Intergenerational mobility with incomplete depreciation of human capital

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Abstract

We analyze the effect of incomplete depreciation of human capital on intergenerational mobility and distributional dynamics by extending a class of influential models. These models assume that human capital completely depreciates after its use by each generation. We show that full depreciation of human capital vastly overestimates social mobility. If human capital depreciates slowly, social mobility is sluggish and inequality becomes more persistent in line with the stylized facts. Using an exact solution for distributional dynamics, we show that social mobility is history dependent. It depends on the state of inequality inherited from the past. Our calibration with plausible rate of depreciation of human capital reproduces more realistic measures of social mobility. We also show that a proportional education subsidy can mitigate the slow social mobility resulting from incomplete depreciation.

Key words:
Intergenerational mobility, inequality persistence, human capital depreciation
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1. Introduction

It is an open question whether the son of a poor farmer will become a highly paid executive manager. The evidence during the last two decades indicates that such intergenerational mobility is slow (Machin, 2004). Clark and Cummins (2012) establish that there is considerable persistence in the wealth status of households in England from 1800 to 2012. They predict that it will take another two hundred years to complete the process of social mobility.[1] In this paper, we examine the role of human capital depreciation in determining intergenerational mobility. We show that human capital obsolescence or depreciation could be an important factor for social mobility via its role in intergenerational knowledge transfer.

The extant literature dealing with social mobility and inequality assumes complete depreciation of human capital (e.g. Glomm and Ravikumar, 1992, Benabou, 1996, 2000, 2002) and focuses exclusively on the intergenerational knowledge complementarity. This basically shuts down important intergenerational knowledge transmission mechanism such as the substitution possibility between homeschooling and formal schooling because parental human capital is assumed to fully depreciate or become obsolete after its use. However, it is evident that knowledge transmit from generation to generation with little or no formal investment.

In this paper, we consider a more general human capital technology which allows for both complementarity and substitutability of human capital knowledge. In the production function for children's human capital, in the model, parental human

[1] Although social mobility is a broader notion of change in social status, we use this term in a narrower sense to indicate intergenerational mobility.

capital enters twice, first multiplicatively with respect to investment in education as effective time (home schooling) and then linearly because of incomplete depreciation of human capital. In both cases children receive part of the human capital of their parents. But with the home schooling mechanism the human capital of the parents and schooling are more of complementary while in the incomplete depreciation case the two are substitutes.

The substitutability of knowledge has implications for mobility in a sense that if old knowledge dies hard, it lowers individual incentive for schooling and training to acquire new knowledge. A lower rate of depreciation depresses current investment because the adult carries forward human capital from the previous generation. On the other hand, greater depreciation or obsolescence of human capital promotes more schooling and training.[2] Through this channel it facilitates greater social mobility. A son of a poor farmer can then quickly transform into a professor through formal schooling. This transmission channel of human capital obsolescence in determining social mobility and inequality has not been well explored in the inequality and social mobility literature.[3] This is exactly where the present paper contributes.

To examine the role of obsolescence of human capital, we extend a standard

[2]There are two principal drivers of human capital depreciation: (i) technical obsolescence and (ii) economic obsolescence. See Rosen (1975), de Grip and van Loo (2002) and de Grip (2006) for a discussion on human capital obsolescence. The former is due to changes that originate in individuals' personal circumstances such as ageing, illness, injury while technological progress accounts for the latter. Our focus in this paper is on the latter.

[3]A sparse literature exists which indirectly corroborates our hypothesis that high depreciation of human capital could promote more schooling. Murillo (2011) studies Spanish workers and find that depreciation of human capital is greater for workers with higher educational attainment. Similar evidence is found by Raymond and Roig (2004). Although these studies do not necessarily allude to a cause-and-effect relationship between human capital depreciation and schooling, it at least motivates a theoretical investigation of such a relationship.

model of Benabou, by adding incomplete depreciation of human capital. Using the Clark and Cumins (2012) estimate of social mobility as our target, we show that the extant models of human capital (e.g. Benabou 1996, 2000, 2002) substantially overestimate social mobility due to the assumption of full depreciation of human capital. On the other hand, our calibrated model with plausible rate of depreciation of human capital could reproduce the Clark and Cumins (2012) estimate of social mobility. The implication of our study is that the observed low social mobility in Clark and Cumins (2012) could be due to slow obsolescence of human capital.

In our model, human capital is the only reproducible capital as in Loury (1981). Adults differ in terms of initial human capital and receive a warm-glow utility from investing in child's education in the spirit of Galor and Zeira (1993). The credit market is imperfect as in Loury (1981), Banerjee and Newman (1993), Galor and Zeira (1993) and Benabou (2000, 2002).[4] Adults cannot borrow from the credit market to remedy the initial deficiency of human capital. The only way capital-poor adult can catch up with the rich is by investing in human capital through schooling. Initial differences in human capital and credit market imperfection give rise to a cross-sectional inequality which transmits from one generation to another. Our model has the standard convergence property that absent idiosyncratic luck, poor catch up with the rich in the long run and inequality vanishes.[5] How fast this

[4] A considerable literature has focused on the role of credit market imperfection in perpetuating inequality and thus it has direct and indirect implications for social mobility (e.g., Loury, 1981, Galor and Zeira, 1993, Benabou, 1996, Mulligan, 1997, Bandyopadhyay and Tang, 2011 among others).

[5] If there is a cross sectional difference in luck, the inequality in the long run will be driven by difference in luck (idiosyncratic shock) as in Becker and Tomes (1979). To focus only on social mobility (which is a property of transitional dynamics) we assume that everybody has the same luck

convergence occurs determines intergenerational mobility.

We show that incomplete depreciation gives rise to heterogeneous investment propensities. This heterogeneity disappears if depreciation is full and then the well known Solow identical saving rule prevails. When depreciation is partial, all agents invest less because of the substitution possibility between old and new knowledge. However, such a decline in investment affects poor more because poor have a higher marginal return to investment. It is thus difficult for the poor to bridge the initial inequality which results in a slower social mobility. On the other hand, the long-run growth rate rises because the undepreciated human capital boosts the prospective gross return to capital.

We derive a novel closed form expression for distributional dynamics showing that the social mobility is history dependent. Societies that inherit high inequality also show slow mobility. This property is consistent with the recent finding of Clark (2013) that social mobility is lower in more unequal economies. To the best of our knowledge, our closed form solution for distributional dynamics showing this history dependent social mobility is new in the literature. The extant papers often appeal to the Cobb-Douglas framework for analytical tractability (e.g., Benabou 1996, 2000, 2002; Bandyopadhyay and Tang, 2011) which makes the social mobility measure invariant to the inequality history.

Finally, we show that our key result that incomplete depreciation of human capital slows down intergenerational mobility is robust when individuals differ in luck. We

but only differ in terms of initial human capital. In section 4, we extend the model to incorporate idiosyncratic luck differences.

also show that a proportional educational subsidy could mitigate the slow social mobility due to partial depreciation of human capital.

The paper is organized as follows. Section 2 presents the model with its properties. Section 3 provides the quantitative analysis. Section 4 points to some extensions of the model. Section 5 concludes.

2. The model

2.1. Preference and technology

Consider a continuum heterogeneous households $i \in [0, 1]$ embedded in overlapping generations. Each household i consists of an adult of generation t attached to a child. A child only inherits human capital from her parents and does not make any decision as her consumption is already included in that of her parents. Adults are endowed with two units of time that they could use for the production of goods and services, leisure and child rearing. An adult, at date t employs a unit of time inelastically into the production process. The rest of her time is optimally allocated between child rearing and leisure. The time used in goods production translates into h_{it} efficiency units (human capital) for the production of final goods and services to earn income (y_{it}) using the following Cobb-Douglas production function:

$$y_{it} = a h_t^{1-\alpha} h_{it}^{\alpha} \tag{1}$$

where $a > 0$ is simply an exogenous productivity parameter, $\alpha \in (0, 1)$, h_t represents the aggregate stock of knowledge in the spirit of Arrow (1962) and Romer (1986) which the adult faces as given although it is determined by the aggregate dynamics.[6] The child at date t behaves as an adult at $t+1$.

Agents care about their own consumption (c_{it}) and leisure $(1-l_{it})$, as in Garcia and Turnovsky (2008), and receive a "joy of giving" from the human capital stock of their children (h_{it+1}). In other words, the utility of the adult at date t is given by:[7]

$$u\left(c_{it}, h_{it+1}\right) = \ln c_{it}\left(1-l_{it}\right)^{\eta} + \beta \ln h_{it+1} \tag{2}$$

where $0 < \beta < 1$ is the degree of parental altruism, l_{it} is the time allocated for nurturing, h_{it+1} represents the human capital of the offspring of agent i. At the end of the period, parents allocate income between current consumption (c_{it}) and spending on education (s_{it}).

$$c_{it} + s_{it} = y_{it}\left(1-\tau\right) \tag{3}$$

where $y_{it}\left(1-\tau\right)$ is the ith individual disposable income and τ is the flat rate tax.[8]

2.2. Human Capital Technology

The human capital is the only reproducible input in our model. The schooling technology specifies how the stock of human capital of parents (h_{it}), their spending

[6] Such a technology basically means that there is private diminishing returns but social constant returns to human capital.

[7] The choice of a logarithmic utility function and altruistic agents with a "joy of giving" motive is merely for simplicity. Also see Glomm and Ravikumar (1992), Galor and Zeira (1993), Saint-Paul and Verdier (1993) and Benabou (2000) for similar settings.

[8] A flat rate income tax τ which is wastefully spent is introduced in this model to aid the calibration.

on child's schooling (s_{it}) and their time (l_{it}) spent on nurturing the child shapes the offspring's human capital. Specifically, we consider the follwoing functional form for the human capital production function:

$$h_{it+1} = a_2(l_{it}h_{it})^{1-\theta}\left[\{(1-\delta)h_{it} + s_{it}\}\right]^{\theta} \quad (4)$$

where $\{\theta, \delta, l_{it}\} \in (0,1)$. The term ($l_{it}h_{it}$) may be used to capture home schooling in quality time; a knowledgeable parent is better equiped in promoting the learning of her child. In contrast, $(1-\delta)h_{it}$ is used to capture some inherited component of human capital, which represents the amount of workable human capital that a child inherits from her parents in the absence of any new investment. If an adult undertakes no investment in her child's education, for instance, unlike Benabou (2000), the child still inherits some workable human capital in proportion to $(1-\delta)h_{it}$.

The main distinction between the two types of intergenerational knoweldge transfer mechanisms is that while one captures more of, the complementarity feature of knowledge, the other reflects its substitutability. The functional form for the human capital technology (4) is novel in a sense that it allows both of such complementarity and substitutability between the old knowledge stock (h_{it}) and the current knowledge flow (s_{it}).

In case of $\delta = 1$, the production function takes the standard form (as in Glomm and Ravikumar, 1992, Benabou, 1996, 2000, 2002, de la Croix and Michel, 2002, p.260 among many other) and the intergenerational link is established only through intergenerational complementarities. The intergenerational elasticity $1-\theta$ determines this link.[9] On the other hand, if $\theta = 1$ (and $l_{it} = a_2 = 1$), the investment technology reverts to the standard linear depreciation rule (i.e., $h_{it+1} = (1-\delta)h_{it} + s_{it}$) as in Mankiw et al. (1992). Therefore, (4) is a standard capital accumulation function in its general form that accounts for both complementarity and substitutability of human capital.

The depreciation cost parameter (δ) in the human capital production function is the main focus of this paper. We argue that a higher rate of depreciation (or obsolescence) of old human capital promotes social mobility by promoting more investment in schooling.

2.3. Initial distribution of human capital

At the beginning, each adult of the initial generation is endowed with human capital h_{i0}. The distribution of h_{i0} takes a known probability distribution,

$$\ln h_{i0} \sim N(\mu_0, \sigma_0^2) \tag{5}$$

and it evolves over time along an equilibrium trajectory.[10]

2.4. Equilibrium

In equilibrium, all individuals behave optimally and the aggregate consistency conditions hold.

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[9] The parameter θ may have different interpretations; as the determinant of the curvature of the marginal return to investment it can be ascribed to a convex capital adjustment cost (see, e.g., Lucas and Prescott, 1971, Basu, 1987, Hercowitz and Sampson, 1991 and Basu et al., 2012).

[10] Similar lognormal distribution of human capital wealth is applied in Glomm and Ravikumar (1992), Benabou (2000, 2002) and de la Croix and Michel, (2002, p.266), which provides a closed form solution to the model.

Optimality: Given h_{it} and h_t, an adult of cohort t solves the following maximization problem, obtained by substituting (3) and (4) into (2),

$$\max_{s_{it}} \left\{ \ln \left(\left(y_{it} \left(1-\tau\right) - s_{it} \right) \left(1-l_{it}\right)^{\eta} \right) + \beta \ln \left(a_2 \left(l_{it} h_{it} \right)^{1-\theta} \left(\left(1-\delta\right) h_{it} + s_{it} \right) \right)^{\theta} \right\} \quad (6)$$

An adult's optimal investment decision constitutes both new investment plus a replacement of depreciated capital:

$$s_{it} = \left(\left(1-\tau\right) \theta \beta y_{it} - \left(1-\delta\right) h_{it} \right) / \left(1+\theta\beta\right) \quad (7)$$

$$l_{it} = \beta \left(1-\theta\right) / \left(\beta \left(1-\theta\right) + \eta \right) \quad (8)$$

Parents thus allocate a constant fraction of their time for nurturing the learning of their offspring. Because of the substitutability between old and new knowledge, adults invest less in schooling if they inherit more old knowledge $(1-\delta)h_{it}$. A lower rate of depreciation depresses current investment across the board because it lowers the marginal benefit of investment. To see this clearly, check from the first order condition for investment that equates the marginal utility cost of investment and the corresponding marginal utility benefit:

$$1/\left((1-\tau) y_{it} - s_{it} \right) = \beta\theta / \left((1-\delta) h_{it} + s_{it} \right) \quad (9)$$

For a given h_{it} and h_t, a lower δ depresses the marginal benefit of investment (the right hand side of (9)) discouraging individual investment propensity.

Note that in general such models allow disinvestment ($s_{it} \leq 0$) although the optimal individual human capital accumulation is always positive.[11] In this case, some adults consume more than their income at the expense of a depleted human capital of their children although the optimal human capital of the offsprings always remain positive (as we see later). In our calibration reported later, the extreme scenario of disinvestment in children does not arise and the steady state investment ratio remains positive for plausible parameter values.

Aggregate Consistency: (i) $c_t \equiv \int c_{it} di$, $s_t \equiv \int s_{it} di$, $y_t \equiv \int y_{it} di$, $h_t \equiv \int h_{it} di$ where the left hand side variable in each of them means the aggregate.[12] (ii) The aggregate budget constraint is thus given by:

$$c_t + s_t = y_t (1 - \tau) \tag{10}$$

2.4.1. Incomplete depreciation and investment propensity

Incomplete depreciation has a nontrivial effect on individual saving propensities in the model. If $\delta = 1$, saving (or investment) propensity is constant and the same across agents:

[11] Real life examples in human capital disinvestment include sending ones offspring to work as child labour in less developed countries without undertaking any investment in schooling or child abuse. This could arise in the model if adults are very myopic (low β), TFP is too low or the tax rate is too high.

[12] We use the operators $\int$ and E interchangeably in the text to denote aggregation across individuals.

$$s/y = (1-\tau)\theta\beta/\left(1+\theta\beta\right) \tag{11}$$

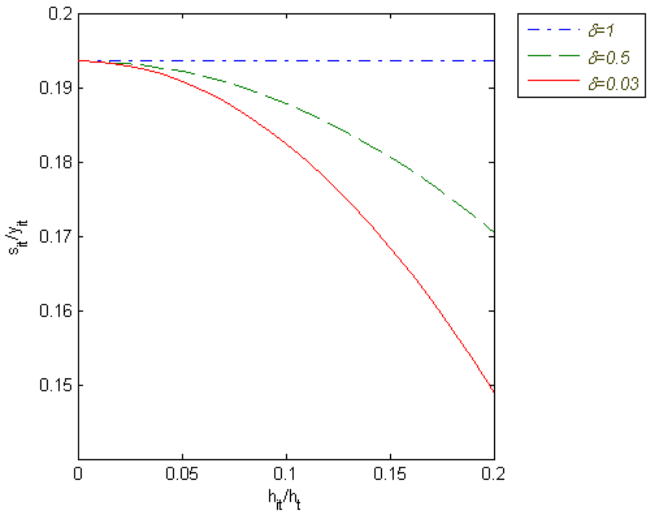
If $\delta \neq 1$ (and $\alpha \neq 1$), however, the saving propensity differs across agents whereas s_{it}/y_{it} is decreasing in h_{it}/h_t.

$$s_{it}/y_{it} = \left(a\theta\beta(1-\tau) - (1-\delta)\left(h_{it}/h_t\right)^{1-\alpha}\right)/a\left(1+\theta\beta\right) \tag{12}$$

Under incomplete depreciation, poor invest more than the rich because they have a higher marginal return to investment (given $\alpha < 1$). However, individual and aggregate investment reach the highest if there is complete depreciation of capital. Figure 1 plots the investment propensities of agents differing in their capital stocks to confirm these results.[13]

[13] The model parameters are fixed at the calibrated levels discussed in Section 3.

Figure 1: Saving propensity, individual wealth and incomplete depreciation



2.4.2. Individual optimal human capital accumulation

Based on (1), (4) and (7), the ith adult's optimal human capital accumulation is given by:

$$h_{it+1} = \phi h_{it} \left(1 - \delta + a_1 h_t^{1-\alpha} h_{it}^{\alpha-1}\right)^{\theta} \tag{13}$$

where $a_1 \equiv (1-\tau)\,a$ and $\phi \equiv a_2 \left((\theta\beta)^2 / \left((1+\theta\beta)(\eta+\beta\theta)\right)\right)^{\theta}$. Thus, each offspring's optimal law of motion of human capital is determined by both the parent's human capital (h_{it}) and the aggregate human depreciation (h_t).

2.4.3. Incomplete depreciation and social mobility

To see the importance of depreciation of human capital for social mobility, log-linearize (13) around the balanced growth rate in order to get:

$$\ln \widetilde{h}_{it+1} \simeq \rho \ln \widetilde{h}_{it} \tag{14}$$

where $\widetilde{h}_{it} \equiv h_{it}/h_t$ and,

$$\rho \equiv \partial \ln \widetilde{h}_{it+1}/\partial \ln \widetilde{h}_{it} = 1 - (\theta (1-\alpha) a_1) / (1 - \delta + a_1) \tag{15}$$

If the ith adult is slightly below the average at date t ($h_{it} < h_t$), equation (15) says that her child will inherit this trait only to the extent of ρ. Thus the greater the size of ρ, the slower the mobility. The inverse of ρ is the social mobility used in the literature (e.g., Benabou, 2002). In the case of $\delta = 1$, ρ reduces to $1 - (1-\alpha)\theta$. A lower depreciation rate ($0 < \delta < 1$), however, raises ρ above this value which means that social mobility is slower if δ is lower.

The same point can be made more generally by computing the dynamics of the cross sectional variance of human capital based on (14) which yields

$$\sigma_{t+1}^2 = \rho^2 \sigma_t^2 \tag{16}$$

where $\sigma_t^2 = \mathrm{var}\left(\ln \widetilde{h}_{it}\right) = \mathrm{var}\left(\ln h_{it}\right)$.

Eq. (16) shows how the inequality transmits from one generation to another. Although inequality asymptotically approaches zero,[14] its short run dynamics, the prime measure of social mobility is determined by ρ. The greater the size of ρ, the slower the social mobility which also translates into a more persistent inequality. It

[14]This is intuitive as there are no factors in our model such as an uninsured idiosyncratic shock that lead to a nondegenerate income distribution.

is straightforward to verify that a lower depreciation rate increases this persistence by slowing down this social mobility.[15] A lower depreciation rate aggravates the process of mobility further by lowering ρ. The following proposition summarizes our key result.

Proposition 1. *A lower depreciation rate (δ) makes the social mobility slower and the inequality process more persistent.*

2.5. Social mobility and distributional dynamics: A closed form solution

In the preceding section, the analysis of the relationship between social mobility and the depreciation rate is established in the neighborhood of the balanced growth rate. Thus the results are true locally. We now show that this result also holds globally. Our model allows for a closed form expression for the distributional dynamics in terms of cross sectional variance of human capital (σ_t^2). In addition, we also derive the short run dynamics of the growth rate of human capital, γ_t.

Proposition 2. *Given the initial cross sectional inequality characterized by (5) and (13), the dynamics of inequality and growth are given by the following laws of motion respectively,*

$$\sigma_{t+1}^2 = \theta^2 \ln \frac{\kappa^2 \exp\left(\theta^{-2}\sigma_t^2\right) + a_1^2 \exp\left(b_1 \sigma_t^2\right) + 2\kappa a_1 \exp\left(b_2 \sigma_t^2\right)}{\left(\kappa + a_1 \exp\left(0.5\omega\sigma_t^2\right)\right)^2} \tag{17}$$

and

$$\gamma_{t+1} = \ln \phi + 0.5\left(1/\theta - 1\right)\left(\sigma_t^2 - \sigma_{t+1}^2\right) + \theta \ln\left(\kappa + a_1 \exp\left(0.5\omega\sigma_t^2\right)\right) \tag{18}$$

[15] The dynamics of income inequality ($\sigma_{y,t}^2$) is also identical and can also be derived from (1) and (16):

$$\sigma_{y,t+1}^2 = \rho^2 \sigma_{y,t}^2$$

where

$$
\begin{aligned}
\gamma_{t+1} &\equiv \ln h_{t+1} - \ln h_t \\
\kappa &\equiv 1 - \delta \\
\omega &\equiv (\alpha - 1)\,(2/\theta + \alpha - 2) \\
b_1 &\equiv \omega + (1/\theta + \alpha - 1)^2 \\
b_2 &\equiv 0.5\omega + (1/\theta + \alpha - 1)\,/\theta
\end{aligned}
$$

Proof. See Appendix A. ■

If $\delta = 1$, one confirms that $\sigma_{t+1}^2 = \rho^2 \sigma_t^2$ as in (16). The fact that (13) is loglinear when $\delta = 1$, the loglinearization and the actual solution converge.[16]

The dynamics of inequality is governed by the time path of $\{\sigma_t^2\}_{t=0}^{t=\infty}$ which is determined by its own history. It is not influenced by growth. On the other hand, the growth rate depends on the current and past inequality. The causality thus runs from inequality to growth in this setting. It is evident by the fact that σ_{t+1}^2 is a function of σ_t^2 alone while γ_{t+1} depends on σ_{t+1}^2 and σ_t^2. A higher contemporaneous inequality depresses growth because $\partial\gamma_{t+1}/\partial\sigma_{t+1}^2 < 0$. This inverse relationship is not surprising in a model with imperfect credit market. Since poor have a higher marginal return to investment than the rich and they cannot borrow from the rich due to credit market imperfection, Pareto efficiency cannot be achieved. Therefore,

[16]Note also that when $\theta = 1$ and $\delta = 1$, we get the well known Solow saving rule: $h_{it+1} = y_{it}(1-\tau)a_2\beta/\,(1+\beta)$.

in such an economy higher inequality corresponds to a greater inefficiency and thus translates into lower growth.

2.5.1. History dependent social mobility

The social mobility based on the *exact* solution is the inverse of the gradient of (17). This gradient is given by,

$$\rho_t^2 \equiv \partial\sigma_{t+1}^2 / \partial\sigma_t^2 = f\left(\sigma_t^2\right) \tag{19}$$

when $\sigma_t^2 = 0$ in the steady state, one obtains,

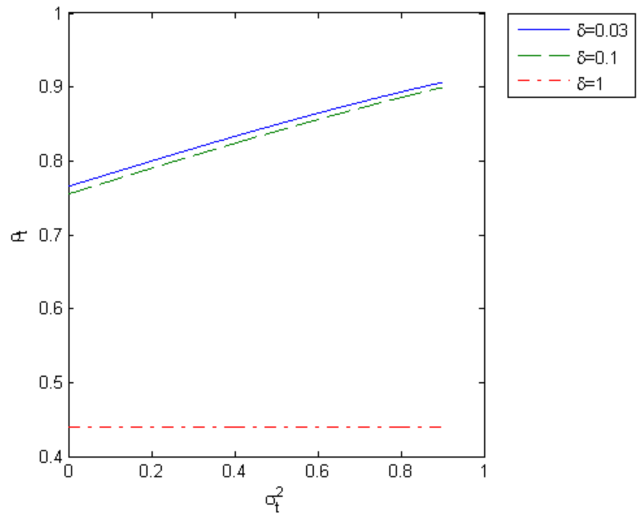
$$\rho_t^2 = \rho^2 = \ \left(1 - \left(\theta\left(1 - \alpha\right) a_1\right) / \left(1 - \delta + a_1\right)\right)^2 \tag{20}$$

which reduces to the loglinearized measure (15). Appendix B presents the derivation of (19).

The exact solution for social mobility (19) reveals a path dependent property which is not seen in the loglinearized version (15). It depends on the current state of inequality, σ_t^2 which is history dependent (see (17)). Figure 2 plots ρ_t against σ_t^2 for alternative values of the depreciation parameter δ. Social mobility is less in a more unequal society which reflects Clark's (2013) empirical finding.[17] Lower depreciation slows down mobility for all inequality states as seen by the comparison (when $\delta = 0.1$ and $\delta = 0.03$). It is noteworthy that for full depreciation ($\delta = 1$) this mobility loses its history dependence property.

[17] See Figures 1 and 2 of Clark (2013).

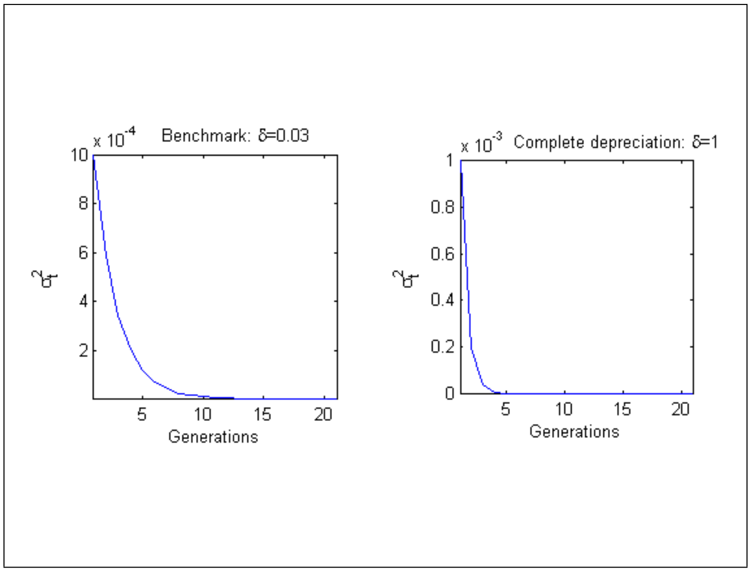
Figure 2: Social mobility versus inequality



2.5.2. Depreciation and distributional dynamics

Figure 3 finally illustrates the distributional dynamics for our exact solution (17) by comparing two economies, one with full depreciation ($\delta = 1$) and the other with incomplete depreciation ($\delta = 0.03$) as fixed in our calibrated economy later on. An incomplete depreciation slows down convergence quite significantly. All these results basically reinforce our key result that the rate of depreciation of human capital could be an important determinant of social mobility and the underlying distributional dynamics.

Figure 3: Incomplete depreciation and the convergence of inequality dynamics



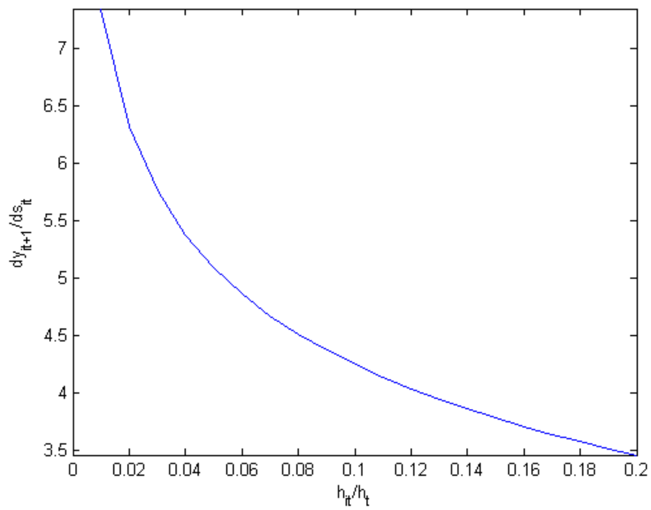
2.5.3. Why does a lower depreciation rate slow down social mobility?

Social mobility in this model is fueled through investment in human capital. Due to diminishing returns, poor households have a higher marginal return to investment than rich. This is shown below where the marginal return to investment ($\partial y_{it+1}/\partial s_{it}$) is decreasing in the relative human capital (h_{it}/h_t):

$$\partial y_{it+1}/\partial s_{it} = \Phi_t \left(h_{it}/h_t\right)^{\alpha-1} \left(1 - \delta + a_1 \left(h_{it}/h_t\right)^{\alpha-1}\right)^{\theta\alpha-1} \tag{21}$$

Appendix C provides the derivation of (21). Figure 4 plots (21) for a given Φ_t.

Figure 4: Incomplete Depreciation and Individual Saving rate



When credit market is missing, agents' investment opportunities are limited to the human capital in hand. Capital-poor agents with higher marginal return to investment try to equalize the differences in wealth by investing more in human capital. A lower rate of depreciation of human capital depresses adult's optimal investment in the child because the adult has already passed some human capital to her child (see eq. (7)). When investment is cut back, the resulting loss of output suffered by the poor is greater because poor have a higher marginal return to investment as seen in Figure 4. This makes it more difficult for the poor to exploit their productivity advantage through investment. This difficulty in catching up is reflected in a slower social mobility.

2.6. Incomplete depreciation, and long-run growth

The long-run growth rate is determined by setting $\sigma^2 = 0$ in (18):

$$\gamma = \ln \phi + \theta \ln (1 - \delta + a_1) \tag{22}$$

A lower δ unambiguously promotes growth. The intuition behind this result is that a lower depreciation boosts the steady state gross marginal product of human capital $(1 - \delta + a_1)$.

To sum up: a lower depreciation cost dampens investment propensity of all agents slowing down social mobility although long-run growth rate is higher. In the next section, we undertake a quantitative analysis of the model to illustrate that incomplete depreciation has nontrivial effect on the magnitude of social mobility.

3. Calibrating social mobility

In this section, we establish using a calibrated version of our model that full depreciation of human capital considerably overestimates social mobility. We first fix some of the model parameters at the conventional levels. There are eight parameters, namely β, η, a, a_2, α, δ, θ and τ. Assuming a psychological discount factor of 0.96, we set $\beta = 0.96^{30} \approx 0.3$, in a period of 30 years (de la Croix and Michel, 2002, p.255).[18] The income tax rate is set at $\tau = 0.3$ reflecting an average 30% income tax. The TFP parameter is normalized at $a = 1$. We set $\eta = 0.24$ letting adults to spend half of their time in child nurturing; the rest, for relaxing. In this case, the

[18] A psychological discount factor of 0.96 matches a 4.17 percent rate of time preference ρ in an infinite lived agent model. That is, $\beta = 1/(1+\rho) = 1/(1+.0417) = 0.96$.

investment specific technology scale parameter a_2 is fixed at 4.39 to target a long-run annual average growth rate of about 2 percent.[19] Regarding θ, we take Glomm's (1997) estimate of 0.8 as a baseline. The baseline value of δ is taken from Mankiw et al. (1992). Table 1 summarizes the baseline parameter values.

Table 1: Baseline parameter values

Preference and technology parameters:	$\beta = 0.3$, $\eta = 0.24$, $a = 1$, $a_2 = 4.39$
Production parameters:	$\alpha = 0.3$, $\theta = 0.8$, $\delta = 0.03$
policy parameter:	$\tau = 0.3$

Given that the central focus of the paper is on the schooling technology (4) with special emphasis on the depreciation parameter δ, we compute the social mobility for a range of δ and θ values. Table 2 reports the results of such a sensitivity analysis. Starting from the baseline values $\delta = 0.03$ and $\theta = 0.8$, a higher depreciation rate raises social mobility. For a full depreciation economy ($\delta = 1$), ρ reaches the lowest value, the maximum mobility. When $\theta = 1$, the investment technology (4) reduces to a standard linear form and the mobility is maximum for a given δ. Clark and Cummins (2012) get ρ estimates in the range (0.7 and 0.8). Table 2 reports that our model estimates of ρ fall in the range of Clark and Cummins for δ between 0.03 and 0.15 and θ between 0.8 and 0.9. Similar picture emerges when we alter δ and θ values in a finer grid which is reported in the three dimensional graph in Figure 5.

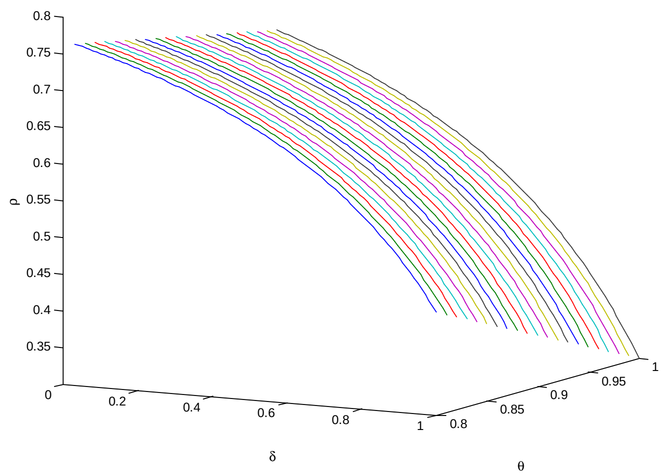
[19] If adults spend full time in child rearing, then $\eta = 0$. In this case, $a_2 = 2.52$ leads to a long-run growth rate of 2%.

Note that a full depreciation economy vastly overestimates mobility even though we take the highest estimate of mobility from Clark and Cummins. The bottom-line of this sensitivity analysis is that incomplete depreciation of human capital ($0 < \delta < 1$) is crucial in reproducing the observed degree of social mobility.

Table 2: Effects of depreciation cost on social mobility for different values of θ

Depreciation cost (δ)	$\theta = 0.8$	$\theta = .9$	$\theta = 1$
0.03	0.7653	0.7359	0.7066
0.05	0.7680	0.7391	0.7101
0.10	0.7550	0.7244	0.6938
0.13	0.7503	0.7191	0.6879
0.15	0.7471	0.7155	0.6839
1	0.4400	0.3700	0.3000

Figure 5: Estimate of ρ at different values of δ and θ



4. Extensions

4.1. Idiosyncratic luck

Until now we assumed that individuals are the same except for initial distribution of human capital. In the long run everybody attains the same human capital through investment in education, when poor households ultimately catch the rich ones due to diminishing returns to human capital investment. The long run distribution of human capital is thus degenerate when everybody becomes identical. What happens when adults also differ in terms of innate ability or luck? Our key result that incomplete depreciation of human capital results in slow mobility continues to hold when individuals differ in luck. The only difference is that the long run distribution of human capital will not degenerate; it is determined only by the cross sectional variance of luck.

To see this, assume that individuals also differ in terms of productivity where the production function becomes:

$$y_{it} = a\varphi_{it} \left(h_t\right)^{1-\alpha} \left(h_{it}\right)^{\alpha} \tag{23}$$

Individuals are thus subject to an i.i.d. idiosyncratic productivity shocks (φ_{it}) which drive their total marginal productivity. φ_{it} follows the process: $\ln \varphi_{it} \sim N(-\upsilon^2/2, \upsilon^2)$. In this case, optimal saving is given by (7) where y_{it} is now given by (23). Thus, the optimal individual human capital accumulation function becomes,

$$h_{it+1} = \phi h_{it} \left(1 - \delta + a_1 \varphi_{it} h_t^{1-\alpha} h_{it}^{\alpha-1}\right)^{\theta}. \tag{24}$$

Loglinearizing (24) around a balanced growth path (following similar procedure as above) where all agents are identical in terms of luck $\varphi_{it} = \varphi = 1$ leads to:[20]

$$\ln h_{it+1} \simeq \rho \ln \widetilde{h}_{it} + \chi \ln \widetilde{\varphi}_{it} = \rho \ln \widetilde{h}_{it} + \chi \ln \varphi_{it} \tag{25}$$

where $\widetilde{\varphi}_{it} \equiv \varphi_{it}/\varphi_t = \varphi_{it}$ and $\chi \equiv \theta a_1/\left(1 - \delta + a_1\right) \in (0, 1)$. Therefore, there are no changes in terms of mobility, which is still determined by ρ, but the dynamics of cross-section inequality is different from that of the determinant economy:

[20] Algebraic derivation is omitted for brevity and available upon request from the authors.

$$\sigma_{t+1}^2 = \rho^2 \sigma_t^2 + \chi^2 \upsilon^2. \tag{26}$$

More importantly, inequality converges now to a non-degenerate steady-state distribution (σ^2):

$$\sigma^2 = \chi^2 \upsilon^2 / \left(1 - \rho^2\right). \tag{27}$$

4.2. Education subsidy

Our model demonstrates that the social mobility is less in economies with lower depreciation of human capital. A proportional education subsidy can help the intergenerational mobility in such a scenario through boosting investment in schooling. To see it, stick to our original set up without luck. Think of a flat rate education subsidy ψ which lowers the cost of schooling s_{it} proportionally for all agents financed by a consumption τ_t^c.[21] The budget constraint (3) changes to:

$$c_{it} \left(1 + \tau_t^c\right) + s_{it}(1 - \psi) = y_{it} \tag{28}$$

Assume that the government balances the budget by setting an average tax rate τ_t^c such that

$$\tau_t^c c_t = \psi s_t \tag{29}$$

[21] We replace the income tax by consumption tax following Benabou (2002) who used a non-distortionary consumption tax to finance education subsidy.

Each agent takes ψ and τ_t^c as parametrically given. The optimal investment function now changes to:[22]

$$s_{it} = \left(y_{it}\theta\beta / \left(1-\psi\right) - \left(1-\delta\right) h_{it}\right) / \left(1+\theta\beta\right) \tag{30}$$

Considering (4), we have the optimal human capital accumulation under education subsidy:

$$h_{it+1} = \phi h_{it} \left(1 - \delta + \left(1-\psi\right)^{-1} a h_t^{1-\alpha} h_{it}^{\alpha-1}\right)^{\theta} \tag{31}$$

Loglinearizing around a balanced growth path where all agents are identical, the intergenerational mobility is given by the inverse of ρ_s:[23]

$$\rho_s = 1 - \frac{\theta a \left(1-\alpha\right)}{\left(1-\delta\right)\left(1-\psi\right) + a} \tag{32}$$

It is straightforward to verify that $\partial \rho_s / \partial \psi < 0$. A higher education subsidy thus promotes social mobility. The effect of subsidy on mobility works via the undepreciated capital stock in our model. Thus, an education subsidy, ψ can be applied to mitigate the slowdown of social mobility caused by lower depreciation.

[22] Due to the log utility functional form, the consumption tax rate τ_t^c does not appear in the optimal decision rule.

[23] Derivation of (31) and (32) is omitted for brevity and available upon request from the authors.

5. Conclusion

The key result of this paper is that slow obsolescence or depreciation of human capital inhibits social mobility and gives rise to persistent economic inequality. This point is not well recognized in the extant social mobility literature because of the standard assumption of full depreciation of human capital. We establish this result by introducing incomplete depreciation of human capital in a variant of the model of Benabou (2000, 2002). Agents are only heterogenous in terms of the initial stock of human capital. Credit market imperfection prevents the poor to equalize this initial difference through borrowing from the rich. The acquisition of human capital through schooling is the principal vehicle of social mobility. Using a novel closed form analytical solution of distributional dynamics, we show that when human capital depreciates slowly, the process of intergenerational mobility considerably slows down and inequality becomes a more persistent process. The closed form solution for social mobility shows that social mobility is slower in economies with greater inequality, which is consistent with some recent empirical studies. Our calibration exercise shows that social mobility can be vastly overestimated if human capital depreciates fully. Our key result that social mobility is slower in a low depreciation economy is robust even though we extend the model by adding idiosyncratic difference in luck. A proportional education subsidy financed by consumption tax could alleviate the slow social mobility due to the lack of obsolescence of old knowledge.

Our model can be extended in different directions. For instance, one can introduce physical capital and examine the effect of labour displacing technical progress on social mobility when human capital has incomplete depreciation. Also, throughout this paper we assume that economic obsolescence of human capital is exogenous. A future extension would be to endogenize depreciation via innovation which gives rise to "creative destruction" of knowledge.

Appendix

A. Proof of Proposition 2

In this section we derive (17) from (13). We can also rewrite (13) as

$$(h_{it+1})^{\varsigma} = \phi^{\varsigma} \left(h_{it}^{\varsigma} \kappa + \epsilon_t h_{it}^{\varkappa+\varsigma} \right) \tag{A.1}$$

where $\varsigma \equiv 1/\theta$, $\varkappa \equiv \alpha - 1$, $\kappa \equiv 1 - \delta$ and $\epsilon_t \equiv a_1 h_t^{1-\alpha}$.

Recall that first h_{it} is assumed to have lognormal distribution:

$$\ln h_{it} \sim N(\mu_t, \sigma_t^2) \tag{A.2}$$

And, from a normal-lognormal relationship, we have:

$$\mathrm{E}\left[h_{it}\right] \equiv h_t = e^{\mu_t + 0.5\sigma_t^2} \tag{A.3}$$

$$\mathrm{var}\left[h_{it}\right] = \left(e^{\sigma_t^2} - 1\right) e^{2\mu_t + \sigma_t^2} \tag{A.4}$$

If $h_{it}$is lognormal, then h_{it}^z is also lognormal for any constant z. Thus:

$$\mathrm{E}\left[h_{it}^z\right] = h_t^z e^{0.5\sigma_t^2 z(z-1)} \tag{A.5}$$

$$\mathrm{var}\left[h_{it}^z\right] = h_t^{2z} e^{\sigma_t^2 z(z-1)} \left(e^{z^2\sigma_t^2} - 1\right) \tag{A.6}$$

We now simply apply (A.5) and (A.6) to derive the following important relations that we use later on:

$$\mathrm{E}\left[h_{it+1}^{\varsigma}\right] = h_{t+1}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t+1}^2} \tag{A.7}$$

$$\mathrm{E}\left[h_{it}^{\varsigma}\right] = h_{t}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t}^2} \tag{A.8}$$

$$\mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right] = h_{t}^{\varsigma+\varkappa} e^{0.5(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_{t}^2} \tag{A.9}$$

$$\mathrm{E}\left[h_{it}^{2\varsigma+\varkappa}\right] = h_{t}^{2\varsigma+\varkappa} e^{0.5(2\varsigma+\varkappa)(2\varsigma+\varkappa-1)\sigma_{t}^2} \tag{A.10}$$

$$\mathrm{var}\left[h_{it+1}^{\varsigma}\right] = h_{t+1}^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_{t+1}^2}\left(e^{\varsigma^2\sigma_{t+1}^2}-1\right) \tag{A.11}$$

$$\mathrm{var}\left[h_{it}^{\varsigma}\right] = h_{t}^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_{t}^2}\left(e^{\varsigma^2\sigma_{t}^2}-1\right) \tag{A.12}$$

$$\mathrm{var}\left[h_{it}^{\varsigma+\varkappa}\right] = h_{t}^{2(\varsigma+\varkappa)} e^{(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_{t}^2}\left(e^{(\varsigma+\varkappa)^2\sigma_{t}^2}-1\right) \tag{A.13}$$

Then, aggregate (A.1) from both sides to derive the aggregate human capital:

$$\mathrm{E}\left[h_{it+1}^{\varsigma}\right] = \phi^{\varsigma}\,\mathrm{E}\left[h_{it}^{\varsigma}\kappa + \epsilon_t h_{it}^{\varsigma+\varkappa}\right] = \phi^{\varsigma}\left\{\kappa\,\mathrm{E}\left[h_{it}^{\varsigma}\right] + \epsilon_t\,\mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right]\right\} \tag{A.14}$$

Plugging (A.7), (A.8) and (A.9) into (A.14):

$$\begin{aligned} h_{t+1}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t+1}^2} &= \phi^{\varsigma}\left\{\kappa h_t^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_t^2} + \epsilon_t h_t^{\varsigma+\varkappa} e^{0.5(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2}\right\} \\ &= \phi^{\varsigma} h_t^{\varsigma}\left\{\kappa e^{0.5\varsigma(\varsigma-1)\sigma_t^2} + a_1 e^{0.5(\varsigma(\varsigma-1)+\varsigma\varkappa+\varkappa(\varsigma+\varkappa-1))\sigma_t^2}\right\} \end{aligned}$$

Thus, the aggregate human capital accumulation function is given by:

$$h_{t+1}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t+1}^2} = \phi^{\varsigma} h_t^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_t^2}\left\{\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right\} \tag{A.15}$$

The growth rate (18) is derived by taking the log from both sides of (A.15).

To derive the distributional dynamics, take the variance from both sides of (A.1):

$$\begin{aligned}
\operatorname{var}\left[(h_{it+1})^{\varsigma}\right] &= \phi^{2\varsigma} \operatorname{var}\left[h_{it}^{\varsigma}\kappa + \epsilon_t h_{it}^{\varsigma+\varkappa}\right] \\
&= \phi^{2\varsigma}\left[\kappa^2 \operatorname{var}\left[h_{it}^{\varsigma}\right] + \epsilon_t^2 \operatorname{var}\left[h_{it}^{\varsigma+\varkappa}\right] + 2\kappa\epsilon_t \operatorname{cov}\left(h_{it}^{\varsigma}, h_{it}^{\varsigma+\varkappa}\right)\right] \qquad \text{(A.16)}
\end{aligned}$$

Using (A.8), (A.9), and (A.10), the covariance term is computed as follows:

$$\begin{aligned}
\operatorname{cov}\left(h_{it}^{\varsigma}, h_{it}^{\varsigma+\varkappa}\right) &= \mathrm{E}\left[h_{it}^{\varsigma} h_{it}^{\varsigma+\varkappa}\right] - \mathrm{E}\left[h_{it}^{\varsigma}\right] \mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right] \\
&= \mathrm{E}\left[h_{it}^{2\varsigma+\kappa}\right] - \mathrm{E}\left[h_{it}^{\varsigma}\right] \mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right] \\
&= h_t^{2\varsigma+\varkappa} e^{0.5(2\varsigma+\varkappa)(2\varsigma+\varkappa-1)\sigma_t^2} - h_t^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_t^2} h_t^{\varsigma+\varkappa} e^{0.5(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2} \\
&= h_t^{2\varsigma+\varkappa} e^{0.5(\varsigma(\varsigma-1)+(\varsigma+\varkappa)(\varsigma+\varkappa-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2} - 1\right) \qquad \text{(A.17)}
\end{aligned}$$

Then, plugging (A.11), (A.12), (A.13) and (A.17) into (A.16) yields:

$$\begin{aligned}
& h_{t+1}^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_{t+1}^2}\left(e^{\varsigma^2\sigma_{t+1}^2} - 1\right) \\
&= \phi^{2\varsigma}\left[\begin{array}{c}
\kappa^2 h_t^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_t^2}\left(e^{\varsigma^2\sigma_t^2} - 1\right) \\
+\epsilon_t^2\left\{h_t^{2(\varsigma+\varkappa)} e^{(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2}\left(e^{(\varsigma+\varkappa)^2\sigma_t^2} - 1\right)\right\} \\
+2\kappa\epsilon_t\left\{h_t^{2\varsigma+\varkappa} e^{0.5(\varsigma(\varsigma-1)+(\varsigma+\varkappa)(\varsigma+\varkappa-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2} - 1\right)\right\}
\end{array}\right]
\end{aligned}$$

or,

$$
\begin{aligned}
& h_{t+1}^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_{t+1}^2}\left(e^{\varsigma^2\sigma_{t+1}^2}-1\right) \\
& = \phi^{2\varsigma} h_t^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_t^2}\left[\begin{array}{c}
\kappa^2\left(e^{\varsigma^2\sigma_t^2}-1\right) \\
+\epsilon_t^2\left\{h_t^{2\varkappa} e^{\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\left(e^{(\varsigma+\varkappa)^2\sigma_t^2}-1\right)\right\} \\
+2\kappa\epsilon_t\left\{h_t^{\varkappa} e^{0.5(\varkappa(2\varsigma+\varkappa-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2}-1\right)\right\}
\end{array}\right]
\end{aligned}
$$

Finally, substituting (A.15) into the above, we get :

$$
\begin{aligned}
& \phi^{2\varsigma} h_t^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_t^2}\left\{\kappa+a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right\}^2\left(e^{\varsigma^2\sigma_{t+1}^2}-1\right) \\
& = h_t^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_t^2}\phi^{2\varsigma}\left[\begin{array}{c}
\kappa^2\left(e^{\varsigma^2\sigma_t^2}-1\right) \\
+\epsilon_t^2\left\{h_t^{2\varkappa} e^{\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\left(e^{(\varsigma+\varkappa)^2\sigma_t^2}-1\right)\right\} \\
+2\kappa\epsilon_t\left\{h_t^{\varkappa} e^{0.5(\varkappa(2\varsigma+\varkappa-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2}-1\right)\right\}
\end{array}\right]
\end{aligned}
$$

or,

$$
\begin{aligned}
& \left\{\kappa+a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right\}^2\left(e^{\varsigma^2\sigma_{t+1}^2}-1\right) \\
& = \left[\begin{array}{c}
\kappa^2\left(e^{\varsigma^2\sigma_t^2}-1\right) \\
+(a_1)^2\left\{e^{\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\left(e^{(\varsigma+\varkappa)^2\sigma_t^2}-1\right)\right\} \\
+2\kappa a_1\left\{e^{0.5(\varkappa(2\varsigma+\varkappa-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2}-1\right)\right\}
\end{array}\right]
\end{aligned} \tag{A.18}
$$

since $\epsilon_t \equiv a_1 h_t^{1-\alpha}$ and $\varkappa \equiv \alpha - 1$.

Considering,

$$\left(\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right)^2 = \kappa^2 + 2\kappa a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2} + (a_1)^2 e^{(\varkappa(2\varsigma+\varkappa-1))\sigma_t^2}$$

further simplifying (A.18) gives

$$e^{\varsigma^2\sigma_{t+1}^2} = \frac{\kappa^2 e^{\varsigma^2\sigma_t^2} + (a_1)^2\left(e^{\varkappa(2\varsigma+\varkappa-1)\sigma_t^2} e^{(\varsigma+\varkappa)^2\sigma_t^2}\right) + 2\kappa a_1\left(e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2} e^{\varsigma(\varsigma+\varkappa)\sigma_t^2}\right)}{\left(\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right)^2}$$

Alternatively,

$$e^{\theta^{-2}\sigma_{t+1}^2} = \frac{\kappa^2 e^{\theta^{-2}\sigma_t^2} + {a_1}^2\left(e^{\left[(\alpha-1)(2/\theta+\alpha-2)+(1/\theta+\alpha-1)^2\right]\sigma_t^2}\right) + 2\kappa a_1\left(e^{[0.5(\alpha-1)(2/\theta+\alpha-2)+(1/\theta+\alpha-1)/\theta]\sigma_t^2}\right)}{\left(\kappa + a_1 e^{0.5(\alpha-1)(2/\theta+\alpha-2)\sigma_t^2}\right)^2}$$

after substituting $\varsigma \equiv 1/\theta$, $\varkappa \equiv \alpha - 1$. Or,

$$e^{\theta^{-2}\sigma_{t+1}^2} = \frac{\kappa^2 e^{\theta^{-2}\sigma_t^2} + {a_1}^2\left(e^{\left(\omega+\lambda^2\right)\sigma_t^2}\right) + 2\kappa a_1\left(e^{(0.5\omega+\lambda/\theta)\sigma_t^2}\right)}{\left(\kappa + a_1 e^{0.5\omega\sigma_t^2}\right)^2} \tag{A.19}$$

where

$$\omega \equiv (\alpha-1)(2/\theta+\alpha-2) < 0,\ \lambda \equiv 1/\theta + \alpha - 1 > 0$$

as given by (17).

B. Social mobility: exact solution

The social mobility (ρ_t) is time varying and is derived by simply taking the first derivative of (17):

$$
\begin{aligned}
\rho_t^2 &\equiv \partial\sigma_{t+1}^2/\partial\sigma_t^2 \\
&= \left(\frac{\kappa^2\theta^{-2}\exp(\theta^{-2}\sigma_t^2)+a_1^2 b_2\exp(b_2\sigma_t^2)+2\kappa a_1 b_3\exp(b_3\sigma_t^2)}{\kappa^2\exp(\theta^{-2}\sigma_t^2)+a_1^2\exp(b_2\sigma_t^2)+2\kappa a_1\exp(b_3\sigma_t^2)} - \frac{a_1\omega\exp(0.5\omega\sigma_t^2)}{\kappa+a_1\exp(0.5\omega\sigma_t^2)}\right)\theta^2
\end{aligned}
\tag{B.20}
$$

If $\sigma_t^2 = 0$, (B.20) reduces to (15). Also, if $\delta = 1$, then $\rho_t = \rho = 1 - (1-\alpha)\,\theta$, which is constant.

C. Derivation of the marginal return of investment

The marginal return to individual investment (21) is computed as follows:

$$
\partial y_{it+1}/\partial s_{it} = \left(\partial y_{it+1}/\partial h_{it+1}\right)\left(\partial h_{it+1}/\partial s_{it}\right) \tag{C.21}
$$

From (1) and (4):

$$
\partial y_{it+1}/\partial s_{it} = \theta\alpha a a_2\left(h_{it+1}/h_{t+1}\right)^{\alpha-1}\left((1-\delta+s_{it}/h_{it}\right)^{\theta-1} \tag{C.22}
$$

Plugging (1), (4), (7) and (18) into the above, one obtains:

$$
\begin{aligned}
\partial y_{it+1}/\partial s_{it} &= \theta\alpha a a_2\left(a_2 h_{it}/h_{t+1}\right)^{\alpha-1}\left(1-\delta+s_{it}/h_{it}\right)^{\theta\alpha-1} \\
&= \theta\alpha a\phi^{\alpha-1/\theta}a_2^{1/\theta}\left(h_{it}/h_{t+1}\right)^{\alpha-1}\left(1-\delta+a_1\left(h_{it}/h_t\right)^{\alpha-1}\right)^{\theta\alpha-1} \\
&= \theta\alpha a\phi^{\alpha-1/\theta}a_2^{1/\theta}\left(h_{it}/\left(h_t\exp\left(\gamma_{t+1}\right)\right)\right)^{\alpha-1}\left(1-\delta+a_1\left(h_{it}/h_t\right)^{\alpha-1}\right)^{\theta\alpha-1} \\
&= \Phi_t\left(h_{it}/h_t\right)^{\alpha-1}\left(1-\delta+a_1\left(h_{it}/h_t\right)^{\alpha-1}\right)^{\theta\alpha-1}
\end{aligned}
$$

since $h_{t+1} = h_t \exp\left(\gamma_{t+1}\right)$ and,

$$\Phi_t \equiv \theta\alpha a\phi^{\alpha-1/\theta}a_2^{1/\theta}\exp\left(\left(1-\alpha\right)\gamma_{t+1}\right) \tag{C.23}$$

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