RAWLSIAN ALLOCATION IN QUEUEING AND SEQUENCING PROBLEM

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Abstract. In this paper we analyze the implication of a particular kind of allocation rule called Rawlsian allocation rule on queueing and sequencing problems. We find that in case of queueing problems, Efficient allocation rules are Rawlsian but the converse is not true. For a particular class of Rawlsian allocation rule we characterize the unique class of transfer rule that ensures non-manipulability. Also in case of a situation where there is private information in multiple dimension, we find that a the particular kind of Rawlsian allocation rule equipped with a suitable transfer rule works as a panacea.

Keywords: Queueing problems, Sequencing problems, Strategyproofness, Rawlsian allocation.

JEL Classification: C72,D82

1. Introduction

A queueing or sequencing environment captures a situation where a finite set of agents wish to avail a service provided by a single server. The nature of service can be homogeneous or heterogeneous and so be the processing times for the service. The server can serve exactly one agent at a time, as a result agents are served sequentially. Agents who are waiting incurs a waiting cost. According to Maniquet [11] queueing model captures many economic situations.

Queueing or sequencing models are extensively analyzed from incentive and axiomatic point of view. When there is private information from the agents' point of view then we have an incentive problem. For example, if the planner's objective is to ensure efficiency of allocation then there is a mismatch between individual objective and that of planner's. Our approach of analysis is also from incentive point of view, where we assume agents have quasi-linear preferences, waiting costs are linear with time. To justify our assumptions, we mention some of the important works in this literature.

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With quasi-linear preferences, it is possible to design mechanisms that satisfy strategyproofness and efficiency of decision. This is due to Hölmstrom's[8] on the uniqueness of the class of Vickrey-Clarke-Gorves (VCG) mechanisms.[1] Suijs[7] and Mitra[3] showed that linearity of cost structure is a crucial assumption to ensure 'first best',that is, efficiency of decision, dominant strategy incentive compatibility and budget balancedness.

Analyzing queueing and sequencing problem under the criterion of efficiency of decision is well-studied in the existing literature. Some recent works like Mishra and Mitra [1] has shown some deviation in this respect. These papers are addresses a few questions form incentive view point by finding necessary restrictions on the possible allocation rules. One question can be asked in this aspect: What is the implication of Rawlsian allocation rule in this context? Or can we have a strategyproof, Rawlsian mechanism? We have partially answered this question in this paper.

As we move on by increasing the level of private information, we find that a particular class of Rawlsian allocation actually makes our way to resolve the problem of individual manipulability. Thus we show although it is impossible to have a mechanism that induces efficiency of decision along with strategyproofness in multidimensional private information setup, it is still possible to guarantee strategyproofness with a subclass of Rawlsian allocation rule.

This paper has been arranged in the following way. In Section 2 we formally introduce the model and add necessary definitions. In Section 3 we use several examples to motivate and illustrate the situations. Then we state and prove our characterization results. Lastly, with Section 4, we draw our conclusions.

2. Model

Consider the set of agents $N = \{1, \ldots, n\}$ with a single machine. Each individual has a some work to be executed by the machine. The nature of this work may be homogeneous or heterogeneous . The machine can process exactly one job at a time. Let $\forall i \in N$, $s_i \in \Re_{++}$ where s_i denotes the processing time of ith agent. Each agent is identified with a waiting cost $\theta_i \in \Re_{++}$, the cost of waiting per unit of time. The profile of waiting costs of the set of all agents is typically denoted by $\theta = (\theta_1, \ldots, \theta_n) \in \Re^n_{++}$. For any $i \in N$, θ_{-i} denotes the profile $(\theta_1 \ldots \theta_{i-1}, \theta_{i+1}, \ldots \theta_n) \in \Re^{n-1}_{++}$. An allocation of n jobs can be done in many ways. Let $\Sigma(N)$ denote the set of all possible queue of structure

[1]See Vickery[12], Clarke[9], Groves[10].

agents in N. An allocation rule is a mapping $\sigma : \Re^n_+ \rightarrow \Sigma(N)$ that specifies for each profile $\theta \in \Re^n_{++}$ an allocation(rank) vector $\sigma(\theta) \in \Sigma(N)$. Agent i's position is denoted by $\sigma_i(\theta)$ which is an input of the vector $\sigma(\theta)$. Given $\sigma(\theta) \in \Sigma(N), \forall i \in N, P_i(\sigma(\theta)) = \{j \in N | \sigma_j(\theta) < \sigma_i(\theta)\}$ denotes the set of predecessors of i and similarly $P'_i(\sigma(\theta)) = \{j \in N | \sigma_j(\theta) > \sigma_i(\theta)\}$ denotes the set of successors of i. Agent i's waiting time is denoted by $S_i(\sigma(\theta))$ and corresponding waiting cost is $S_i(\sigma(\theta))\theta_i$. A transfer rule is a mapping $t : \Re^n_+ \rightarrow \Re^n$ that specifies for each profile $\theta \in \Re^n_{++}$ a transfer vector $t(\theta) = (t_i(\theta), \ldots, t_n(\theta)) \in \Re^n$. We assume that the utility function of each agent $i \in N$ is quasi-liner and is of the form $U_i(\sigma(\theta), t_i(\theta), \theta_i) = -S_i(\sigma(\theta)\theta_i + t_i(\theta)$, where $t_i(\theta)$ is the monetary transfer of agent to i.

2.1. **Queueing situation.** In the existing literatures the structure of queueing problem is specified by the following situations:{i} $\forall i, j \in N, s_i = 1$[2]: {ii} $\forall i \in N, S_i(\sigma(\theta)) = (\sigma_i(\theta) - 1)$. So utility of a general agent i is $U_i(\sigma(\theta), t_i(\theta), \theta_i) = -(\sigma_i(\theta) - 1)\theta_i + t_i(\theta)$. A queueing game is denoted by $\Omega = \langle N, \Re^n_{++} \rangle$.

2.2. **Sequencing.** The literature puts no such restrictions over processing time in case of sequencing problems. So there is heterogeneity in processing time . In Sequencing problem $\forall i \in N, S_i(\sigma(\theta)) = (s_i + \sum\limits_{j \in P_i\sigma(\theta)} s_j)$. A sequencing game is denoted by $\Omega = \langle N, \Re^n_{++}, \Re^n_{++} \rangle$.

Definition 1. $\forall \theta \in \Re^n_{++}$, a queue $\sigma(\theta) \in \Sigma(N)$ is efficient if $\sigma(\theta) \in \arg \min\limits_{\sigma(\theta) \in \Sigma(N)} \sum_{i=1}^n S_i(\sigma(\theta))\theta_i$.

The implication of efficiency is that agents are ranked according to the non-increasing order of their waiting costs (that is, if $\theta_i \geq \theta_j$ under a profile θ, then $S_i(\sigma(\theta)) \leq S_i(\sigma(\theta))$). Moreover, there are profiles for which more than one rank vector is efficient. For example, if all agents have the same waiting cost, then all rank vectors are efficient. Therefore, we have an efficiency correspondence. In this paper we consider a particular efficient rule (that is, a single valued selection from the efficiency correspondence). For our efficient rule, we use the following *tie breaking rule*: if $i < j$ and $\frac{\theta_i}{s_i} = \frac{\theta_j}{s_j}$ then $S_i(\sigma(\theta)) < S_i(\sigma(\theta))$. This tie breaking rule guarantees that, given a profile $\theta \in \Re^n_{++}$, the efficient rule selects a single rank vector from $\Sigma(N)$.

[2]as long as the processing time of agents are homogeneous,we are in the regime of queueing games since we can always normalize the processing time

Definition 2. A mechanism(σ, t) is efficient (EFF) if for all announced profile

$$\theta \in \Re^n_{++}, \sigma(\theta) \in \arg\min_{\sigma \in \Sigma(N)} \sum_{i=1}^{n} S_i(\sigma(\theta))\theta_i.$$

Efficiency here basically implies minimization of aggregate waiting cost.

A mechanism is (σ, t) constitutes of an allocation rule σ and a transfer rule t. We are interested in strategy proof mechanism for the queueing/sequencing problem.

A Rawlsian allocation is in general perceived as an allocation which is best when seen from point of minimum valued agent. In queueing or sequencing since the valuation is negative (because it is cost), so it turns out to be an allocation which minimizes the burden of the agent who is incurring maximum cost. The definition of Rawlsian allocation is as follows:

Definition 3. A mechanism (σ, t) is Rawlsian (RA) if for all announced profile

$$\theta \in \Re^n_{++}, \min_{\sigma(\theta) \in \Sigma(N)} \max_{j \in N} S_j(\sigma(\theta))\theta_j.$$

Definition 4. In a mechanism (σ, t), the allocation rule σ is Non-increasing in Own Type (NOT) if:
$\forall \theta_{-i} \in \Re^{n-1}_{++}, \forall i \in N, S_i(\sigma(\theta_i, \theta_{-i}))$ is non-increasing in θ_i .

Definition 5. A mechanism(σ, t) is strategy-proof (SP) if $\forall i \in N, \forall \theta_i, \theta'_i \in \Re_{++}$ and $\forall \theta_{-i} \in \Re^{n-1}_{++}$ we have,
$-S_i(\sigma(\theta_i, \theta_{-i}))\theta_i + t_i(\theta_i, \theta_{-i}) \geq -S_i(\sigma(\theta'_i, \theta_{-i}))\theta_i + t_i(\theta'_i, \theta_{-i})$.

It means for any agent truthful reporting is weakly dominates false reporting irrespective of other players report.

Definition 6. A mechanism (σ, t) satisfies *budget balancedness* (BB) if $\forall \theta \in \Re^n_{++}, \ \sum_{i \in 1}^{n} t_i(\theta) = 0$.

The profile θ and θ' are *S-variants* if $\forall i \in N \setminus S, \theta_i = \theta'_i$.

Definition 7. A mechanism (σ, t) is *weak group strategyproof*(WSP) if for all *S-variants* θ, θ' $U_i(\sigma(\theta), t_i(\theta), \theta_i) \geq U_i(\sigma(\theta'), t_i(\theta'), \theta_i)$ for at least one $i \in S$.

This implies as long as all the group member are not strictly better off by deviating from their true profile, such group will not be formed.

Definition 8. A mechanism (σ, t) is *pair-wise group strategyproof* (PWSP) if for all *S-variants* θ, θ' where $|S| = 2$,
$U_i(\sigma(\theta), t_i(\theta), \theta_i) \geq U_i(\sigma(\theta'), t_i(\theta'), \theta_i)$ for at least one $i \in S$.

This implies pair of agents deviates from their true profile by jointly misreporting if an only if they are both strictly better off from the situation when they truthfully reports.

3. Result

Before going further into the detail of the implications of Rawlsian allocation let us fix the regarding what Rawlsian allocation in general is and what is it in case of queueing and sequencing problems:

Example 1. What do we mean by Rawlsian allocation rule?

Consider the set of three agents $N = \{1, 2, 3\}$ and allocation set $A = \{a, b, c\}$. The table below describes the valuation of every agents under the three different allocations:

Agent \ Alternative	a	b	c
1	17	8	13
2	12	15	7
3	9	10	9
Min(value)	9	8	7

Maximum of minimum (value) = 9. Hence the corresponding allocation 'a' is the Rawlsian allocation.

The example above depicts a very general situation and shows how to Rawlsian allocation in that general situation. But what happens when we specifically deal with queueing or sequencing setup?

Consider a profile $\theta = (\theta_1, \theta_2, \theta_3)$ such that $\theta_1 \geq \theta_2 \geq \theta_3$ and $s = (s_1, s_2, s_3)$. The following table describes all possible allocations and each agents processing cost under each of this allocations:

Agnt \ Alt	(1,2,3)	(1,3,2)	(2,1,3)	(2,3,1)	(3,1,2)	(3,2,1)
1	$s_1\theta_1$	$s_1\theta_1$	$(s_1 + s_2)\theta_1$	$(s_1 + s_2 + s_3)\theta_1$	$(s_1 + s_3)\theta_1$	$(s_1 + s_2 + s_3)\theta_1$
2	$(s_1 + s_2)\theta_2$	$(s_1 + s_2 + s_3)\theta_2$	$s_2\theta_2$	$s_2\theta_2$	$(s_1 + s_2 + s_3)\theta_2$	$(s_1 + s_2 + s_3)\theta_1$
3	$(s_1 + s_2 + s_3)\theta_3$	$(s_1 + s_3)\theta_3$	$(s_1 + s_2 + s_3)\theta_3$	$(s_2 + s_3)\theta_3$	$s_3\theta_3$	$s_3\theta_3$
Max(cost)						

For more illustrations we use several examples below.

Example 2. A queueing problem: Consider $N = \{1, 2, 3\}$; $\theta = (\theta_1, \theta_2, \theta_3)$ where $\theta_1 = 5$, $\theta_2 = 4$, $\theta_3 = 3$ and $s = (s_1, s_2, s_3)$ where $s_1 = 1, s_2 = 1$ and $s_3 = 1$.

The following table describes all possible allocations and each agents processing cost under each of this allocations:

Agnt \ Alt	(1,2,3)	(1,3,2)	(2,1,3)	(2,3,1)	(3,1,2)	(3,2,1)
1	5	5	10	15	10	15
2	8	12	4	4	12	8
3	9	6	9	6	3	3
Max(cost)	9	12	10	15	12	15

Minimum of maximum(cost) = 9. Hence the corresponding allocation '(1,2,3)' is the Rawlsian allocation. **Note that cost = -value**.

Example 3. A sequencing problem: Consider $N = \{1, 2, 3\}$; $\theta = (\theta_1, \theta_2, \theta_3)$ where $\theta_1 = 10$, $\theta_2 = 5$, $\theta_3 = 1$ and $s = (s_1, s_2, s_3)$ where $s_1 = 1, s_2 = 2$ and $s_3 = 3$.

The following table describes all possible allocations and each agents processing cost under each of this allocations:

Agnt \ Alt	(1,2,3)	(1,3,2)	(2,1,3)	(2,3,1)	(3,1,2)	(3,2,1)
1	10	10	30	60	40	60
2	15	30	10	10	30	25
3	6	4	6	5	3	3
Max(cost)	15	30	30	60	40	60

Minimum of maximum(cost) = 15. Hence the corresponding allocation '(1,2,3)' is the Rawlsian allocation.

Example 4. Another sequencing problem : Consider $\theta = (100, 5, 3)$ and $s = (1, 2, 3)$

Agnt \ Alt	(1,2,3)	(1,3,2)	(2,1,3)	(2,3,1)	(3,1,2)	(3,2,1)
1	100	100	300	600	400	600
2	15	30	10	10	30	25
3	18	12	18	15	3	3
Max(cost)	100	100	300	600	400	600

Minimum of maximum(cost) = 100. Hence corresponding allocation rules '(1,2,3) and (1,3,2)' are Rawlsian allocations. **But note that '(1,2,3)' is the only efficient allocation rule for this example.**

Example 5. What will be a Rawlsian/Efficient allocation if $\theta = (15, 75, 18)$ and $s = (3, 5, 6)$?

Here a Rawlsian allocation is the queue arrangement where 2nd agent(75) is in 1st position, 3rd agent(18) is in 2nd position and 1st agent(15)is in first position.
But to find Efficient allocation rule in this case, we calculate $(\frac{\textbf{waiting cost}}{\textbf{processing time}})$ for each agent and then rank them in decreasing order of magnitude. So 2nd agent (75;5) is in 1st position, 1st agent (15;3) is in and position and 3rd agent (18;6) is in 3rd position of the queue.

Notice that if processing time is homogeneous across agents then Efficient allocation rule is also Rawlsian. But the converse is not true.

Remark 1. **(i)** In all these queueing or sequencing examples the allocations, where agents are ranked according to the decreasing value of their waiting costs, are in fact always Rawlsian while the ranking in the above specified manner is not necessary to ensure a Rawlsian allocation(see Example 4).

(ii) To decide over efficient allocation rule we need both the informations: agents waiting time and processing time. But to allocate agents in Rawlsian method information on processing time is not necessary.

(iii) Efficient allocation rule tries to minimize the aggregate servicing cost while Rawlsian allocation minimizes the burden of maximum cost bearer.

3.1. **Unidimensional private information case:** Here we have private information regarding agents waiting cost but each agents processing time (of their own work that is to be executed by the server) is publicly known.

Proposition 1. *Sufficient condition for an allocation to be Rawlsian is that the agents must be ordered in the queue with decreasing value of their waiting cost.*

Proof. Take any $\theta \in \Re^n_{++}$(privately known) such that $\theta_1 \geq \theta_2 \ldots \geq \theta_n$(note that this we can do without loss of generality) and also $s = (s_1, s_2, \ldots, s_n) \in \Re^n_{++}$(publicly known) . The above proposition claims that $\sigma^*(\theta) = (1, 2, \ldots, n)$ is a Rawlsian allocation.
With $|N| = n$ agents we can have $n!$ different allocations among which $\sigma^*(\theta)$ is the one.

Agnt \ Alt	$\sigma^*(\theta) = (1,2,\ldots,n)$	(...)	(...)	(...)	$\sigma(\theta)$	(...)
1	$s_1\theta_1$					
2	$(s_1+s_2)\theta_2$					
3	$(s_1+s_2+s_3)\theta_3$					
$\vdots$						
i	$(\sum\limits_{j\in P_i(\sigma^*(\theta))} s_j + s_i)\theta_i$					
$\vdots$						
n	$(\sum\limits_{j\in P_n(\sigma^*(\theta))} s_j + s_n)\theta_n$					
Max(cost)	$(\sum\limits_{j\in P_i(\sigma^*(\theta))} s_j + s_i)\theta_i$					

The above table there are $n!$ coloumns each one devoted for on of the every possible $n!$ allocations and there are $|N| = n$ rows devoted to each of the players. To find Rawlsian allocations we need to find the maximum entry for each of the coloumn and then we select minimum of all these maximum costs/entries. Then the allocation for which we get the maxmin entry is the Rawlsian one.

We suppose that the maximum entry for the coloumn with allocation $\sigma^*(\theta)$ is due to agent i and the cost is $(\sum\limits_{j\in P_i(\sigma^*(\theta))} s_j + s_i)\theta_i$. Consider $A_i^* = \{P_i(\sigma^*(\theta)) \cup i\} \subset N$. Since any general allocation rule $\sigma(\theta)$ involves each of the n agents, so $\forall\sigma(\theta) \in \sum(N)$ define (an ordered set) set $B(\sigma(\theta)) = \sigma(\theta) \setminus \{j | j \notin A_i^*\}$ such that $|B(\sigma(\theta))| = |A_i^*|$ and all the elements of $B(\sigma(\theta))$ are also elements of A_i^* and vice versa only difference between $B(\sigma(\theta))$ and A_i^* is they are ordered set and unordered set respectively. We denote the collection of all such $B(\sigma(\theta))$ as $\sum B(\sigma(\theta))$. Let us define the function $\xi : \sum B(\sigma(\theta)) \to A_i^*$ where $\xi(B(\sigma(\theta))) = l$(say) and $l \in A_i^*$ is the last element of the ordered set $B(\sigma(\theta))$. So the total cost for l in $\sigma(\theta)$ is at least $(\sum\limits_{j\in P_i(\sigma^*(\theta))} s_j + s_i)\theta_l$.

Since $\theta_i = \min A_i^*$, $(\sum\limits_{j\in P_i(\sigma^*(\theta))} s_j + s_i)\theta_i \leq (\sum\limits_{j\in P_i(\sigma^*(\theta))} s_j + s_i)\theta_l$.

Hence the above proposition is proved.

□

Now the allocation where agents are placed in the queue by decreasing values of their waiting costs then the allocation is Rawlsian one. We call this specific kind of Rawlsian allocation as RA^*. Now the next relevant question is , what is the transfer rule that induces RA^* to be strategyproof(SP).

In our earlier work "Incentive and normative analysis on sequencing problem." [4], we have uniquely characterized the necessary transfer rule that induces any non-increasing in own type (NOT) allocation rule into SP. So to apply our earlier result in this setup, we need to show RA^* is NOT.

Lemma 1. *RA^* is NOT.*

Proof. Consider a profile $(\theta_i, \theta_{-i}) \in \Re^n_{++} \ni \theta_1 \geq \theta_2 \geq \ldots \geq \theta_i \geq \ldots \geq \theta_n$. So $P_i(\sigma(\theta)) = \{1, 2, \ldots, i-1\}$. Now let $\theta'_i > \theta_i$. So there can be two possibles cases.
Case(i) $\theta'_i > \theta_i \ni \theta'_i \leq \min\limits_{j \in P_i(\sigma(\theta))} \theta_j$:
Then $P_i(\sigma(\theta)) = P_i(\sigma(\theta'_i))$. Hence $S_i(\sigma(\theta_i, \theta_{-i})) = S_i(\sigma(\theta'_i, \theta_{-i})$.
Case(i) $\theta'_i > \theta_i \ni \theta'_i > \min\limits_{j \in P_i(\sigma(\theta))} \theta_j$:
Then $P_i(\sigma(\theta)) \subseteq P_i(\sigma(\theta'_i))$ and $P_i(\sigma(\theta)) \neq P_i(\sigma(\theta'_i$. Hence $S_i(\sigma(\theta_i, \theta_{-i})) > S_i(\sigma(\theta'_i, \theta_{-i})$.
Hence the Lemma is proved.

□

Consider a sequencing problem $\Omega = \langle N, \Re^n_{++}, \Re^n_{++} \rangle$. So for any profile $\theta \in \Re^n_{++}$(where θ_is are privately known) such that $\theta_1 \geq \theta_2 \ldots \geq \theta_n$ and $s = (s_1, s_2, \ldots, s_n) \in \Re^n_{++}$(where s_i s are publicly known). Consider the allocation rule $\sigma^*\theta$ is RA^* and the transfer rule is:

$$\forall i \in N, \;\; t_i(\theta) = h_i(\theta_{-i}) - \sum_{j \in P'_i(\sigma^*(\theta))} s_j \theta_j. \tag{1}$$

Proposition 2. *RA^* is SP if and only if the transfer rule is given by equation 2.*

Proof. RA^* is non-increasing in own type allocation rule (by Lemma 1). Hence we can apply Theorem (1) of our earlier work [4], which proves the above Proposition.[3] □

Remark 2. A mechanism (σ, t) is EFF and SP if and only if $\forall \theta \in \Re^n_{++}$ the allocation rule minimizes the aggregate cost and $\forall i \in N$, $t_i(\theta) = h_i(\theta_{-i}) - s_i \sum_{j \in P'_i(\sigma^*(\theta))} \theta_j$. But a mechanism $(\sigma(, t)$ is RA^* and SP if and only if $\forall \theta \in \Re^n_{++}$ $\sigma(\theta) = \sigma^*(\theta)$ and $\forall i \in N, \; t_i(\theta) = h_i(\theta_{-i}) - \sum_{j \in P'_i(\sigma^*(\theta))} s_j \theta_j$.

[3]For details of the proof see Incentive and normative analysis on sequencing problem.[4]

Remark 3. In case of queueing games $\Omega = \langle N, \Re^n_{++} \rangle$ the transfer rule along with RA^* that guarantees SP is of the following form:

$$\forall i \in N, \;\; t_i(\theta) = h_i(\theta_{-i}) - \sum_{j \in P_i'(\sigma^*(\theta))} \theta_j. \tag{2}$$

which is basically an alternative representation of VCG[4]mechanism. Note that the above equation is directly follows if in equation 2 we assume $\forall i \in N, s_i = 1$. In other words, for queueing games a mechanism (σ, t) which is RA^*+SP, is also EFF+SP and this is not necessarily true for sequencing games as $RA^* \neq$ EFF in that case.

Proposition 3. *In case of queueing problems RA+SP+BB mechanism is possible.*

Proof. Note that RA^* is a Rawlsian allocation (by Proposition 1). In case of queueing games RA^*= EFF. Hence by the result of Mitra 2001 [2], the proposition is proved. ☐

3.2. **Multidimensional private information case:** Here we have situations where the sequencing problem is $\Omega = \langle N, \Re^n_{++}, \Re^n_{++} \rangle$. Any profile $\theta \in \Re^n_{++}$(where θ_i s are privately known) such that $\theta_1 \geq \theta_2 \ldots \geq \theta_n$ and $s = (s_1, s_2, \ldots, s_n) \in \Re^n_{++}$(also s_is are privately known).

What is non-manipulability or strategyproofness (SP) in this set-up?

Definition 9. A mechanism (σ, t) is strategy-proof(SP) if $\forall i \in N$, $\forall \theta_i, s_i, \theta_i', s_i' \in \Re_{++}$ and $\forall \theta_{-i}, s_{-i} \in \Re^{n-1}_{++}$, we have
$U_i(\sigma(\theta, s), t_i(\theta, s); \theta_i, s_i) \geq U_i(\sigma(\theta, s), t_i(\theta, s'); \theta_i', s_i')$.

In this situation it is impossible to have EFF + SP mechanism. Can RA^* helps achieve SP in such situation? The answer is 'yes!'.

The idea is as follows: When we use RA^* as allocation rule, we are basically allocating agents on the basis of waiting costs only. So incomplete information about the other dimension that is, processing time have no role to play. Now if we use the transfer rule $t_i(\theta, s)$ given by equation 2 an agent by his own can not do better in terms of transfer by manipulating his processing time, since it has no contribution in the transfer function.So only channel with which an agent can manipulate is his waiting cost. But we have seen (by Proposition 2) for any given $s \in \Re^n_{++}$ and $\theta_(- i) \in \Re^{n-1}_{++}$ truthful revelation is weakly dominant strategy when the transfer rule is given by equation 2. So he has no incentive to manipulate his private informations.

[4]See Vickery[12], Clarke[9], Groves[10].

Proposition 4. *When there is private information in multiple(2) dimension RA^* allocation rule induces a mechanism(σ^*, t) in SP if* $\forall\theta, s \in \Re^n_{++}, \forall\theta_{-i} \in \Re^{n-1}_{++}$ *and* $\forall i \in N; t_i(\theta, s) = h_i(\theta_{-i}) - \sum_{j \in P'_i(\sigma^*(\theta))} s_j\theta_j$.

The proof is fairly obvious hence omitted.

4. Conclusion

This paper is exclusively on the backdrop of queueing and sequencing games. In this particular context we have highlighted the situation when Efficient allocation is indifferent from Rawlsian allocation, when one implies another,etc. Thereafter we completely characterize a particular class of Rawlsian allocation rule called RA^*. Having well-acquainted with the fact that when there is private information in more than one dimension it is impossible to induce Efficiency along with strategyproofness we ventilate on a secondary solution, that is, we can replace Efficiency by RA^* and then look for strategyproofness and we show such a mechanism a possible.

Although we offer kind of a secondary alternative mechanism to ensure non-manipulability by any agent on his own, such mechanism can not sustain non-manipulability by group. Hence the next question is, do we have a group-strategyproof mechanism where the allocation rule is RA^*? We plan to look into this aspect in future.

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