Dynamic Contracting in Micro-finance: Progressive Lending

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Abstract

This paper examines progressive lending in micro-finance whereby, contingent on repayment, the loan size increases over time. Despite being a near universal feature in most micro-finance loan schemes, progressive lending has largely been ignored in the literature. We examine a dynamic model with continuous time framework, and there is compulsory savings with the MFI. We demonstrate that as long as the fraction of forced savings is not too small, the optimal contract involves progressive lending. More precisely initially loan size increases over time and after a certain point of time it becomes constant over time. Further the paper shows that the individual's utility decreases even when she can save but not with the MFI. Also, competition among MFI-s affects the poor adversely, even in absence of double dipping and the increase in the fraction of forced savings and interest rate on deposit benefit the individual.

1 Introduction

Over the last few decades micro-finance institutions (henceforth MFIs) all over the world has challenged the traditional thinking that the poor villagers are *unbankable*, given that they do not possess any collateral. It has been shown that MFIs has not only reached millions of poor all over the world, the repayment rates are often also exceptionally high(Armendariz and Morduch 2005). While there is some debate over the extent of long term impact of micro-finance, that it has helped the poor in several dimensions is not in question.[1]

In this paper we focus on the various institutional aspects of micro-finance loans. Typically micro-finance loans have many interesting features, e.g. joint liability, sequential lending, progressive lending, etc. While the first wave of literature focussed on static aspects like joint liability, subsequently dynamic aspects like sequential lending, immediate and frequent repayment, etc. were taken up (see subsection 1.1 for a discussion). However, the literature has been mostly silent on a feature that is near universal, i.e. progressive lending, whereby loan sizes increase over time (contingent on repayment of course)(Morduch 1999, Armendariz and Morduch 2005, Rutherford 2001)[2]. Our basic objective is to provide a simple theoretical explanation of this feature, based on dynamic incentives. Moreover, we shall address this issue in the presence of compulsory savings, whereby a fraction of income generated from the MFI loan is kept as deposit with the concerned MFI.

[1]Lately, positive effects of Microfinance have largely been questioned. Banerjee et al. in 2013 did not found any effect of microcredit on poverty indicators, such as household spending and the number of children going to school. Roodman and Morduch (2009) reported that there were fatal flaws in many leading studies which concluded that microcredit reduced poverty. However, in his rigorously researched book Roodman(2011) wrote "...Each client's experience with microfiance is unique... The success of microfinance is real, if subtler than generally understood...".

[2]Bandhan, SKS India, Annapurna Microfinance (P) Ltd., Spandan India, BRAC in Bangladesh(source: respective websites) are only some examples where loan size increases over time.

Since the income of the poor people are not only low but also volatile they need financial services which allow them to save. And, contrary to the saying that poor people are "too poor to save", they want to save and they do save. But at the same time, they find it difficult to maintain the discipline to save and no formal institute allow them to save. To cater that demand there are many informal savings devices and services[3] which allow them to save up. But they are inadequate,unevenly distributed and poor people have limited access to them, so they are even ready to pay high prices to save up[4]. Despite huge demand and positive impact on the welfare of poor people, only a handful of MFI-s provide savings service. Safesave in Bangladesh, Vivekananda Sevakendra-O-Sishu Uddyan (VSSU) in India, Bolivian village banking MFI Crédito con Educacíon Rural, Finca Nicaragua are some examples of MFI-s which take forced savings (Westley 2004). In this paper, we have shown that providing compulsory savings service is welfare improving and it is highest when it is provided by the MFI.

Turning to the formal framework, we examine a dynamic model with continuous time framework. There is a single MFI which can lend at every instant to a borrower and objective of the MFI is to maximise the borrower's welfare. There is ex post moral hazard in that the borrower can default on its loan. However, in case of repayment she has to deposit a fraction of her net income with the MFI. In the basic model we have assumed that there is no other savings instrument available, thus the accumulated deposit constitutes the only source of savings. There is a target level of savings, so that once this target is achieved, the borrower can use it to connect to the formal sector, which yields her a large lifetime utility. Alternatively, we can interpret this lifetime utility as coming out of the financial stability achievable once your savings reach a certain level.

We solve for the optimal loan contract where the MFI designs the contract so as to maximise the present value of lifetime utility of the borrower, subject to the feasibility constraint (i.e. the target savings amount is reached), and the sequence of dynamic incentive constraints ensuring that the borrower does not have any incentive to default at any point. We demonstrate that as long as the fraction of forced savings is not too small, the optimal contract involves progressive lending. More precisely initially loan size increases over time and after a certain point of time it becomes constant over time.

After the main analysis, we have compared the situation with that where the borrower has outside savings option, we have found that the borrower's utility decreases in this modified set up. Lastly, we have done some comparative statics analyses. We have observed that competition among MFI-s, captured through increase in cost of capital which in turn increases the amount to be repaid, decreases borrower's utility, even when it is possible to restrict double dipping. We have found that borrower's utility increases with increase in interest rate on deposit and also with increase in the fraction of the forced savings.

1.1 Literature Review

Traditionally the literature has focused on group lending and joint liability. Group lending and joint liability has been extensively analysed, among others, Ghatak (1999), Guinanne and Ghatak (1999), etc. The subsequent literature has focussed more on dynamic schemes, e.g. immediate and frequent repayment, sequential financing, etc. While sequential lending has been analysed by Roy Chowdhury (2005, 2007) and Aniket (2004), immediate and frequent repayment has been analysed by Chowdhury et al. (2014), Jain and Mansuri (2003) and Fischer and Ghatak (2010).

This paper continues with this theme of dynamic incentives. One of the earliest papers which addresses the role of dynamic incentives in microfinance is Morduch (1999), who reports that the most practised dynamic incentive is increasing loan size over time. Tedeschi (2006) has shown how the welfare of the borrowers will increase if the punishment phase is decreased from infinity to a finite time period. We have followed traditional approach where punishment phase is infinite, however since there is no uncertainty in production so in equilibrium it does not affect the borrower. Also we have allowed (forced) savings hence MFI lends for only finite period of time.

[3] ROSCA, ASCA etc

[4] like Jyothi in Vijaywada. Dwellers of neighbouring slum, where there is no Jyothi at work envy Jyothi's client(Rutherford 2009)

Progressive lending in microfinance has been addressed by Shapiro (2012). In his paper the uncertainty comes from the borrowers' patience level and lenders are unaware of that. He found that loan size will increase over time however in all equilibria but one all borrowers default.

Ghosh and Ray (2001) analysed a credit market consisting of two types of borrowers - namely good and bad, where good borrowers always repay, and bad borrowers never do. In their mechanism, progressivity helps in screening good types from the bad. The optimal contract has two phases - new and old. The profit maximising lender offers the former contract to new borrowers whereas the latter is offered to those borrowers who have repaid loans in the previous period(s) successfully. The loan amount of new phase is lower than that of old phase. However, since bad borrowers never repay so new phase consists of only one period, the defaulters do not get any future loan and those who repay enter into old phase. Our paper does not rely on asymmetric information, and moreover, the borrowers are strategic, rather than always good, or always bad, in that they will default if it suits them, but not necessarily always. We find that the progressivity is an incentivising device for such borrowers.

In a very generalised infinite horizon model Ray (2002) has shown that optimally the contract periods can be divided into two phases, initial phase and mature phase. Both the lender and borrower are taken to be profit maximising. The length of the initial phase depends on the lender's bargaining power, in this phase the lender will enjoy the entire surplus and loan size will increase over time. In the mature phase, the loan size and the entire agreement sequence settles down to the stationary terms under the borrower's best sequence.

While the optimal contract in this paper has similarities with that in Ray (2002), there are several differences. First, in our paper the lender (MFI) is benevolent so the objective here is to maximise borrower's payoff. Second, unlike Ray (2002), we have allowed for (forced) savings. Hence, the MFI lends only for a finite period of time.

Rutherford(2009), in his book, has extensively studied the lives of poor people and he has found that they do need "basic personal financial intermediation[5]".

In one randomized study of microsavings Dupas and Robinson(2009) found that it reduces average poverty, among female vendors in a rural Kenyan market.

Ashraf, Karlan and Yin in their series of papers have shown that access to commitment savings product increases the welfare and disciplne of savings though it diminishes over time.

Aniket(2011) has addressed savings in microfinance in a theoretical model. In that setting two individuals form a group endogenously, one individual(the borrower) gets loan from the MFI and the other individual(the saver) saves with MFI and that saving, along with the loan from MFI, is used to finance the borrower's project. The wealth threshold for being a borrower is greater than that for being a saver. In equilibrium, their will be negative assortative matching within the group along wealth lines. Also, the paper shows that under some assumptions the MFI-s that offering both borrowing and saving opportunities are able to reach poorer individuals than MFI-s that offer only borrowing opportunities. Our paper addresses individual lending and saving. And also shows that the utility of the poor is more when the savings technology is offered by MFI, in compare to any other outside savings opportunity.
This is the first paper, to the best of our knowledge, which addresses savings in microfinance in individual lending set up.

The rest of the paper is organized as follows. Section 2 describes the economic environment and Section 3 presents the equilibrium outcomes. Section 4 compares the situation where outside savings option is available to the borrower, but it is not possible to save with the MFI. In Section 5 some comparative statics analyses have been namely the effect of competition among MFI-s, change in the fraction of compulsory savings and change in the interest rate on deposit. Section 6 concludes.

[5] services which enable poor people to convert their small savings into usefully large lump sums

2 The Economic Environment

The economy consists of one borrower, one micro-finance institute (henceforth MFI), and a bank, all risk neutral. The borrower does not have any money or asset which she can use as collateral. Since the bank, which is in the formal sector, requires collateral to approve a loan, the borrower is not eligible for formal sector loans. The borrower can, however, obtain collateral free loans from the MFI, and once she has sufficient funds, can approach the formal sector bank.

Thus the borrower can potentially be either in the informal, or in the formal sector. Production in the informal sector involves *ex post* moral hazard, in that the borrower can decide to default even though observability of output is not an issue. In case of default, there is limited liability in that the only penalty that can possibly be imposed is the stopping of future loans, and the confiscation of any amount the borrower may have deposited with the MFI.

The borrower has access to a deterministic production function $f(\cdot)$ that satisfies the usual assumptions:

Assumption 1 $f(\cdot) > 0$; $f'(\cdot) > 0$; $f''(\cdot) < 0$; $f'(0) = \infty$ *and* $f'(\infty) = 0$.

Thus the production function is increasing, strictly concave, smooth and satisfies the Inada conditions. Given A1, the efficient scale of investment k^*, where

$$k^* = \underset{k}{\operatorname{argmax}}[f(k) - k],$$

is well defined.

Turning to the formal sector, let S denote the amount of collateral required to get a loan from the bank. Once the borrower obtains a loan, this connects her to the formal sector, with the possibility of future loans, etc. Let V denote the present discounted value of the life time utility from getting connected to the formal sector (gross of S). Alternatively, S can be interpreted as the minimal amount required so as to achieve a measure of financial stability and consumption smoothing, with V denoting the lifetime discounted utility from such savings.[6]

We consider a dynamic framework where time, denoted by t, is continuous and goes on for ever. Let r denote the real rate of interest, where $r > 0$. At the beginning, i.e. at $(t = 0)$, the MFI announces a loan scheme $\langle \{k_t\}_{t=0}^{\infty}, T\rangle$, where k_t denotes the amount loaned at every $t \leq T$, where T denotes the termination date of this loan scheme. This contract also includes that at every t the borrower has to deposit $\alpha(\in [0, 1])$ part of her net income. The borrower either accepts or rejects this loan scheme, with the game ending in case the borrower rejects.

If she accepts, then at every t, $0 < t \leq T$:

- Subject to there having been no default till this point, the MFI makes a loan of k_t to the borrower, which yields an instantaneous output of $f(k_t)$ to the borrower.
- The borrower then decides whether to repay or not:
  - In case the borrower decides to repay, the MFI obtains k_t towards loan repayment, α part of the remaining amount, i.e. $f(k_t) - k_t$, is deposited with the MFI. Any amount deposited attracts an interest at the rate r. And the rest i.e. $(1 - \alpha)[f(k_t) - k_t]$ is consumed by the borrower instantaneously.
  - In case the borrower defaults, she enjoys the entire current income $f(k_t)$, but does not get any loans in the future. Moreover, the entire amount she deposited with the MFI till that date, is confiscated by the MFI. Further the MFI terminates the contract, and the borrower does not get any loan from that point onwards. There is no savings technology available, thus the borrower's lifetime utility, in case of default at t, is $f(k_t)$.

[6] Roodman (2011) suggests that such consumption smoothing is an important motivation behind joining MFIs.

At this point note that the framework involves compulsory savings[7]. The borrower has to deposit α portion of her net income with the MFI. This serves two purposes. First, of course, the amount deposited helps to incentivise repayment, and thus is beneficial to the MFI (and ultimately to the borrower as well). Second, such deposits also provide a direct service to the borrower, in the sense that it serves as a savings technology to the borrower, who may have either no other savings technology available to her, or even if such technologies are available, commitment problems may prevent her from utilising them. For simplicity, we have assumed that α i.e. the proportion of the net income to be deposited is *exogenously* given. We shall relax both these assumptions in follow up work.

The MFI is assumed to be benevolent, and maximises the lifetime utility of the borrower.

3 Equilibrium

Consider a scheme $\langle \{k_t\}_{t=0}^{\infty}, T\rangle$. The objective of the MFI is to maximise the borrower's utility, subject to (i) the feasibility condition (FC) that the amount of savings accumulated by the borrower exceeds S by the end of the scheme (so that she can move to the formal sector), and (ii) the dynamic incentive compatibility constraints that at no $t \leq T$, the borrower has an incentive to default.

Therefore the problem of the MFI is to:

$$\underset{<\{k_t\}_{t=0}^{\infty},T>}{\text{Maximize}} \qquad \int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt - S + V\right]$$

Subject to:

$$\text{FC:} \quad \int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt \geqslant S, \tag{1}$$

$$\text{DIC:} \quad \int_t^T e^{-r(t'-t)}\Big[(1-\alpha)[f(k_{t'})-k_{t'}]\Big]dt' + e^{-r(T-t)}\left[\int_0^T e^{r(T-t'')}\alpha[f(k_{t''})-k_{t''}]dt'' - S + V\right] \geqslant f(k_t); \quad \forall t \leqslant T. \tag{2}$$

3.1 Optimal Contract

We assume that the net gain from moving to the formal sector that is, the difference in lifetime continuation payoff and collateral required to move to the the formal sector is sufficiently high.

Assumption 2 $f(k^*) - k^* < r(V-S)$.

This is reasonable in that once a borrower is connected to the formal sector, she can access more productive technologies, greater legal and state protection, etc.

Now we are in a position to state our first proposition.

Proposition 1 *Let A1 and A2 hold. Then the feasibility constraint (FC) binds at the optimal contract.*

Proof:
Step 1. We first prove that the borrower's utility decreases with T, so that

$$\frac{d}{dT}\left[\int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt - S + V\right]\right] < 0.$$

[7]Bolivian village banking MFI Crédito com Educacíon Rural, Finca Nicaragua take forced savings.

Note that, $e^{-rT}\Big[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt - S + V\Big] = \int_0^T e^{-rt}\alpha[f(k_t)-k_t]dt - e^{-rT}\Big[V-S\Big]$. Thus,

$$\int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt+e^{-rT}\Big[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt-S+V\Big] = \int_0^T e^{-rt}[f(k_t)-k_t]dt-e^{-rT}\Big[V-S\Big]$$

$$\begin{aligned}
& \frac{d}{dT}\Bigg[\int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-rT}\Big[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt - S + V\Big]\Bigg] \\
= \; & \frac{d}{dT}\Bigg[\int_0^T e^{-rt}[f(k_t)-k_t]dt - e^{-rT}\Big[V-S\Big]\Bigg] \\
= \; & e^{-rT}[f(k_T)-k_T] - re^{-rT}(V-S) \\
= \; & e^{-rT}\Big[[f(k_T)-k_T] - r(V-S)\Big]. \qquad (3)
\end{aligned}$$

Clearly, given A2, it must be that $f(k_T) - k_T - r(V-S) < 0$, so that the RHS of (3) is negative.[8]

Step 2. Now suppose the FC does not bind. Consider another scheme that is identical to the original scheme, but ends at $T' < T$. For T' close to T, FC will be satisfied, and the borrower's utility increases by the preceding argument. Finally, DIC holds for all $t \leq T'$, as the original scheme satisfied the DIC and $T' < T$ (we mimic the argument in step 1 to prove this). ■

Intuitively, if the borrower continues with the MFI after building the required collateral, his savings will go up but at the cost of delay in joining the formal sector. Due to this delay, present value of continuation payoff decreases and this waiting cost becomes more than the gain from extra savings.

Hence, the problem of the MFI becomes:

$$\underset{<\{k_t\}_{t=0}^{\infty},T>}{\text{Maximize}} \quad \int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-rT}V$$

Subject to:

$$\text{FC:} \quad \int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt = S, \qquad (4)$$

$$\text{DIC:} \quad \int_t^T e^{-r(t'-t)}\Big[(1-\alpha)[f(k_{t'})-k_{t'}]\Big]dt' + e^{-r(T-t)}V \geqslant f(k_t); \quad \forall t \leqslant T. \qquad (5)$$

Let the optimum scheme be denoted by $<\{k_t^*\}_{t=0}^{\infty}, T^*>$, where T^* is the time required to save S under this scheme, i.e.

$$\int_0^{T^*} e^{r(T^*-t)}\alpha[f(k_t^*)-k_t^*]dt = S.$$

Before proceeding further let us introduce

Definition 1 $\mathbf{k_{It}}$ *is the amount of loan for which the DIC at t binds.*

Now, we are in a position to introduce our next proposition.

Proposition 2 *Consider the optimal scheme* $<\{k_t^*\}, T^*>$. *Then* $k_t^* = \min\{k_{It}, k^*\}$ *for almost all t.*

Proof: There can be two situations:

Either $k_{It} \leqslant k^*$. In *Step1.* we will show that, in this case, $k_t^* = k_{It}$ almost everywhere.

[8] $f(k^*) - k^* = \text{Max}[f(k) - k]$ i.e. $f(k_T) - k_T \leqslant f(k^*) - k^*$. Therefore, when A2 holds $f(k_T) - k_T < r(V-S)$.

Or, $k_{It} > k^*$. In *Step2.* we will show that, in this case, $k_t^* = k^*$ almost everywhere. And, from these two steps we can conclude that $k_t^* = \min\{k_{It}, k^*\}$ for almost all t.

Step1. Consider the set $\mathcal{T} = \{t \leqslant T^* : k_{It} \leqslant k^*\}$. We want to show that $k_t^* = k_{It}$ for almost all $t \in \mathcal{T}$ Observe that, any $k_t^* > k_{It}$ will not satisfy DIC, so here, it is sufficient to prove the following claim. The claim is that the measure of the set $\mathcal{M} = \{t \in \mathcal{T} : k_t^* < k_{It}\}$ is zero.

Suppose not. Then $\exists M (\subsetneq \mathcal{M}) = \{t \leqslant T' < T^* : t \in \mathcal{T} \text{ and } k_t^* < k_{It}\}$ such that the measure of $M > 0$. We then construct another scheme $< \{k_t'\}, T >$ such that:

$$k_t' = k_t^* \qquad \forall t \leqslant T^* \text{and}\, t \notin M$$
and $k_t' \in (k_t^*, k_{It})$ $\forall t \in M$.

By construction, $< \{k_t'\}, T >$ satisfies DIC.

Moreover, $\forall t \in M$, we have that $k_t^* < k_t' < k^*$[9] and consequently $[f(k_t') - k_t'] > [f(k_t^*) - k_t^*]$. Further, $\forall t \leqslant T^*$and $t \notin M$, we have that $k_t' = k_t^*$, and therefore $[f(k_t') - k_t'] = [f(k_t^*) - k_t^*]$. Therefore,

$$\int_0^{T^*} e^{r(T^*-t)} \alpha[f(k_t') - k_t']dt > \int_0^{T^*} e^{r(T^*-t)} \alpha[f(k_t) - k_t]dt = S.$$

Since time is continuous, $\exists \Delta' > 0$ such that $\int_0^{T^*-\Delta'} e^{r(T^*-\Delta'-t)} \alpha[f(k_t') - k_t']dt \geqslant S$ and $\Delta' < T^* - T'$. Thus, for Δ' small enough, we have constructed another scheme $< \{k_t'\}, T >$ that satisfies DIC and FC, and ends earlier than T^*. Thus, by the argument in Proposition 1, T^* can't be the optimum.

Step2. Consider the set $\mathcal{S} = \{t \leqslant T^* : k_{It} > k^*\}$. We want to show that $k_t^* = k^*$ for almost all $t \in \mathcal{S}$

i) The claim is that the measure of the set $\mathcal{N} = \{t \in \mathcal{S} : k_t^* > k^*\}$ is zero.

Suppose not. Then $\exists N (\subsetneq \mathcal{N}) = \{t \leqslant T'' < T^* : t \in \mathcal{S} \text{ and } k_t^* > k^*\}$ such that the measure of $N > 0$. We then construct another scheme $< \{k_t''\}, T >$ such that:

$$k_t'' = k_t^* \qquad \forall t \leqslant T^* \text{and}\, t \notin N$$
and $k_t'' \in (k^*, k_t^*)$ $\forall t \in N$.

By construction, $< \{k_t''\}, T >$ satisfies DIC[10].

Moreover, $\forall t \in N$, we have that $k^* < k_t'' < k_t^*$and consequently $[f(k_t'') - k_t''] > [f(k_t^*) - k_t^*]$. Further, $\forall t \leqslant T^*$and $t \notin N$, we have that $k_t'' = k_t^*$, and therefore $[f(k_t'') - k_t''] = [f(k_t^*) - k_t^*]$. Therefore,

$$\int_0^{T^*} e^{r(T^*-t)} \alpha[f(k_t'') - k_t'']dt > \int_0^{T^*} e^{r(T^*-t)} \alpha[f(k_t) - k_t]dt = S.$$

Since time is continuous, $\exists \Delta'' > 0$ such that $\int_0^{T^*-\Delta''} e^{r(T^*-\Delta''-t)} \alpha[f(k_t'') - k_t'']dt \geqslant S$ and $\Delta'' < T^* - T''$. Thus, for Δ'' small enough, we have constructed another scheme $< \{k_t''\}, T >$ that satisfies DIC and FC, and ends earlier than T^*. Thus, by the argument in Proposition 1, T^* can't be the optimum.

ii) The claim is that the measure of the set $\mathcal{P} = \{t \in \mathcal{S} : k_t^* < k^*\}$ is zero.

[9] Remember $\forall t \in M$ and $k_{It} \leqslant k^*$ and By construction $k_t^* < k_t' < k_{It}$

[10] k_t^* satisfies DIC, and $\forall t \leqslant T^*$ $k_t'' \leqslant k_t^*$

Suppose not. Then $\exists P(\subsetneq \mathcal{P}) = \{t \leqslant T^{'''} < T^* : t \in \mathcal{S} \text{ and } k_t^* < k^*\}$ such that the measure of $P > 0$. We then construct another scheme $< \{k_t^{'''}\}, T >$ such that:

$$k_t^{'''} = k_t^* \qquad \forall t \leqslant T^* \text{and}\, t \notin P$$

and $k_t^{'''} \in (k_t^*, k^*)$ $\forall t \in P$.

By construction, $< \{k_t^{'''}\}, T >$ satisfies DIC[11].

Moreover, $\forall t \in P$, we have that $k_t^* < k_t^{'''} < k^*$and consequently $[f(k_t^{'''}) - k_t^{'''}] > [f(k_t^*) - k_t^*]$. Further, $\forall t \leqslant T^*$and $t \notin P$, we have that $k_t^{'''} = k_t^*$, and therefore $[f(k_t^{'''}) - k_t^{'''}] = [f(k_t^*) - k_t^*]$. Therefore,

$$\int_0^{T^*} e^{r(T^*-t)}\alpha[f(k_t^{'''}) - k_t^{'''}]dt > \int_0^{T^*} e^{r(T^*-t)}\alpha[f(k_t) - k_t]dt = S.$$

Since time is continuous, $\exists \Delta^{'''} > 0$ such that $\int_0^{T^*-\Delta^{'''}} e^{r(T^*-\Delta^{'''}-t)}\alpha[f(k_t^{'''}) - k_t^{'''}]dt \geqslant S$ and $\Delta^{'''} < T^* - T^{'''}$.

Thus, for $\Delta^{'''}$ small enough, we have constructed another scheme $< \{k_t^{'''}\}, T >$ that satisfies DIC and FC, and ends earlier than T^*. Thus, by the argument in Proposition 1, T^* can't be the optimum.

From Steps 1 and 2 we can conclude that $k_t^* = \min\{k_{It}, k^*\}$ for almost all t. ■

Intuitively, the MFI wants to design the scheme in such a way that the borrower can connect with the formal sector as soon as possible. Thus ideally the MFI would like to lend an amount of k^* at every instant as this maximises the amount saved, enabling the borrower to achieve her savings target. However, due to ex post moral hazard problem, the MFI cannot lend more than $f^{-1}(e^{-r(T^*-t)}V)$ at every $t \leqslant T^*$.

So, the problem of the MFI becomes:

$$\underset{<\{k_t\}_{t=0}^{\infty}, T>}{\text{Maximize}} \qquad \int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t) - k_t]\Big]dt + e^{-rT}V$$

Subject to:

$$\text{FC:} \qquad \int_0^T e^{r(T-t)}\alpha[f(k_t) - k_t]dt = S, \tag{6}$$

$$\text{DIC:} \qquad k_t^* = \min\{k_{It}, k^*\} \text{for almost all } t \leqslant T^* \tag{7}$$

3.2 Dynamics of loan size

Now we are in a position to address our main finding of this paper. We will see that if fraction of forced savings is not too small then loan size will be non-decreasing over time. Hence, the next assumption.

Assumption 3 $1 - \alpha < r$

Also, we will assume that V is large, which will be used in the following lemma.

Assumption 4 $f(k^*) < V$.

Before stating our main result we need to introduce another definition.

[11]Remember, $k_t^{'''} \leq k^* < k_{It}$ $\forall t \leqslant T^*$

Definition 2 *Let $\hat{t}(T^*)$ solve*

$$k_{I\hat{t}} = k^*,$$

i.e. $\hat{t}(T^*)$[12] *denotes the t such that the maximum permissible loan according to the DIC at t is equal to the efficient amount of loan.*

Proposition 3 Characterisation of Loan Size
Given A3 and A4 there will be weak progressive lending for almost all $t \in [0, T^*]$.
Precisely, loan size will increase for almost all $t \in [0, \hat{t}(T^*))$ and then it will become constant over time.

To prove this we need the following lemmas.

Lemma 1 *Given A3 k_{It} is increasing over time $\forall t \in [0, T^*]$.*

Proof: From Definition 1 we know that

$$\text{DIC:} \qquad \int_t^T e^{-r(t'-t)}\Big[(1-\alpha)[f(k_{t'}) - k_{t'}]\Big]dt' + e^{-r(T-t)}V = f(k_{It}); \qquad \forall t \leqslant T^*$$

So, to prove that k_{It} is increasing over time $\forall t \in [0, T^*]$, it is sufficient to prove that L.H.S. of DIC is increasing over time[13] for any arbitrary $t (\in [0, T^*])$, given A3.

Notice L.H.S. of DIC at t $=(1-\alpha)\int_t^T e^{-r(t'-t)}[f(k_{t'}) - k_{t'}]dt' + e^{-r(T-t)}V$
$= (1-\alpha)e^{rt}\int_t^T e^{-rt'}[f(k_{t'}) - k_{t'}]dt' + e^{-r(T-t)}V$

Differentiating with respect to t
$(1-\alpha)r\int_t^T e^{-r(t'-t)}[f(k_{t'}) - k_{t'}]dt' - (1-\alpha)e^0[f(k_t) - k_t] + re^{-r(T-t)}V$
$= (1-\alpha)r\int_t^T e^{-r(t'-t)}[f(k_{t'}) - k_{t'}]dt' - (1-\alpha)[f(k_t) - k_t] + re^{-r(T-t)}V$
$= r\Big[(1-\alpha)r\int_t^T e^{-r(t'-t)}[f(k_{t'}) - k_{t'}]dt' + e^{-r(T-t)}V\Big] - (1-\alpha)[f(k_t) - k_t]$
$= rf(k_t) - (1-\alpha)[f(k_t) - k_t]$
where the last equality comes from DIC.

Given A3 $rf(k_t) - (1-\alpha)[f(k_t) - k_t] > 0$[14]. Hence the result. ■

Lemma 2 *Given Assumption 3 and Assumption 4:*

1. *$\hat{t}(T^*) < T^*$, $\forall S$.*
2. *$k_{It} > k^*$, $\forall t > \hat{t}(T^*)$.*

Proof:

1. Observe that, $k_{IT} = f^{-1}((1-\alpha)[f(k_T) - k_T] + V) > k^*$ where the last inequality comes from A4[15]. And as we have observed in the earlier Lemma k_{It} increases monotonically with t given Assumption 3. Also, k_{It} is continuous. Hence, $\hat{t}(T^*) < T^*$, $\forall S$.
2. It is straight forward from the previous argument. ■

[12] In *Lemma1* we will prove that given A3 k_{It} is increasing over time $\forall t \in [0, T^*]$, so $\hat{t}(T^*)$ is unique.
[13] By A1 $f'(\cdot) > 0$
[14] Note $f(k_t) > f(k_t) - k_t$
[15] By A4 $V > f(k^*)$ and $(1-\alpha)[f(k_T) - k_T] > 0$, thus $(1-\alpha)[f(k_T) - k_T] + V > f(k^*)$

Now we are in a position to prove Proposition 3.

Proof of Proposition 3: From Lemma 2 we know $k_{It} < k^*$, $\forall t < \hat{t}(T^*)$, hence $k_t^* = k_{It}$ for almost all $t < \hat{t}(T^*)$ and from Lemma 1 we know that k_{It} is increasing $\forall t \leqslant \hat{t}(T^*)(< T^*)$. Hence, loan size will increase for almost all $t < \hat{t}(T^*)$.
And from $\hat{t}(T^*)$ onwards $k^* = \min\{k^*, k_{It}\}$ hence $k_t^* = k^*$ for almost all $t \in [\hat{t}(T^*), T^*]$.
Hence, the result. ■

4 Oustside Savings Option

Suppose the MFI does not provide the savings service, but the individual has access to some other savings technology. The individual saves α part of her income at every instant of time, interest rate is r. And, the individual cannot withdraw the money before T.

So, the problem of the MFI, here, is to:

$$\underset{<\{k_t\}_{t=0}^{\infty}, T>}{\text{Maximize}} \qquad \int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt - S + V\right]$$

Subject to:

$$\text{FC}'' : \quad \int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt \geqslant S, \tag{8}$$

$$\begin{aligned}\text{DIC}'' : \quad & \int_t^T e^{-r(t'-t)}\Big[(1-\alpha)[f(k_{t'})-k_{t'}]\Big]dt' + e^{-r(T-t)}\left[\int_0^T e^{r(T-t'')}\alpha[f(k_{t''})-k_{t''}]dt - S + V\right] \\ & \geqslant f(k_t) + e^{-rT}\int_0^t e^{r(T-t)}\alpha[f(k_{t''})-k_{t''}]dt; \qquad \forall t \leqslant T. \end{aligned} \tag{9}$$

Following the argument of Proposition 1 and 2 we can observe, in this case also, that FC $''$ will bind at optimum and $k_{st}^* = \min\{k_{sIt}, k^*\}$[16].

Proposition 4 *Borrower's utility decreases when outside savings option is available.*

Proof: Observe that, $k_{sIt} < k_{It} \forall t$. So, $T_s^* > T^*$. Hence, the borrower's utility is more when the MFI offers the savings service. ■

This result is quite intuitive. When the borrower saves with MFI, she has more incentive to repay as otherwise she will lose the entire amount she saved till that period. On the other hand, when she saves anywhere else, she gets back the saved amount after T period, so incentive to repay decreases which in turn decreases the amount of loan in equilibrium, thus the time required to save S increases, hence the utility of the borrower decreases.

5 Comparative Statics

In this section we will explore the effect of changes in the values of different parameters on borrower's welfare. We will start with the case where the MFI faces competition, then we will move to the case where the fraction of forced savings changes and then we will change the interest rate on deposit.

[16] $< \{k_{st}^*\}_{t=0}^{\infty}, T_s^* >$, where T_s^* is the time required to save S under this scheme. And k_{sIt} is the amount of loan for which the DIC $''$ at t binds.

5.1 Competitive MFI

Now, suppose the MFI faces competition, such that the cost of capital increases, so even a benevolent MFI will not be able to charge zero interest rate in order to sustain. Let for k amount of capital the borrower has to repay ck where $c > 1$. However, we are assuming that double dipping is not possible. We are assuming this just to focus exclusively on the effect of change in cost of capital, that is even if it is possible to restrict the villagers to borrow from more than one MFI-s, what will be the effect of increase in competition among MFI-s.

In this set up we will define the efficient scale of investment k_c^* as

$$k_c^* = \underset{k}{\text{argmax}}[f(k) - ck]^{17}$$

So, in this setup the problem of the MFI-s becomes:

$$\underset{<\{k_t\}_{t=0}^{\infty}, T>}{\text{Maximize}} \qquad \int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t) - ck_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t) - ck_t]dt - S + V\right]$$

Subject to:

$$\text{FC}': \quad \int_0^T e^{r(T-t)}\alpha[f(k_t) - ck_t]dt \geqslant S, \tag{10}$$

$$\text{DIC}': \quad \int_t^T e^{-r(t'-t)}\Big[(1-\alpha)[f(k_{t'}) - ck_{t'}]\Big]dt' + e^{-r(T-t)}\left[\int_0^T e^{r(T-t'')}\alpha[f(k_{t''}) - ck_{t''}]dt - S + V\right] \geqslant f(k_t); \quad \forall t \leqslant T \tag{11}$$

Clearly, when A2 holds the inequality $f(k_c^*) - ck_c^* < r(V - S)$ also holds[18].

So, our next proposition is:

Proposition 5 *Let A1 and A2 hold. Then the modified feasibility constraint (FC$'$) binds at the optimal contract.*

Proof: The proof of this proposition is same as the proof of Proposition1.

Step 1. We first prove that the borrower's utility decreases with T, so that

$$\frac{d}{dT}\left[\int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t) - ck_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t) - ck_t]dt - S + V\right]\right] < 0.$$

Note that, $e^{-rT}\Big[\int_0^T e^{r(T-t)}\alpha[f(k_t) - ck_t]dt - S + V\Big] = \int_0^T e^{-rt}\alpha[f(k_t) - ck_t]dt - e^{-rT}\Big[V - S\Big]$. Thus,

$$\int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t) - ck_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t) - ck_t]dt - S + V\right] = \int_0^T e^{-rt}[f(k_t) - ck_t]dt - e^{-rT}\Big[V - S\Big]$$

$$\begin{aligned}
&\frac{d}{dT}\left[\int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t) - ck_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t) - ck_t]dt - S + V\right]\right] \\
&= \frac{d}{dT}\left[\int_0^T e^{-rt}[f(k_t) - ck_t]dt - e^{-rT}\Big[V - S\Big]\right] \\
&= e^{-rT}[f(k_T) - ck_T] - re^{-rT}(V - S) \\
&= e^{-rT}\Big[[f(k_T) - ck_T] - r(V - S)\Big].
\end{aligned} \tag{12}$$

[17]Observe , $f'(k_c) = c > 1 = f'(k)$ and since $f(\cdot)$ is a concave function $k_c^* < k$

[18]By definition $f(k^*) - k^* > f(k_c^*) - k_c^* > f(k_c^*) - ck_c^*$ where the last inequality comes from the fact that $c > 1$

Clearly, given A2, it must be that $f(k_T) - ck_T - r(V-S) < 0$, so that the RHS of (12) is negative.[19]

Step 2. Now suppose the FC does not bind. Consider another scheme that is identical to the original scheme, but ends at $T' < T$. For T' close to T, FC will be satisfied, and the borrower's utility increases by the preceding argument. Finally, DIC holds for all $t \leq T'$, as the original scheme satisfied the DIC and $T' < T$ (we mimic the argument in step 1 to prove this). ■

Let the optimum scheme be denoted by $< \{k^*_{ct}\}_{t=0}^{\infty}, T^*_c >$, where T^*_c is the time required to save S under this scheme, i.e.

$$\int_0^{T^*_c} e^{r(T^*_c - t)} \alpha[f(k^*_{ct}) - ck^*_{ct}]dt = S.$$

So, now the problem of the MFI is:

$$\underset{<\{k_t\}_{t=0}^{\infty},T>}{\text{Maximize}} \qquad \int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t) - ck_t]\Big]dt + e^{-rT}V$$

Subject to:

$$\text{FC}\,' : \quad \int_0^T e^{r(T-t)} \alpha[f(k_t) - ck_t]dt = S, \tag{13}$$

$$\text{DIC}\,' : \quad \int_t^T e^{-r(t'-t)}\Big[(1-\alpha)[f(k_{t'}) - ck_{t'}]\Big]dt' + e^{-r(T-t)}V \geqslant f(k_t); \quad \forall t \leqslant T. \tag{14}$$

Like before, we will introduce the following definition and after that the proposition.

Definition 3 $\mathbf{k_{cIt}}$ *is the amount of loan for which the modified dynamic incentive compatibility constraint (DIC$'$) binds at time period t .*

Proposition 6 *Consider the optimal scheme* $< \{k^*_{ct}\}, T^*_c >$. *Then* $k^*_{ct} = \min\{k_{cIt}, k^*_c\}$ *for almost all t.*

Proof: Mimicing the argument of proof of Proposition 2, this proposition can be proved.

There can be two situations:

Either $k_{cIt} \leqslant k^*_c$. Just like before it can be shown that, in this case, $k^*_{ct} = k_{cIt}$ almost everywhere.

Or, $k_{cIt} > k^*_c$. Similarly mimicing the previous argument it can be shown that, in this case, $k^*_{ct} = k^*_c$ almost everywhere.

And, from these two steps we can conclude that $k^*_{ct} = \min\{k_{cIt}, k^*_c\}$ for almost all t. ■

So, the problem of MFI becomes:

$$\underset{<\{k_t\}_{t=0}^{\infty},T>}{\text{Maximize}} \qquad \int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t) - ck_t]\Big]dt + e^{-rT}V$$

Subject to:

$$\text{FC}\,' : \qquad \int_0^T e^{r(T-t)} \alpha[f(k_t) - ck_t]dt = S, \tag{15}$$

$$\text{DIC}\,' : \qquad k^*_{ct} = \min\{k_{cIt}, k^*_c\} \ \text{ for almost all } t \leqslant T^*_c \tag{16}$$

Now, we need to introduce another definition:

[19] $f(k^*_c) - ck^*_c = \text{Max}[f(k) - ck]$ i.e. $f(k_T) - ck_T \leqslant f(k^*_c) - ck^*_c$. And, as we have observed when A2 holds the inequality $f(k^*_c) - ck^*_c < r(V-S)$ also holds. Therefore, when A2 holds $f(k_T) - ck_T < r(V-S)$.

Definition 4 *Let $\hat{t}_c(T_c^*)$ solve*

$$k_{cI\hat{t}_c} = k_c^*,$$

i.e. $\hat{t}_c(T_c^*)$ denotes the t such that the maximum permissible loan according to the DIC$'$ at t is equal to the efficient amount of loan.

Proposition 7 *Given A3 and A4 there will be weak progressive lending for almost all $t \in [0, T_c^*]$. Precisely, loan size will increase for almost all $t \in [0, \hat{t}_c(T^*))$ and then it will become constant over time.*

To prove this we will first prove the following lemmas.

Lemma 3 *Given A3 k_{cIt} is increasing over time $\forall t \in [0, T_c^*]$.*

Proof: From Definition 3 we know that

$$\text{DIC}': \quad \int_t^T e^{-r(t'-t)}\Big[(1-\alpha)[f(k_{t'}) - ck_{t'}]\Big]dt' + e^{-r(T-t)}V = f(k_{cIt}) \quad \forall t \leqslant T_c^*$$

So, to prove that k_{cIt} is increasing over time $\forall t \in [0, T_c^*]$, it is sufficient to prove that L.H.S. of DIC$'$ is increasing over time[20] for any arbitrary $t (\in [0, T_c^*])$, given A3.

Notice L.H.S. of DIC$'$ at t $= \int_t^T e^{-r(t'-t)}\Big[(1-\alpha)[f(k_{t'}) - ck_{t'}]\Big]dt' + e^{-r(T-t)}V$
$= (1-\alpha)e^{rt}\int_t^T e^{-rt'}[f(k_{t'}) - ck_{t'}]dt' + e^{-r(T-t)}V$

Differentiating with respect to t
$(1-\alpha)r\int_t^T e^{-r(t'-t)}[f(k_{t'}) - ck_{t'}]dt' - (1-\alpha)e^0[f(k_t) - ck_t] + re^{-r(T-t)}V$
$= (1-\alpha)r\int_t^T e^{-r(t'-t)}[f(k_{t'}) - ck_{t'}]dt' - (1-\alpha)[f(k_t) - ck_t] + re^{-r(T-t)}V$
$= r\Big[(1-\alpha)r\int_t^T e^{-r(t'-t)}[f(k_{t'}) - ck_{t'}]dt' + e^{-r(T-t)}V\Big] - (1-\alpha)[f(k_t) - ck_t]$
$= rf(k_t) - (1-\alpha)[f(k_t) - ck_t]$
where the last equality comes from DIC$'$.

Given A3 $rf(k_t) - (1-\alpha)[f(k_t) - ck_t] > 0$[21]. Hence the result. ■

Lemma 4 *Given A3 and A4:*

1. *$\hat{t}_c(T_c^*) < T_c^*$, $\forall S$.*
2. *$k_{cIt} > k_c^*$, $\forall t > \hat{t}_c(T_c^*)$.*

Proof:

1. Observe that, $k_{cIT} = f^{-1}((1-\alpha)[f(k_T) - ck_T] + V) > k_c^*$ where the last inequality comes from A4[22]. And as we have observed in the earlier Lemma k_{cIt} increases monotonically with t given A3. Also, k_{cIt} is continuous. Hence, $\hat{t}_c(T_c^*) < T_c^*$, $\forall S$.
2. It is straight forward from the previous argument. ■

[20] By A1 $f'(\cdot) > 0$
[21] Note $f(k_t) > f(k_t) - ck_t$
[22] By A4 $V > f(k^*) > f(k_c^*)$ and $(1-\alpha)[f(k_T) - ck_T] > 0$, thus $(1-\alpha)[f(k_T) - k_T] + V > f(k^*)$

Now we are in a position to prove Proposition 7.

Proof of Proposition 7: From Lemma 4 we know $k_{cIt} < k_c^*$, $\forall t < \hat{t}_c(T_c^*)$, hence $k_{ct}^* = k_{cIt}$ for almost all $t < \hat{t}_c(T_c^*)$ and from Lemma 3 we know that k_{cIt} is increasing $\forall t \leqslant \hat{t}_c(T_c^*)(< T^*)$. Hence, loan size will increase for almost all $t < \hat{t}_c(T_c^*)$.
And from $\hat{t}_c(T_c^*)$ onwards $k_c^* = \min\{k_c^*, k_{cIt}\}$ hence $k_{ct}^* = k_c^*$ for almost all $t \in [\hat{t}_c(T_c^*), T_c^*]$.
Hence, the result. ■

Welfare Analysis:

Proposition 8 *Utility of the borrower decreases as competition among MFI-s increases.*

Proof: Observe, $k_{ct}^* < k_t^*$ $\forall t$[23]. Hence, $T^* < T_c^*$ And we have already observed that borrower's utility decreases with T. Hence, welfare of the borrower will decrease as competition increases, which is captured by c. ■

This result is quite intuitive in that as competition increases the cost of capital increases so the borrower has to repay more for same amount of loan, so the net income of the borrower decreases. Hence, per period savings decreases, so the time required to save S increases, hence, that means the borrower remains in the poverty trap for longer period of time. Hence, the borrower's utility decreases.

5.2 Change in the fraction of forced savings

In this section we will observe the effect of increase in the fraction of compulsory savings, that is α on borrower's utility.

The problem of the MFI is to

$$\underset{<\{k_t\}_{t=0}^{\infty}, T>}{\text{Maximize}} \quad \int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt - S + V\right]$$

Subject to:

$$\text{FC:} \quad \int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt \geqslant S, \tag{17}$$

$$\text{DIC:} \quad \int_t^T e^{-r(t'-t)}\Big[(1-\alpha)[f(k_{t'})-k_{t'}]\Big]dt' + e^{-r(T-t)}\left[\int_0^T e^{r(T-t'')}\alpha[f(k_{t''})-k_{t''}]dt'' - S + V\right] \geqslant f(k_t); \quad \forall t \leqslant T. \tag{18}$$

As earlier the at equilibrium it will boil down to:

$$\underset{<\{k_t\}_{t=0}^{\infty}, T>}{\text{Maximize}} \quad \int_0^T e^{-rt}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-rT}V$$

Subject to:

$$\text{FC:} \quad \int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt = S, \tag{19}$$

$$\text{DIC:} \quad k_t^* = \min\{k_{It}, k^*\} \text{for almost all } t \leqslant T^* \tag{20}$$

Now, suppose α, the fraction of forced savings increases, the following proposition summarizes the effect of that increase on borrower's utility.

[23] $k_{cIt} < k_{It}$ and $k_c^* < k^*$. Hence $\min\{k_c^*, k_{cIt}\}$ <$\min\{k^*, k_{It}\}$

Proposition 9 *Given A2, utility of a borrower increases with α*[24].

Proof:
Step1 Firstly, we will prove that as α increases optimum T, that is, time required to save S decreases.
So, we will show: $\left.\frac{dT}{d\alpha}\right|_S < 0$
Differentiating FC with respect to α we get

$$\frac{dT}{d\alpha} = -\frac{S}{\alpha^2[f(k_T) - k_T]} < 0$$

So, as α increases T decreases. Let, the new optimum time to save S be T^*_α

Step2 Now, we will show that dynamic incentive compatibility constraint(DIC) will be relaxed with increase in α. For that we will show that L.H.S. of DIC will increase.

Note that,

$$\begin{aligned}\text{L.H.S. of DIC} &= \int_t^T e^{-r(t^{'}-t)}\Big[(1-\alpha)[f(k_{t^{'}}) - k_{t^{'}}]\Big]dt^{'} + e^{-r(T-t)}V \\ &= e^{rt}\int_t^T e^{-rt^{'}}(1-\alpha)[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} + e^{rt}\left[\int_0^T e^{-rt^{'}}\alpha[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} - S\right] + e^{-r(T-t)}V \\ &= e^{rt}\int_0^T e^{-rt^{'}}[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} - e^{rt}\int_0^t e^{-rt^{'}}(1-\alpha)[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} - e^{rt}S + e^{-r(T-t)}V\end{aligned}$$

where the first equality comes from FC.
Differentiating DIC with respect to α we get

$$\begin{aligned}& e^{rt}e^{-rT}\big[f(k_T) - k_T\big]\frac{dT}{d\alpha} + e^{rt}\int_0^t e^{-rt^{'}}[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} - re^{-r(T-t)}V\frac{dT}{d\alpha} \\ =& e^{-r(T-t)}\frac{S}{\alpha^2}\left[\frac{rV}{f(k_T) - k_T} - 1\right] + e^{rt}\int_0^t e^{-rt^{'}}[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} \\ =& e^{-r(T-t)}\frac{S}{\alpha^2[f(k_T) - k_T]}\Big[rV - [f(k_T) - k_T]\Big] + e^{rt}\int_0^t e^{-rt^{'}}[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} \\ >& 0\end{aligned}$$

where the last inequality comes from *A2* [25].

So k_{It} will increase $\forall t \leq T^*_\alpha$, so $\hat{t_\alpha}(T^*_\alpha)$ will decrease, hence, the borrower will start getting k^* faster. So, that will further decrease T and so on. And from *Step1* of *Proposition1* we know that the utility of the borrower increases as T decreases. Hence the result. ■

[24]However, note that we are getting this result due to the particular form of utility function. In more realistic situation minimum amount of consumption is required for subsistence. So, the borrower would have valued consumption more when α is very high, in that case borrower's utility will decrease with increase in α. More, precisely there would exist a threshold value of α such that utility of the borrower will decrease if α is increased beyond that threshold. We are not using that here, as it will complicate the analysis without adding anything to the qualitative result of this paper.

[25]By A2 $r(V-S) > f(k^*) - k^* \geq f(k_T) - k_T$. So, $rV > f(k_T) - k_T$ And, $e^{rt}\int_0^t e^{-rt^{'}}[f(k_{t^{'}}) - k_{t^{'}}]dt^{'} > 0$.

Intuitively, this result is quite plausible. As the fraction of forced savings increases the borrower's instantaneous consumption decreases but his savings increases and he can acheive V faster, so if V is sufficiently large(captured through A2), the instantaneous loss will be much lesser than the gain from obtaining V faster. So, the utility of every instant increases which in turn relaxes the dynamic incentive compatibility constraint, hence per period loan amount (weakly) increases. Hence, the borrower will be able to acheive the required savings sooner, which will increase her utility and so on. Hence, the result.

5.3 Change in the interest rate on deposit

In this section we will observe the effect of increase in the interest rate on deposit on borrower's utility. For that we will first differentiate between interest rate on deposit and discount rate of the borrower. Let, interest rate on deposit be r like before and discount rate of the borrower be δ.
So, in this set up the problem of the MFI is:

$$\underset{<\{k_t\}_{t=0}^{\infty},T>}{\text{Maximize}} \quad \int_0^T e^{-\delta t}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-\delta T}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt - S + V\right]$$

Subject to:

$$\text{FC}'' : \quad \int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt \geqslant S, \tag{21}$$

$$\text{DIC}'' : \quad \int_t^T e^{-\delta(t'-t)}\Big[(1-\alpha)[f(k_{t'})-k_{t'}]\Big]dt' + e^{-\delta(T-t)}\left[\int_0^T e^{r(T-t'')}\alpha[f(k_{t''})-k_{t''}]dt - S + V\right] \geqslant f(k_t); \quad \forall t \leqslant T. \tag{22}$$

In this set up the modified form of assumption 2 is **Assumption** $2'$ $f(k^*) - k^* < \delta(V-S)$.

Proposition 10 *Given A2, FC$''$ will bind at optimum.*

Proof: Step 1. We first prove that the borrower's utility decreases with T, so that

$$\frac{d}{dT}\int_0^T e^{-\delta t}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-\delta T}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt - S + V\right] < 0.$$

Now, observe

$$\begin{aligned}
&\frac{d}{dT}\int_0^T e^{-\delta t}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-\delta T}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt - S + V\right] \\
=&e^{-\delta T}(1-\alpha)[f(k_T)-k_T] - \delta e^{-\delta T}\left[\int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt - S + V\right] + e^{-\delta T}\alpha[f(k_T)-k_T] \\
=&e^{-\delta T}\left[\big[f(k_T)-k_T-\delta(V-S)\big] - \delta\int_0^T e^{r(T-t)}[f(k_t)-k_t]dt\right]
\end{aligned}$$

Clearly, given A2$'$, it must be that $f(k_T) - k_T - \delta(V-S) < 0$, so that the RHS of (14) is negative.[26]
Now, following *Step2* of *Proof of Proposition1* we can similarly conclude that FC$''$ will bind at optimum.■

[26] $f(k^*) - k^* = \text{Max}[f(k)-k]$ i.e. $f(k_T) - k_T \leqslant f(k^*) - k^*$. Therefore, when A2$'$ holds $f(k_T) - k_T < \delta(V-S)$. And also $\delta\int_0^T e^{r(T-t)}[f(k_t)-k_t]dt > 0$

DIC has not changed, so the problem of MFI remains:

$$\underset{<\{k_t\}_{t=0}^{\infty},T>}{\text{Maximize}} \qquad \int_0^T e^{-\delta t}\Big[(1-\alpha)[f(k_t)-k_t]\Big]dt + e^{-\delta T}\Big[V-S\Big]$$

Subject to:

$$\text{FC}'' : \quad \int_0^T e^{r(T-t)}\alpha[f(k_t)-k_t]dt = S, \tag{23}$$

$$\text{DIC}'' : \quad k_t^* = \min\{k_{It}, k^*\}\text{for almost all } t \leqslant T^*. \tag{24}$$

Now, let r is changed. The effect of change in r on borrower's welfare is summarised in the following proposition.

Proposition 11 *Borrower's utility increases with increase in the rate of interest on deposit.*

Proof:
Step1 Firstly, we will prove that as α increases optimum T, that is, time required to save S decreases.

So, we will show: $\left.\frac{dT}{dr}\right|_S < 0$

Differentiating FC$''$ with respect to r we get

$$\begin{aligned}\frac{dT}{d\alpha} &= -\frac{rS}{\alpha[f(k_T)-k_T]}\\ &< 0\end{aligned}$$

So, as r increases T decreases. Let, the new optimum time to save S be T_r^*

Step2 Now, we will show that dynamic incentive compatibility constraint(DIC) will be relaxed with increase in r. For that we will show that L.H.S. of DIC will increase with increase in r.

Differentiating DIC with respect to α we get

$$\begin{aligned}&\Bigg[e^{-\delta(T-t)}(1-\alpha)\big[f(k_T)-k_T\big] - \delta e^{-\delta(T-t)}V\Bigg]dT\\ =&\frac{e^{-\delta(T-t)}rS}{\alpha[f(k_T-k_T]}\Big[\delta V - [f(k_T)-k_T]\Big] + e^{-\delta(T-t)}rS\\ >&0\end{aligned}$$

where the last inequality comes from $A2'$ [27].

So k_{It} will increase $\forall t \leq T_r^*$, so $\hat{t_r}(T_r^*)$ will decrease, hence, the borrower will start getting k^* faster. So, that will further decrease T and so on. And from *Step1* of *Proposition1* we know that the utility of the borrower increases as T decreases. Hence the result. ■

Intuitively, it is quite obvious. As interest rate on deposit increases the borrower can fulfill the savings requirement faster and thus he gets V sooner. Hence, her utility increases with the increase in rate of interest on deposit.

[27] By A2$'$ $\delta(V-S) > f(k^*) - k^* \geq f(k_T) - k_T$. So, $\delta V > f(k_T) - k_T$ And, $e^{-\delta(T-t)}rS > 0$.

6 Conclusion

In this paper we have addressed two important institutional aspects of microfinance - progressive lending and compulsory savings. In a dynamic framework with continuous time, we have shown that the optimal contract involves progressive lending, with compulsory savings helping the poor to build collateral, which they can use to connect with the formal sector, and consequently escape the poverty trap. This analysis underlines the importance of progressive lending, as well as forced savings. It has been argued that MFIs should provide savings technology to its clients so that they can "turn small, regular savings into usefully large sums"which can be used for "investment, gaining ownership of assets and managing consumption". The analysis here suggests that micro-finance and micro-savings need to be studied in unison, and provides a starting point of this analysis.

Next note that the analysis can be extended to allow for multiple borrowers. We show that even in this framework, the optimal contract involves terminating the contract as soon as the minimal amount of savings has been achieved. In the presence of credit contained MFIs, which is realistic, the MFI will have an even greater incentive to do so, so that the loan amount can be used for lending to other borrowers. Thus the analysis goes through.

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