\theta}$ , the net expected return from rolling over is negative even if all others roll over. Therefore not rolling over is the dominant strategy in this case.

For  $\theta \in [\underline{\theta}, \bar{\theta}]$ , whether investment proceeds depends on the size of the total funds raised. In this interval a coordination problem arises, and would give rise to multiple equilibria when  $\theta$  is common knowledge.

## 2.3 Incomplete Information and Signals

The state of fundamentals  $\theta$  is drawn from a uniform distribution on the interval  $[0, 1 + \Delta]$ ,  $\Delta > 0$ . Each lender receives a signal  $x$  of  $\theta$  in period 1, where  $x$  is uniform on  $[\theta - \varepsilon, \theta + \varepsilon]$ . Conditional on the true state  $\theta$ , the signals are independently and identically distributed. After receiving the signal, lenders simultaneously decide whether to roll over their loan or withdraw. The decisions made result in aggregate loan rollover of  $L(\theta)$ .

Finally, a technical requirement. To ensure that the signal intervals are well defined at the lower boundary of the relevant range of fundamentals, I assume that  $\underline{\theta} = 1/r \geq 2\varepsilon$  and  $1 + \Delta - \bar{\theta} \geq 2\varepsilon$ . Note that the latter is possible assuming  $\Delta > 2\varepsilon$  since  $\bar{\theta}$  is at most 1.

### 3 Equilibrium

I consider monotone equilibria (as I show later on, there are no other types of equilibria). In a monotone equilibrium, there is a threshold  $x^*$  such that agents roll over their loan if and only if  $x \geq x^*$ . The aggregate size of the loan is the mass of agents who receive  $x \geq x^*$ . Thus

$$L(\theta) = \begin{cases} 0 & \text{if } \theta < x^* - \varepsilon, \\ \frac{\theta + \varepsilon - x^*}{2\varepsilon} & \text{if } x^* - \varepsilon \leq \theta < x^* + \varepsilon, \\ 1 & \text{if } \theta \geq x^* + \varepsilon. \end{cases}$$

Clearly, total investment increases in  $\theta$ .

First, given any signal cutoff  $x^*$ , let us calculate the threshold  $\theta^*$  such that successful investment occurs if and only if  $\theta \geq \theta^*$ . This is given by

$$L(\theta^*) = 1 - \psi \quad (3)$$

Solving,

$$x^* = (2\psi - 1)\varepsilon + \theta^* \quad (4)$$

Next, given that the project earns a high return if and only if  $\theta \geq \theta^*$ , let us calculate the signal cutoff  $x^*$ . The expected payoff of an agent with signal  $x$  from a loan rollover is

$$\begin{aligned} V(\theta^*, x) &= \Pr(\theta \geq \theta^* | x) E(\theta r | \{\theta \geq \theta^*, x\}) \\ &\quad + \Pr(\theta < \theta^* | x) E(p\theta r + (1-p)\theta | \{\theta < \theta^*, x\}) - 1, \\ &= \frac{1}{2\varepsilon} \int_{\theta^*}^{x+\varepsilon} \theta r d\theta + \frac{1}{2\varepsilon} \int_{x-\varepsilon}^{\theta^*} (p\theta r + (1-p)\theta) d\theta - 1, \\ &= \frac{1}{4\varepsilon} \left( (1-p)(r-1)((x-\varepsilon)^2 - \theta^2) \right) + xr - 1. \end{aligned}$$

For any given  $\theta^*$ ,  $x^*$  solves the indifference condition

$$V(\theta^*, x^*) = 0. \quad (5)$$

Using equation (4) to write  $x^*$  in terms of  $\theta^*$ , and substituting in the expression for  $V(\theta^*, x^*)$  from above, then using (5), we get a single equation in  $\theta^*$ :

$$\frac{(1-p)(r-1)(\varepsilon^2 + (\theta^* + (2\psi - 1)\varepsilon)^2 - \theta^{*2})}{4\varepsilon} + \frac{1}{2}(\theta^* + (2\psi - 1)\varepsilon)(1 + r + p(r - 1)) - 1 = 0$$

Solving, we get following (unique) rollover cutoff. The details of the derivation are relegated to the appendix.

$$\theta^* = \frac{1 + (1 - \psi)^2(1 - p + pr)\varepsilon - \psi^2\varepsilon r}{r - (1 - \psi)(1 - p)(r - 1)} \quad (6)$$

I show next that the unique monotone equilibrium identified above is also the only equilibrium irrespective of the strategies considered. In particular, investing for and only for  $x \geq x^*$  is the only strategy that survives iterative elimination of strictly dominated strategies. This dominance-solvability is similar to several other applications of coordination games exhibiting strategic complementarities in payoffs. The proof is exactly similar to the uniqueness proof in Morris and Shin (2003, 2004) and omitted.

**Proposition 1.** *The monotone strategy of investing if and only if  $x \geq x^*$  is the only strategy that survives iterative elimination of strictly dominated strategies, and therefore the equilibrium in monotone strategies is the unique equilibrium.*

Next, taking the limit as  $\varepsilon \rightarrow 0$ , we get the rollover cutoff:

$$\theta^* = \frac{1}{r - (1 - \psi)(1 - p)(r - 1)} \quad (7)$$

At  $\psi = 0$ ,  $\theta^* = 1 \equiv \bar{\theta}$ . Further, for  $\psi = 1$  or  $p = 1$ ,  $\theta^* = 1/r \equiv \underline{\theta}$ . Further,  $\theta^*$  is clearly decreasing in  $\psi$  as well as  $p$ . It follows that for any  $0 < \psi < 1$  and/or  $0 < p < 1$ ,  $\underline{\theta} < \theta^* < \bar{\theta}$ .

### 3.1 Equilibrium Market Haircut

The market required stable funds for any state of fundamentals  $\theta$  is the minimum stable funding the intermediary must itself provide in order for successful coordination of lending to take place. In other words, it is the minimum haircut to ensure that  $\theta \geq \theta^*$ .

Since  $\theta^*$  is decreasing in  $\psi$ , the haircut is smallest if we can ensure coordination is just successful at  $\theta$ . If we support coordination at  $\theta$  by making  $\theta > \theta^*$ , we can lower the intermediary's own stable funding and still ensure that  $\theta \geq \theta^*$ . Therefore the market haircut necessary to ensure  $\theta \geq \theta^*$  is given by the solution for  $\psi$  to  $\theta = \theta^*$ , where  $\theta^*$  is given by equation (7).

Solving, market required haircut for coordination to be successful at state  $\theta$  is  $\psi^M(\theta)$  is given by

$$\psi^M(\theta) = 1 - \frac{r\theta - 1}{(1-p)(r-1)\theta}$$

### 3.2 Minimum Market Required Haircut

The market required haircut calculated above depends on the promised loan interest factor  $r$ . This is presumably determined by supply and demand factors in the market for funds outside the scope of the model. Even though  $r$  is exogenous to the model, we can determine an upper bound for  $r$  which gives us a lower bound for the coordination threshold  $\theta^*$ , as well as a minimum market required haircut to sustain coordination at any given  $\theta$

For any  $\theta \geq \theta^*$ , the gross payoff of an intermediary is given by  $\theta(R - (1-k)r)$  whenever the project succeeds, and 0 otherwise. Since the intermediary invests  $\psi$ , its gross payoff in the success state must exceed  $\psi$  as otherwise the intermediary would necessarily receive a strictly negative net expected payoff for any  $\theta \geq \theta^*$ . Thus a necessary condition for the intermediary to run the project at all is<sup>3</sup>

$$\theta^*(R - (1-\psi)r) \geq \psi.$$

Changing the inequality to an equality and solving for  $r$  gives us the following upper bound on  $r$ :

$$r_{\max} = 1 + \frac{R-1}{1-\psi(1-\psi)(1-p)}.$$

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<sup>3</sup>In equilibrium we must have  $\pi_I(\theta^*) \geq 0$ . Suppose, on the contrary, that  $\pi_I(\theta^*) < 0$  in equilibrium. Then there is a positive measure of values of  $\theta \geq \theta^*$  for which  $\pi_I(\theta) < 0$ . Note also that both  $\theta^*$  and  $\pi_I(\cdot)$  are decreasing in  $r$ . If the intermediary sets a slightly lower  $r$ , its payoff would increase for all values of  $\theta \geq \theta^*$ , and further,  $\theta^*$  would increase a little, therefore reducing the measure of the set of values of  $\theta$  for which payoff is negative. Thus such a reduction in  $r$  is strictly profitable for the intermediary. This contradiction proves that in equilibrium we must have  $\pi_I(\theta^*) \geq 0$ . Given this, an upper bound on  $r$  is given by  $\theta^*(R - r(1-\psi)) - \psi = 0$ .

Using this upper bound, we can get a lower bound on the value of  $\theta^*$ , denoted by  $\theta_{\min}^*$ :

$$\theta_{\min}^* = \frac{1 - \psi(1 - \psi)(1 - p)}{R - (1 - p)(1 - \psi)(R - (1 - \psi))}.$$

We can now calculate the minimum market required haircut to sustain coordination at any given  $\theta$  using the same logic as in the derivation of  $\psi^M(\theta)$  above. Since  $\theta_{\min}^*$  is decreasing in  $\psi$ , the haircut is smallest if we can ensure coordination is just successful at  $\theta$ . If we support coordination at  $\theta$  by making  $\theta > \theta_{\min}^*$ , we can lower the intermediary's own stable funding and still ensure that  $\theta \geq \theta_{\min}^*$ . Therefore the minimum haircut necessary to ensure  $\theta \geq \theta_{\min}^*$  is given by the solution for  $\psi$  to  $\theta = \theta_{\min}^*$ . From this, we get

$$\psi_{\min}^M(\theta) = \frac{1}{2(1 - \theta)} \left( A - \sqrt{\frac{(1 - p)A^2 + 4(1 - \theta)((R - (1 - p)(R - 1))\theta - 1)}{1 - p}} \right)$$

where

$$A = (1 - \theta) + \theta(R - 1)$$

### 3.3 Inefficiency

The upper bound for  $r$  derived above also gives a lower bound for the inefficiency generated in the loan rollover problem considered here. A lower bound on the inefficiency generated by the coordination problem among lenders is given by

$$\max\{0, \theta_{\min}^* - \theta_{fb}\}.$$

**Proposition 2.** *If  $\psi < 1$  so that there is some short term borrowing,  $\theta_{\min}^* > \theta_{fb}$ . In other words, an inefficiency arises from coordination. As  $\psi \rightarrow 1$ ,  $\theta_{\min}^* - \theta_{fb} \rightarrow 0$ .*

**Proof:** It is straightforward to see that  $\lim_{\psi \rightarrow 1} \theta_{\min}^* = 1/R = \theta_{fb}$ . This proves the second part of the proposition. Next, note that

$$\frac{\partial \theta_{\min}^*}{\partial \psi} = -\frac{1}{D^2} ((1 - p)(R - 1)((1 - \psi)(2 - (1 - p)(1 - \psi)))) < 0,$$

where  $D = R - (1 - \psi)(1 - p)(R - (1 - \psi))$ . Further,  $\theta_{fb}$  is independent of  $\psi$ . Therefore, as  $\psi$  falls from 1,  $\theta_{\min}^*$  rises above  $\theta_{fb}$ . This proves the first part of the proposition. ||

Thus efficiency is achieved only when the bank funds the entire project from own stable funds without any short term borrowing. Whenever some short term borrowing is required, so that there is a positive amount requiring to be rolled over, there is an inefficiency arising from the coordination problem.

#### 4 Ambiguity and Sudden Jumps in Market Required Haircut

In this section I show that if  $p < 1$  for a financial institution (the market is not certain that the institution would be rescued if it became illiquid in period 1), when environment is characterized by ambiguity (signal is ambiguous) and agents are ambiguity averse, coordination breaks down completely.

Suppose some investors face a slight ambiguity about the quality of their own signal. Specifically, suppose some investors think that their signal might have some (arbitrarily small) bias relative to the signals received by others. In other words, such an investor  $i$  regards the signal as being

$$x_i = \theta + \beta_i + \varepsilon_i$$

where  $\beta_i \in [-b, b]$ , and  $b > 0$ . In other words, each buyer believes that the signal they receive is drawn from some distribution from a *set* of distributions with means  $\theta + \beta_i$ , where the bias  $\beta_i$  is distributed over  $[-b, b]$ . Note that  $b$  can be arbitrarily small, so that the extent of the ambiguity could be vanishingly small.

The preferences of the buyers is represented by the maxmin expected utility model of Gilboa and Schmeidler (1989). Following their model, ambiguity averse investors maximize their minimum expected utility over the set of possible signal distributions. Here, the minimizing signal distribution for investor  $i$  is the one with mean  $\theta + b$ . Investor  $i$  therefore chooses the cutoff  $x_i$  to maximize expected utility conditional on signal  $x_i = \theta + b + \varepsilon_i$ .

$$u_i(x_i, \theta^*) = \frac{1}{2\varepsilon} \int_{\theta^*}^{x_i - b + \varepsilon} \theta r d\theta + \frac{1}{2\varepsilon} \int_{x_i - b - \varepsilon}^{\theta^*} (p\theta r + (1 - p)\theta) d\theta - 1$$

Integrating and rearranging terms, we get

$$u_i(x_i, \theta^*) = \frac{1}{4}(r-1)(1-p) \left( \frac{(x_i - b)^2 - \theta^{*2}}{\varepsilon} + \varepsilon \right) + \frac{1}{2}(1-p+r(1+p))(x_i - b) - 1 \quad (8)$$

Let  $y^*$  be the rollover signal threshold for all others. We want to see the relation between  $x_i$  and  $y^*$ . In the standard case without ambiguity there would be a common threshold  $x_i = y^*$ . In this case,  $\theta^*$  and  $y^*$  are such that  $L(\theta^*|y^*) = 1 - \psi$ . Using equation (4), this implies,

$$\frac{\theta^* + \varepsilon - y^*}{2\varepsilon} = 1 - \psi.$$

Solving, we get  $\theta^* = y^* + \varepsilon - 2\psi\varepsilon$ . Substituting the value of  $\theta^*$ , we get payoff in terms of  $x_i$  and  $y^*$ :

$$\hat{u}_i(x_i, y^*) = \frac{1}{4}(1-p)(r-1) \left( \frac{(x_i - b)^2 - y^{*2}}{\varepsilon} - 2y^*(1-2\psi) - \varepsilon(1-2\psi)^2 + \varepsilon \right) + \frac{1}{2}(x_i - b)(1-p+r(1+p)) - 1 \quad (9)$$

Suppose  $p < 1$ . If  $x_i - b > y^*$ , for small  $\varepsilon$ , the payoff becomes large and positive, so that  $x_i$  must be lower in equilibrium. Thus  $x_i - b \not> y^*$ . Also, if  $x_i - b < y^*$ , for small  $\varepsilon$ , the payoff is large and negative, so that the cutoff  $x_i$  of investor  $i$  must be higher. Therefore  $x_i - b \not< y^*$ . It follows that the only value of  $x_i$  compatible with equilibrium is given by  $x_i - b = y^*$ , implying  $x_i = y^* + b$ .

In other words, investor  $i$  invests above a signal cutoff that is above the cutoff of others. If all others invest above cutoff  $y^*$ ,  $i$  invests above cutoff  $y^* + b$ . But since everyone behaves this way, it is impossible to have an interior solution for  $y^*$ . In other words, coordination only succeeds when it is a dominant strategy to rollover, i.e. when  $\theta \geq \bar{\theta}$ . Otherwise coordination fails. Thus for the entire set of fundamentals for which coordination matters, coordination collapses.

Note that this is a discontinuous collapse. As shown earlier, without ambiguity there is an interior threshold  $\theta^*$  beyond which coordination succeeds. Starting from this position if we introduce even the slightest degree of ambiguity in signals, coordination collapses completely and  $\theta^*$  jumps to  $\bar{\theta}$ .

Any financial intermediary for which the rescue probability is  $p = 1$ , coordination is irrelevant. However, for any other value of  $p$ , coordination collapses under any degree of ambiguity and there is a run by short term investors.

The calculations and discussion above prove the following result.

**Proposition 3.** *Suppose  $b_i > 0$  for all  $i$  so there is some (possibly arbitrarily small) ambiguity across all lenders. For any  $p < 1$  coordination breaks down completely.*

Next, suppose a fraction  $\alpha \in [0, 1]$  of lenders are sensitive to ambiguity. It follows from the result above that if there is any ambiguity in signals, this fraction of agents would not be useful for coordination - if others already establish some success threshold  $\theta^*$ , each ambiguity-sensitive agent would want to rollover debt beyond a cutoff above  $\theta^*$ . However,  $\theta^*$  itself must be established by relying on the fraction  $(1 - \alpha)$  of ambiguity-insensitive agents only. It follows that equation (3) becomes

$$(1 - \alpha)L(\theta^*) = 1 - \psi.$$

Since  $L(\theta^*) \leq 1$ , it is clear that coordination cannot succeed if  $\alpha > \psi$ . This proves the following result:

**Proposition 4.** *For any  $\alpha > \psi$ , coordination breaks down completely.*

#### 4.1 Ambiguity and Haircut

An interior coordination cutoff  $\hat{\theta}^*$  can be obtained if  $\alpha < \psi$ .

As noted above, equation (3) now becomes

$$(1 - \alpha)L(\hat{\theta}^*) = 1 - \psi.$$

Equation (5) is unchanged. Following the same steps as above, we get the rollover threshold:

$$\hat{\theta}^* = \frac{1 - \alpha}{r(1 - \alpha) - (r - 1)(1 - \psi)(1 - p)}.$$

The maximum gross payment to debt holders in period 2 if the project succeeds is

$$\hat{r}_{\max} = \frac{\psi(1 - \psi)(1 - p) - R(1 - \alpha)}{\psi(1 - \psi)(1 - p) - (1 - \alpha)}.$$



Finally, the lowest value of cutoff that can be sustained given any specific  $\alpha$  is

$$\hat{\theta}_{\min}^* = \frac{\psi(1-\psi)(1-p) - (1-\alpha)}{(R-1+\psi)(1-\psi)(1-p) - R(1-\alpha)}.$$

Figure 1 below plots  $\hat{\theta}_{\min}^*$  as a function of  $\psi$  for different values of  $\alpha$ . For  $\alpha = 0$ ,  $\hat{\theta}_{\min}^*$  coincides with  $\theta_{\min}^*$  calculated in the case without ambiguity. The figure also shows the case of  $\alpha = 0.4$ . Now, for  $\psi < 0.4$  coordination breaks down completely, and then for higher values of  $\psi$ , we get lower values of  $\hat{\theta}_{\min}^*$ .

We can also use this to find the minimum haircut needed to sustain coordination at any given  $\theta$ . The least-demanding way of sustaining any given  $\theta$  is to achieve it as the cutoff  $\hat{\theta}^*$ .

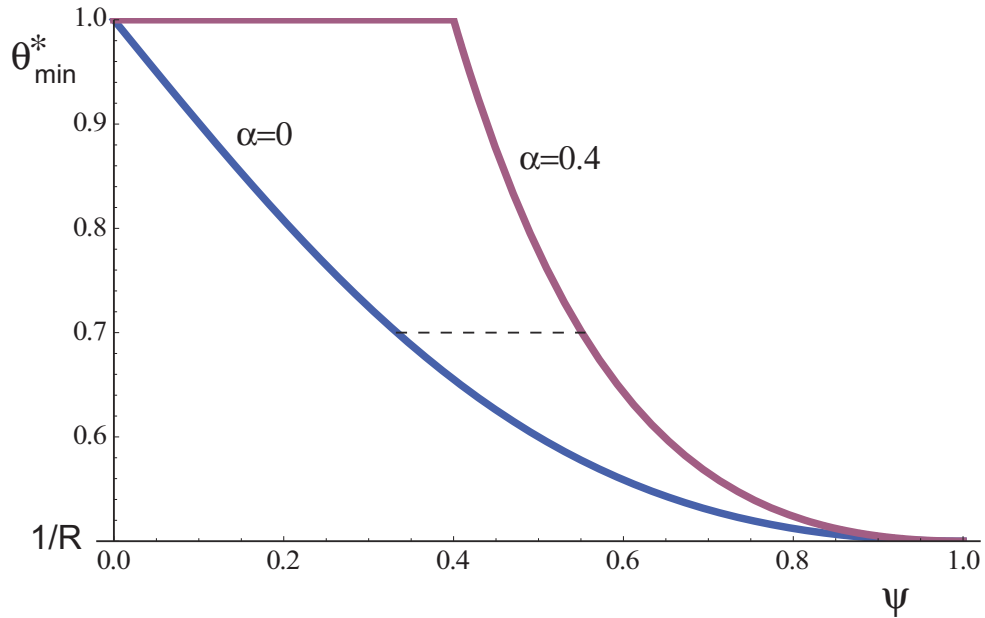


Figure 1: The lowest possible coordination threshold  $\hat{\theta}_{\min}^*$  as a function of  $\psi$  for different values of  $\alpha$ .

Given any  $\alpha$ , the Y-axis shows the lowest value of  $\hat{\theta}^*$  that can be sustained for any  $\psi$ . We can invert this to find the lowest value of haircut needed to sustain any given  $\theta$  as  $\hat{\theta}^*$ . This can be done simply by setting  $\theta = \hat{\theta}_{\min}^*$  and solving for  $\psi$ . In the picture, if we take any  $\theta$  on the Y-axis, then the corresponding value of  $\psi$  on the X-axis is the lowest value of haircut that can support  $\theta$  as  $\hat{\theta}^*$ .

The dashed line shows, given a particular  $\theta$  (in this case  $\theta = 0.7$ ), the minimum haircut required to sustain coordination at 0.7 for different values of  $\alpha$ . If  $\alpha$  is 0, this is around 0.33. However if  $\alpha = 0.4$  (so that 40% of lenders are ambiguity-sensitive), at the advent of ambiguity the minimum haircut to sustain  $\theta = 0.7$  jumps up to around 0.55. Thus given that some fraction of lenders are ambiguity averse, there is a sudden increase in haircut at the advent of ambiguity. Of course, if  $\alpha$  exceeds the available stable funds  $\psi$ , the haircut jumps up all the way to 100%.

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