Agricultural productivity and industrialization: A reformulation*

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Abstract

In this paper we examine the role of agricultural productivity on the process of industrialization in an economy. We show that an improvement in agricultural productivity may not always facilitate industrialization by releasing labor from agriculture to industry. In fact, when agriculture is highly productive, a further improvement in its productivity can reduce the size of the industrial sector. However, when agriculture is initially low productive, size of the industrial sector is positively related to agricultural productivity. This makes a case for an inverted-'U' shaped relationship between agricultural productivity and size of the industrial sector. However, welfare effects of productivity improvement are always positive.

***JEL* Classification:** F43, O11, O41. **Key Words:** Agricultural productivity, Manufacturing productivity, Sectoral sizes, Welfare.

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1 Introduction

The role of agricultural productivity on economic development has been one of the major issues of discussion in development economics. The question of how an improvement in the productivity of agriculture is related to the process of industrialization and structural change in an economy has intrigued many generations of economists and policy makers. The broad consensus is that an improvement in agricultural productivity should relocate labor from agriculture to industry and thereby facilitate the process of industrialization. The logic behind this argument follows from the fact that an improvement in agricultural productivity requires less labor to produce the same amount of agricultural good. Thus productivity improvement pushes labor out of this sector. In addition to this, income in-elasticity of the demand for agricultural good imply that increased income associated with productivity improvement mostly get spent on industrial goods. Thus non-homotheticity in preferences between agricultural and industrial goods makes a strong case for a positive linkage between labor saving technical change in agriculture and the size of the industrial sector. These explanations are already prevalent in the literature and make much intuitive sense specially in an economy that is closed to the rest of world (see Matsuyama (1992)).

However, empirical evidences linking agricultural productivity and industrial development is mixed. According to many economic historians, an improvement in agricultural productivity raises the wage rate making labor costly to be hired by industry (see, Mokyr (1976), Field (1978) and Wright (1979)). This scarcity of cheap labor prohibits local industry to flourish. Historically, Belgium and Switzerland was not much productive in agriculture compared to Netherlands. However the spectacular growth in industrial sector came first in the former two countries and later to Netherlands (see Mokyr (2000)). Economic historians calls this the Law of Comparative Advantage.

In recent empirical studies by Foster and Rosenzweig (2004, 2008), it has been shown that, in the context of rural Indian economy, agriculturally more productive regions were not accompanied by an expansion of rural industry. In fact, industrial diversity were present in those areas where agriculture was less productive. In their own words - "*Our results are striking and, to our minds, unequivocal. Growth in income from the nonfarm sector in rural India over the last 30 years has been substantial, and the primary source of this growth, the expansion of rural industry, is not predicated on expansion of local agricultural productivity. Indeed, as would be anticipated by a model in which rural industry producing tradable goods seeks out low-wage areas, factory growth was largest in those areas that did not benefit from enhancement of local agricultural productivity growth over the study period.*" (Foster and

Rosenzweig (2004), pp. 541).

In a more recent study, Bustos et. al. (2013) provided direct empirical evidences on the effects of agricultural productivity on industrial development. They studied the recent widespread adoption of new agricultural technologies in Brazil. When agricultural productivity growth came in the form of adoption of genetically engineered soybean seeds (which they call 'labor-saving' technological change), it leads to an employment growth in the industrial sector. However, in case of adoption of second-harvest maize (which they call 'labor-biased' technological change), agricultural productivity growth leads to a reduction in industrial employment. Thus the effect of productivity improvement in agriculture depends on the factor-bias of technical change and no uniform view exists in the empirical literature.

These empirical evidences along with the historical records of industrialization highlight that improvement in agricultural productivity may not always induce industrial development. In this paper we ask - why in some cases industrial development follows after an agricultural revolution while in other cases it does not? Why agricultural productivity improvement might be an inducement for industrialization even in a small open economy? Its not that these questions have not been asked in the previous literature. However, our results are very different from the received wisdom of the literature.

We developed a simple two-sector economy with non-homothetic preferences between agriculture and industrial goods. We show that an improvement in agricultural productivity may or may not lead to an expansion of the industrial sector. The result depends on the initial productivity level of agriculture. When agriculture is less productive to begin with, an improvement in its productivity is associated with an expansion of the industrial sector. This is made possible by the relocation of labor from agriculture to industry. However, when agriculture is already much productive, any further improvement in its productivity attracts labor from industry to join in agriculture. This leads to a decline in the size of industrial sector and the diversity of its products. Thus there is an inverted-U shaped relationship between agricultural productivity and the size of the industrial sector in our model. These results are in line with Foster and Rosenzweig's (2004, 2008) observation in the context of rural economy in India.

In our model, the preference structure is *CES* - allowing for broad generality in substitution possibility between the two goods. An improvement in agricultural productivity raises the wage rate as well as the relative price of the manufacturing good. These two effects combined together generates positive demand for agricultural good (call this the 'demand' effect). Had this been the only channel, size of the agricultural sector would have grown up to meet to the higher demand. However, with productivity improvement, it now requires less labor to produce the same amount of agricultural output. With subsistence consumption of food, this effect tends to reduce the size of the agricultural sector (call this the 'supply' effect, since this effectively releases labor from the subsistence sector). When initial productivity in agricultural is low, a large fraction of the labor force are already engaged in the subsistence sector. In this case, an improvement in productivity releases sufficient labor from agriculture to join in industry so that the 'supply' effects dominates the 'demand' effect and size of the agricultural sector shrinks with its productivity improvement. Exactly opposite happens when agricultural productivity is sufficiently high to begin with so that relatively smaller fraction of population is working in the subsistence sector. These are broad intuitions behind the 'U'-shaped relationship mentioned earlier.

We are not the first one to build a two-sector model in order to see the linkages between agricultural productivity and industrial development. In fact, the (theoretical) literature in this line took-off with a classic contribution by Matsuyama (1992). Our questions are very much similar to those in Matsuyama, however, results are very different. Matsuyama (1992) modelled a two-sector economy with non-homothetic preferences. He showed that, under closed economy assumption, an improvement in agricultural productivity leads to an increase in the size of the industrial sector. Larger size of the industrial sector then leads to a higher rate of growth of the economy (due to the presence of learning-by-doing kind of technological progress). In our model, however, the size of the industrial sector may or may not grow due to productivity improvements in agriculture.[1] Additionally, under the assumption of a small open economy, we show that agricultural productivity improvement may facilitate industrialization by drawing labor from agriculture to industry. In Matsuyama, this result is exactly the opposite in a small open economy.

Eswaran and Kotwal (2002) used a small open economy to examine the linkage between agricultural productivity and industrialization within the presence of a service sector. The motivating questions in their paper are very similar to ours. In fact, the openning line of their paper asks the following question - *"Is high agricultural output (per capita) a help or a hindrance to industrialization?"* They show that at a high enough level of agricultural productivity, a further increase in agricultural productivity leads to industrialization. Agricultural productivity growth can therefore facilitate industrialization even in a small open economy. Though our model is primarily based on closed economy assumption, we show similar results hold in an open economy even without any service sector and its non-tradebility

[1] Matsuyama (1992) used a *CES* preference in appendix B (pp.332) of his paper. Our paper uses similar preferences. However, our modelling choice for the manufacturing sector is different from Matsuyama (1992).

as assumed in their paper.

Our paper is closely related to a recent paper by Francisco and Markus (2011) where the authors studied labor relocation out of agriculture due to technological improvement in both agricultural and non-agricultural sector. An improvement in agricultural productivity pushes labor out of agriculture and into the industry (they call it 'push factor'). With an improvement in productivity in the non-agriculture sector, labor is attracted toward this sector away from agriculture (they call it 'pull factor'). They provided a simple two sector model in closed economy to study the relative strength of these two effects on structural change. Their major focus is on the movement of the relative price of the manufacturing goods which they relate to its historical trend (as observed in time-series data). They show that the relative price of the manufacturing good always increases due to an equal proportionate increase in sectoral productivities. Our result differs from them. We show that, an equal proportionate increase in the productivity of both sectors lead to a (unambiguous) decline in the relative price of manufactures. This is exactly opposite to their result and has further implication to the empirical part of their paper. Our result indicate that, during the year 1840 -1920 when the relative manufacturing price is shown to have a definite negative trend, it need not be the case where 'labor-pull' effect dominated. In fact, it may very well be that the sectoral productivities have grown at similar rates.[2]

In another recent paper, Gollin and Rogerson (2014) consider the issue of transport costs and subsistence agriculture in a closed economic system. They find out different channels that can lead to greater allocation of labor to the agricultural sector. One of these channels is the lower agricultural productivity. They showed that improvement in agricultural productivity, though have overall negative impact on the share of labor engaged in agriculture (a result similar to Gollin et al. (2002)), may actually increase the labour share in agriculture in the nearby-city region. However, welfare impact is large (in their calibration exercises) due to an improvement in agricultural productivity. In our work, part of the workforce in agriculture are engaged in producing subsistence food. Any productivity improvement would unambiguously reduce the size of this subsistence production. Yet overall size of agriculture can be higher due to large demand effects owing to adverse terms-of-trade movement in agriculture.

Kogel T. and Alexia P. (2001) used a two sector general equilibrium model to study the effects of agricultural productivity improvement on fertility and economic growth. They

[2] In appendix B (pp.154) of their paper, Francisco and Markus (2011) extended the model using a *CES* preferences. They show that their basic results survive under this generalization given that the elasticity of substitution parameter is not too large. Our results are free from that restriction and we put emphasis on the agricultural productivity parameter in making any conditional statement.

show that higher productivity in the traditional sector brings about population growth that induces endogenous productivity growth in the industrial sector. This way the economy can have sustained economic growth. Our work resembles similarity with them by the fact that we also model industrial sector with endogenous product variety produced by monopolistic competition. However, we abstract from growth and fertility related issues and focus only on the sectoral sizes due to productivity changes.

Rest of our paper is organized as follows. Section 2 lays down the basic model. Comparative static results are provided in section 3. Section 4 deals with the issues of trade and section 5 discusses the welfare impact of productivity improvement. Finally section 5 concludes the paper.

2 The model

2.1 Preliminaries

There are L individuals in an economy each endowed with one unit of labor. Each of them earns a competitive wage denoted by w. There are two sectors in the economy - agriculture and industry (or, manufacturing, we will use these two terms interchangeably). Labors are freely mobile equalizing the wage rate across sectors. Agricultural production is done under perfect competition while industrial sector is characterized by both monopolistic competition (in the intermediate goods production sector) and perfect competition (in the final goods sector). A representative agent's utility maximization problem is given by

$$\max_{c_A, c_i} \quad U = \left[b(c_A - \gamma)^\theta + c_M^\theta\right]^{\frac{1}{\theta}}; \quad c_A > \gamma, \quad \theta \in (0,1) \tag{1}$$

$$\text{subject to} \qquad p_A c_A + p_M c_M = w, \tag{2}$$

Here c_A and c_M are the consumption of agricultural good and industrial good respectively. The subsistence level of consumption of the agricultural good is given by γ (> 0). The parameter b (> 0) in preference captures any biasness for agricultural good. Wage income (denoted by w) is the only source of income in our model. The elasticity of substitution (denoted by ϵ) between the two goods in preferences is given by $\epsilon \equiv \frac{1}{1-\theta} \in (1, \infty)$.

We normalize the price of the agricultural good to unity (i.e., $p_A \equiv 1$). Then p_M represents the relative price of the manufacturing good (or, manufacturing terms-of-trade). The utility maximization problem gives us the following first order condition;

$$c_A = \gamma + c_M \big(bp_M\big)^{\epsilon}.$$

Since c_A and c_i's are individual consumption level, multiplying by L to both sides of the above condition gives us the following;

$$Lc_A = L\gamma + Lc_M \big(bp_M\big)^{\epsilon}.$$

Let us denote the aggregate production of the agricultural good by x_A and that of manufacturing good by x_M. Then, market clarence (i.e., $Lc_A = x_A$ and $Lc_M = x_M$) gives us

$$x_A = L\gamma + x_M \big(bp_M\big)^{\epsilon}. \tag{3}$$

The term $L\gamma$ in eq. (3) represents the aggregate subsistence consumption of food in the economy.

2.2 Production

Labor is the only factor of production in the economy. We assume that agricultural goods are produced using the following technology,

$$x_A = AL_A. \tag{4}$$

It requires L_A amount of labor to produce x_A amount of agricultural goods and A is a measure of sectoral productivity. Since the subsistence consumption of food in the economy is given by $L\gamma$, it requires that only $\frac{L\gamma}{A}$ workers are engaged in subsistence production. We assume that the economy is sufficiently large so that the following inequality always holds ture:

$$L > \frac{L\gamma}{A}. \tag{5}$$

Note that in an economy of size L, only $\frac{L\gamma}{A}$ workers are engaged in the subsistence sector. Then the fraction of population engaged in the subsistence sector is simply given by $\frac{\gamma}{A}$. This is free from any scale effect. In an economy where A is very low, a large fraction of the work force are engaged in subsistence production while the opposite is true in an economy with higher productivity in agriculture. This matches well with the empirical observation made by Gollin and Rogerson (2014) with respect to the economy of Uganda regarding the size of its subsistence production sector.

Our agricultural production technology in eq. (4) along with the normalization of agricultural price imply that the wage rate is determined by the productivity parameter, i.e.,

$$w = A. \tag{6}$$

With free mobility of workers across sectors, the same wage rate applies to everywhere and this becomes the per-capita income in this economy.

The production of the manufacturing goods requires intermediate inputs. These inputs are aggregated using a CES technology to produce the final manufacturing good. There are n number of different intermediate inputs. The production technology of the final good is given by the following.

$$x_M = M\big(\sum_{i=1}^{n} z_i^{\delta}\big)^{\frac{1}{\delta}}; \quad \delta \in (0,1). \tag{7}$$

Here z_i is the amount of ith intermediate input used in the production of the final good. We denote the elasticity of substitution between any two intermediate inputs by σ. Then $\sigma \equiv \frac{1}{1-\delta} > 1$. The parameter M captures the productivity in the manufacturing sector. For higher values of M, the same amount of intermediate inputs produce more of the final goods. Thus the process of assembling different inputs into the final goods production becomes more efficient with higher values of M.

We assume that $\sigma \geq \epsilon$, i.e., the elasticity of substitution among different intermediate inputs in the production is larger than the elasticity of substitution between agriculture and industry in the consumption. If we had allowed the industrial goods in the utility function (i.e., c_M in eq. (1)) to be an aggregate of n different consumption varieties, the assumption $\sigma \geq \epsilon$ would simply mean that industrial goods are more substitutable among themselves in consumption that they are as a whole with the agricultural good. As an example, it make sense to assume that two varieties of car are more substitutable to each other than car as a whole with rice or wheat. However, none of our results would change with such a modification by allowing intermediate inputs to be regarded as consumption varieties in the utility function. We, however, choose to work with intermediate input varieties and impose the assumption that $\sigma \geq \epsilon$.

The production of the final good is done under perfect competition. Let π_M denote the profit and p_i be the price per unit of ith intermediate input. Then profit maximization in the final manufacturing good sector can be given by

$$\operatorname*{Max}_{z_i \geq 0} \pi_M = p_M x_M - \sum_{i=1}^{n} p_i z_i; \quad \text{subject to } eq. \text{ (7)}.$$

This solves for the following demand functions for the intermediate inputs,

$$z_i = \frac{p_i^{-\sigma}\left(\sum_{j=1}^{n} z_j p_j\right)}{\sum_{j=1}^{n} p_j^{1-\sigma}}; \quad \forall i \in [1, n]. \tag{8}$$

The above demand function along with the condition that profit must be zero ($\pi_M = 0$) under perfect competition ensure that the price of the final manufacturing good becomes

$$p_M = \frac{1}{M}\left(\sum_{j=1}^{n} p_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{9}$$

Note that an exogenous improvement in productivity parameter, M, leads to a decrease in price. Also an increase in the number of intermediate inputs leads to an efficiency gain in the manufacturing sector. This later gain is purely due to the specialization effect - as the number of inputs grow, each being more specialized leads to an overall efficiency gain in the production process.

2.3 Intermediate inputs

Each variety of the intermediate inputs is being produced by a monopoly producer. The production of variety i needs both fixed cost (denoted by α) as well as marginal cost (denoted by β). The production function of *ith* intermediate good is given by

$$L_i = \alpha + \beta z_i,$$

where L_i is the amount of labor hired by *ith* producer. The producer faces the following profit maximization problem;

$$\underset{p_i}{\text{Max}}\, \pi_{z_i} = p_i z_i - (\alpha + \beta z_i) w.$$

subject to the demand function for her product, z_i, given in eq. (8). While maximizing profit, each producer takes p_M as given even though it is affected by the choice of p_i (see eq. (9)). The profit maximization problem along with free entry in the intermediate input production sector gives the following solutions of price and quantity.

$$p_i = \frac{A\beta}{\delta}, \quad \text{[using } w = A \text{ by (6)];}$$
$$z_i = \frac{\alpha\delta}{(1-\delta)\beta}; \quad \forall i.$$

Aggregate employment in the intermediate goods sector which is equivalent to manufacturing employment is denoted by L_M and is given by

$$L_M = \sum_{i=1}^{n} L_i = \frac{n\alpha}{1-\delta}. \tag{10}$$

With these solutions, aggregate production of the final manufacturing good (in eq. (7)) and the price index (in eq. (9)) takes the following form,

$$x_M = M\frac{\alpha\delta}{(1-\delta)\beta} n^{\frac{1}{\delta}}; \tag{11}$$
$$p_M = \frac{A\beta}{M\delta} n^{-\left(\frac{1}{\delta}-1\right)};. \tag{12}$$

Note that, labors are not directly employed in the final manufacturing good production (see eq. (7)). However indirectly they are employed through intermediate goods production. To see this, let us re-write eq. (11) using (10) as follows

$$x_M = L_M \frac{M\delta}{\beta} n^{\frac{1}{\delta}-1}.$$

Then the marginal productivity of labors in the final manufacturing goods sector is given by

$$\frac{\partial x_M}{\partial L_M} = \frac{M\delta}{\beta} n^{\frac{1}{\delta}-1}; \tag{13}$$

and multiplying this expression with p_M in (12), called value marginal productivity, one gets back the wage rate, A. This verifies that wage rate is equalized across sectors. Note that the marginal productivity expression is increasing in n. We will later show that an increase in L_M is associated with higher n. Then higher size of the manufacturing sector is associated with higher marginal productivity of its labor.[3] This would imply that any increase in size of this sector would imply lower (relative) price of manufacturing. We shall develop this important result later (see proposition 3) and here we want to build an intuitive idea behind

[3]This is in contrast with neo-classical production function where, due to diminishing marginal productivity assumption, larger size reduces marginal productivity.

that result.

Finally the labor market clearing condition can be given by the following.

$$L_A + L_M = L.$$

Using eqs. (4) and (10), above equation can be re-written as

$$x_A = AL - \frac{An\alpha}{1-\delta}. \tag{14}$$

Next, using equations (11), (12) and (14), we can re-write eq. (3) as follows

$$L - \frac{n\alpha}{1-\delta} = \frac{L\gamma}{A} + n^{\frac{\delta-\theta}{\delta(1-\theta)}} \left(\frac{A}{M}\right)^{\epsilon-1} \frac{\alpha\delta}{(1-\delta)\beta} \left(\frac{\beta}{\delta}\right)^{\epsilon} b^{\epsilon}. \tag{15}$$

This is the final equation that solves for n uniquely. To see this, note that the left hand side of this equation is a decreasing function of n. The right hand side is clearly an increasing function of n since $\delta \geq \theta$ (by assumption). Then these two curves must intersect to each other exactly once. This proves that there is a unique solution for n.

3 Comparative statics

3.1 Agricultural Productivity

3.1.1 Case: $\delta = \theta$

We first assume a special case that $\delta = \theta$. With this the right hand side in eq. (15) does not depend on n and it becomes much simpler to perform comparative static exercises. Later we will generalize on this assumption and show that our basic insight will remain valid. When $\delta = \theta$, we rewrite eq. (15) as

$$L - \frac{n\alpha}{1-\delta} = \frac{L\gamma}{A} + \left(\frac{A}{M}\right)^{\epsilon-1} \frac{\alpha\delta}{(1-\delta)\beta} \left(\frac{\beta}{\delta}\right)^{\epsilon} b^{\epsilon}. \tag{16}$$

Define a new term T such that $T \equiv \frac{\alpha\delta}{(1-\delta)\beta} \left(\frac{\beta}{\delta}\right)^{\epsilon} b^{\epsilon}$. The existence of a unique solution of n (call this n^*) in eq. (16) is guaranteed under the following condition.

$$L > \frac{L\gamma}{A} + \left(\frac{A}{M}\right)^{\epsilon-1} T.$$

For sufficiently large value of L this condition is likely to hold true. Next we see how n^* is affected by a change in A. We obtain the following comparative static results.

$$\frac{dn^*}{dA}\begin{cases} > 0 & \text{if } A < \left(\frac{L\gamma}{T(\epsilon-1)}\right)^{\frac{1}{\epsilon}} M^{\frac{\epsilon-1}{\epsilon}} \\ = 0, & \text{if } A = \left(\frac{L\gamma}{T(\epsilon-1)}\right)^{\frac{1}{\epsilon}} M^{\frac{\epsilon-1}{\epsilon}} \\ < 0 & \text{if } A > \left(\frac{L\gamma}{T(\epsilon-1)}\right)^{\frac{1}{\epsilon}} M^{\frac{\epsilon-1}{\epsilon}}. \end{cases} \tag{17}$$

The sign of $\frac{dn^*}{dA}$ depends on the initial productivity of the agricultural sector. When Agricultural productivity is below a critical level, given by $\left(\frac{L\gamma}{T(\epsilon-1)}\right)^{\frac{1}{\epsilon}} M^{\frac{\epsilon-1}{\epsilon}}$, any further improvement of it leads to a relocation of labor from agriculture to industry. This increases the size of the industrial sector along with its product variety. However, if we begin with sufficiently higher productivity in agriculture, its further improvement would attract labor from industry to join in agriculture. This would raise the relative size of agricultural sector at the cost of industry.

Intuitively, an increase in A brings in two opposite effects on the size of the industrial sector. First, with an increase in A, the subsistence amount of agricultural goods can now be produced with fewer labor. This tends to lower the size of agricultural sector. Second, an improvement in A raises the wage rate and the terms-of-trade moves in favour of industry. Both of these raise the demand for food due to substitutability in preferences. For sufficiently lower values of A, agriculture is predominantly characterized by subsistence production. Then an improvement in A leads to a large relocation of labor so that the first effect dominates the second and the size of this sector goes up with productivity.[4] Opposite happens when A is relatively high to begin with. We thus get an inverted-U shaped relationship between A and n^*.

To show the plausibility of an inverted 'U' shape relationship between agricultural productivity (A) and the size of industrial sector (proxied by n^*), we run a simulation exercise with parameters taking values: $L = 1000, \alpha = 1, \beta = 0.1, \delta = \theta = .67, \gamma = 0.01, b = 2, A = 0.13, M = 1$. With these parameter values, eq. (16) solves for $n^* = 310$. Starting from this initial situation, we allow parameter A to increase its value by 0.1 and record the corresponding values of n^*. The graph of this relationship is shown in figure 1 below. As evident, an increase in A raises the number of intermediate inputs initially. As A is increased further (beyond the value 2.0274 in this example), n^* starts falling. Since n^* and the size of manufacturing sector is one-to-one (see eq. (10)), similar relationship would hold between A and

[4] See that the partial derivative with respect to A of the first term in the RHS of eq. (16) takes very high negative values for lowers values of A, i.e., $\frac{d}{dA}\left(\frac{L\gamma}{A}\right) \to -\infty$ as $A \to 0$.

the size of the industrial sector (L_M).

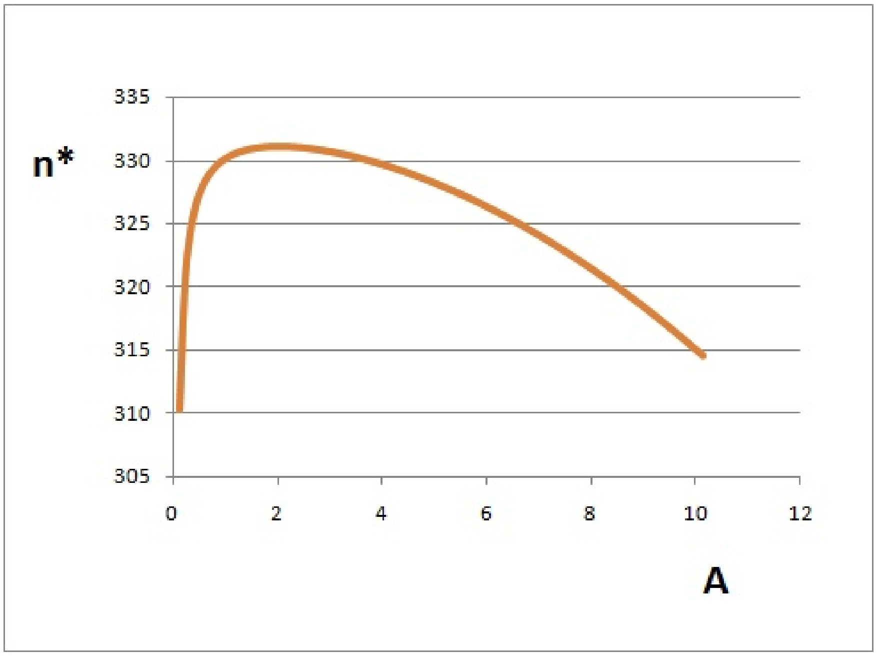

Figure 1: Agricultural productivity (A) and manufacturing diversity (n^*) (case: $\delta = \theta$).

3.1.2 Case: $\delta > \theta$

In this case, the elasticity of substitution takes different values in consumption and production. The relevant equation that now solves n is given by eq. (15). As we see, the qualitative results will be the same here as in earlier case. Intuitively, the substitution effect that raises the demand for food now gets stronger with $\delta > \theta$.[5] So, relative to the $\theta = \delta$ case, more people are now engaged in the food production sector. This tends to lower the value of n that solves eq. (15) (compared to n that solves eq. (16)). In figure 2 below, we report the comparative static results of increasing A on n^*. We take the same parameter values as earlier except for $\delta = 0.9$ and $\theta = 0.67$. When n^* is falling, it is falling at a faster rate compared to the $\delta = \theta$ case. Clearly, the inverted-'U' shape result is preserved. We now combine the results of previous two subsections in the following proposition:

Proposition 1. *An improvement in agricultural productivity may or may not increase the size of the industrial sector. When Agricultural productivity is already very high (low), any further of its improvement lowers (raises) the size of the industrial sector.*

[5]To see this, note that the second term in the RHS of eq. (15) is now increasing in n.

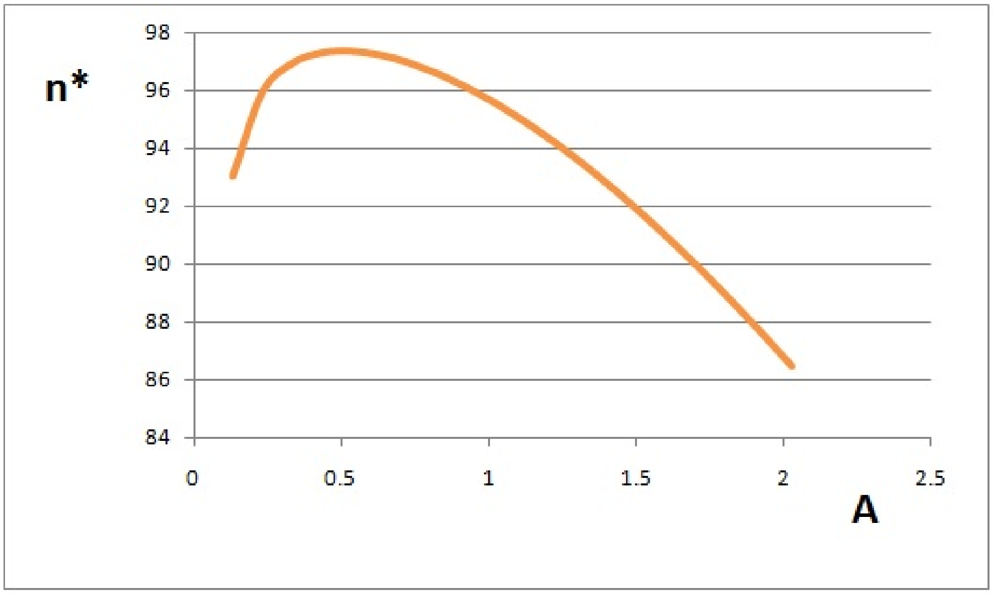

Figure 2: Agricultural productivity (A) and manufacturing diversity (n) (case: $\delta > \theta$)

One can also interpret these results in proposition 1 in terms of size of the subsistence sector. From our model it is following that in an economy (or, region) where agricultural productivity is low, a large fraction of its people are working in the subsistence sector (i.e., higher values of $\frac{\gamma}{A}$). From here, an improvement in A raises the size of the industrial sector by bringing in more diversity in the varieties it offers. Opposite happens in an economy with relatively smaller size of the subsistence production. These results are in line with the observations made by Foster and Rosenzweig (2004, 2008) reported in the introduction. Next we see how any change in manufacturing productivity affects sectoral sizes.

3.2 Manufacturing Productivity

An improvement in manufacturing productivity (M) moves the terms-of-trade in favour of agriculture (i.e., p_M goes down - see eq. (12)). With relative price of the manufacturing going down, its demand goes up. This draws in additional labor from the agricultural sector. A larger size of the industrial sector facilitates entry of even larger intermediate varieties. Real wage, measured in terms of manufacturing goods, goes up. These results can be verified using eq. (15). One can easily see that an increase in M lowers the right hand side of (15). Then n has to go up to bring back equality.

It is interesting to note that a proportionate increase in M and A, keeping $\frac{A}{M}$

constant, raises n unambiguously (see eq. (15)).[6] It is only in the special case where $\epsilon = 1$ that any proportionate change in sectoral productivities does not induce structural change. One can also see that an increase in the productivity level of intermediate manufacturing production (i.e., lower values of α or β) has similar effects to an increase in M. We summarize these results in the following proposition.

Proposition 2. *(a) An improvement in productivity of the final (or, intermediate) industrial good always raises the size of the industrial sector and real wages.*
(b) Any proportionate increase in productivity level of agriculture and manufacturing keeping their relative productivity unchanged raises (lowers) the size of the industrial (agricultural) sector.

It is possible to draw some conclusion on the movement of terms-of-trade (p_M) due to proportionate change in productivities. Let us denote the growth rate in productivities as $\hat{A}$ $(= \frac{\dot{A}}{A})$ and $\hat{M}$ $(= \frac{\dot{M}}{M})$ in the agricultural and nonagricultural sector respectively where a 'dot' over a variable denotes its time rate of change, i.e., $\dot{A} = \frac{d}{dt}A$. Then from eq. (12) we get

$$\hat{p_M} = \hat{A} - \hat{M} - \left(\frac{1}{\delta} - 1\right)\hat{n} < \hat{A} - \hat{M}. \tag{18}$$

Here the last inequality follows under the assumption that $\hat{n} > 0$ (note that $\delta \in (0, 1)$). In proposition 2 we have already established that an equal proportionate change in productivities would lead to an increase in n. Then we can establish the following result.

Proposition 3. *An equal proportionate increase in the productivity level of agriculture and manufacturing would unambiguously lead to a decrease in the relative price of manufacturing good.*

The proof of this result follows immediately from eq. (18); simply put $\hat{A} = \hat{M}$ (> 0) in (18) and note that $\hat{p_M} < 0$ since $\hat{n} > 0$ (see proposition 2, part (b)). Intuitively, when productivities are growing at an equal pace in both sectors, there are continuous relocation of labors going on in the direction from agriculture to industry. This raises the size of the manufacturing sector and makes the production of larger varieties of intermediate goods possible. Larger n reduces the price of the final manufacturing good (see eq.(12)). This would unambiguously lead to higher welfare as can be seen in the next section.

[6] In eq. (15), when both A and M are growing at the same rate, their ratio, $\frac{A}{M}$, is remained constant. But the right hand side of (15) is falling due to an increase in A. Then the left hand side also has to fall. This is possible only when n goes up. But this will raise the right hand side again (since $\delta > \theta$). Finally an equilibrium will be reached at a higher value of n. Note that n can never fall as that will make the right hand side of (15) to fall in value while the left hand side to rise in value, never bringing an equality in both sides.

Our results in proposition 3 are different from Francisco and Markus (2011). In their paper, relative price of the manufacturing good always increases due to an equal proportionate increase in sectoral productivities.[7] Alternatively, a decrease in relative price of manufacturing must mean a faster rate of productivity growth in the manufacturing sector than in agricultural sector in their model. In their empirical observation, they document that relative price of manufacturing goods are having a definite negative trend during the period 1840 to 1920 while the labor share in agriculture is falling continuously (see their fig-1). They relate to this fact that nonagricultural productivity must have grown at a higher rate than agricultural productivity (i.e., 'labor pull' effect dominated) until the year 1920.

However, in our model, we do not need to assume faster technological progress in nonagricultural sector to get declining trend in relative price of manufacturing. In fact, even with equal productivity improvement in both sectors (i.e., $\hat{A} = \hat{M}$), eq. (18) implies that p_M should fall simply due to relocation of labor from agriculture to industry. Thus declining share of agricultural labor is associated with falling relative price of the manufacturing goods in our model. This is very much consistent with the historical data as documented in Francisco and Markus (2011) without having to assume dominating 'labor pull' effects. To explain the rising trend in relative prices after World War II (as seen in the historial data), the main driver has to be faster rate of agricultural productivity growth (or, dominating 'labor push' effect) as mentioned in Francisco and Markus (2011).

4 Trade

In case of small open economy one could reasonably assume that p_M is given by the world market, making $\hat{p_M} = 0$ in (18) (in the home country). Then resource allocation is determined purely by the expression of p_M in eq. (12) in the home economy, i.e., n is determined. So, to explain structural change in the form of declining labor share in agriculture (i.e., to explain $\hat{n} > 0$), it has to be the case that $\hat{A} - \hat{M} > 0$. Thus in an small open economy, faster productivity improvement in agriculture is very much consistent with migration of labor from agriculture to industry.

[7] See their eq. (18) pp. 134 and the associated paragraph explaining the result. They explained the result by saying that *"... decreases in the relative price of manufactures are unambiguously associated with faster technological change in the nonagricultural sector, i.e., they indicate that the labor pull effect dominates. If the relative price rises, the situation is less clear. An equal proportionate increase in the productivity of both sectors induces* ***an increase*** *in the relative price of manufactures, ..."* (Francisco and Markus (2011), pp.134-135). The last line in this quotation is just the opposite to the result that we obtained here in proposition 3, which is - an equal proportionate increase in the productivity of both sectors induces ***a decrease*** in the relative price of manufactures. (emphasis in underlined text)

This result is in contrast to Matsuyama (1992) and Francisco and Markus (2011) where faster productivity improvement in agricultural leads to migration of labor from industry to agriculture under small open economy assumption. To see the source of this difference, let us look at more closely to one of the fundamental assumptions of these class of models which is wage equalization across sectors. In Matsuyama (1992) and Francisco and Markus (2011), this assumption lead to the following condition:[8]

$$AG'(L_A) = p_M\big(MF'(1 - L_A)\big);$$

where agricultural production function is $Y^A = AG(L_A)$ and manufacturing production function is $Y^M = MF(1 - L_A)$ with the standard neoclassical properties. The left hand side is the value marginal productivity of labor in agriculture and right hand is the same in manufacturing. Then, given p_M, faster productivity improvement in agriculture raises the wage rate there which takes away labor from industry. In our model, the same condition can be given by (using eq.(13))

$$A = p_M\left(\frac{M\delta}{\beta}n^{\frac{1}{\delta}-1}\right).$$

Here, given the value of p_M, an improvement in $\frac{A}{M}$ must increase n and thereby giving the direction of migration from agriculture to industry. Thus increasing returns in the manufacturing sector is the source of this difference. The result that agricultural productivity growth can lead to industrialization in a small open economy is provided in Eswaran and Kotwal (2002). However, in their model, the service sector plays a crucial role.

Instead of small open economy, we now assume that there is a foreign country with productivity level A^* and M^* in agriculture and industry respectively ('*' variable denotes foreign). Assume that all other parameters are the same in both home and foreign and that only final goods are tradable. Initially, the condition is such that the following inequality holds.

$$A > A^*;\ M > M^* \text{ and } \frac{A}{M} = \frac{A*}{M*}.$$

Thus home is (absolutely) more productive in both agriculture and in manufacturing but there are no comparative productivity advantage for home. Then eqs.(12) and (16) immediately imply that (assuming $\delta = \theta$ case)[9]

$$n > n^* \text{ and } p_M < p_M^*.$$

[8]See eq.(4) in both Francisco and Markus (2011) and Matsuyama's (1992) paper.

[9]Similar results would be obtained for $\delta > \theta$ case where the relevant equation is (15). However, we focus here on eq.(16) for simplicity and clarity of results.

Thus initial size of the industrial sector is larger in home compared to foreign and home would export manufacturing to the foreign. Here trade pattern is purely determined by the absolute advantage. It is also easy to see that when home has a comparative (but not absolute) advantage in agriculture (i.e., $\frac{A}{M} > \frac{A*}{M*}$ and $A = A^{*}$ holds), trade pattern will be such that home should export agricultural good and import manufacturing from the foreign. In this case, lower (absolute) manufacturing productivity in home indicates that manufacturing goods are relatively more costly there compared to foreign.

5 Welfare

To see the welfare effect of productivity changes, we find out the following demand functions:

$$c_M = \frac{A-\gamma}{p_M\left(1+b^{\epsilon}p_M^{\epsilon-1}\right)};$$
$$c_A = \gamma + \frac{(bp_M)^{\epsilon}\,(A-\gamma)}{p_M\left(1+b^{\epsilon}p_M^{\epsilon-1}\right)}.$$

Using these expressions in eq. (1), the indirect utility function takes the following form.

$$U = (A-\gamma)\left(p_M^{1-\epsilon}+b^{\epsilon}\right)^{\frac{1}{\epsilon-1}}. \tag{19}$$

Note that per-capita welfare depends positively on the productivity term (A) and negatively on terms-of-trade term (p_M, since $\epsilon > 1$). With an increase in A, terms-of-trade always goes up, i.e., manufacturing goods become relatively costly. Then, apparently, it seems from eq. (19) that the welfare is ambiguous since its first term on the right hand side goes up while the second term goes down. However, in figure 3 below, we plot per capita welfare (U) with respect to A for the same parameter values as in subsection 3.1.2 ($\delta > \theta$ case). It is clear that improvement in productivity (A) dominates the terms-of-trade effect and welfare is increasing in A.

With an improvement in manufacturing productivity (M), welfare is affected only through the terms-of-trade movement. An increase in M always raises n^{*} (see prop 2), and hence, unambiguously lowers p_M (see eq. (12)). Then it is clear that welfare should always go up due to an improvement in manufacturing productivity.

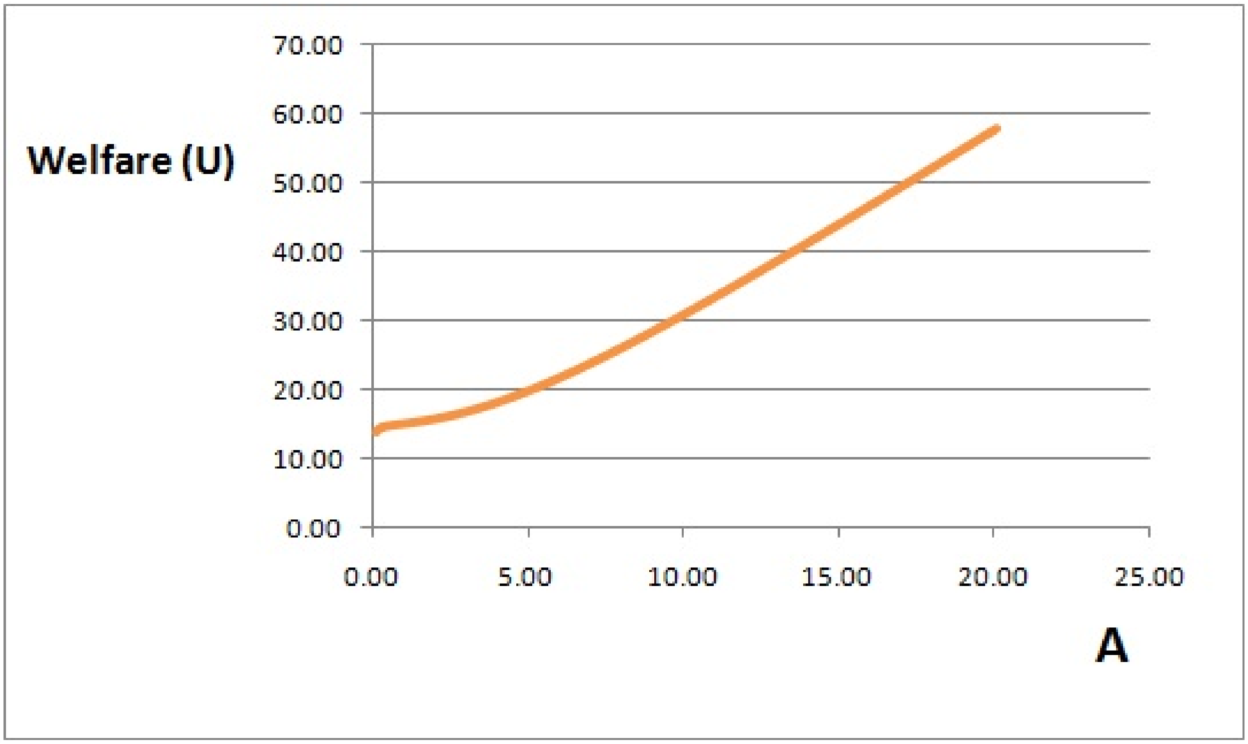

Figure 3: Agricultural productivity (A) and per capita welfare (U) (case: $\delta > \theta$)

6 Conclusion

In this paper we take a fresh look at an old issue in development economics literature - relationship between agricultural productivity and industrialization. We show that an improvement in agricultural productivity may lead to an expansion of the agricultural sector at the cost of manufacturing. This possibility takes place when agriculture is already much productive in an economy, or, where subsistence food production sector is relatively small. However, when agriculture is less productive to begin with, further improvement in its productivity can lead to an expansion of the manufacturing sector. Thus, there is an inverted-U shaped relationship between agricultural productivity and size of the industrial sector.

We also show that an improvement in manufacturing sector productivity always draws in more labour into this sector. However a proportional improvement in both agricultural and manufacturing productivity such that their relative productivity does not change, leads to a relocation of labor from agriculture to manufacturing. Welfare is always positively related to productivity improvement across sectors. These results are shown using a broad class of substitutability in preferences between agriculture and manufacturing goods.

Our modelling structure is simple. It is possible to extend the analysis in many directions. One could bring-in learning-by-doing driven growth into this framework and see how an improvement in productivity affects economic growth. One could also introduce transport costs into the framework by explicitly modelling the location of production. We leave these for our future research work.

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