Procurement Contracts and Adaptation Costs

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ABSTRACT

We analyze the procurement contracts used by private and public sector entities and the associated incentive structures. We explicitly model designing, organization and implementation stages. We show complementarities between the efforts put at different stages, and their consequences for completeness of contract and 'adaptation costs'. Without using assumption of asymmetric information among the contracting parties, we show efficiency enhancing role of designing efforts. Role of ex-post monitoring is also analyzed. The comparative statics in the paper offer new insights for real world phenomena.

1 Introduction

Procurement is a significant activity all over the world. Both public and private sector entities contract for procurement of several kind of goods and services including infrastructure. Various types of procurement contracts are used for the purpose. In this paper, we model incentive structure induced by used procurement contracts used in the real world, including, 'Design-Bid-Build' (DBB) and 'Design and Build' (DB) contracts and their different variants like 'Item Rate' (IR) contracts as well as the cost sharing contracts.

Procurement contracts for infrastructure and several other goods require the contractor to execute construction activities as a part of the project. The real world construction projects face several kind of uncertainty during the implementation phase. As a result, on several occasions the construction plans and designs have

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to be changed. Moreover, the mutually agreeable changes have be negotiated by buyer and the contractor. Bajari et al 2014, using the data on California highway projects, showed that costs of adapting to such changes are as high as 11% of the contract price. But these 'Adaptation Costs' are not adequately modeled and analyzed in the existing theoretical literature on procurement contracts. The literature on construction engineering suggests that efforts put in at various stages of project development play role in reducing these costs. Therefore, these costs may not be exogenous, and can be influenced by the parties involved in the project.

In this paper, we model the efforts that can be put by the buyer and the contractor. We analyze the effect these efforts on various project costs, including the costs of implementing mid-way changes during the construction phase of the project. We show that the buyer can affect these costs through better planning and contractor by better organization of different work tasks needed for the implementation cost[1]. on adaptation costs.

Even if the concerned parties are risk neutral, the benefit from these efforts are shown to be falling in the uncertainty, the probability and the magnitude of adaptations made. Planning and designing effort can in-fact, affect these uncertainties. In addition, there can be complementarities between the two efforts but these are not at all modeled in the existing literature. Better planning is expected to aid in better organization. We show that the analysis of such complementarities offer new insights into the observed real life phenomena which are not explained by existing literature.

The public sector entities are generally required to provide all of the relevant information to the bidder contractors. In such a scenarios, both of the contracting parties are expected to have similar information about the project. Moreover, empirical literature has shown that in case of infrastructure and construction projects, ex-ante all parties, the buyer and the contractor, face same uncertainties and have symmetric information (see Bajari and Tadelis 2001). Nonetheless, since 1980's the procurement contracts have been analyzed under different asymmetric information frameworks, ranging from static adverse selection and moral hazard to their dynamic counterparts, with and without commitment. If information is taken to be symmetric, then existing models imply no efficiency role of initial planning. In contrast, we assuming symmetric information throughout. Still, we show that initial planning and completeness of contract has important efficiency implications both of which are endogenous to the model.

One would expect different contracts to provide different incentives to the contracting parties. As a result, the adaptation costs are also expected to vary with different contracts. However, these aspects have not been analyzed in the existing literature. We model these costs in what we consider is a novel way and analyze them for widely used contracts; including 'Item Rate' (IR) contracts. These contract have

[1]Bajari and Lewis 2011 have considered the case where work rate adapts to productivity shocks given that contracts have time incentives. But they did not consider initial planning and its impact on adaptation cost.

not been rigourously analyzed in the existing literature.

We compare Design and build (DB) and Design-bid-build (DBB) contracts. Latter includes cost sharing contracts including cost plus and fixed price contracts. We show that for given design, cost sharing (including cost plus) contracts are dominated by item rate contracts which are in turn dominated by DB contracts but this dominance no longer holds when change in design is allowed for.

For all contracts, initial planning and organization effort and the adaptation costs are increasing in the learning of the designer and falling in the technical complexity of the project. In contrast to result in existing literature that there will always be under-investment in initial planning (Ganuza 2007), we show that over-investment is also possible.

Moreover, we consider different compensation schemes at the time of renegotiation. In addition to Nash bargaining, widely used cost sharing and market rate pricing are also considered. Nash bargaining based compensation at the time of renegotiation implies that initial contract price will increase with probability of renegotiation. But Bajari et al 2014 have shown that in fact opposite is true. This phenomenon is explained by the model if compensation scheme is based on cost sharing or market rates. We show that the relationship between organization effort and ex-post bargaining power of the contractor is monotonic while that between initial planning (and expected cost) and ex-post bargaining power is non-monotonic.

We show that the incentives provided to designers also have effect on incentives of the contractor and vice-versa. If contract is cost plus, i.e., bad incentives for contractor, then planning will also be low leading to more incomplete contract. Thus C+ contracts are found to be the most incomplete of all. While IR and cost sharing contracts may be excessively complete. Finally, we also show that initial planning and ex-post monitoring are substitutes. These pattern find support in the empirical literature.

In section 2, we present the model. In section 3, the expressions for expected costs are derived, social optimization problem is solved and first best level of designing and organization effort are computed. Section 4 characterizes equilibria of different contracts and provides comparative statics results. Section 5 and 6 we allow for change in design and the model the associated 'destruction cost'. Section 7 concludes. Proves are relegated to appendix.

2 Model

Suppose buyer/sponsor wants to procure a project[2]. Procurement of a project involves various stages. It starts with the **planning and designing** stage of the project. The planning and designing is characterized by several activities. The first

[2]Below we will use buyer and sponsor interchangeably. The procurement of a good, a service or construction and implementation of an engineering project, all will be called a 'project'. The distinction will be made below from time to time as and when necessary.

is the description of the 'buyer's requirement'. The buyer's requirement specifies the 'output' to be delivered and the associated quality standards. The output can be as simple as stationery, buses, wagons or as complex as military equipments, infrastructure projects, like roads, railways etc. For an expressway project, the output generally specifies the length, the number of traffic lanes, the number and location of cross-section, by-passes, under-passes, over-passes, cross-section, toll-plazas, major, medium and small bridges, service roads, etc. Renovation may specify the extent of the repair to be done.

Once the output features of the project are decided at $T = 0$, the next task is the choice of 'design' to implement that output. A 'design' is an engineering plan which specifies the list of engineering works/tasks (popularly known as work-items) and provides the description of work procedures, sequencing of different works required to complete the project. Thus it specifies what is to be done and how it is to be done. For example, a typical road project requires many works to be done, such as, construction of embankment, construction of subgrade, building of earthen and concrete shoulders, fixing of drainage spouts, laying of boulder apron, among many others. The table lists a total of 78 major and 26 minor activities for a bridge work corresponding to a given output and given design.

Let $\{\omega_i | i \in [0, \overline{W}]\}$ denote the set of all possible works needed to be performed for the given design and for the given output. Each work item is indexed by a real number $i \in [0, \overline{W}]$. Plausibly, $0 < \overline{W}$. They are ordered in increasing order of complexity, i.e., a work item with the larger index represents greater complexity[3]. For mathematical convenience, we will represent work items by their corresponding indices. So equivalently, we will denote the set of work items by $[0, \overline{W}]$.

Before construction actually begins, the quantities and per unit costs of the project work items are estimated. For example, a project may be estimated to require construction of 4500 cum of embankment, 6000 cum of subgrade and earthen shoulder, 10,000 sqm of bituminous prime coat etc. So each work item is required to be performed in different magnitudes with different unit of measurement. This is what we call 'quantities'. So for the example above, the expected quantity measured in cubic meters, for the work item- construction of embankment, is 4500. Let q_w^e denotes the *estimated* quantity of the w^{th} work-item/activity and c_w^e denotes the *estimated* per unit cost of the w^{th} work-item/activity.

But given the design/engineering plan, the actual cost of a project work invariably turns out to be different from its estimated value. This can happen either because the actual quantities or per-unit costs or both turn out to be different from their estimated values which in turn is the result of imperfect estimation techniques available to the designer. Let q_w^a denote the *actual* quantity of the w^{th} work-item/activity, and c_w^a

[3]As shown below, the set of work items may vary across different project designs and across different buyer's requirements/output features.

denote the *actual* per unit cost of the w^{th} work-item/activity[4].

Following the literature and to keep analysis simple, we assume that the quantities and cost related contingencies become observable to the buyer and the contractor at the beginning of construction, i.e., at $T = 2$. In particular, at $T = 2$, q_w^a as well as c_w^a become known for each $w \in [0, \overline{W}]$; at $T = 0$, there is uncertainty about both the quantities and their per unit costs for each work item $w \in [0, \overline{W}]$.

However, at the beginning of the project designing stage, say at $T = 1$, the designers put in effort to estimate the values of yet unknown vectors $\mathbf{q^a} = (q_w^a)$ and $\mathbf{c^a} = (c_w^a)$. Let d denotes the unverifiable but observable designing effort. As a result of this effort, the designers get publicly observable signals of quantity-relevant and cost-relevant states of nature. Alternatively put, the effort d produces signals/estimates of quantities, and their respective per-unit costs. More importantly, d affects the precision of these signals. The effort d and other efforts modeled below in the paper are measured by their respective costs.

Now we explicitly model the effect of this planning and designing effort d and the relation between expected and actual quantities and per unit costs. Let us first discuss the **quantity relevant states of nature** [5]. The vector of actual quantities $\mathbf{q^a}$ depends on the state of nature, i.e., the conditions on the project site. For example, the type of optimum mixture of the concrete and bitumen required, the kind of foundations needed for flyovers, etc., depend on the quality of soil at the project site. Initially the quantities of concrete and bitumen are specified to attain a given mix but if the optimum mixture needed ex-post turns out to be different, then their actually used quantities will also be different. Similar will be true for the material needed for foundation. Formally speaking, at $T = 0$, $\mathbf{q^a}$ is a random variable. In the discussion that follows, a letter with $\tilde{}$ stands for the random variable while letter without $\tilde{}$ corresponds to a particular realization of that random variable. So, $\tilde{q}_w^a$ denotes the variable representing the actual quantity of work item w. Suppose, for each work w, the set of possible values for $\tilde{q}_w^a$ is $(\underline{q}_w, \overline{q}_w)$ where $\underline{q}_w \geq 0$ and $\overline{q}_w > \underline{q}_w$ $\forall w \in [0, \overline{W}]$. Assume that it has a prior distribution denoted by $G_{q_w}(q_w^a)$ with associated density $g_{q_w}(q_w^a)$ defined on the set of possible values, $(\underline{q}_w, \overline{q}_w)$. The joint prior distribution (respectively density function) of actual quantities is given by $G_q(\mathbf{q}^a)$ (respectively $g_q(\mathbf{q^a})$). Correlation across different work items are allowed. Denote the corresponding expected value by μ_{q_w} and variance by $\sigma_{q_w}(\mathbf{q^a})$ for each work item w.

Let, $\tilde{q}_w^s$ denote the signal of $\tilde{q}_w^a$ received, say at $T = 1$, as a result of planning effort. Before $T = 1$, only the prior distribution is known and can be used for

[4]Bajari et al 2014, based on data on California highway projects, found value of quantity overrun (defined as $(q_w^a - q_w^e)c_w^e$) to vary from -9,462,806 dollars to 1,699,937 dollars.

[5]The information structure taken here, for both quantity and cost relevant states of nature, is adapted from the literature (See, for eg., Ganuza and Penalva (2010)).

decision making[6].

The signal is also defined on the open interval $(\underline{q}_w, \overline{q}_w)$. The relation between $\tilde{q}_w^s$ and $\tilde{q}_w^a$ is stochastic. Specifically, $\tilde{q}_w^s$ is a noisy signal of $\tilde{q}_w^a$, for each $w \in [0, \overline{W}]$. The distribution of the signal given that the actual quantity vector is $\mathbf{q^a}$ depends on the designing effort, the technical complexity of the project and the experience of the planners with designing. Let the technical complexity of the project be denoted by τ and the experience of designers with project planning and designing be denoted by l .

Note that complexity, τ is a project specific characteristic. Also learning level, l is industry or institution specific. So they are given exogenously for the project at hand. On the other hand, effort level d is endogenous and for a given project, may vary with different contractual forms, as shown below.

Let $F_q(\mathbf{q^s}|\mathbf{q^a}, d, \tau, l)$ (respectively, $f_q(\mathbf{q^s}|\mathbf{q^a}, d, \tau, l)$) be the joint distribution (respectively, the joint density function) of the signal vector conditional on the actual quantity vector $\mathbf{q^a}$, the level of planning effort d, the complexity of the project τ and the learning level of the sponsor l. Thus given d, τ and l, a signal is represented by the family of distributions $\{F_q(\mathbf{q^s}|\mathbf{q^a})\}_{\mathbf{q^a} \in \tilde{\mathbf{q}}^\mathbf{a}}$ where these distributions vary with different actual quantities vector $\mathbf{q^a}$. Let $F_{q_w}(q_w^s|\mathbf{q^a}, d, \tau, l)$ (respectively, $f_{q_w}(q_w^s|\mathbf{q^a}, d, \tau, l)$) be the individual distribution (respectively, the density function) of the signal for a work item w given the actual quantity vector $\mathbf{q^a}$. We allow for these distributions also to be correlated, so that the joint distribution function may or may not be equal to the product of these individual distributions for a given work item w. d, l and τ affects the precision of these work items in the sense described below. Plausibly, assume that the signal is an unbiased estimate of actual quantities, i.e. $E[\tilde{q}_w^s|\mathbf{q^a}, d, \tau, l] = q_w^a$.

Now let us discuss the **cost relevant states of nature** and the different components of cost. Plausibly, the cost of a work item has two principal components- a) the cost of inputs (material, labor, capital, etc.) which is the product of per unit input cost and the quantities as well as b) the cost of organizing and managing different tasks. Literature has not dissected these components but we have opened this black box. Both components are shown to be equally important and play different roles in the analysis below.

Now let us model the first component. Since quantity of a work item is already analyzed, now we consider the per unit input cost of a work item. Let the actual per

[6]Before $T = 1$, a very crude signal of quantity and per unit cost may be available based on which it is decided whether to undertake the project or not. The signal $\tilde{q}_w^s$ will be a more informative signal if positive planning effort is put. We assume throughout that the benefits from the project is so high that project will be implemented in every state of nature. However its design and output features may vary from what was initially decided to be implemented.

unit input cost be denoted by κ_w^a. Like quantity, per unit input cost is also stochastic. For each w, let the set of possible values of $\tilde{\kappa}_w^a$ be $(\underline{\kappa}_w, \overline{\kappa}_w)$ where $\underline{\kappa}_w \geq 0$ and $\overline{\kappa}_w > 0$. Assume that $\tilde{\kappa}_w^a$ has a prior distribution denoted by G_{κ_w} with associated density g_{κ_w}. Let the corresponding joint distribution (resp. density function) for actual per unit input cost of all work items be given by G_κ (resp. g_κ). Denote the corresponding expected value by μ_{κ_w} and variance by $\sigma_{\kappa_w}(\kappa^{\mathbf{a}})$. Like for quantities, we allow for per unit cost across work items to be correlated. Indeed, they are correlated for 2 reasons- a) a macroeconomic nationwide shock will result in actual per unit cost for some work items to be correlated. They may move in same direction (like effect of inflation) or in opposite directions (price of some inputs may increase while for another input may decrease). b) Some work items may involve use of common inputs and thus their cost will move in same direction.

As a result of planning effort d, for each w, buyer receives the signal of $\tilde{\kappa}_w^a$ denoted by $\tilde{\kappa}_w^s$ which is also distributed on the open interval $(\underline{\kappa}_w, \overline{\kappa}_w)$. Due to imperfect estimation techniques, actual and estimated per unit costs may turn out to different [7] [8]. Let $F_{\kappa_w}(\kappa_w^s|\kappa^{\mathbf{a}}, d, \tau, l)$ (respectively, $f_{\kappa_w}(\kappa_w^s|\kappa^{\mathbf{a}}, d, \tau, l)$) be the distribution (respectively, the density function) of the signal given the actual per unit input cost vector $\kappa^{\mathbf{a}}$. $\tilde{\kappa_w^s}$ is a noisy signal of $\tilde{\kappa_w^a}$, for each $w \in [0, \overline{W}]$. We assume that the signal is an unbiased estimate of the actual per unit input cost, i.e., $E(\tilde{\kappa}_w^s|\kappa^{\mathbf{a}}, d, \tau, l) = \kappa_w^a$.

We take quantity and cost relevant states of nature to be independent, and so F_{κ_w} and F_{q_w} are independent distributions. Note that we allow for interdependence of quantity and cost signals across work items. Thus joint posterior distribution of quantity (resp. per unit input cost) may or may not equal $\prod_w (F_{q_w})(q_w^a|\mathbf{q}^{\mathbf{e}}, d, \tau, l, G_q)$ (resp. $\prod_w (F_{\kappa_w})(\kappa_w^a|\kappa^{\mathbf{e}}, d, \tau, l, G_\kappa)$).

Now we consider the second component of cost, i.e., the organization cost. The engineering literature on projects[9] suggests that organization of project works is crucial for the construction costs. The benefit from a given level of organizational effort depends on the efficiency of the entity responsible for the construction and implementation of project tasks, generally a contractor. Let us denote this organization cost

[7] The signal can be based on 'blue book' prices and the expectation over future market conditions, for example, those related to inflation, exchange rate (in case, some inputs are imported), demand and supply conditions of a given input, etc. The blue book prices are derived from past bids and/or the current market prices.

[8] Note that we have allowed for optimization of input mix given relative input prices for each item w.

[9] For e.g., see Potocan et al 2012, Ameh and Osegbo 2011, Ahuja et al 1994, Jonas and Soderlund 2004 and Masten et al 1991. A phrase from Jonas and Soderlund 2004 on page 187 reads as follows: "...there is a need for a purposeful organization effort and a high need of coordination in order to execute a number of tasks/activities.". Masten et al 1991 JLEO estimated organization costs to be as high as 14% of the total cost. The benefit/savings from better organization are also shown to be substantial.

by $\bar{\bar{\kappa}}_w$. Since we take all the bidders to be homogenous having same innate ability, this cost is independent of the identity of the party implementing the project. So if it is not feasible to reduce cost through better organization, then cost of a work item is given by $\kappa_w^a.q_w^a + \bar{\bar{\kappa}}_w$. It represents the cost associated with inefficient way of organizing and managing tasks. A part of the cost is determined by market conditions and cannot be influenced by the entity responsible for the construction. But the other part of the cost can be influenced by the contractor in two ways.

1) Putting effort in better management and organization and coordination of men and material will lead to smooth functioning of the construction process and thus reduce wastage.

2) Putting effort in finding and implementing the most cost efficient input combination to produce a given quantity of work item.

Let, x denotes this effort. It is unverifiable but observable effort and is put by the entity responsible for building and construction of the project facility. It reduces construction costs through better organization of works and through optimal choice of input combination. It is undertaken at time $T = 3/2$, i.e., before actual construction starts [10].

x represents the expenditure or efforts in organization of works, searching and securing supply of inputs, manpower, etc [11]. By improving the efficiency of work, x reduces construction costs [12]. x also represent the use of most cost efficient combination of inputs to complete a work item/activity. In literature,[13] finding and implementing such a combination is taken to be effortless and costless. We have opened this black box and has modeled such an effort/investment formally[14] But the total benefit from x depends on the divergence between expected quantity of a work item, and its finally used actual quantity. In textbook microeconomics also, the optimal input combination depends on the level of output produced. Initially, the contractor chooses the most cost efficient combination based on expected output to be produced but if actual level of output turns out to be different, then switching to now best input combination is not without friction. Bajari et al 2014 has shown that the cost of re-organizing work in case ground conditions/quantities of work items turns out

[10]The contractor can also undertake cost reducing effort, say y, during the construction phase. We will discuss the implication of it from time to time. Since uncertainty over actual quantity and cost gets realized at the beginning of the construction phase, benefit from y won't depend on planning and designing effort d.

[11]Alternatively, instead of taking aggregate x, we could have taken vector $(\mathbf{x}_w)$ for each work item w. In real world, contractor puts effort to organize all work items needed for the completion of the project rather than doing it separately for each work item. So our contention is that taking aggregate x is more close to how planning and organizing is actually done by the contractor.

[12]We assume that the effort x is a *design-specific* investment by the contractor, in the sense described later.

[13]For eg., see Bajari and Tadelis 2001, Ganuza 2007, Laffont and Tirole 1993.

[14]Mandell and Nilsson 2010 has described the importance of it in determining the optimality of Unit Price and FP contracts.

to be different from expectation is quite huge[15] Specifically, for work w, x reduces construction costs by

$$\kappa_w^1(x)\left[\kappa_w^0 - |q_w^e(.) - q_w^a|\right]$$

Let us denote it by $k_w^a(x, q_w^e(.), q_w^a)$[16] where [17] [18]
$\kappa_w^1(0) = 0$, i.e., if no cost reducing effort is put, then there is no saving.
$\kappa_w^0.\kappa_w^1(x)$ represents the reduction in cost of work item w due to effort x if actual quantity used turns out to be equal to the expected quantity, i.e., $q_w^a = q_w^e(.)$. It is the deterministic part of the cost saving. We assume that

κ_w^0 is high enough so that term in bracket is positive $\forall q_w^e \& \forall q_w^a$.
$\kappa_w^1(x).\left[|q_w^e(.) - q_w^a|\right]$ is the stochastic component of the cost saving and represents reduction in cost saving due to unfulfilled expectations over the quantities of work items. It represents the cost involved in adapting expected circumstances to the actually realized circumstances.

We assume that

$$\forall w \in [0, \overline{W}], \frac{\partial \kappa_w^1(x)}{\partial x} > 0$$

.
That is, construction cost decreases[19] with the management-effort by the contractor. Also $\frac{\partial \kappa_w^1(0)}{\partial x} = \infty$.

Further we assume that$\forall w$, $\frac{\partial^2 \kappa_w^1(x)}{\partial x^2} < 0$.

Remarks

Suppose there are many contractors hired by the project sponsor to implement the project. The cost of wrong expectations modeled above also reflects the coordination problem between these contractors. Suppose different contractors are allocated different work items. If quantity of a work item changes, then quantity of other work

[15]Papers from construction engineering, for eg., Ogunlana et al. 1996, Achuenu and Kolawole 1998, Apolot et al and Menon and Rahman 2013 also shows importance of difference in estimated and actual quantities and shortage of materials in explaining high cost and cost overruns.

[16]Below we will derive formally, the posterior distribution of actual quantity and the associated expected quantity given the signal vector. It will be shown to be a function of d, l and τ.

[17]For simplicity, we have taken a linear function but our qualitative results will continue to hold for any increasing and weakly convex function, i.e., $\left[\frac{\psi(|q_w^e(d,\tau,l,G_q) - q_w^a|)}{q_w^a}\right]$ where ψ is an increasing (may or may not be linear) and weakly convex function.

[18]In the text, we have taken divergence proportionate to the actual quantity. Note that this implies that equal absolute divergence on either side of quantity signal q_w^e do not have symmetric effect. For example, suppose $q_w^e = 5$. Now if $q_w^a = 3$, then $\frac{|q_w^e - q_w^a|}{q_w^a} = 2/3$. But if $q_w^a = 7$, then $\frac{|q_w^e - q_w^a|}{q_w^a} = 2/7$. But given convex order, our qualitative results will continue to hold even if we take absolute divergence or if we take divergence proportionate to the quantity signal q_w^s. Only mild assumption on the function ψ is needed, i.e., it should be sufficiently convex, specially for small values of q_w^a.

[19]Our results will continue to hold even if cost decreases strictly for just a non-empty subset of work items.

item can also change. There can be sequence in performance of different tasks. Such interdependence between distribution of quantity signals across work items is allowed for in our paper. Similar is true for cost signals.

All the above are common knowledge to the buyer and to the contractor. The parties involved, i.e., the buyer/sponsor and the contractor are risk neutral.

Timeline

- T=0 (*Project Planning stage*): Buyer chooses the output features of the project.
- T=1 (*Project Designing stage*)
  - effort d is put in by the project designer;
  - signals for quantity, q^s_w and per unit input cost κ^s_w for each work item w are received and the estimates $q^e_w(.)$ and $\kappa^e_w(.)$ are arrived at.
- T= $\frac{3}{2}$ (*Investment in cost reducing effort x by the construction contractor*).
- T=2 (*Construction Starts*)
  - q^a_w and c^a_w become known for each $w \in [0, \overline{W}]$
- T=3 (*Project Completion and Payment Made*).

The timing for the *Tendering and Awarding of the Contract* depends on the specific contractual form and will be specified later.

3 Expected Costs and First best levels

Now given the output requirement, and given the chosen design, we analyze how the choice variables x and d are chosen[20]. In order to solve for the first best or equilibrium values of the endogenous variables of the model, d and x, the decision makers has to derive the posterior distribution of the actual quantity given the signal and the expected total costs and cost savings.

After receiving the quantity signal vector $\mathbf{q^s}$, the decision makers will update their belief over the actual quantities. The posterior joint density of the actual quantity vector $\mathbf{\tilde{q}^a}$ given the prior distribution and the information contained in the signal, denoted by $\hat{f}_q(\mathbf{q^a}|\mathbf{\tilde{q}^s} = \mathbf{q^s}, d, \tau, l, G_q)$ can now be derived using Bayes rule.

$$\begin{aligned}
\hat{f}_q(\mathbf{q^a}|\mathbf{\tilde{q}^s} = \mathbf{q^s}, d, \tau, l, G_q) &= \frac{g_q(\mathbf{q^a}) f_q(\mathbf{q^s}|\mathbf{q^a}, d, \tau, l)}{\int_{\mathbf{\tilde{q}^a}} g_q(\mathbf{q^a}) f_q(\mathbf{q^s}|\mathbf{q^a}, d, \tau, l)) d(\mathbf{q^a})} \\
&= \frac{g_q(\mathbf{q^a}) f_q(\mathbf{q^s}|\mathbf{q^a}, d, \tau, l)}{f_q(\mathbf{q^s}|d, \tau, l)}.
\end{aligned}$$

[20]Later we will analyze the more general case when engineering design and/or the buyer's requirement needs to be changed ex-post.

where the denominator gives the marginal density of the quantity signal vector.

Similarly we can derive the individual posterior distributions for the actual quantity of a given work item w, $\hat{f}_{q_w}(q_w^a|\tilde{\mathbf{q}}^{\mathbf{s}} = \mathbf{q}^{\mathbf{s}}, d, \tau, l, G_q)$. For notational simplicity, let the posterior distribution of actual quantity conditional on the received signal be given by:
$F_{q_w}(q_w^a|\mathbf{q}^{\mathbf{s}}, d, \tau, l, G_q) \equiv \hat{F}_{q_w}(q_w^a|\tilde{\mathbf{q}}^{\mathbf{s}} = \mathbf{q}^{\mathbf{s}}, d, \tau, l, G_q)$ with associated density
$f_{q_w}(q_w^a|\mathbf{q}^{\mathbf{s}}, d, \tau, l, G_q) \equiv \hat{f}_{q_w}(q_w^a|\tilde{\mathbf{q}}^{\mathbf{s}} = \mathbf{q}^{\mathbf{s}}, d, \tau, l, G_q)$
The estimate of quantity for a work item w is formed using this posterior distribution. Specifically, it is the mean of the posterior distribution. As mentioned before, let $q_w^e(\mathbf{q}^{\mathbf{s}}, d, \tau, l, G_q) \equiv E[q_w^a|\mathbf{q}^{\mathbf{s}}, d, \tau, l, G_q]$ denotes the expected quantity of work item w and $\sigma_{q_w}(\mathbf{q}^{\mathbf{s}}, d, \tau, l, G_q)$ denotes the variance of the actual quantity of work item w, $\tilde{q}_w^a$, given the signal vector $\mathbf{q}^{\mathbf{s}}$ and its precision, i.e., given d, τ and l.

Note that the posterior distribution uses two sets of information. One is the information provided by the prior distribution G_q and the other is the information contained in the signal. The weight attached to the latter in derivation of posterior distribution depends on the informativeness of the signal with more weight being attached if the signal is more reliable.

Let us consider a special case where both the prior distribution of actual quantity and the marginal distribution of the signal are the same and are flat, i.e., $\forall(\mathbf{q}^{\mathbf{a}}, \mathbf{q}^{\mathbf{s}})$, $g_q(\mathbf{q}^{\mathbf{a}}) = f_q(\mathbf{q}^{\mathbf{s}}|d, \tau, l) = \bar{a}$, say, where $\bar{a}$ is some constant. Then $f_q(\mathbf{q}^{\mathbf{a}}|\mathbf{q}^{\mathbf{s}}, d, \tau, l) = f_q(\mathbf{q}^{\mathbf{s}}|\mathbf{q}^{\mathbf{a}}, d, \tau, l))$. Given unbiasedness of the signal, we have $q_w^e \equiv E[q_w^a|\mathbf{q}^{\mathbf{s}}, d, \tau, l] = q_w^s$, i.e., the expected quantity is equal to signal itself. In this case, prior distribution is not at all informative and the only information available is contained in the signal. Thus entire weight is attached to the signal giving expected quantity being same as the signal.

As mentioned before, we plausibly assume that the informativeness of the signal depends on the level of d. Specifically we assume that given $\mathbf{q}^{\mathbf{s}}, \tau$ and l, the posterior distribution associated with higher d has lower expected absolute divergence between the actual values and the expected quantity. That is, for given $\mathbf{q}^{\mathbf{s}}, \tau, l$ and $d > d' \geq 0$, we have

$$E[(|\tilde{q}_w^a - q_w^e(.)|)|\mathbf{q}^{\mathbf{s}}, d', \tau, l, G_q] > E[(|\tilde{q}_w^a - q_w^e(.)|)|\mathbf{q}^{\mathbf{s}}, d, \tau, l, G_q]$$

Similar conditions hold for τ and l. Higher l implies greater precision while higher τ implies lower precision[21]. In case, $q_w^e = q_w^s$ which will be true if prior distribution

[21] For higher τ, even if the description of less complex work items remains same but even then the estimates of their quantities may be less precise. For example, the construction of granular sub base is more likely to result in more varied quantities (cubic meter units) due to greater uncertain soil conditions. Also the description of excavation and resurfacing may remain same but now it will be more difficult to get precise estimates of their expected units (See Baccarini 1996). So, the expected absolute mean divergence will increase strictly with τ. But as shown below, same may not be true for estimated per unit input cost as in this case, variability of macroeconomic and local market conditions play a more important role.

is not informative as shown above, then this condition is weaker than second order stochastic dominance property. Latter requires the expected value of any convex function of actual quantity to be falling in d and l and rising in τ.

Assume

$$F_{q_w}(q_w^a|\mathbf{q^s},\infty,\tau,l,G_q) = \begin{cases} 0 & \text{if } q_w^a \in (\underline{q}_w, q_w^s); \\ 1 & \text{if } q_w^a \in [q_w^s, \overline{q}_w). \end{cases}$$

That is, as designing effort approaches infinity, the quantity signal becomes perfectly informative. In this case, $q_w^e = q_w^s = q_w^a$. As the precision of the signal increases, $q_w^e(.)$ gets closer to q_w^s [and both gets closer to q_w^a] and approaches q_w^s [resp. q_w^a] as signal gets perfectly informative.

At $T = 0$, only the prior distribution of the actual quantity and the marginal distribution of the signal can be used by the parties to choose planning effort d. But at $T = 1$ when the signal is received, the posterior distribution and the estimate of the actual quantity will be computed and used by the contractor and the sponsor.

Note that to reduce notational thickness, we have used same function F_{q_w} for the conditional distribution of the signal given actual quantities, the marginal distribution of the signal and the posterior distribution of actual quantities given the signal. For example, $F_{q_w}(q_w^s|\mathbf{q^a},d,\tau,l)$ denotes the conditional distribution of signal given actual quantities, $F_{q_w}(q_w^s)$ denotes the marginal distribution of the signal and $F_{q_w}(q_w^a|\mathbf{q^s},d,\tau,l,G_q)$ denotes the posterior distribution of actual quantities given the signal. Similar thing is done with the variance function. The function we are referring to will be clear both from the context and from the argument of the function.

As an illustration, let us analyze the case when the signal is **linearly distributed**. In this case, with probability p, the signal is equal to the actual quantity, i.e., $\tilde{q}_w^s = q_w^a$ and with the remaining probability, the signal is pure noise and is distributed independently of the actual quantity. In the latter case, it is also distributed according to the prior distribution G_{q_w}. In this case, the estimated quantity is given by $q_w^e = pq_w^s + (1-p)\mu_{q_w}$. So q_w^e is an increasing function of q_w^s. Posterior distribution associated with higher p has lower expected absolute divergence from mean as compared to that with lower p [22]. Plausibly, we assume that p, which is a measure for the preciseness of the signal, is increasing in d and l and decreasing in τ. Thus q_w^e is also a function of d, l and τ. It increases with d if $q_w^s > \mu_{q_w}$ and it decreases with d if $q_w^s < \mu_{q_w}$. Its marginal distribution is same as the prior distribution.

Similar to the case of quantity, the posterior density of the actual per unit input cost $\tilde{\kappa_w^a}$ given the signal κ_w^s, can also be derived using Bayes rule. Let the corresponding distribution and the density function be denoted by $F_{\kappa_w}(\kappa_w^a|\kappa^{\mathbf{s}},d,\tau,l,G_\kappa)$ and $f_{\kappa_w}(\kappa_w^a|\kappa^{\mathbf{s}},d,\tau,l,G_\kappa)$ respectively. Let the corresponding joint posterior distribution (resp. density function) be denoted by
$F_\kappa(\kappa^{\mathbf{a}}|\kappa^{\mathbf{s}},d,\tau,l,G_\kappa)$ (resp. $f_\kappa(\kappa^{\mathbf{a}}|\kappa^{\mathbf{s}},d,\tau,l,G_\kappa)$).

Like quantity estimates, d, τ and l also affect the precision of cost estimates.

[22] For details, see Ganuza and Penalva (2010).

Assume

$$F_{\kappa_w}(\kappa_w^a|\kappa^{\mathbf{s}},\infty,\tau,l,G_\kappa) = \begin{cases} 0 & \text{if } \kappa_w^a \in (\underline{\kappa}_w,\kappa_w^s); \\ 1 & \text{if } \kappa_w^a \in [\kappa_w^s,\overline{\kappa}_w). \end{cases}$$

That is, if very high effort is put, then per unit input cost can be estimated without any error. Given $d' > d \geq 0$ [$l' > l$], we assume that the distribution $F_{\kappa_w}(\kappa_w^a|\kappa^{\mathbf{s}},d',\tau,l,G_\kappa)$ [resp. $F_{\kappa_w}(\kappa_w^a|\kappa^{\mathbf{s}},d,\tau,l',G_\kappa)$] has lower expected absolute mean divergence as compared to the distribution $F_{\kappa_w}(\kappa_w^a|\kappa^{\mathbf{s}},d,\tau,l,G_\kappa)$ [resp. $F_{\kappa_w}(\kappa_w^a|\kappa^{\mathbf{s}},d,\tau,l,G_\kappa)$]. For τ, it is less plausible that complexity of the project will affect the precision of per unit cost estimates as argued before. But for symmetry, we will take weak inequalities for τ, i.e., given $\tau > \tau'$, we assume that $F_{\kappa_w}(\kappa_w^a|\kappa^{\mathbf{s}},d,\tau',l,G_\kappa)$ has weakly lower expected absolute mean divergence as compared to the distribution $F_{\kappa_w}(\kappa_w^a|\kappa^{\mathbf{s}},d,\tau,l,G_\kappa)$.

Let $\kappa_w^e(\kappa^{\mathbf{s}},d,\tau,l,G_\kappa) \equiv E[\kappa_w^a|\kappa^{\mathbf{s}},d,\tau,,G_\kappa]$ denotes the expected per unit input cost of work item w and $\sigma_{\kappa_w}(\kappa^{\mathbf{s}},d,\tau,l,G_\kappa)$ denotes the variance of the actual per unit input cost of work item w, $\tilde{\kappa}_w^a$, given the signal vector $\kappa^{\mathbf{s}}$ and its precision, i.e., given d, τ and l.

So far we have talked about the scenario after signal is received. Now before the signal is received, only marginal distributions are available. As mentioned earlier, the marginal distribution of signal $\tilde{q}_w^s$ is given by

$$F_{q_w}^{q_w^s}(q_w^s) = \int_0^{q_w^s}\int_0^{\overline{q}_0}...\int_0^{\overline{q}_{\overline{W}}} f_{q_w}(x|\mathbf{q^a},d,\tau,l)g_q(\mathbf{q^a})d\mathbf{q^a}dx$$

To economize on notations, define $F_{q_w}(q_w^s) \equiv F_{q_w}^{q_w^s}(q_w^s)$. Let the associated density be given by $f_{q_w}(q_w^s)$[23]. Let $F_q(\mathbf{q^s})$ (resp. $f_q(\mathbf{q^s})$) denote the joint marginal distribution of signal vector $\tilde{\mathbf{q}}^{\mathbf{s}}$. Using law of iterated expectations, we get

$$E[\tilde{\mathbf{q}}^{\mathbf{s}}] = E_{\mathbf{q^a}}[E_{\mathbf{q^s}}[\tilde{\mathbf{q}}^{\mathbf{s}}|\mathbf{q^a}]]$$

Given unbiasedness, we have

$$E_{\mathbf{q^a}}[E_{\mathbf{q^s}}[\tilde{\mathbf{q}}^{\mathbf{s}}|\mathbf{q^a}]] = E_{\mathbf{q^a}}[\tilde{\mathbf{q}}^{\mathbf{a}}]$$

i.e.,

$$E[\tilde{\mathbf{q}}^{\mathbf{s}}] = \mu_{\mathbf{q}}$$

Let $\sigma_{q_w}(\mathbf{q^s})$ denotes the variance of the quantity signal for work item w. Similarly, applying law of iterated expectations again gives

$$E_{\mathbf{q^s}}[\mathbf{q^e}|\mathbf{q^s}]] \equiv E_{\mathbf{q^s}}[E_{\mathbf{q^a}}[\tilde{\mathbf{q}}^{\mathbf{a}}|\mathbf{q^s}]] = E_{\mathbf{q^a}}[\tilde{\mathbf{q}}^{\mathbf{a}}] = \mu_{\mathbf{q}}$$

[23]Plausibly, we assume that d,τ and l do not affect the ex-ante marginal distribution of the signal. This is because when the actual quantities are not known, there is no information at all. This assumption is also standard in the literature. See Ganuza and Penalva 2004,2010; Ganuza 2007, Johnson and Myatt 2006

i.e., expected value of expected quantity given the signal is $\mu_{\mathbf{q}}$[24].

Similarly, let the marginal distribution of per unit input cost signal for a work item w, $\tilde{\kappa}^s_w$ be given by $F_{\kappa_w}(\kappa^{\mathbf{s}}_{\mathbf{w}})$ and the associated density by $f_{\kappa_w}(\kappa^{\mathbf{s}}_{\mathbf{w}})$. The corresponding joint marginal distribution (resp. the density function) of per unit input costs for all work items is denoted by $F_\kappa(\kappa^{\mathbf{s}})$ (resp. $f_\kappa(\kappa^{\mathbf{s}})$). Again using law of iterated expectations, for a given work item w, we get

$$E[\tilde{\kappa}^s_w] = E[\tilde{\kappa}^e_w] = \mu_{\kappa_w}$$

Let $\sigma_{\kappa_w}(\kappa^{\mathbf{s}})$ be the associated variance function.

In the following discussion, we will be suppressing the prior distribution $\mathbf{G} = \{G_q, G_\kappa\}$ in the argument.

Given the above derived posterior distributions, in this section, we derive the expression for expected construction cost for two cases: 1) when x is not feasible and 2) when x plays a significant role in reducing per unit costs. There are two variants of expected total construction costs depending on at what time we are taking expectation. This is because the information available to the parties differ across time periods. First one is after d is put and quantity and per unit input signals are received and the estimates are arrived at. This is ex-post to planning and designing stage and is relevant in making optimal/equilibrium choice of x at $T = 3/2$. We will denote it by $C^e_{[0,\overline{W}]}(x|\mathbf{q}^{\mathbf{s}}, \kappa^{\mathbf{s}}, d, \tau, l) \equiv E_{\mathbf{q}^{\mathbf{a}},\kappa^{\mathbf{a}}}[C^a_{[0,\overline{W}]}|\mathbf{q}^{\mathbf{s}}, \kappa^{\mathbf{s}}, d, \tau, l]$. At this time d is fixed and its cost is sunk, so we take it to be inclusive of cost x but exclusive of cost of d. The other useful measure of expected cost is the ex-ante expected cost before d is put and signals are received, i.e., expected cost at $T = 0$. This is needed in making choice of planning and designing effort d. It is denoted by $EC^e_{[0,\overline{W}]}(x, d, \tau, l)$ and is calculated as $E_{\mathbf{q}^s,\kappa^{\mathbf{s}}}[C^e_{[0,\overline{W}]}(x|\mathbf{q}^{\mathbf{s}}, \kappa^{\mathbf{s}}, d, \tau, l)] + d$. Equivalently, we can derive it by integrating over the signal vectors (keeping in mind that expected values given the signal is also a function of d, l and τ.). It is inclusive of cost of both d and x.

3.1 x is not feasible

The actual cost of the initial design (ignoring the cost of planning effort d) is given by

$$C^a_{[0,\overline{W}]}(.) = \int_0^{\overline{W}} [c^a_w \times q^a_w]\, dw,$$

where as mentioned before, c^a_w is actual per unit cost. For notational convenience, we will drop the subscript $[0, \overline{W}]$ in the discussion that follows. Since $c^a_w = \kappa^a_w$ in the

[24]We can also derive the marginal distribution of $\mathbf{q}^{\mathbf{e}}$, denoted by $F_q(\mathbf{q}^{\mathbf{e}}|d, l, \tau)$. Even if the prior distribution and the marginal distribution of the signal does not depend on d, l and τ, the marginal distribution of the estimated quantity do depend on this. This is because they affect the weight attached to the distribution of the signal vis a via the prior distribution.

absence of x, we get

$$C^a(.) = \int_0^{\overline{W}} [\kappa_w^a \times q_w^a]\, dw,$$

Given that F_{κ_w} and F_{q_w} are independent, we get the expected input cost for a work item w, given quantity and per unit input cost signals and thus given the estimates, as $E_{\kappa_w^a, q_w^a}(\kappa_w^a q_w^a | \mathbf{q}^\mathbf{s}, \kappa^\mathbf{s}) = E_{\kappa_w^a}(\kappa_w^a | \kappa^\mathbf{s}).E_{q_w^a}(q_w^a | \mathbf{q}^\mathbf{s}) = \kappa_w^e(\kappa^\mathbf{s}, .).q_w^e(\mathbf{q}^\mathbf{s}, .)$. Thus the total expected cost becomes $C^e(\mathbf{q}^\mathbf{s}, \kappa^\mathbf{s}, d, \tau, l)$

$$= \int_0^{\overline{W}} \kappa_w^e(\kappa^\mathbf{s}, .).q_w^e(\mathbf{q}^\mathbf{s}, .)dw$$

The total expected cost, inclusive of the cost of planning effort d, and given the signals, is given by

$$TC^e(\mathbf{q}^\mathbf{s}, \kappa^\mathbf{s}, d, \tau, l) = C^e(\mathbf{q}^\mathbf{s}, \kappa^\mathbf{s}, d, \tau, l) + d$$

Taking the expectation of above across the quantity and per unit costs signals, we get the ex-ante expected cost as $EC^e(x, d, \tau, l)$:

$$\begin{aligned} &= \int_0^{\overline{W}} \int_{\kappa^\mathbf{s}} \int_{\mathbf{q}^\mathbf{s}} \kappa_w^e(\kappa^\mathbf{s}, .)q_w^e(\mathbf{q}^\mathbf{s}, .)dF_{q_w}(\mathbf{q}^\mathbf{s})dF_{\kappa_w}(\kappa^\mathbf{s})dw + d \\ &= \int_0^{\overline{W}} \mu_{\kappa_w}.\mu_{q_w} dw + d \end{aligned}$$

Note that planning effort d, complexity of the project τ and learning of the designer l do not affect expected cost in this case. When x is not feasible, d provides no gains and has no efficiency implications. Thus first best level of d denoted by d^* will be zero. It will also be true in equilibrium for any contract given that parties are risk neutral.

x is expected not to play any role for procurement of standard goods which can be either directly bought from the market or through competitive tender procedure. For example, office stationery, buses, etc. In this case, there is no organization of work tasks during post-contracting phase and thus no need for initial planning.

3.2 x is feasible

x is expected to play an important role where good is uniquely produced to suit buyer's needs and where production happens in post contracting phase and takes time to complete. When there are different work tasks to be implemented, need for organization arises.

Normalizing the organization cost $\bar{\bar{\kappa}}_w$ to zero, we have the actual cost for a work-item w given the estimates of quantity as

$$\begin{aligned} C_w^a(x, q_w^e, q_w^a, \kappa_w^a) &= \kappa_w^a q_w^a - \kappa_w^1(x)\left[\kappa_w^0 - |q_w^e - q_w^a|\right] \\ &= \kappa_w^a q_w^a - K_w^a(x, q_w^e, q_w^a) \end{aligned}$$

where for each $w \in [0, \overline{W}]$, $\kappa_w^a q_w^a$ is interpreted as the actual cost in the absence of effort x, i.e, if x is not modeled, then actual cost is $\kappa_w^a q_w^a$. It also represents the expensive way of performing a work item when work is organized inefficiently and no effort is put in better organization.

Let $C_w^e(x|\mathbf{q^s}, \kappa^{\mathbf{s}}, d, \tau, l, \mathbf{G})$ be the expected cost of work item w given the signal and the corresponding estimates of quantity and per unit input costs and where $\mathbf{G} = \{G_q, G_\kappa\}$. Given the independence of quantity and per unit input cost signals, it is given by

$$\begin{aligned} &= \kappa_w^e(\kappa^{\mathbf{s}}, .)q_w^e(\mathbf{q^s}, .) - \kappa_w^1(x)\left[\kappa_w^0 - E_{q_w^a}\left[(|q_w^e(\mathbf{q^s}, .) - q_w^a|)|\mathbf{q^s}, d, \tau, l, G_q\right]\right] \\ &= \kappa_w^e(\kappa^{\mathbf{s}}, .)q_w^e(\mathbf{q^s}, .) - K_w^e(x|\mathbf{q^s}, d, \tau, l, G_q) \end{aligned}$$

where $K_w^e(x|\mathbf{q^s}, d, \tau, l, G_q)$ represents saving in the expected cost due to effort x of work item w given the quantity signal. Note that q_w^e in the above expression corresponds to the signal received q_w^s and expectation is taken at a time when both are given.

Now the actual cost of the initial design given the quantity estimates and the realized quantity and per unit input costs becomes $C_{[0,\overline{W}]}^a(x, .)$

$$= \int_0^{\overline{W}} [\kappa_w^a q_w^a - \kappa_w^1(x)(\kappa_w^0 - (|q_w^e - q_w^a|))]dw \tag{1}$$

Actual/realized cost saving on account of x, denoted by $K^a(x, .)$, is given by:

$$K_{[0,\overline{W}]}^a(x, .) = \int_0^{\overline{W}} [\kappa_w^1(x)(\kappa_w^0 - (|q_w^e - q_w^a|))]dw$$

Now taking the expected value of (1), given the vector of quantity signals $\mathbf{q^s}$ and the per unit input cost signals $\kappa^{\mathbf{s}}$, and using their independence, we get the total expected cost. Note that the expected input cost is same as the case when x is not feasible. But now there is expected cost saving on account of x also. Thus as derived earlier for a given work item w, the total expected cost is given by $C_{[0,W]}^e(x|\mathbf{q^s}, \kappa^{\mathbf{s}}, \tau, l, d)$ becomes

$$\begin{aligned} &= \int_0^{\overline{W}} [\kappa_w^e(\kappa^{\mathbf{s}}, .)q_w^e(\mathbf{q^s}, .) - \kappa_w^1(x)(\kappa_w^0 - E_{q_w^a}((|q_w^e(\mathbf{q^s}, .) - q_w^a|)|\mathbf{q^s}, \tau, l, d))]dw + x \qquad (2) \\ &= \int_0^{\overline{W}} \kappa_w^e(\kappa^{\mathbf{s}}, .)q_w^e(\mathbf{q^s}, .)dw - K^e(x|\mathbf{q^s}, \tau, l, d) + x \end{aligned}$$

where

$\int_0^{\overline{W}} \kappa_w^e(\kappa^{\mathbf{s}}, .)q_w^e(\mathbf{q^s}, .)dw$ represents the total expected input cost given the signals in absence of cost reducing effort x,

$K^e(x|\mathbf{q^s}, \tau, l, d)$ represents the *total expected cost saving* due to effort x given the quantity signal, and equals

$$K^e(x|\mathbf{q^s}, \tau, l, d) = \int_0^{\overline{W}} [\kappa_w^1(x)(\kappa_w^0 - E_{q_w^a}((|q_w^e(\mathbf{q^s}, .) - q_w^a|)|\mathbf{q^s}, \tau, l, d))]dw$$

So total expected cost, given the vector of quantity and per unit input cost signals can also be written as:

$$TC^e_{[0,\overline{W}]}(x|\mathbf{q^s},\kappa^{\mathbf{s}},\tau,l,d) = \int_0^{\overline{W}} \kappa^e_w(\kappa^{\mathbf{s}},.)q^e_w(\mathbf{q^s},.)dw - K^e(x|\mathbf{q^s},\tau,l,d) + x + d$$

Analyzing the expected cost saving given the quantity signal vector, we have

$$\frac{\partial K^e(x|\mathbf{q^s},\tau,l,d)}{\partial x} = \int_0^{\overline{W}} \frac{\partial \kappa^1_w(x)}{\partial x}(\kappa^0_w - E_{q^a_w}((|q^e_w(\mathbf{q^s},.) - q^a_w|)|\mathbf{q^s},\tau,l,d))dw$$

and

$$\frac{\partial^2 K^e(x|\mathbf{q^s},\tau,l,d)}{\partial x^2} = \int_0^{\overline{W}} \frac{\partial^2 \kappa^1_w(x)}{\partial x^2}(\kappa^0_w - E_{q^a_w}((|q^e_w(\mathbf{q^s},.) - q^a_w|)|\mathbf{q^s},\tau,l,d))dw$$

Given that $\kappa^1_w(x)$ is increasing and is concave in x, we have, $\frac{\partial K^e(x|\mathbf{q^s},\tau,l,d)}{\partial x} > 0$ and $\frac{\partial^2 K^e(x|\mathbf{q^s},\tau,l,d)}{\partial x^2} < 0$, i.e., $K^e(x|\mathbf{q^s},\tau,l,d)$ is also increasing and concave in x.

We can also define total expected cost even before quantity and per unit input cost signals are received. As mentioned before, it is denoted by $EC^e_{[0,\overline{W}]}(x,\tau,l,d)$. Taking expectation of (2) over the quantity and the per unit input cost signals, we get $EC^e_{[0,\overline{W}]}(x,\tau,l,d)$,

$$\begin{aligned} &= \int_0^{\overline{W}} \int_{\kappa^{\mathbf{s}}} \int_{\mathbf{q^s}} \kappa^e_w(\kappa^{\mathbf{s}},.).q^e_w(\mathbf{q^s},.)dF_q(\mathbf{q^s})dF_\kappa(\kappa^{\mathbf{s}})dw \\ &- \int_0^{\overline{W}} \int_{\mathbf{q^s}} \kappa^1_w(x)(\kappa^0_w - E_{q^a_w}((|q^e_w(\mathbf{q^s},.) - q^a_w|)|\mathbf{q^s},\tau,l,d))]dF_q(\mathbf{q^s})dw \end{aligned} \tag{3}$$

As mentioned earlier, use of iterated expectations implies $E[\tilde{q}^e_w] = \mu_{q_w}$ and $E[\tilde{\kappa}^e_w] = \mu_{\kappa_w}$. So we get

$$EC^e_{[0,\overline{W}]}(x,\tau,l,d) = \int_0^{\overline{W}} \mu_{\kappa_w}\mu_{q_w}dw - \int_0^{\overline{W}} \int_{\mathbf{q^s}} K^e(x|\mathbf{q}^s,d,\tau,l)dF_q(\mathbf{q^s})dw$$

where $\int_0^{\overline{W}} \mu_{\kappa_w}\mu_{q_w}dw$ represents the expected total input cost before quantity and per unit input cost signals are received.
$\int_0^{\overline{W}} \int_{\mathbf{q^s}} K^e(x|\mathbf{q^s},d,\tau,l)dF_q(\mathbf{q^s})dw$ denotes the expected cost saving before receiving the signals. Let us denote it by $EK^e(x,d,\tau,l)$. As mentioned before, $dF_q(\mathbf{q^s})$ is the joint marginal distribution of quantity signal vector $\mathbf{q^s}$. This is before any new information, in addition to that contained in prior distribution, is received for the actual quantities. Note that input cost does not depend on d, l and τ. Expected cost

saving is affected by these and thus is the main focus of the paper.
So ex-ante total expected cost becomes:

$$EC^e_{[0,\overline{W}]}(x,\tau,l,d) = \int_0^{\overline{W}} \mu_{\kappa_w}\mu_{q_w} dw - EK^e(x,d,\tau,l) + x + d$$

Suppose that for the given buyer's requirement and given the design, expected social benefit be given by B. So, net expected social benefit after estimates of quantity and per unit input costs are arrived at, is given by $B - TC^e_{[0,\overline{W}]}(x|\mathbf{q^s},\kappa^{\mathbf{s}},\tau,l,d)$. The choice variables of the model, d and x, are chosen so as to maximize the net social benefit from the project given the information available at that point in time, and given the complexity of the project τ and the learning level of the project designer l. The solution to the social optimization problem is obtained by backward induction. Since x is put after planning effort d and signals and estimates are known, we will first solve for the first best response function of x, $x^*(\mathbf{q^s},d,\tau,l)$ given the level of d[25]. Then given this response function, we will solve for the first best level of d, $d^*(\tau,l)$. Note that the first best values of both d and x depend on the parameters of the model, τ and l.

3.2.1 First Best level of x

Note that x is put at $T = 3/2$, i.e., after the planning effort d is put and the signals over quantity and per unit input cost are received and estimates are determined but before the actual cost $C^a_{[0,\overline{W}]}$ is known. So the first best level of x minimizes the total expected cost or equivalently, maximizes the total expected cost saving on account of x, $K^e(x|\mathbf{q^s},\tau,l,d)$, given the designing effort d. So given that the received signal vector is $\mathbf{q^s}$ and $\mathbf{q^e}$ is the corresponding estimated quantity vector, it solves the following optimization problem:

$$\max_x \{K^e(x|\mathbf{q^s},\tau,l,d) - x\}$$

i.e.,

$$\max_x \quad \{\int_0^{\overline{W}} [\kappa^1_w(x)(\kappa^0_w - E_{q^a_w}((|q^e_w(\mathbf{q^s},.) - q^a_w|)|\mathbf{q^s},\tau,l,d))]dw - x\}$$

FOC:

$$\frac{\partial K^e(x|\mathbf{q^s},\tau,l,d)}{\partial x} = 1$$

i.e.,

$$\int_0^{\overline{W}} [\frac{\partial \kappa^1_w(x)}{\partial x}(\kappa^0_w - E_{q^a_w}((|q^e_w(\mathbf{q^s},.) - q^a_w|)|\mathbf{q^s},\tau,l,d))]dw = 1 \qquad (4)$$

[25]Since expected cost saving depends on these, so will be $x^*(.)$ as shown below.

Let $x^* = x^*(\mathbf{q^s}, \tau, l, d)$ solves (4). Given that $K^e(x|\mathbf{q^s}, \tau, l, d)$ is concave in x as shown above, SOC is met and we have $x^*(.)$ to be unique and positive. The FOC above shows that expected total cost of the project is falling in x.

Suppose there are two designers, $\{1, 2\}$, and both report same expected quantities and per unit input costs. Let the first best organization effort, expected cost saving and expected total cost for the project designed by designer i be denoted by attaching subscript i. Then we have the following result:

Lemma 1 *For any given realization of quantity signal, we have that ceteris paribus, the first best level of x and total expected cost saving will be higher for the project by the designer with greater effort, with greater experience and the one implementing less technical project and thus with greater precision of the quantity estimates.*

Proof: i) Given the signal $\mathbf{q^s}$, $x_1^*(., d_1)$ solves

$$\int_0^{\overline{W}} [\frac{\partial \kappa_w^1(x)}{\partial x}(\kappa_w^0 - E_{q_w^a}((|q_w^e(\mathbf{q^s}, .) - q_w^a|)|\mathbf{q^s}, \tau, l, d_1))]dw = 1$$

and $x_2^*(., d_2)$ solves

$$\int_0^{\overline{W}} [\frac{\partial \kappa_w^1(x)}{\partial x}(\kappa_w^0 - E_{q_w^a}((|q_w^e(\mathbf{q^s}, .) - q_w^a|)|\mathbf{q^s}, \tau, l, d_2))]dw = 1$$

If $d_1 > d_2$, then given the increased precision of quantity signals, we have

$$E_{q_w^a}((|q_w^e(\mathbf{q^s}, .) - q_w^a|)|\mathbf{q^s}, \tau, l, d_1) < E_{q_w^a}((|q_w^e(\mathbf{q^s}, .) - q_w^a|)|\mathbf{q^s}, \tau, l, d_2)$$

Given the concavity of $\kappa_w^1(x)$, this implies that $x_1^*(., d_1) > x_2^*(., d_2)$.
Now expected cost saving for the first designer is $K_1^e(x_1^*|\mathbf{q^s}, \tau, l, d_1)$

$$= \kappa_w^1(x_1^*)(\kappa_w^0 - E_{q_w^a}((|q_w^e(\mathbf{q^s}, .) - q_w^a|)|\mathbf{q^s}, \tau, l, d_1))]dw$$

and $K_2^e(x_2^*|\mathbf{q^s}, \tau, l, d_2)$ is

$$= \kappa_w^1(x_2^*)(\kappa_w^0 - E_{q_w^a}((|q_w^e(\mathbf{q^s}, .) - q_w^a|)|\mathbf{q^s}, \tau, l, d_2))]dw$$

Now given that $x_1^* > x_2^*$ and that $\kappa_w^1(x)$ is increasing in x and $E_{q_w^a}((|q_w^e(\mathbf{q^s}, .) - q_w^a|)|\mathbf{q^s}, \tau, l, d_1) < E_{q_w^a}((|q_w^e(\mathbf{q^s}, .)-q_w^a|)|\mathbf{q^s}, \tau, l, d_2)$, we get $K_1^e(x_1^*|\mathbf{q^s}, \tau, l, d_1) > K_2^e(x_2^*|\mathbf{q^s}, \tau, l, d_2)$.
B) Similar arguments as above give
$l_1 > l_2$ implies $x_1^*(\mathbf{q^s}, \tau, l_1, d) > x_2^*(\mathbf{q^s}, \tau, l_2, d)$, and $K_1^e(x_1^*|\mathbf{q^s}, \tau, l_1, d) > K_2^e(x_2^*|\mathbf{q^s}, \tau, l_2, d)$
And $\tau_1 < \tau_2$ implies $x_1^*(\mathbf{q^s}, \tau_1, l, d) > x_2^*(\mathbf{q^s}, \tau_2, l, d)$, and $K_1^e(x_1^*|\mathbf{q^s}, \tau_1, l, d) > K_2^e(x_2^*|\mathbf{q^s}, \tau_2, l, d)$.

□

Thus even if two designers same quantity signals, the optimal level of x and implied expected cost saving will be different if they have put different effort. This is because signal received after higher d is more reliable and actual quantity is more likely to be closer to it leading to low adaptations. This increases marginal benefit from x inducing its higher value. The expected cost saving will be higher due to 2 reasons: 1) greater cost saving for given organizational effort x (direct effect) and 2) higher x put in equilibrium due to increased marginal benefit (indirect effect). It also works through the effect of d, l and τ on the precision of quantity signals.

Corollary 1 *For any given realization of quantity signals and for any given realization of expected input cost, we have that ceteris paribus, the total expected cost will be lower for the project by the designer with greater effort, with greater experience and the one implementing less technical project and thus with greater precision of the quantity estimates.*

Proof: The total expected cost is expected input cost minus the expected cost saving. Since given the estimates, the former is fixed and does not depend on d, l and τ, we have the required result.

□

Thus even if two designers arrive at same estimate of expected input costs, their estimate of total expected cost will be different if they put different effort. This is due to 2 reasons: 1) higher level of x being induced and 2) higher expected cost saving.

Next we consider the effect of d (and l and τ) on expected organizational effort (defined below). Note that first best level of x depends on the realization of the quantity signal. At the time when d is put, the signal is not yet received. Thus we should also consider the effect of d on average value of x^*, on ex-ante expected cost savings and on ex-ante total expected expected cost before the receipt of signals.

The following proposition extends the earlier result for the case where quantity and per unit input cost signals are not yet received. Now expected first best level of x before receiving the signals is increasing in d and l and decreasing in τ. Here expectation is taken over all possible signals values. Also given that x respond optimally to d, i.e., given $x^*(.)$, we have ex-ante cost savings [and ex-ante expected cost] to be rising [resp. falling] in the designing effort and the learning level of the project designer and falling [resp. rising] in the complexity of the project.

Let us define average/expected organizational effort in first best as $\bar{x}^*(d, l, \tau) = \int_{\mathbf{q^s}} x^*(\mathbf{q^s}, d, l, \tau) dF_q(\mathbf{q^s})$. Now we have the following proposition.

Proposition 1 *Expected first best level of x, $\bar{x}^*(.)$ is rising in d and l and falling in τ. Given that x responds optimally, ex-ante expected cost saving rises in d and l and falls in τ and ex-ante total expected cost falls in d and l and rises with τ[26].*

[26] Ganuza 2007 has also shown that ex-ante expected total cost is falling in planning effort and is rising in project complexity when there is asymmetric information. We have explained the same

The above proposition says that the first best reaction function of average cost reducing investment, $\bar{x}^*(d,l,\tau)$, is increasing in d and l, and falling in τ. This is because given any realization of quantity signal, higher d and l increases the marginal benefit of x by increasing the precision of the quantity estimates, thereby leading to low adaptation and thus increased expected benefit from x. Similar is true for ex-ante expected cost saving and ex-ante total expected cost via the effect of d, l and τ on the precision of quantity estimates.

3.2.2 First Best level of d

As mentioned earlier, planning effort d is put at $T = 1$. After it is put, then quantity and per unit cost signals are received, the precision of which depend on the level of d. Thus the first best level of d maximizes the ex-ante expected net social benefit before the quantity and the per unit input cost signals are received. Equivalently it minimizes the ex-ante expected cost. It also takes into account the effect of d on the marginal gain from x and how x^* changes with it. So the first best level of d denoted by $d^*(\tau,l)$ solves

$$\min_d \{EC^e_{[0,\overline{W}]}(d, x^*(d,.)|\tau,l) + x^*(d,.) + d\}$$

i.e.,

$$\{\min_d \quad \int_0^{\overline{W}} [\mu_{\kappa_w}\mu_{q_w} - \int_{\mathbf{q^s}} \kappa^1_w(x^*(.))(\kappa^0_w - E_{q^a_w}((|q^e_w(\mathbf{q^s},.) - q^a_w|)|\mathbf{q^s},\tau,l,d))dF_q(\mathbf{q^s})]dw\}$$

FOC:

$$-\frac{\partial EC^e_{[0,\overline{W}]}(x^*(.),\tau,l,d)}{\partial d} = 1$$

i.e.,

$$\int_0^{\overline{W}} \int_{\mathbf{q^s}} \kappa^1_w(x^*(\mathbf{q^s},.))\frac{\partial E_{q^a_w}((|q^e_w(\mathbf{q^s},.) - q^a_w|)|\mathbf{q^s},\tau,l,d)}{\partial d} dF_q(\mathbf{q^s})dw = -1 \qquad (5)$$

Note the term involving $\frac{\partial x^*}{\partial d}$ gets dropped due to FOC of x (Envelope theorem). Note that $\frac{\partial EC^e_{[0,\overline{W}]}(x^*(.),\tau,l,d)}{\partial d} < 0$. We assume $d^*(.)$ to be positive. This will be true, in particular, if $\lim_{d\to 0} \frac{\partial E_{q^a_w}((|q^e_w(\mathbf{q^s},.)-q^a_w|)|\mathbf{q^s},\tau,l,d)}{\partial d} = \infty$ [27].

phenomena without assuming asymmetric information. Ganuza 2007 does not consider any cost reducing organizational effort and thus does not allow for any complementarity with the designing effort. Also it does not take into account the effect of learning.

[27] When we allow for initial design to be complete, then $\lim_{d\to 0} \frac{\partial W}{\partial d} = \infty$ where W measures the specification level also gives $\lim_{d\to 0} \frac{\partial EC^e_{[0,\overline{W}]}(\tau,l,d)}{\partial d} = \infty$ and $d^*(.)$ to be positive.

SOC:

$$\int_0^{\overline{W}} \int_{\mathbf{q^s}} \kappa_w^1(x^*(\mathbf{q^s},.)) \frac{\partial^2 E_{q_w^a}((|q_w^e(\mathbf{q^s},.) - q_w^a|)|\mathbf{q^s},\tau,l,d)}{\partial d^2} dF_q(\mathbf{q^s})dw$$
$$+ \quad [\int_0^{\overline{W}} \int_{\mathbf{q^s}} \frac{\partial \kappa_w^1(x^*(\mathbf{q^s},.))}{\partial x} \frac{\partial x^*}{\partial d} \frac{\partial E_{q_w^a}((|q_w^e(\mathbf{q^s},.) - q_w^a|)|\mathbf{q^s},\tau,l,d)}{\partial d} dF_q(\mathbf{q^s})dw$$

The intuition behind the second term: As planning effort d increases, the marginal benefit from cost reducing investment x increases as greater precision leads to lower adaptation. This leads to increase in its first best optimal value. But as x increases, marginal benefit from d increases further. Thus the problem may not be convex $\forall d$ (and $\forall l, \tau$). This implies that there can be more than one critical points. We assume that global optimum exists. Since d^* is positive and there is no upper bound on its value, so optimum will be among these critical points. At this global optimum, the problem will be convex locally and thus second order condition will be satisfied. Thus the above expression, say $C_d(.)$, is positive.

Now we get the following proposition:

Proposition 2 *The first best level of d, $d^*(\tau, l)$ and of expected x, $\bar{x}^*(d^*(l,\tau), l, \tau)$ is increasing in learning level l and decreasing in project complexity τ.*

Above proposition says that first best level of planning effort d^* is increasing in l and decreasing in τ, and accounting for this that d^* varies with these parameters, the expected first best level of x, $\bar{x}^*(d^*(l,\tau), l, \tau)$ is also increasing in l and decreasing in τ. Note that earlier we talked about how response function of x behaves given d does not respond to changes in the parameters of the model, d and τ. Now we are talking about the first best level of x given that d also responds optimally to the parameters of the model.

Thus even if cross partial derivatives with respect to d, τ and l are zero, we have the positive relationship between d^* and l working through the effect on optimal choice of x, i.e., x^* which works due to their effect on the precision of quantity signal, as shown in Proposition 1. Increase in learning increases precision of quantity signals, thereby reducing the extent of adaptation required. It, in turn, leads to increase in marginal benefit from x and thus increase marginal benefit from d. Opposite holds for τ.

Proposition 3 *Given that d and x are at their first best values, $d^*(\tau, l)$ and $x^*(\mathbf{q^s}, d, \tau, l)$, we have ex-ante expected cost saving to be increasing in l and falling in τ and ex-ante expected cost to be falling in l and rising in τ[28].*

The above proposition says that expected costs are falling in the learning level of the project designer and rising in project complexity. This is because of their effect

[28] As mentioned before, the results will hold qualitatively if we take plausible situation where efforts are complementary. That is, marginal gain from designing effort d increases with experience and falls with complexity of the project.

on the precision of quantity estimates and thus, the subsequent effect on adaptation cost. A higher l may be interpreted as better experience of private or government sponsor for the project of a given sector. For projects which are procured by both private and public sponsor, the one with greater experience will perform better. So for example, if there is reason to believe that private sector is more experienced, then DB contracts will have advantage in this regard.

In real world, it is generally seen that government first take simpler, less complex projects and then venture into more complex projects. One reason could be to have better learning as time passes. This empirical finding can be explained by our model. We will get this result if experience l and complexity τ are complementary, i.e., marginal benefit from learning is higher for more complex projects and if this effect is sufficiently large. In other words, if higher learning can mitigate the effect of greater complexity. Formally, suppose $l' > l$ and $\tau' > \tau$, where $l' - l$ represents better learning over time and τ' (τ) is the index of complexity of the more (less) complex project. Then government will postpone more complex project in favor of less complex project if following is true [29] :

$$EC^e_{[0,\overline{W}]}(d^*(.), x^*(.), \tau', l') + EC^e_{[0,\overline{W}]}(d^*(.), x^*(.), \tau, l)$$
$$< EC^e_{[0,\overline{W}]}(d^*(.), x^*(.), \tau', l) + EC^e_{[0,\overline{W}]}(d^*(.), x^*(.), \tau, l')$$

i.e.,

$$EC^e_{[0,\overline{W}]}(d^*(.), x^*(.), \tau', l') - EC^e_{[0,\overline{W}]}(d^*(.), x^*(.), \tau', l)$$
$$< EC^e_{[0,\overline{W}]}(d^*(.), x^*(.), \tau, l') - EC^e_{[0,\overline{W}]}(d^*(.), x^*(.), \tau, l)$$

i.e., $\frac{\partial^2 EC^e_{[0,\overline{W}]}(d^*(.),x^*(.),\tau,l)}{\partial\tau\partial l} < 0$

Proposition 4 *Given that d and x are at their first best values, $d^*(\tau, l)$ and $x^*(\mathbf{q^s}, d, \tau, l)$ and respond optimally to the parameters of the model, l and τ, then we have the following result:*
If $\forall q$, $\frac{\partial^2 E_{q^a_w}((|q^e_w(\mathbf{q^s},.)-q^a_w|)|\mathbf{q^s},\tau,l,d)}{\partial\tau\partial l} < 0$ and it is sufficiently large, then $\left[\frac{\partial EC^e_{[0,\overline{W}]}(d^*(.),x^*(.),\tau,l)}{\partial\tau\partial l} < 0\right]$.

If x is fixed at some level and is not a choice variable, then we get $\frac{\partial C^e_{[0,\overline{W}]}(d^*(.),x^*(.),\tau,l)}{\partial\tau\partial l} < 0$ if cross partial $\frac{\partial^2 E_{q^a_w}((|q^e_w(\mathbf{q^s},.)-q^a_w|)|\mathbf{q^s},\tau,l,d)}{\partial\tau\partial l}$ is negative. If the contract is a C+ contract, then x will be zero for all values of τ and l. In this case, $\frac{\partial^2 E_{q^a_w}((|q^e_w(\mathbf{q^s},.)-q^a_w|)|\mathbf{q^s},\tau,l,d)}{\partial\tau\partial l} < 0$ will straightforwardly imply that $\frac{\partial C^e_{[0,\overline{W}]}(d^*(.),x^*(.),\tau,l)}{\partial\tau\partial l} < 0$. That is, if the benefit from learning, in terms of getting more precise signals, is lower for more complex projects, then complex projects will be taken up later.

[29]For simplicity, we assume that efforts d and x are chosen optimally.

4 Contracts and Equilibria

There are two widely used definitions of cost overruns:

- **Absolute cost overruns**- It is equal to the absolute amount in money by which the actual cost differs from the estimated cost and is denoted by CO. Given that initially estimated cost is denoted by IC, we have $CO = C^a_{[0,\overline{W}]}(.) - IC(.)$.

- **Proportionate cost overruns**- It is equal to the absolute CO divided by the initial estimate of cost and is denoted by CO_p. So $CO_p = \frac{C^a_{[0,\overline{W}]} - IC(.)}{IC(.)}$

As mentioned before, depending on the information available at the time when estimates are formed, there are two variants of estimated cost: one is before any detailed planning is done, i.e., before d is put and signals over quantity and per unit input cost are received. In this case, initially estimated cost is $EC^e(.)$. The other is after planning and designing has been done and as a result more precise signals are received. In this case, $C^e(\kappa^{\mathbf{s}}, \mathbf{q}^{\mathbf{s}}, .)$ is the estimated cost. Similarly, average/expected [absolute] cost overrun also has different variants depending on the estimated cost and whether the average is taken over the actual quantities given the signals or over the signals also. In principle, three variants of average absolute and proportionate each are possible, giving in total six variants. When the initial cost estimate is $C^e(.)$, then average can be taken either before or after the signals. If it is taken after the signals, then average CO [proportionate CO] will be denoted by $CO^e(., C^e)$ [resp. $CO^e_p(., C^e)$] and if it is taken over signals also, then it will be denoted by $ECO^e(., C^e)$ [resp. $ECO^e_p(., C^e)$]. If ex-ante cost estimates are used, then we will get $ECO^e(., EC^e)$ [resp. $ECO^e_p(., EC^e)$].

Now we analyze initial estimate of cost. For the purpose of preparation of cost estimates for the project, the work items required to complete the project as per the chosen design has to be identified and defined/described ex-ante before the construction begins. These estimates are used to get approval for the project and at the time of inviting bids for the implementation of the project. Bids are invited for the the identified and specified work items[30]. Works will be implemented during the construction phase.

For the given design of the project, all works may or may not be specified initially before the start of construction. Specification of a work item involves three things- a) identification of a work item, b) description of a work item in bid documents and also c) description of all the fine details for the purpose of the contract. There are 2 sources which combine to give incomplete design - a)engineers miss out on some work items and b) even if these are not missed, some work items may not be described as their exact features may not be known (more likely to happen with highly complex work

[30]In case of Design Bid and Build contracts, the buyer will invite bids from the contractor. While in case of Design and Build contracts, the contractor will delegate the work to subcontractors.

items). Let, $W_s = \{\omega_i | i \in [0, W]\}$ be the set of works which are initially specified[31], where $0 \leq W \leq \overline{W}$. So the set of specified work items is represented by $W_s = [0, W]$,[32] i.e., the initial specified design covers the first W works. Let us denote the 'level of specification' of the initial design by the highest index of specified work items, i.e., W. So the set of unspecified work items, denoted by W_{ns} becomes $-W_s = (W, \overline{W}]$. If $W < \overline{W}$, then we say that the design of the project is 'incompletely specified'. Remaining more complex works, $(W, \overline{W}]$, are left unspecified.

The initially specified design is also called 'the scope of the project'. Thus the scope of a project is comprised of the set W_s and their demanded quantities and work procedure to be followed to execute these work items.

Plausibly, the informativeness of the signal also depends on whether the work item w is specified initially or not. The signals are less precise if a work item is unspecified as compared to the case when it is specified. Similar to above we define precision in terms of expected absolute mean divergence. So for a work item w, the posterior distribution $F_{q_w}(q_w^a | \mathbf{q^s}(.), d, \tau, l)$, when it is specified, has lower expected absolute mean divergence as compared to the distribution when it is not initially specified. It implies that for a given work item w, $\forall (d, \tau, l)$[33]

$$E((|q_w^e(.) - \tilde{q}_w^a|)|\mathbf{q^s}, d, \tau, l)_{w \in W_{ns}} > E((|q_w^e(.) - \tilde{q}_w^a|)|\mathbf{q^s}, d, \tau, l)_{w \in W_s}$$

Similar property holds for the per unit input cost signal.

Now we can define $D = \frac{W}{\overline{W}}$ where $D \in [0, 1]$ and is a measure of the *completeness of the project design*. D increases as completeness of initial design increases. When all work items can be specified, then $D = 1$ and when the set of unspecified work items is non-empty, then $D < 1$.

Lemma 2 *If x is not feasible, and the completeness of design is given exogenously, then latter will have no effect on ex-ante expected total cost and thus on d^* which is zero in this case.*

So completeness of design has no efficiency implications in this case. But it plays crucial role when x is feasible as shown in the following lemma.

Proposition 5 *If x is feasible, then as design completeness increases, we have that:*
i) the first best level of x, i.e., $x^*(.)$, given any realization of signals, and expected x^* before receiving the signals increase.
ii) Given that x is at first best, for any given realization of quantity and per unit input signals, expected cost saving increases while expected total cost falls.
iii) Given that x responds optimally, first best level of d, i.e., $d^*(.)$ increases.

[31] Since specification of tasks requires costly effort as argued below, so there will be no over-specification of the design.

[32] Since works are ordered in increasing order of complexity, so if work item W is specified, then all work items w such that $0 \leq w \leq W$ are also specified.

[33] Ameh and Osegbo 2011 , Makulsawatudom et al. (2004), Simpeh 2012, Cantarelli et al 2010 and Aibinu and Jagboro (2006) talk about incomplete drawings and its implications for productivity.

iv)Given that d and x are at their first best values, ex-ante expected cost saving increases while ex-ante expected total cost falls.

Comparing average value of x^* in the two cases of complete and incomplete design, we find that it is lower in the incomplete design case. In fact, we have the result that it falls with the incompleteness of initial design. This is because marginal benefit from x is lower if a work item is unspecified which in turn is due to poor planning for such work items.

Given that only W work items can be identified, defined and described ex-ante before construction begins, so the initial cost estimates are for the set of works $[0, W]$ only. It is either $C^e_{[0,W]}(.)$ or $EC^e_{[0,W]}(.)$. When construction begins, the contractor and the sponsor come to know the ground conditions and the details and the exact nature of the left out, more complex, work items now become known at no further cost. As mentioned earlier, the actual cost of works is observed at $t = 2$. At this time, the design will be completed, i.e., the remaining $(W, \overline{W}]$ works are also incorporated in the design and completed during construction phase. Any change made to the initial scope of the project is termed as 'change in scope'. Thus addition of the earlier left out work item results in change in scope[34]. Ignoring the arguments, we get actual absolute cost overrun as

$$CO = C^a_{[0,\overline{W}]} - IC_{[0,W]}(.)$$

where $IC(.) \in \{C^e(.), EC^e(.)\}$. Note that $CO \lesseqqgtr 0$ due to uncertainty over quantities and per unit input costs of different work items.
Thus different variants of average CO are given by

$$\begin{aligned} CO^e(., C^e) &= C^e_{[0,\overline{W}]} - C^e_{[0,W]} = C^e_{(W,\overline{W}]} \\ ECO^e(., C^e) &= E_{\mathbf{q^s},\kappa^\mathbf{s}}[CO^e(., C^e)] = EC^e_{(W,\overline{W}]} \\ ECO^e(., EC^e) &= EC^e_{[0,\overline{W}]} - EC^e_{[0,W]} = EC^e_{(W,\overline{W}]} \end{aligned}$$

Similarly, realized proportionate cost overrun is given by

$$CO^P = \frac{C^a_{[0,\overline{W}]} - IC_{[0,W]}(.)}{IC_{[0,W]}(.)}$$

Alternatively, we can define CO^P as

$$CO^P = \frac{C^a_{[0,\overline{W}]}}{IC_{[0,W]}(.)}$$

[34]The documents published by the National Highway Authority of India (NHAI) and used for tendering road and highway projects defines change in scope as "13.1.2 Change of Scope shall mean: (a) change in specifications of any item of Works; (b) omission of any work from the Scope of the Project...(c) any additional work, Plant, Materials or services which are not included in the Scope of the Project, including any associated Tests on completion of construction." In the discussion so far, we have considered only the last part (c), i.e., addition of work items for design completion. In later extensions of the model, where change in design and change in output demanded by the buyer is allowed for, there we will analyze all the three cases of change in scope.

Different variants of average proportionate CO can be defined analogously. They are

$$\begin{aligned} CO_p^e(., C^e) &= \frac{C^e_{[0,\overline{W}]} - C^e_{[0,W]}}{C^e_{[0,W]}} = \frac{C^e_{(W,\overline{W}]}}{C^e_{[0,W]}} \\ ECO_p^e(., C^e) &= E_{\mathbf{q^s},\kappa^\mathbf{s}} \left[\frac{C^e_{(W,\overline{W}]}}{C^e_{[0,W]}} \right] \\ ECO_p^e(., EC^e) &= \frac{EC^e_{[0,\overline{W}]} - EC^e_{[0,W]}}{EC^e_{[0,W]}} = \frac{EC^e_{(W,\overline{W}]}}{EC^e_{[0,W]}} \end{aligned}$$

Lemma 3 *Irrespective of whether x is feasible or not, we have all variants of both expected absolute and proportionate cost overruns to be nonnegative. Additionally*
a) If the initial design can be completely specified, i.e., $D = 1$ then all variants of both average absolute and proportionate cost overruns are zero.
b) If initial design specification is incomplete, i.e., $D < 1$, then all variants give average cost overruns to be positive.

Empirical literature, as reviewed above shows the presence of positive cost overruns on average. Ameh and Osegbo 2011 , Makulsawatudom et al. (2004), Simpeh 2012, Cantarelli et al 2010 and Aibinu and Jagboro (2006) talk about incomplete drawings and its implications for productivity and cost overruns. The underlying dynamics between incomplete drawings and positive cost overruns on average can be explained by the model.

An implication of this is that when initial design is complete, i.e., when $W = \overline{W}$, then irrespective of whether cost reducing investment x is feasible or not, then average CO, both absolute and proportionate, are zero. Empirical literature[35] shows that cost overruns in case of construction projects, like building of road, railways, residential buildings etc. are found to be systematically positive. Plausibly in these projects, $W < \overline{W}$ as these are complex projects where all the work items cannot be specified at the beginning. On the other hand, simple projects, for eg., involving procurement of standard machinery, buses, stationery seldom suffer from overruns. The good is well specified in this case and thus initial specification of the project is almost complete[36] i.e., $W = \overline{W}$. Thus we have provided an explanation for varying magnitude of COs across sectors based on degree of specification of the good.

Now we analyze how average absolute cost overruns changes with planning and organization effort and with the parameters of the model. Later we will analyze the proportionate cost overruns.

[35] For example, see Singh 2010, Ganuza 2007. Flyvjberg et al 2010 gives an overview of the empirical literature on cost overruns.

[36] Cost overruns in this case is generally due to change in buyer's requirement ex-post. Change in initial design and change in scope will be analyzed in later sections.

Proposition 6 *Given that initial design is incomplete and is given exogenously, i.e., $D < 1$, then*
i) if x is not feasible, then all variants of expected absolute cost overruns are decreasing in the completeness of design but are invariant to d, l and τ.
ii) if x is feasible, then all variants of expected absolute cost overruns are decreasing in design completeness W, x, d and l and increasing in τ. It holds even if d and x responds to the parameters of the model, W, l and τ given that they are monotonically increasing in W, d and l and monotonically decreasing in τ. In particular, it holds when they respond optimally.

It follows since for a work item w, $C^e_w(.)$ and $EC^e_w(.)$ are decreasing in x, d and l and increasing in τ as shown above. Also x^* is shown to be increasing in W, d and l and decreasing in τ and d^* is increasing in W and l and falling in τ. So their indirect effect via effect on decision/endogeous variables x and d also moves in same direction as the direct effect. Above proposition says that good organization and management by the contractor during the pre-construction stage helps in reducing average CO[37]. We give explanation for varying level of average CO across projects for a given sector. If buyer is more learned and experienced and/or if project is technically less complex, then CO will be low.

The above proposition says that in first best and for monotonic contracts, i.e., where x and d changes monotonically with the parameters of the model, (elsewhere we show that cost sharing contracts (including fixed price and cost plus), item rate contracts and Design and Build contracts satisfy these properties) absolute cost overrun is decreasing in l and d and increasing in τ. As demonstrated in the literature using case studies and questionnaire surveys, our paper shows formally that cost overrun is falling in initial planning and management and organization skills of the contractor. So our paper gives a rigorous theoretical model to explain the empirical findings and anecdotal evidence on cost overruns. As earlier, we have two variants of this result. First, when d is fixed and x responds to it according to reaction $x^i(.)$ and second, when d is also chosen endogenously as a function of l and τ.

The above proposition holds even if we take ex-post perspective perspective, i.e., the cost estimates are already made, and so $IC(.) = \bar{C}$. This is because they have similar effect on total expected costs also. Later we show that this distinction becomes crucial. The results in the above proposition works through two channels- one is the direct effect and the other is the indirect effect through reaction function of x. For all the contracts above, including first best, x responds positively to d and l and negatively to τ. But empirical literature mentions a contract which is also widely used but which gives reverse reaction by x as shown below. This is 'Maximum Guarantee Price (MGP)'. In case of such a contract, the buyer defines a maximum price, say M,

[37]Note that investment y which will be undertaken during construction stage and which affects the cost of the complete design will also result in fall average CO. It is true irrespective of whether it is a one-time investment or is put on daily basis

and the payment structure is such that it is a C+ contract till cost is below M and after the cost shoots up that level, then it is a fixed price contract with fixed payment M. Formally, we have

$$P = \begin{cases} TC^a & \text{if } TC^a_{[0,\overline{W}]}(x) < M; \\ M & \text{if } TC^a_{[0,\overline{W}]}(x) > M. \end{cases}$$

where P denotes the agreed price and $TC^a_{[0,\overline{W}]}(x)$ denotes the actual total cost of the project. As mentioned earlier, $TC^a_{[0,\overline{W}]}(x)$ is given by

$$TC^a_{[0,\overline{W}]}(x) = \int_0^{\overline{W}} [\kappa^a_w q^a_w - \kappa^1_w(x)(\kappa^0_w - (|q^e_w - q^a_w|))]dw + x + d$$

Its distribution depends on d, l and τ with expected total cost being a decreasing function of d and l and an increasing function of τ as shown above. Now we analyze how the distribution of actual total varies with different values of d. We are interested in the probability that actual cost shoots up a given level M because only in that case the benefit from cost saving from x accrues to the contractor. Let the distribution of actual total cost be given by $F_{TC}(\tilde{TC}^a|x, d, l, \tau)$ with support $(0, \overline{TC}(x, d, l, \tau)$. Given d, l and τ, an increase in x decreases the expected cost and shifts the entire distribution of actual cost to the left. Thus the above probability is falling in x as actual cost is falling in x.

Given the assumptions so far, it is not possible to compare this probability for different values of d, l and τ. If the distribution of actual total cost satisfies first order stochastic dominance property with respect to these variables, then we get this probability to be falling in d and l and increasing in τ as shown below. The distribution F is said to first order stochastically dominates (FOSD) distribution G of a random variable v, if $F(v) \leq G(v)$ for every v. If we suppose $F_{TC}(\tilde{TC}^a|x, d', l, \tau$ FOSD $F_{TC}(\tilde{TC}^a|x, d'', l, \tau$ where $d'' > d'$, then $1 - F_{TC}(M|x, d', l, \tau) \geq 1 - F_{TC}(M|x, d'', l, \tau)$, i.e., the the probability of actual cost being higher than M is higher if d is smaller. Similar condition can be imposed with respect to l and τ.

In this case, x is chosen to solve the following optimization problem:

$$\max_x \{prob(TC^a(x) > M)K^e(x|\mathbf{q^s}, \tau, l, d) - x\}$$

FOC:

$$prob(TC^a(x) > M)\frac{\partial K^e(x|\mathbf{q^s}, \tau, l, d)}{\partial x} + K^e(x|\mathbf{q^s}, \tau, l, d)\frac{\partial[prob(TC^a(x) > M)]}{\partial x} = 1$$

Let $x^{MGP}(.) \geq 0$ solves this.

Derivating the above FOC with respect to d delivers

$$C_x.\frac{\partial x^{MGP}}{\partial d} + prob(TC^a(x^{MGP}) > M)\frac{\partial^2 K^e(x^{MGP}|\mathbf{q^s}, \tau, l, d)}{\partial x \partial d} + \frac{\partial K^e(x^{MGP}|\mathbf{q^s}, \tau, l, d)}{\partial x} \frac{\partial[prob(TC^a(x^{MGP}) > M)]}{\partial d} + \frac{\partial prob(TC^a(x^{MGP}) > M)}{\partial x}\frac{\partial K^e(x^{MGP}|\mathbf{q^s}, \tau, l, d)}{\partial d} = 0$$

where C_x represents the second order derivative with respect to x and is negative at the equilibrium value x^{MGP}. This delivers $\frac{\partial x^{MGP}}{\partial d}$ as:

$$\begin{aligned} &= -\frac{1}{C_x}\{prob(TC^a(x^{MGP}) > M)\frac{\partial^2 K^e(x^{MGP}|\mathbf{q^s},\tau,l,d)}{\partial x \partial d} + \frac{\partial K^e(x^{MGP}|\mathbf{q^s},\tau,l,d)}{\partial x} \\ &+ \frac{\partial[prob(TC^a(x^{MGP}) > M)]}{\partial d} + \frac{\partial prob(TC^a(x^{MGP}) > M)}{\partial x}\frac{\partial K^e(x^{MGP}|\mathbf{q^s},\tau,l,d)}{\partial d}\} \end{aligned}$$

Now the first term on the right hand side is positive but the later two are negative. Thus $\frac{\partial x^{MGP}}{\partial d} \lesseqgtr 0$. If it is negative, then expected absolute CO may increase with d. Similar derivations will give such ambiguity with respect to l and τ also.

Ganuza requires imperfect competition and asymmetric information to explain varying level of CO. It shows that CO varies with different market structure with higher CO in less competitive market. We give an alternative explanation in terms of incomplete specification which varies with the kind of product to be procured. We do not need imperfect competition and/or asymmetric information to get the result.

4.1 Cost overruns for the government/buyer

The total final payment made by the buyer and received by the contractor under contract i is $P_i + P'_i$. That is, the total actual project cost for the buyer, CB^a, is

$$CB^a = P_i + P'_i$$

where as mentioned before P_i is the initially agreed price and P'_i is the payment at the time of renegotiation if initial contract is i. Assuming competitive bidding and rational expectations about the work quantities and their costs and the renegotiation process (over remaining works items) later, the initial price P_i will be such that it gives

$$= E[CB^a] = P_i + E[P'_i] = C^e_{[0,\overline{W}(\tau)]} \tag{6}$$

In case of item rate contract, the initial bid vector $(\mathbf{b}_w)$ where $w \in [0, W(d,\tau,l)]$ satisfies the above equality. The estimated cost for buyer before inviting bids is the expected cost of the initially specified design, $C^e_{[0,W(d,\tau,l)]}$. Initial estimate for the cost is for the specified initial design. This is what we see in practice, from government reports.

In view of (6), above results with respect to initial cost, absolute COs and proportionate COs also hold for cost overrun for the buyer.

Above discussion assumes that initial design remains optimal ex-post so that due to incomplete description of design, expected cost overruns are positive under design-bid-build (DBB) contracts. In the next section, we allow for change in design.

5 Change in design

So far we have assumed that given the output features, there is just one design comprising of work items $\{\omega_i | i \in [0, \overline{W}]\}$ and the associated expected quantities and per unit costs. The project will be completed if these work items are implemented. But in reality, once buyer's requirement is fixed, then the next task is the choice of design from the set of available feasible designs. The entity responsible for the design inspects the project site. Given observable site conditions and design relevant state of nature, suppose design D becomes relevant. The initial design is chosen/specified assuming a particular design-relevant state of nature. During construction if the state of nature remains same as expected, then no change in the initial design is needed. In this case, the benefit from design D is given by B. Let the expected cost from this design be given by $EC^e(x, d, \tau, l)$ as described in the previous sections.

So far we have considered the case that the initially specified design is also optimal ex-post. But in practice, the initial design may need to be changed. Later it may be realized that the actual site condition/design relevant state of nature is different from that expected, so design needs to be changed for given output features of the project. If the realized state of nature turns out to be different, then the benefit from design D will be 0 and it cannot be implemented. So change in design is needed to achieve the same output.

Initial design may also need to be changed if buyer changes the output demanded or quality standards. Thus a change in design can be needed for two reasons: One is when during the construction stage, the actual ground condition turns out to be different from what was initially expected and thus warrant a change in engineering design to achieve the demanded output. The other is when buyer's need change during the course of time and a different output gives him/her much larger benefit while the benefit from initial design is quite low. Let a different design $\hat{D}$ becomes relevant/implements the new output and gives benefit B. Let the probability of change in design[38], on either count, whether it be change in engineering plan or change in output demanded, be given by $(1 - \pi)$.[39] Let the expected cost from this design be given by $\hat{EC}^e(x, d, \tau, l)$. As mentioned earlier, a design specifies what is to be done and how it is to be done. Anything that changes the initial engineering plan will be treated as change in design. Specifically, a 'change in design' comprises of a) change in the set of work items (including addition/deletion of work items) and/or b)

[38]Note that even if the probability of change in engineering plan for given output depends on the project characteristics, the buyer can influence the probability of need for changed output. We will analyze this in later sections.

[39]Note that when change in design is not a possibility, then D is a measure of both incomplete design and incomplete contract. When we allow for change in design, then D measures the degree of incompleteness of the initial design, and both D and π are measures of completeness of the initial contract. The contract has to be renegotiated both for completion of initially specified design and for change in design.

change in work procedures and sequencing of tasks, and/or c) change in the quantities demanded of the initially specified work items. Note that change in design without changing the output features of the project will affect only the first two components, while change in buyer's requirement may affect all the three components.

The actual input cost for the changed design, denoted by $\hat{C}_0^a$, is given by

$$\hat{C}_0^a = \int_0^{\overline{\hat{W}}} \hat{\kappa}_{\hat{w}}^a . \hat{q}_{\hat{w}}^a d\hat{w}$$

where,
$[0, \overline{\hat{W}}]$ denotes the index of work items for the changed design. The set of all possible work items $\{\hat{\omega}_j | j \in [0, \overline{\hat{W}}]\}$ needed to be performed for the changed design is taken to be completely general. Some new work items may be added while some existing work items may need to be dropped.
$\hat{q}_{\hat{w}}^a$ denotes the actual quantities of the $\hat{w}$th work-item for the alternative design.
$\hat{\kappa}_{\hat{w}}^a$ denotes the actual per unit input cost of the $\hat{w}$th work-item for the alternative design.
Let, $\hat{q}_{\hat{w}}^s$ (resp. $\hat{\kappa}_{\hat{w}}^s$) denote the independent, publicly observable signal and let $\hat{q}_{\hat{w}}^e(.)$ (resp. $\hat{\kappa}_{\hat{w}}^e(.)$) denote the corresponding expected value of $\hat{q}_{\hat{w}}^a$ (resp. $\hat{\kappa}_{\hat{w}}^a$) given the signal.

Note that for the work items which are needed for both the designs, i.e., $\omega \in \{\omega_i | i \in [0, \overline{W}]\} \bigcap \{\hat{\omega}_j | j \in [0, \overline{\hat{W}}]\}$, per unit input cost, both actual and the estimate will be the same as for the initial design. But even for these work items, the actual and the estimated/demanded quantities may be different due to changed design. As mentioned before, even if set of work items for the initial and the changed design are same, a change in quantities demanded comprises a change in design. So for a work item w', $\hat{q}_{w'}^e$ may not equal $q_{w'}^e$. Thus $\hat{q}_{w'}^a$ may also be different from $q_{w'}^a$. For example, take the case of increase in the scale of the project. Suppose the scale has doubled. In this case, the set of work items remain the same, i.e., $\omega \in \{\omega_i | i \in [0, \overline{W}]\} = \{\hat{\omega}_j | j \in [0, \overline{\hat{W}}]\}$ but now for a work item w, $\hat{q}_w^e = 2q_w^e$. This constitutes a change in design as per our formulation. Similarly, decrease in project scale, addition and/or deletion of work items, and change in sequence for performance of various work items (i.e., change in work procedure) also constitute a change in design. A change in design is quite different from actual quantities being different from the expected quantities. As argued above, latter can happen even for the given design due to imperfect estimation techniques. On the other hand, a change in design involves substantial change and re-organization. Due to re-organization of work, the effectiveness of design specific effort x will be lower and this is the focus of our analysis.

Note that change in scope is a broader term compared to change in design. As mentioned, any change made to the initially specified design constitutes a change in scope. Thus it comprises of both completion of the initial design and change in design which may be either for the same output or for different output. Now the question arises: If we see change in scope, how do we say whether it is completion of initial design or that the design has changed. First note that for given ground condition, a change in design is expected to lead to change in set of work items, i.e., both addition and deletion of work items. Thus if there is just addition of work items to produce the same output, i.e., without changing the buyer's requirement, then it is more likely that initial design is completed now. On the other hand, if there is deletion of work items, or simultaneous deletion and addition of work items, or change in output demand by the buyer, then it is the case of change in design.

To the extent that some work items may overlap for the two designs, the benefit of planning for the initial design will be helpful in getting precise estimates of per unit costs for those work items. But even if same work items are there, the distribution of quantities are likely to be different now, i.e., they may be demanded in different quantities now. So the contractor cannot use the information gained for the initial design with respect to quantities. Also the set of the work items may be different now. There is no planning for the work items which are added in the alternate design. Thus d will be less informative for alternate design compared to the case if planning is done exclusively for the alternate design, in the sense of convex order. Note that even when the initial design is incomplete, at least first $W(d,\tau,l)$ work items have more precise estimates, but for the alternate design, estimates are very crude for all the work items.

So the expected input cost for the changed design is given by:

$$\hat{C}^e_0(\hat{\mathbf{q}}^{\mathbf{s}}, \hat{\kappa}^{\mathbf{s}}, d, l, \tau) = \int_0^{\overline{\hat{W}}} \hat{\kappa}^e_{\hat{w}}(\hat{\kappa}^{\mathbf{s}}, .).\hat{q}^e_{\hat{w}}(\hat{\mathbf{q}}^{\mathbf{s}}, .)d\hat{w}$$

As mentioned earlier, at $t = 2$, i.e., at the beginning of construction phase, the state of the nature and the actual costs and benefits get realized. So whether change in design is needed or not becomes known at this time. Later we will also consider the case when need for change in design is realized during the construction phase and when work on initial design is partially done.

A change in design of the project affects construction costs in two ways. One, it may affect the set of work items and/or the quantities of the existing work items, and therefore the input costs as already described above. Two, it affects the gains from the organizational effort x. Since the works and resources are organized for the initial design, it seems plausible to argue that x is more effective in decreasing the construction cost for the initial design compared to the alternate design. Formally, effort x is a *design-specific* investment by the contractor[40]. Let us denote actual cost

[40]Papers from construction engineering, for eg., Ameh and Osegbo 2011, Mansfield et al., 1994, Al-

saving due to effort x for the changed design by $\hat{K}^a(x,.)$ and expected cost saving after signals are received by $\hat{K}^e(x|\hat{\mathbf{q}}^\mathbf{s}, d, \tau, l)$. So the total expected cost for the changed design given the signals is given by

$$\hat{C}^e(x|\hat{\kappa}^\mathbf{s}, \hat{\mathbf{q}}^\mathbf{s}, d, \tau, l) = \int_0^{\overline{\hat{W}}} \hat{\kappa}^e_{\hat{w}}(\hat{\kappa}^\mathbf{s}, .).\hat{q}^e_{\hat{w}}(\hat{\mathbf{q}}^\mathbf{s}, .)d\hat{w} - \hat{K}^e(x|\hat{\mathbf{q}}^\mathbf{s}, d, \tau, l)$$

The total ex-ante expected cost for the changed design, before the signals are received is given by

$$\hat{EC}^e(x, d, \tau, l) = \int_0^{\overline{\hat{W}}} [\hat{\mu}_{\kappa_{\hat{w}}}.\hat{\mu}_{q_{\hat{w}}} - \int_{\hat{\mathbf{q}}^\mathbf{s}} \hat{K}^e(x|\hat{\mathbf{q}}^\mathbf{s}, d, \tau, l)d\hat{F}_{\hat{q}_{\hat{w}}}(\hat{\mathbf{q}}^\mathbf{s})]d\hat{w}$$

Given that x is design-specific, we have $\forall(\mathbf{q}^\mathbf{s}, \hat{\mathbf{q}}^\mathbf{s})$:

$$\frac{\partial K^e(x|\mathbf{q}^\mathbf{s}, d, \tau, l)}{\partial x} > \frac{\partial \hat{K}^e(x|\hat{\mathbf{q}}^\mathbf{s}, d, \tau, l)}{\partial x} \geq 0$$

i.e.,

$$\frac{\partial K^e(x|\mathbf{q}^\mathbf{s}, d, \tau, l)}{\partial x} - \frac{\partial \hat{K}^e(x|\hat{\mathbf{q}}^\mathbf{s}, d, \tau, l)}{\partial x} > 0$$

That is, the marginal gain from design specific investment x is non-negative and is higher for the initial design. This works through two channels- one because of better planning resulting in more precise quantity estimates for the initial design and the other due to design specificity of x.
Also $\hat{K}^e(x|\hat{\mathbf{q}}^\mathbf{s}, d, \tau, l)$ is concave in x, i.e., the marginal gain is falling. For background discussion and further results, see appendix.

For exposition, we started with the case when there is just one alternate design if the ground condition is different from expected. Now we generalize it to $n-1$ alternate ground conditions. Let the expected cost from these designs be $C^e_i(x|\mathbf{q}^\mathbf{s}_\mathbf{i}, \kappa^\mathbf{s}_\mathbf{i}, d, \tau, l) = C^e_{0i}(.) - K^e_i(x|\mathbf{q}^\mathbf{s}_\mathbf{i}, d, \tau, l)$ where $i \in \{2, 3, , ..., n\}$ and
$\mathbf{q}^\mathbf{s}_\mathbf{i}$ is the vector of quantity signals and $\kappa^\mathbf{s}_\mathbf{i}$ is the vector of per unit input cost signals for alternate design i.
$C^e_{0i}(.) = \int_0^{\overline{W_i}} \kappa^e_{wi}(\kappa^\mathbf{s}_\mathbf{i}, .).q^e_{wi}(\mathbf{q}^\mathbf{s}_\mathbf{i}, .)d(wi)$ represents expected input cost of alternate design i
$K^e_i(x|\mathbf{q}^\mathbf{s}_\mathbf{i}, d, \tau, l)$ represents expected cost saving for design i. Let the probability that design i becomes relevant ex-post be π_i. So now $1-\pi$ equals $\pi_2 + ... + \pi_n$.
$\hat{K}^e(x|\mathbf{q}^\mathbf{s}_\mathbf{2}, ..., \mathbf{q}^\mathbf{s}_\mathbf{n}, d, \tau, l)$ now represents expected cost saving from the alternate designs and equals

$$\pi_2 K^e_2(x|\mathbf{q}^\mathbf{s}_\mathbf{2}, d, \tau, l) + ... + \pi_n K^e_n(x|\mathbf{q}^\mathbf{s}_\mathbf{n}, d, \tau, l)$$

Momani 2000, Xiao and Proverbs 2002, Frimpong et al. 2003 (due to weather conditions), Kikwasi 2012, , Achuenu and Kolawole 1998, Simpeh 2012 also points to the design specificity of organization and management effort in post contracting stage before the start of construction. They talk about falling labour productivity in the instances of change in design.

Thus expected cost $\hat{C}^e(x|\kappa^{\mathbf{s}}_{\mathbf{2}}, \mathbf{q}^{\mathbf{s}}_{\mathbf{2}}, ..., \kappa^{\mathbf{s}}_{\mathbf{n}}, \mathbf{q}^{\mathbf{s}}_{\mathbf{n}}, d, \tau, l)$ now becomes

$$\pi_2 C^e_2(x|\mathbf{q}^{\mathbf{s}}_{\mathbf{2}}, \kappa^{\mathbf{s}}_{\mathbf{2}}, d, \tau, l) + ... + \pi_n C^e_n(x|\mathbf{q}^{\mathbf{s}}_{\mathbf{n}}, \kappa^{\mathbf{s}}_{\mathbf{n}}, d, \tau, l)$$

Given that each cost saving function satisfies the above derived properties (since derivations given in the appendix apply to each alternate design), so $\hat{K}^e(x|\mathbf{q}^{\mathbf{s}}_{\mathbf{2}}, ..., \mathbf{q}^{\mathbf{s}}_{\mathbf{n}}, d, \tau, l)$ also continues to satisfy these properties. Let the vector of quantity signals for the alternate designs be denoted by $\hat{\mathbf{q}}^{\mathbf{s}}$ and that of per unit costs by $\hat{\kappa}^{\mathbf{s}}$ and let $\hat{\mathbf{q}}^{\mathbf{e}}(\hat{\mathbf{q}}^{\mathbf{s}}, .)$ (resp. $\hat{\kappa}^{\mathbf{e}}(\hat{\kappa}^{\mathbf{s}}, .)$) denote the corresponding expected value vector.

Allowing for change in design, the expected net social benefits are now given by

$$\mathcal{B}^e(.) = \pi(B - C^e(x|\mathbf{q}^{\mathrm{s}}, \kappa^{\mathrm{s}}, d, \tau, l)) + (1-\pi)(B - \hat{C}^e(x|\hat{\mathbf{q}}^{\mathrm{s}}, \hat{\kappa}^{\mathrm{s}}, d, \tau, l)) - x - d$$

and the expected total costs are given by $\mathcal{C}^e(x|\mathbf{q}^{\mathrm{s}}, \kappa^{\mathrm{s}}, \hat{\mathbf{q}}^{\mathrm{s}}, \hat{\kappa}^{\mathrm{s}}, d, \tau, l, \pi) = \pi C^e(.) + (1-\pi)\hat{C}^e(.)$:

$$\begin{aligned}
&= \pi[C^e_0(.) - K^e(x|\mathbf{q}^{\mathrm{s}}, d, \tau, l)] + (1-\pi)[\hat{C}^e_0(.) - \hat{K}^e(x|\hat{\mathbf{q}}^{\mathrm{s}}, d, \tau, l)] + x + d \\
&= [\pi C^e_0(.) + (1-\pi)\hat{C}^e_0(.)] - [\pi K^e(x|\mathbf{q}^{\mathrm{s}}, d, \tau, l) + (1-\pi)\hat{K}^e(x|\hat{\mathbf{q}}^{\mathrm{s}}, d, \tau, l)] \\
&+ \ x + d
\end{aligned}$$

Let $\mathcal{K}^e(x|\mathbf{q}^{\mathrm{s}}, \hat{\mathbf{q}}^{\mathrm{s}}, d, \tau, l, \pi) = \pi K^e(x|\mathbf{q}^{\mathrm{s}}, d, \tau, l) + (1-\pi)\hat{K}^e(x|\hat{\mathbf{q}}^{\mathrm{s}}, d, \tau, l)$ denote the total expected cost saving after receiving the signals.

Similarly the ex-ante expected cost before receiving quantity and per unit input cost signals is given by

$$E\mathcal{C}^e(x, d, \tau, l, \pi) = \pi EC^e(x, d, \tau, l) + (1-\pi)\hat{EC}^e(x, d, \tau, l)$$

Let $E\mathcal{K}^e(x, d, \tau, l, \pi) = \pi EK^e(x, d, \tau, l) + (1-\pi)\hat{EK}^e(x, d, \tau, l)$ denote the total expected cost saving before receiving the signals.

5.1 Social Optimization Problem (SOP)

Again the model is solved using backward induction. First the optimal level of x as a function of d, $x^*(d, .)$ is computed. Then, first best level of planning and designing effort d^* is chosen taking into account the subsequent effect on x.

First best level of x

Note that x is put at $t = 3/2$, i.e., before it is known whether change in design is needed or not. Ignoring the terms independent of x in $\mathcal{B}^e(.)$ expression, $x^*(\mathbf{q}^{\mathrm{s}}, \hat{\mathbf{q}}^{\mathrm{s}}, d, \pi, \tau, l)$ solves

$$\max_x \{\pi K^e(x|\mathbf{q}^{\mathrm{s}}, d, \tau, l) + (1-\pi)\hat{K}^e(x|\hat{\mathbf{q}}^{\mathrm{s}}, d, \tau, l) - x\}$$

Note that d is given at the time of choosing x. Also note that we are allowing for more general possibility that the benefit from x for the alternate design is non-negative and thus is taken into account by the contractor in his/her optimization problem [41]. FOC:

$$\pi \frac{\partial K^e(x|\mathbf{q^s}, d, \tau, l)}{\partial x} + (1-\pi)\frac{\partial \hat{K}^e(x|\hat{\mathbf{q}}^\mathbf{s}, d, \tau, l)}{\partial x} = 1$$

Again $x^*(\mathbf{q^s}, \hat{\mathbf{q}}^\mathbf{s}, d, \tau, l)$ is unique and positive due to strict concavity of the optimization problem.

Extending the earlier results to allow for change in design, we get

Proposition 7 *i) For any given realization of quantity signals for the initial and alternate designs, we have that ceteris paribus, the first best level of* x *and total expected cost saving will be higher for the project with greater probability of initial design being optimal, and for the project by the designer with greater effort, with greater experience and the one implementing less technical project and thus with greater precision of the quantity estimates.*
ii)For any given realization of quantity signals and for any given realization of expected total input costs for all designs, we have that ceteris paribus, the total expected cost will be lower for the project by the designer with greater effort, with greater experience and the one implementing less technical project and thus with greater precision of the quantity estimates. The total expected cost falls in the probability of initial design being optimal, i.e., π *if given the signals, the expected input cost for the alternate design is at least as large as that of initial design.*
iii)Expected first best level of x *is rising in* d *and* l *and* π *and falling in* τ*. Given that* x *responds optimally, ex-ante expected cost saving rises in* d *and* l *and* π *and falls in* τ *and ex-ante total expected cost falls in* d *and* l *and rises with* τ*. Ex-ante total expected cost falls in* π *if ex-ante expected input cost for the alternate design is at least as large as that of initial design.*

The above proposition extends the earlier results to allow for cases when change in design is possible. It also says that optimal cost reducing effort and expected cost savings is increasing in contract completeness measured by probability of no change in design, π. It also gives sufficient condition for total expected cost to be falling in probability of no change in design. If expected input cost of the alternative design is (weakly) larger than that of the initial design, then total expected cost is increasing in the probability of change in design. So for increase in scope and for addition of work items, this will be obviously true. But even if the expected input cost for both

[41]For investment y which is put during the construction phase, there will not be any cost due to changed design (in terms of reduction in cost saving) like for x and it will not be affected by planning effort d . This is because it will be specific to the finally implemented design as it is put after the finally implemented design is known. Since it will be put ex-post to any renegotiation, thus it does not depend on the compensation principle used at the time of renegotiation. It will be shown that y will be put optimally for IR, FP and DB contract (i.e., for variants of fixed price contracts) but there will be under-investment in it in case the initial contract is a cost sharing contract (including cost plus contract).

the designs are comparable, then also we get this result. However the necessary and sufficient condition for this to hold is that total expected cost (input cost minus cost saving due to effort x) is higher for the alternate design.

First best level of d

Again first best level of d minimizes the ex-ante total expected cost before the signals are received. So $d^*(\tau, l, \pi)$ solves

$$\min_d \{E\mathcal{C}^e(x^*(d,.), d, \tau, l, \pi) + d\}, i.e.,$$

$$\min_d \{\pi(EC^e_{[0,\overline{W}]}(x^*(d,.), d, \tau, l)) + (1-\pi)(\hat{EC}^e_{[0,\hat{\overline{W}}]}(x^*(d,.), d, \tau, l)) + d\}$$

FOC:

$$-\pi \frac{\partial EC^e(x^*(d,.), \tau, l, d)}{\partial d} - (1-\pi)\frac{\partial \hat{EC}^e(x^*(d,.), d, \tau, l, d)}{\partial d} = 1$$

$$\pi \frac{\partial EK^e(x^*(d,.), d, \tau, l)}{\partial d} + (1-\pi)\frac{\partial \hat{EK}^e(x^*(d,.), d, \tau, l)}{\partial d} = 1$$

Similar to the case for the given initial design, we get:

Proposition 8 *i)The first best level of d, $d^*(\tau, l, \pi)$ and of expected x, $\bar{x}^*(d^*(l,\tau), l, \tau, \pi)$ are increasing in learning level l and π and decreasing in project complexity τ.*
ii)Given that d and x are at their first best values, ex-ante expected cost saving is rising in l and π and falling in τ and ex-ante expected cost is falling in l and and rising in τ. Latter falls with π if ex-ante expected cost for alternate design is at least as large as that for the initial design.

So optimal planning effort is decreasing in the probability of change in design. This happens because of two reasons: a) One is the effect on direct benefit from d in terms of greater increase in cost saving for the initial design as compared to the alternate design. b)The other is the indirect effect through x. x increases the marginal benefit from d and is an increasing function of π itself. Similar argument holds for first best level of expected organization effort.

5.2 Choice of design and its determinants

So far we have assumed that there are just two ground conditions for given output features and that first design is chosen without analyzing in detail the choice of design. Now we will relax both these assumptions. First we will consider the choice of design for 2 ground condition case, i.e., design for the second ground condition can also be chosen initially. Then we will generalize this case to multiple possible ground condition. The factors which will affect this choice of design are analyzed.

Now we will analyze the problem of choice of initial design at $t = 1/2$, i.e., after the determination of buyer's requirement but before d and x are put. Once design is chosen, then planning will be done for that design. So this choice is based on either prior distribution or crude signals of actual quantity and per unit costs being available at that time. As shown earlier, if the first design is chosen optimally which remains optimal ex-post with probability π, then ex-ante expected cost will be

$$\begin{aligned} E\mathcal{C}^e(x,d,\tau,l) &= \pi EC^e(x,d,\tau,l) + (1-\pi)E\hat{C}^e(x,d,\tau,l) \\ &= [\pi EC_0^e + (1-\pi)E\hat{C}_0^e] - [\pi EK^e(x,d,\tau,l) + (1-\pi)E\hat{K}^e(x,d,\tau,l)] + x + d \end{aligned}$$

Alternatively, if second design is to be chosen initially, then x and d will be specific to this design and will be denoted by $\hat{x}$ and $\hat{d}$. The ex-ante expected cost, in this case, will become

$$\begin{aligned} E\hat{\mathcal{C}}^e(\hat{x},\hat{d},\tau,l) &= \pi EC^e(\hat{x},\hat{d},\tau,l) + (1-\pi)E\hat{C}^e(\hat{x},\hat{d},\tau,l) \\ &= [\pi EC_0^e + (1-\pi)E\hat{C}_0^e] - [\pi EK^e(\hat{x},\hat{d},\tau,l) + (1-\pi)E\hat{K}^e(\hat{x},\hat{d},\tau,l)] + \hat{x} + \hat{d} \end{aligned}$$

Now the socially optimal designing effort $\hat{d}^*$ and cost reducing effort $\hat{x}^*$ will minimize $E\hat{\mathcal{C}}^e(\hat{x},\hat{d},\tau,l)$.

Note that irrespective of which design is chosen initially, the expected benefit from the project is always $\pi.B + (1-\pi)B = B$. Thus the magnitude of $E\mathcal{C}^e(\hat{x},\hat{d},\tau,l)$ and $E\hat{\mathcal{C}}^e(\hat{x},\hat{d},\tau,l)$ will determine which design is initially chosen. Given that x, $\hat{x}$, d and $\hat{d}$ are at their first best or equilibrium value of some contract, denoted with superscript i, the first design will be chosen initially for contract i, where $i \in \{FB, IR, FP, C+, CS, DB\}$ iff

$$E\mathcal{C}^e(x^i,d^i,\tau,l) < E\hat{\mathcal{C}}^e(\hat{x}^i,\hat{d}^i,\tau,l)$$

i.e.,

$$\pi EK^e(x^i,d^i,\tau,l) + (1-\pi)E\hat{K}^e(x^i,d^i,\tau,l) - x^i - d^i$$
$$< \pi EK^e(\hat{x}^i,\hat{d}^i,\tau,l) + (1-\pi)E\hat{K}^e(\hat{x}^i,\hat{d}^i,\tau,l) - \hat{x}^i - \hat{d}^i$$

Note that expected input cost, $\pi EC_0^e + (1-\pi)E\hat{C}_0^e$, will remain same no matter which design is initially chosen. What matters for the choice of design is expected total cost saving in the two cases. Then above expression can be re-written as

$$\pi EK^e(x^i,d^i,\tau,l) - \pi EK^e(\hat{x}^i,\hat{d}^i,\tau,l) - x^i - d^i$$

$$> (1-\pi)E\hat{K}^e(\hat{x}^i,\hat{d}^i,\tau,l) - (1-\pi)E\hat{K}^e(x^i,d^i,\tau,l) - \hat{x}^i - \hat{d}^i$$

$$\pi[EK^e(x^i,d^i,\tau,l) - EK^e(\hat{x}^i,\hat{d}^i,\tau,l)] - x^i - d^i$$
$$> (1-\pi)[E\hat{K}^e(\hat{x}^i,\hat{d}^i,\tau,l) - E\hat{K}^e(x^i,d^i,\tau,l)] - \hat{x}^i - \hat{d}^i$$

Suppose the cost saving for the design which is not initially planned for is zero, i.e., $E\hat{K}^e(x,d,\tau,l) = 0 = EK^e(\hat{x},\hat{d},\tau,l)$ for the two design case, and if the expected cost saving is same for two designs if d and x are specific to that design respectively, i.e., $EK^e(x, d, \tau, l) = \hat{EK}^e(\hat{x}, \hat{d}|\tau, l)$, then design for which the probability of remaining optimal ex-post is greater than 1/2 will be chosen. That is, first design is chosen if $\pi > 1/2$ and second design is chosen if $\pi < 1/2$.

Now we generalize this condition to case with n possible ground conditions with associated designs. The probability of design l being ex-post optimal be given by π_l where $i \in \{1, 2, ..., n\}$ and $\pi_1 + \pi_2 + ... + \pi_n = 1$. Let the expected cost saving for design l be given by EK_l^e. Let the designing and cost reducing effort specific to design l be denoted by x_l and d_l. Then design l will be chosen if following condition holds $\forall m \neq l$:

$$\pi_l[EK_l^e(x_l^i, d_l^i, \tau, l) - EK_l^e(x_m^i, d_m^i, \tau, l)] - x_l^i - d_l^i$$

$$> \pi_m[EK_m^e(x_m^i, d_m^i, \tau, l) - EK_m^e(x_l^i, d_l^i, \tau, l)] - x_m^i - d_m^i$$

For simplicity, we have assumed that effect of planning and cost reducing effort for the design not chosen initially is same irrespective of the initially chosen. In particular, for any design l, $EK_l^e(x_m^i, d_m^i, \tau, l)$ is same $\forall m \neq l$. Now we get following determinants of choice of design generalized for n possible ground conditions:

1) If x is not feasible and is not helpful in reducing cost, then it does not matter which design is chosen initially. This is because expected input cost remains same irrespective of the initial choice and there is no adaptation cost of changing design.

2) Note that the presence of adaptation cost associated with change in design implies that $EK_l^e(x_l^i, d_l^i, \tau, l) - EK_l^e(x_m^i, d_m^i, \tau, l) > 0$. Thus the left hand side of above inequality is increasing in π_l[42].Thus ceteris paribus, the higher is the probability that a design remains optimal ex-post, the greater are the chances that that design will be chosen initially. For example, if π_1 is higher, then greater are the chances that first design will be chosen.

3) Suppose the cost saving for the design which is not initially planned for is zero, i.e., $\forall m \neq l$, $EK_l^e(x_m^i, d_m^i, \tau, l) = 0$, and if the expected cost saving is same for all designs if d and x are specific to the respective design, i.e., $EK_l^e(x_l^i, d_l^i, \tau, l)$ is same $\forall l$, then the choice of initial design is determined solely by the probabilities of remaining optimal ex-post. The above condition is then reduced to $\forall m \neq l$: $\pi_l \geq \pi_m$. That is, design which will remain optimal with highest probability will be chosen ex-ante.

4)Ceteris paribus, the greater is the term $EK_m^e(x_l^i, d_l^i, \tau, l)$ where $m \neq l$, the greater are the chances that design l will be chosen ex-ante. Former can happen because of two reasons: the low degree of specificity of investment x_l and also low degree of specificity of designing effort d_l. Thus ceteris paribus, the design for which the effect of x_l and d_l on other designs are higher, the greater are the chances of choosing that design to begin with.

5) Ceteris paribus, the higher is the term $EK_l^e(x_l^i, d_l^i, \tau, l)$, the greater are the chances that design l will be chosen initially. This can happen if cost reducing effort put and/or initial planning done for this design is very fruitful in decreasing expected

[42]In two design case, right hand side will also be falling with π_l as $\pi_m = 1 - \pi_l$.

costs. Latter has an important implication. One way through which d_l has greater effect on cost saving of design l is through greater completeness of initial design, i.e., $D_l(d_l, \tau, l)$ is higher. As shown above cost saving is greater if a work item is initially specified than when it is not. Also if benefit from d is falling in the complexity of work items, we will have that buyer will choose a simple, less complex design to begin with. Both these in turn imply that initially chosen design will look more complete. This is in sync with the empirical findings that initially the buyer starts with a simple more complete design and then later, when uncertainty over quantities and ground conditions get realized, shifts to more complex design or add more complex work items.

Thus the choice of design 1) is a function of π; 2) depends on the specificity of x which is a design specific investment and how important investment x is in reducing cost; and 3) depends on the effect of initial planning, both its specificity and importance in reducing cost. Ceteris paribus, design which is less costly in terms of designing costs and thus look more complete even with low planning effort will be chosen to begin with. Then during construction when uncertainty over quantities and costs gets realized and if site conditions warrant, then switch to more costly design.

Remarks Note that the choice of initial design can also vary with the type of contract used for procurement. For example, in case of C+ contracts, x is zero and if $\kappa_w^1(0) = \hat{\kappa}_{\hat{w}}^1(0) = 0$, then d^{C+} will also be zero. In this case, then any design can be chosen by the buyer to start with irrespective of what is optimal. The choice of design will be analyzed in more detail in 'contracts and equilibria' section.

6 Destruction Cost

In the current formulation of the model, we have 2 contracting periods, first when contract is allocated and second at the beginning of construction phase when uncertainty over actual quantity and cost gets resolved and ground conditions become known. But in reality, ground conditions and need for change in design may be revealed after construction has begun. Let us denote this time by t. As mentioned before, then $\tilde{t}$ will denote the random variable representing the earliest possible technologically feasible time that need for change in design can be potentially realized. For simplicity, we take the length of the construction phase for the initial design to be 1. Let $t \in [0, \bar{t})$ where $\bar{t} < 1$ represents the maximum time for realization of need for change in design[43]. The probability that till time $\bar{t}$, there is no need for change in design, as mentioned before, is π. Let the probability of realization for change in design at time t, given that it has not yet realized, be π_t. Then the probability

[43]If ground conditions are realized at $z = 1$, i.e., at the end of the construction phase, and change in design is needed, then there will not be any renegotiation as the contractor has already completed his/her work as per contract. Then to implement the new design, the sponsor will invite bids for a different contract.

of change in design given that initial design remains optimal till time t_0 is given by $\int_{t_0}^{\bar{t}} \pi_t dt$. Let us denote it by $P(t_0)$. Note that $\int_0^{\bar{t}} \pi_t dt = 1 - \pi$.

Suppose construction proceeds at uniform pace. So till time t, t^{th} portion of the work on initial design is implemented and t fraction of the total cost is incurred. When design needs to be changed, then some of the earlier done work may become redundant for the new design and may need to be destroyed. Maximum destruction cost (DC) denoted by $max[DC^a]$ is when all the work already done goes waste. So it is equal to the total cost incurred so far. Now the need for change may not be detected readily as the state of nature gets realized. That is, there can be delay in detecting the need for change. Let us denote the delay in detection by $\hat{\delta}(.) \geq 0$. It is a function of monitoring effort, denoted by m, which can result in earlier detection. Generally, the buyer monitors the project at regular intervals during the construction stage[44]. Regular monitoring is expected to result in quick detection and adaptation decision by the buyer/government of the changes required. Simpeh 2012 also points out that need for change is not readily identifiable. Detecting errors and the associated need for change as soon as it is technically feasible to know them requires costly effort. So $\hat{\delta}_m < 0$. Also $\hat{\delta}(., m = \infty) = t$, i.e., delay in detection is falling in m and if sufficiently high monitoring effort is put, then delay in detection can be eliminated. Also it is a weakly increasing function of t, $\hat{\delta}_t \geq 0$. If the minimum time needed for realization of need for change is at a later date during the construction stage (i.e., higher the t), then actual detection may also take more time as former may represent difficulty in finding the need for changed design. Let us denote the time when the need for design is actually detected, by $\hat{t}$ where $\hat{t} = t + \hat{\delta}(t, m) \geq t$. So we have $\hat{t}_m < 0$ and $\hat{t}_t > 0$. Since till the time when detection happens, $\hat{t}$ fraction of the total cost of the initially specified design has been incurred, so maximum realized destruction cost given t and m becomes:

$$\begin{aligned} max[DC^a] &= \hat{t}(t,m) C^a_{[0,\overline{W}]} \\ &= \hat{t}(t,m) \left\{ \int_0^{\overline{W}} \left[\kappa^a_w q^a_w - \kappa^1_w(x)(\kappa^0_w - (|q^e_w - q^a_w|)) \right] dw \right\} \end{aligned}$$

Thus maximum expected destruction cost (DC^e), given the signals, denoted by $max[DC^e]$ becomes

$$max[DC^e] = \int_0^{\bar{t}} \hat{t}(t,m)\pi_t dt \int_0^{\overline{W}} \left[\kappa^e_w(.) q^e_w(.) - \kappa^1_w(x)(\kappa^0_w - E_{q^a_w}((|q^e_w(.) - q^a_w|)|\mathbf{q^s}, d, \tau, l)) \right] dw$$

Ex-ante maximum destruction cost can be defined analogously.

[44] In India, NHAI monitors all projects which have initially estimated cost greater than 20 crores on quarterly basis. Projects costing more than 1000 crores called mega projects are monitored on monthly basis.

If some of the implemented work items are common, then it is not necessary to destroy all the work. Suppose change in design results in increase in the size of the project, then there will be no need to destroy any work item implemented so far. In this case, DC will be zero. We can capture this congruence between the two designs by the parameter $1 - \gamma$. Thus γ is the in-congruency parameter. Plausibly γ is an increasing function of the mass of the set a defined as $a = [\omega_0, \omega_{\overline{W}}] - [\hat{\omega}_0, \hat{\omega}_{\overline{\hat{W}}}]$. It is the set of work items comprising the initial design which are not needed for the new design. For $w \in [\omega_0, \omega_{\overline{W}}] \bigcap [\hat{\omega}_0, \hat{\omega}_{\overline{\hat{W}}}]$, i.e., for work items which are common in both the designs, it is an increasing function of $b = \max\{0, q^e_w - \hat{q}^e_w\}$. That is, the extent of reduction in quantities of work items.

Case I For **increase in scope**, i.e., only the quantity of existing work items has increased in the same or different proportion, a is null set and b is zero. So in this case $\gamma(|a|, b) = \gamma(0, 0) = 0$, i.e., no destruction cost.
Case II For **decrease in scope**, i.e., only the quantity of existing work items has decreased in the same or different proportion, a is null set and b is positive. So in this case $\gamma(|a|, b) = \gamma(0, b) > 0$, i.e., destruction cost is positive.
Case III For **addition of new work items**, a is null set and b is zero. So in this case also $\gamma(|a|, b) = \gamma(0, 0) = 0$ and destruction cost is zero.
Case IV For **deduction of work items**, a is positive while b is zero. So in this case $\gamma(|a|, b) = \gamma(|a|, 0) > 0$.
So destruction cost is positive if changed design leads to deduction of work items or reduction in quantities demanded of existing work items.
More precisely, γ is an increasing function of the the set of work items implemented till time t which are redundant for the new design. Let ω_t denote the set of work items implemented till time t. Then it is an increasing of $|a_t|$ where $a_t = \omega_t - [\hat{\omega}_0, \hat{\omega}_{\overline{\hat{W}}}]$. Similarly for b. So γ is a weakly increasing function of t.

Now Actual destruction cost becomes

$$DC^a = \zeta(\hat{t}(t, m), \gamma(t))\pi_t dt.C^a_{[0,\overline{W}]}$$

where $\zeta(.)$ is increasing in both its arguments. Thus $\zeta_t > 0$, $\zeta_m < 0$ and $\zeta_\gamma > 0$. That is, the later it is realized that a change in design is needed, the lower is the monitoring effort and lower the congruence between the initial and the changed design, higher will be the destruction cost. So expected destruction cost becomes

$$DC^e = \int_0^{\bar{t}} \zeta(\hat{t}(t, m), \gamma(t))\pi_t dt.C^e_{[0,\overline{W}]}$$

We have similar expressions for ex-ante expected destruction cost.

Now we consider the renegotiation cost (RC). Even if the ground conditions are detected at time z but the changes may not be readily implemented. There can be delay in implementing the necessary changes. Let us denote it by $\hat{\hat{\delta}}(.) \geq 0$. It can be due to dispute and the time consuming renegotiation[45]. It requires effort to bring about agreement to implement the changes. If monitoring effort is high, then it will be easy to give proves and supporting documents at the time of arbitration in favor of the case and to arrive at a more agreeable cost estimate for the changes. So $\hat{\hat{\delta}}(.)$ is also a decreasing function of m[46]. Also the delay is a weakly increasing function of γ, the in-congruency parameter. If the two design are very different, then it may be more difficult to agree on the price for changes. For already specified work items, it may be easier to get to a more agreeable price as compared to new wok items for which there is no initial bid. But even for former, the total payment is not the product of bid and quantity if quantity varies varies by more than 25%. Rather there is adjustment made (See Bajari et al 2014). For new work items also there may exist blue book data which provides reference price. Thus RC will be weakly increasing the in-congruency parameter. Time taken in renegotiation may also be a weakly increasing function of delay in detection and thus a weakly increasing of t. If it takes more time to detect the necessary changes, it means that the changed design is complex and thus bringing agreement over the required changes and the associated price will also be more difficult. Thus even if ground conditions can be potentially known at time t, they are actually implemented at time $\hat{\hat{t}} = \hat{t}(t,m) + \hat{\hat{\delta}}(t,m,\gamma)$ where $\hat{\hat{\delta}}_t \geq 0, \hat{\hat{\delta}}_m < 0$ and $\hat{\hat{\delta}}_\gamma \geq 0$. Renegotiation cost is an increasing function of the time taken to bring about agreement[47], i.e., $\hat{\hat{\delta}}(.)$. This delay leads to additional costs on three counts:1) Overhead costs, 2) haggling cost and 3)loss of social surplus due to delayed use of output from the project. Plausibly, we take these costs to be proportional to total cost. This delivers expected renegotiation cost to be

$$\begin{aligned} RC &= \int_0^{\bar{t}} \hat{\hat{\delta}}(t,m,\gamma)\pi_t dt . C^e_{[0,\overline{W}]} \\ &= \int_0^{\bar{t}} \hat{\hat{\delta}}(t,m,\gamma)\pi_t dt \int_0^{\overline{W}} \left[\kappa^e_w(.)q^e_w(.) - \kappa^1_w(x)(\kappa^0_w - E_{q^a_w}((|q^e_w(.) - q^a_w|)|\mathbf{q^s}, d, \tau, l))\right] dw \end{aligned}$$

Thus combined destruction cost and renegotiation cost [48] becomes

$$\eta(t,\gamma,m).C^e_{[0,\overline{W}]}$$

[45]There can be renegotiation costs at the time of completion of initial design also. But such costs are not expected to be large as contract generally has some provision for it and buyer has some right to demand these. Even if we allow for this possibility, our results will get further strengthened.

[46]Apolot et al and Menon and Rahman 2013 shows that poor monitoring and control leads to high cost overruns.

[47]For simplicity, we assume that agreement always occurs and that same contractor implements the changed design.

[48]Renegotiation cost may also depend on destruction cost as it is more difficult to come to agreement if DC is huge. Empirical literature, for e.g., Bajari et al 2014 has shown that it is relatively more difficult to successfully renegotiate in case of deductions. Thus renegotiation cost is higher in case of deduction of work items or decrease in scope as compared to additions and increase in scope. Thus destruction cost and renegotiation cost vary in similar way with a and b. This possibility is allowed in the model.

where $\eta(t,\gamma,m) = \int_0^{\bar{t}}[\zeta(t,m,\gamma) + \hat{\hat{\delta}}(t,m,\gamma)]\pi_t dt$. Thus we have $\eta_t > 0$, $h_\gamma > 0$ and $h_m < 0$[49].

So now the total expected cost after receiving the signals becomes $\pi C^e + (1-\pi)\hat{C}^e + \eta(t,\gamma,m)C^e$

$$\begin{aligned}
&= \pi[C_0^e - K^e(x|\mathbf{q^s},d,\tau,l)] + (1-\pi)[\hat{C}_0^e - \hat{K}^e(x|\hat{\mathbf{q}}^{\mathbf{s}},d,\tau,l)] \\
&+ \eta(t,\gamma,m)[C_0^e - K^e(x|\mathbf{q^s},d,\tau,l)] + x + d \\
&= [(\pi C_0^e + (1-\pi)\hat{C}_0^e + \eta(t,\gamma,m))C_0^e] - [\pi K^e(x|\mathbf{q^s},d,\tau,l) \\
&+ (1-\pi)[\hat{K}^e(x|\hat{\mathbf{q}}^{\mathbf{s}},d,\tau,l) + \eta(t,\gamma,m)K^e(x|\mathbf{q^s},d,\tau,l)] + x + d
\end{aligned}$$

Ex-ante expected cost can be similarly derived.

The first best and equilibrium level of efforts d and x under different contracts will change under this scenario but our qualitative results will continue to hold. We get the following result:

Proposition 9 *A) First best levels of x and d are*
i) increasing in the destruction and the renegotiation costs.
ii) increasing in γ and falling in m.
B) Suppose that there are two probability functions π_t and π'_t where latter first order stochastically dominates (FOSD) the former, then ceteris paribus, $d^*(.)$ and $x^*(.)$ will be higher in the latter case.

That is, if the alternate design is very different from the initial design and it is more likely that the need for change will be realized at a later date, the higher will be the cost reducing effort put by the contractor and higher will be the initial planning effort[50]. On the other hand, when buyer puts higher monitoring effort ex-post, then low planning and cost reducing effort will be put. Thus ex-post monitoring and ex-ante planning are substitutes.

[49] Alternatively, one can analyze the problem in terms of sequences and subsequences. Let $T = \{1,2,...,t\}$ denote the finite sequence of time when ground condition gets realized. Consider its subset $Z = \{1,2,...z\}$ where $z \leq t$. Let T_z be a subsequence and is given by an increasing function going from Z to T. It represents the point in time when required changes are actually adopted. Now the transaction cost of change becoming necessary at time period t is an increasing function of the gap it got delayed, i.e., the distance min $[T(.) - t]$ s.t. $T(.) \geq t$. Greater monitoring means that set Z is larger and the distance is smaller. This way of modeling also gives the reduced form result that adaptation cost is falling in level of monitoring.

[50] As mentioned before, the contractor can undertake cost reducing effort y during construction. Suppose it is put at some period $t_y \in [2,3]$. Now there are two possibilities. If $z + 2 > t_y$, i.e., y is put before design is changed, then analysis will be similar to x. If $z + 2 < t_y$, i.e., it is put after design is changed, then it will be specific to the new design and no further renegotiation. So it will be chosen optimally under every contract except the cost sharing contracts including cost plus contract.

6.1 Adaptation cost

Adaptation cost is incurred when set of work items and actual quantity of work item turns out to be different than what is planned for. It equals $\pi A_1 + (1-\pi)A_2 + (1-\pi)(DC + RC)$ where

$$A_1 = \int_0^{W(d,\tau,l)} \kappa_w^1(x) E_{q_w^a}((|q_w^e - q_w^a|)|\mathbf{q^e}, \tau, l, d)dw + \int_{W(d,\tau,l)}^{\overline{W}(\tau)} \kappa_w^1(x) E_{q_w^a}((|q_w^e - q_w^a|)|\mathbf{q^e}, \tau, l, d)dw$$

$$A_2 = \int_0^{\overline{\hat{W}}(\tau)} \hat{\kappa}_w^1(x) E_{\hat{q}_{\hat{w}}^a}((|\hat{q}_{\hat{w}}^e - \hat{q}_{\hat{w}}^a|)|\hat{\mathbf{q}}^{\mathbf{e}}, \tau, l, d))d\hat{w}$$

. We normalize A_i by dividing it by the amount of cost saving in case when actual quantities are same as expected. That is, we take proportionate loss in cost saving due to adaptation, as a proportion of maximum possible cost saving when there is no adaptation. So A_1 is divide by $\kappa_w^0.\kappa_w^1(x)$. Now we will look for its determinants. All four terms and thus total adaptation cost are falling in d and l and rising in τ.

As shown above, the combined destruction and renegotiation cost and thus also total AC are falling in congruency of the two designs and in the monitoring effort and rising in the delay in realization of change in design.

7 Conclusion

to be written.

8 Technical Appendix

Available on request.