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Corruption in Multidimensional Procurement Auctions under Asymmetry

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Abstract

We examine corruption under two kinds of scoring auctions, 'first-score' and 'second-score' when the bidders are heterogeneous in their cost of production. If the procurement agent is also in charge of the verification of the quality, she can allow the supplier to produce the good at a cheaper quality in exchange for a bribe. We find that in our two-bidder set up, the agent will always ask the stronger firm to bribe under the second-score auction mechanism. However, in the first-score auction mechanism, our numerical simulations suggest that the agent will choose to approach the weaker firm.
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1 Introduction

Public procurement can be defined as the acquisition of goods and services by the government in order to improve upon the state of the economy. This includes procurement contracts related to transport, health, education, infrastructure, defence equipments etc. The government or government agencies call for tender proposals, rather than simply offering the contracts on first-come-first-serve basis. Once the submission period is over and the proposals are evaluated, the firm quoting the lowest price is declared the winner and offered the contract. The revenue generated in this manner is high compared to any other non-competitive procurement process where no bidding takes place and the good is bought at a fixed or posted price . As Krishna (2010) points out,

> "The process of procurement via competitive bidding is nothing but an auction, except in this case the bidders compete for the right to sell their products or services. Billions of dollars of government purchases are almost exclusively made this way."

Procurement processes are considered to be of utmost economic importance and are a matter of interest to many researchers across the world. It is estimated that public procurement approximately constitutes about 15-20% of GDP in developed and developing jurisdictions.[1] While auction is considered to be an efficient procurement mechanism, the separation between the auctioneer (a public official) and the buyer (government or any government agency) gives rise to the possibility of corruption in the system. The World Bank defines a corrupt practice as "the offering, giving, receiving, or soliciting, directly or indirectly, of anything of value to influence the action of a public official in the procurement process or in contract execution."[2]

In India, public procurement has been estimated to constitute about 30% of the GDP. With the auctions being intensively used in buying or selling, such corrupt deals can have significant economic consequences. In 2011, India was ranked 95th out of 178 countries in Transparency International's (TI) Corruption Perceptions Index. In their study conducted in India in 2005, TI found that more than 55% of Indians are paying bribes to get their jobs done at the government offices.[3] There has been ample evidence of misconduct and corruption in the procurement processes in India. In a World Bank-Confederation of Indian Industries (CII) survey of 210 private sector firms carried out in 1999, 60% responded that a bribe of 2 to 25% of the price is necessary to secure government contracts.[4]

Public procurement auctions often award contracts on the basis of quality characteristics of the product along with its price. The firms are required to submit two proposals in the form of their bids, a financial and a technical one. The *financial proposal* specifies the price while the *technical proposal* specifies the product in terms of "the cost of operating, maintaining and repairing goods or works, the time for delivery of goods, completion of works or provision of services, the characteristics of the subject matter of the procurement, such as the functional characteristics of goods or works and the environmental characteristics of the subject matter, the terms of payment and of guarantees in respect of the subject matter of the procurement."[5] These technical proposals are reviewed by an evaluation committee and are assigned quality scores/bids (Lengwiler and Wolfstetter, 2006). The auctioneer then has to choose that firm a winner which offers best price and quality combination in the sense of providing "high quality and low price". If the auctioneer who is in charge of the auction is also in charge of the verification of the quality of the good, she can enter into an agreement with one of the suppliers before the bids are placed that allows the supplier to produce the good at a cheaper quality in exchange for a bribe.

In one of the biggest scandals in Indian history, a Swedish company Bofors won the tender to supply the Indian Army with 410 155-mm howitzers in 1984. During the evaluation process, as was revealed much later, Bofors was (illegally) permitted to alter its bid without re-tendering. Eventually, despite the objections of the army and others who preferred the French Sofma gun over Bofors on quality grounds, the order went to Bofors.[6] Another embarrassing scandal was the embezzlement of funds during the Commonwealth Games, 2010. The Central Vigilance Commission(CVC) estimated the misappropriation of funds amounting to about Rs.8000 crore.[7] At least 22 Games-related construction and procurement works carried out by different government agencies were probed by the CVC for alleged financial irregularities along with the use of poor quality raw materials, and many of those involved were arrested.

Professor Bibek Debroy and Laveesh Bhandari estimate that the public officials in India may be extracting almost 1.26 per cent of the GDP through corruption every year. The sectors reported to be highly corrupt are that of transport and real estate among others.[8] In 2006, the then chief minister of Uttar Pradesh, Mulayam Singh Yadav, ordered an inquiry after the United Progressive Alliance (UPA) chairperson Sonia Gandhi's remark on the appalling road conditions in the state. During the inquiry it was found that the quality of the material used in the construction of roads was not as per the contractual agreement. Many officials and contractors were then arrested,and several were blacklisted.[9]

Che (1993) describe a model where both price and the quality score are aggregated by the means of a Scoring Rule designed by the buyer. This kind of a procurement process that is used to buy a differentiated product is often termed as a "Scoring Auction" in the Auction Literature. Che defines three two-dimensional auctions that are called 'first-score', 'second-score' and 'second-preferred-offer' auctions. In a *first-score auction*, "each firm submits a sealed bid and, upon winning, produces the offered quality at the offered price." In a *second-score auction*, the winner is asked to match the second highest score in the auction while in a *second-preferred-offer auction*, the winner is asked to match the exact price-quality combination of the bidder who scored the second highest. Scoring auctions are found to dominate many other procurement processes such as menu auctions, beauty contests, and price-only auctions in terms of expected utility to the buyer even when a supplier's private information about his costs is multidimensional (Asker and Cantillon, 2008).

In the model illustrated in Che (1993), the sellers are assumed to be symmetric in their cost of production. "...Asymmetries are often important in contract bidding. Each potential contractor has essentially the same information about the nature of the project but a different opportunity cost of completing it. Whenever some aspect of these differences is common knowledge, beliefs are asymmetric..."(Maskin and Riley, 2000). Asymmetry amongst firms can be observed due to differences in various aspects such as the location of the firm (local or distant), processing capacity, technology etc (Ramaswami et al., 2009; Banker and Mitra, 2007). One of the crucial findings in Che (1993) is the two-dimensional Revenue Equivalence Theorem. This theorem states that under a *Naive* Scoring Rule which truly reflects seller's preferences, all the three scoring auctions yield the same expected utility to the buyer. This Revenue(Utility) Equivalence of the Scoring Auctions as postulated by Che (1993) breaks down in the presence of asymmetry.

In the example constructed in Chandel and Sarkar (2014) , while the types of both the strong and weak sellers are drawn from a uniform distribution, the strong seller's type distribution is a "stretched" version of that of the weak seller. The bidders are required to submit a two-dimensional bid quoting the level of quality promised to be delivered and the payment in return. These bids are then evaluated using a Scoring Rule that is equivalent to the Buyer's utility from the contract and the firm with the highest score is declared the winner. We find that under the first-score and second-score auction formats, the expected utility to the buyer no longer remains the same when the bidding firms are asymmetric in nature.

We extend the model used in Chandel and Sarkar (2014) to incorporate a procurement agent who is known to be corrupt with some probability. We consider a case where the buyer delegates the task of procuring the desired good to an agent. This act of delegation separating the auctioneer with the buyer provides the possibility of corruption in the system. If the agent who is in charge of the auction is also in charge of the verification of the quality of the good, she can enter into an agreement with one of the suppliers before the bids are placed that allows the supplier to produce the good at a cheaper quality in exchange for a bribe. We find that in our two-bidder setup, the agent will always ask the stronger firm to bribe under the second-score auction mechanism. However, in the first-score auction mechanism, our numerical simulations suggest that the agent will approach the weaker firm to enter into a corrupt agreement. Also, if the buyer is unaware of the agent being corrupt or, in other words, if the procurement agent is able to shield his corrupt activities from the buyer, then the buyer's preference between the first-score or the second-score auction is same as that when the agent is honest.

The rest of the paper is divided into five sections. Section 2 reviews the literature associated with various studies on how corruption takes place in auctions. Section 3 outlines the skeleton of the model used to study corruption under the two scoring auction formats. Section 4 and 5 consist of the workings of the Second-Score and First-Score auctions respectively. Lastly, Section 6 includes the concluding remarks of our paper.

2 Review of Literature

In the auction theory literature, vast as it is, the issue of corruption in procurement auctions, or for that matter in auctions, is addressed only until very recently. The procurement agent can either agree to readjust a bid (Compte et al., 2005; Auriol, 2006) or can compromise on quality of the good provided (Burguet and Che, 2004; Celentani and Ganuza, 2002), in exchange for a bribe. This study falls into the latter category and the model considered draws significantly from the work of Celentani and Ganuza (2002) which studies the impact of competition on corruption in a multidimensional procurement auction when the suppliers are symmetric. Our study also adds to the literature on corruption in procurement auctions by reviewing the model in an asymmetric setting. Further, it contributes to the literature on multidimensional auctions where the bidders bid on different aspects of the good other than the price. Such auctions have been in use for the procurement of goods and services where the quality delivered is crucial, such as, defence equipments, road construction contracts, devices using heavy technology etc.

Compte et al. (2005) show a negative impact of corruption on competition in a first-price auction when the firms are symmetric and bid only in price. They assume that all the bidding firms have an incentive to be corrupt, so they not only participate in price competition, but also in a bribe competition. The suppliers are expected to increase their contract price by that same amount as the bribe offered. However, their study shows that this increment in the price bid would be much greater than the bribe paid to the agent, thereby preventing competitive bidding and enforcing a collusion among the bidders. Burguet and Che (2004) examine a multidimensional procurement process where a firm's cost of production is assumed to be different from its' rival's but is no longer privately known. They observe that when the agent has high manipulation power, bribery leads to inefficient procurement, where the firm providing lower quality of the good wins the contract in exchange of a bribe. It is shown that the optimal scoring rule announced by the buyer is biased against quality, so as to provide lesser discretion to the agent who is information-superior to the principal. In their model, the agent manipulates the quality score by quoting the quality provided by bribing firm to be higher, $(q + m)$ than what it had bid earlier, q. In our proposed study, the agent, if corrupt, *ex ante* chooses the firm she will demand a bribe from. This is contrary to the Burguet and Che (2004) setting where the firms engage in bribe competition while bidding for the contract. Lengwiler and Wolfstetter (2006) examine different kinds of corruption within the procurement mechanisms and classifies them under *Bid Rigging, Bid Orchestration and Distortion of Quality Ranking*. They further propose 'practical' mechanisms in order to restrain these forms of corruption.

There is another kind of corruption, a particular form of bid rigging, where the favoured bidder is offered to match the winning bid in exchange for a bribe. This is known as the "Right of First Refusal"(ROFR). Sometimes ROFR is practised legally and is preferred by both the parties involved. A very good example of this is the tenant contract. Often, at the time of renewal of the contract, the house-owner would offer the residing tenant to match the highest price offered. Lee (2008) examines an asymmetric procurement auction setting where the auctioneer decides to grant the ROFR to one bidder and announces it before the bidding starts. It is shown that if the asymmetry is sufficiently large, then the auctioneer would benefit by favouring a weak bidder. However, this kind of practice is rarely seen in defence contracts where quality is of utmost importance. Also, "this scheme implies that all bidders know about the corruption, and thus entails a large risk of detection. An auctioneer who cares about the risk of detection should consider proposing corruption only to a small set of bidders"(Lengwiler and Wolfstetter, 2006).

Burguet and Perry (2007) examine corruption in asymmetric procurement auctions keeping quality and quantity fixed at a certain level. They argue that the asymmetry in the cost structure, location or internal rules against bribery, might induce a kind of a preference asymmetry amongst firms where some firms might choose to be honest while others might not. So the asymmetry in the structure of the firms, induces the weak firms to be dishonest. In our model, however, even a strong firm has an incentive to bribe as it still can be interested in lowering the quality level required to be delivered. Therefore, in our study, we assume that the firms are asymmetric directly in terms of their production capacity, location etc. Further, it is only realistic that a firm's discretion to be corrupt should be restricted within the procurement framework and not assumed beforehand. Burguet and Perry observe that corruption does not necessarily induce the dishonest bidder to bid less aggressively or induce the honest bidder to bid more aggressively, contrary to what is expected. Also, even with corruption, the strong bidder bids less aggressively than the weak bidder, as expected in an optimal mechanism.

The model used in this paper follows Celentani and Ganuza (2002) which studies the impact of competition on corruption in a multidimensional procurement auction when the suppliers are symmetric. They show that under certain conditions, corruption increases with competition. They construct an optimal mechanism where the firms bid a price-quality combination and the agent evaluates those bids based on a scoring rule. The procurement agent is assigned the task of verification of the quality delivered since she is supposed to have superior information over quality than the buyer. This provides incentives for the agent to accept and verify a lower delivered quality in exchange of a bribe. They report that higher the probability of corruption, lesser should be the weight attached to quality. In other words, with the possibility of corruption the principal should provide lesser discretion to the procurement agent.

3 Model

Consider a buyer who wants to procure an indivisible good from two potential suppliers. Suppose that the principal delegates the execution of the procurement auction to an agent who has "..superior information over the delivered quality"(Celentani and Ganuza 2002). In the case of government contracts, a bureaucrat or a government official is assigned the task of procurement within the guidelines set by the government (principal). The agent then sets up a two-dimensional procurement auction where the participants (suppliers or firms) submit a bid that, along with the payment p, specifies the quality q of the good that the supplier promises to deliver. By assigning certain weight to the quality of the good, the principal, in a way, provides opportunity for the agent to accept a higher price bid on the grounds of superior quality (scope for corruption).

The buyer is assumed to be risk neutral and his utility from the contract $(q,p) \in \mathbb{R}^2_+$ is given by $U(q,p) = V(q) - p$ where we assume that $V'(q) > 0$, $V''(q) < 0, \lim\limits_{q \to 0} V'(q) = \infty$, $lim_{q\to\infty}V'(q) = 0$ in order to ensure an interior solution.

As far as the agent is concerned, she too is assumed to be risk-neutral with the wage normalized to zero. Let the probability of the agent being corrupt is x and that of being honest is $1-x$. For simplicity, it is assumed that the bidding firms are otherwise honest and do not offer a bribe unless the agent asks for it. For now, we assume that while the bidders know that the agent is corrupt with probability x, the buyer/principal believes that the agent in charge of the procurement process is honest.

There are two risk-neutral, (expected) profit maximizing suppliers, labelled strong (s) and weak (w). The cost incurred by a firm in producing the good of the promised quality level q is $c(q, \theta_i)$ where θ_i is the cost/efficiency parameter of the firm and is privately known. The cost function $c(q, \theta_i)$ is assumed to be decreasing and convex in the efficiency component(Branco 1997). Also, let us assume that $c_q > 0, c_\theta < 0, c_{qq} \geq 0, c_{\theta\theta} > 0$, $c_{q\theta} < 0$ and $c_{qq\theta} < 0$.

Expected profit for the firm with type θ after winning the contract is

$$\pi(q,p|\theta) = \{p - c(q,\theta)\}.Prob\,[\,win\,|S(q,p)]$$

Let a scoring rule be a function $S : \mathbb{R}^2 \to \mathbb{R}$ that associates a score $S(q,p)$ to any potential contract (q,p) between the buyer and a supplier. The Scoring Rule used by the buyer to evaluate the two-dimensional bids submitted in the auction is of the quasi-linear form i.e. $S(q,p) = s(q) - p$ where $s(.)$ is increasing at least for $q \leq argmax_q\, s(q) - c(q,\theta)$ (Che 1993). A *Naive* Scoring Rule is the one that reveals the buyer's true preferences i.e. $S(q,p) = U(q,p)$ or some monotone transformation of U.

3.1 Timing of Events:

1. Nature selects the efficiencies θ_s, θ_w. It is common knowledge that θ_s, θ_w are drawn independently from the uniform distributions on $[\underline{\eta}, \eta_s]$ and $[\underline{\eta}, \eta_w]$ respectively where $\eta_w < \eta_s$ i.e. F_w (first-order) stochastically dominates F_s over $[\underline{\eta}, \eta_w]$

$$F_s(\theta) = \frac{\theta - \underline{\eta}}{\eta_s - \underline{\eta}}\ \forall\, \theta \in [\underline{\eta}, \eta_s] \text{ and } F_w(\theta) = \frac{\theta - \underline{\eta}}{\eta_w - \underline{\eta}}\ \forall\, \theta \in [\underline{\eta}, \eta_w]$$

2. Let the probability of the agent being corrupt is x and that of being honest is $1 - x$.
3. Both the firms privately learn their own efficiency parameters $\theta_i \; \forall \; i = s, w$.
4. The principal publicly announces the scoring rule along with the auction format i.e. either *first-score* or the *second-score* is to be used without observing the privately valued θ_i.
5. The procurement agent then has to acquire the good through this auction mechanism. *Tie-breaking rule*: If both the firms achieve the highest score, the winner is determined through a random draw. The game would then proceed as follows;
   (A) If the agent is honest, then the auction process goes about in the usual manner. The firms are requested to place the price and quality bids. The winning firm is selected based on the scoring rule. It is paid the quoted price p and is asked to produce the good at the specified quality level q, in case of the first-score auction. If the second-score auction mechanism is announced, the firm is assigned that price and quality combination that generates the second-highest score in the auction. The agent verifies that the delivered quality is indeed q and certifies it.

   (B) If the agent is corrupt, he demands a bribe from one of the bidders before the bidding can take place. Denote the bribe paid as B_s if the agent decides to extort the bribe from the strong bidder and as B_w if she decides to extort it from the weak bidder. The firm, if it accepts the offer, is declared the winner under the pretext that it offers the best price and quality combination. It then receives a payment as per the contract, produces the good at a lower quality level q_C than the quality level specified in the mechanism, and pays out the bribe to the agent, all not necessarily in the same order. The agent then verifies that the quality delivered is q instead of q_C and certifies it.

   (C) If the firm rejects the offer, the agent does not get any other chance to contact another firm. The procurement process is managed honestly as described earlier.

We assume that the corrupt arrangement is detected with a probability γ and both the agent and the bidder are penalized with penalties P^A and P^B respectively.

As seen in the timing of events in our model, we have assumed that the probability of the agent being corrupt is exogenously given, i.e. x. This is a deviation from Celentani and Ganuza (2002) framework where the agent's decision to be corrupt is endogenous and depends on the cost of corruption that he/she incurs, β, known to follow a uniform distribution over a fixed interval, $[0, \bar{\beta}]$. Now, this decision to be corrupt is made even before the agent is matched with a bidder. However, it must be so that when the agent is matched with a bidder and learns its type, it still remains profitable for the agent to engage in corrupt transaction with that firm. Otherwise, the agent would choose not to ask for a bribe even though β is sunk. To ensure that this holds, Celentani and Ganuza (2002) uses Assumption 2 and henceforth derive the probability of corruption. We avoid making any such assumption in our model and assume that the agent is corrupt with some exogenously given probability which assists in isolating the effect of asymmetry amongst bidders on the agent's choice structure.

After the buyer announces which scoring auction is to be used, the dishonest agent decides to approach either of the firms for a bribe. He or she offers the firm to replace its bid tuple afterwards with the one that ensures it to be declared the winner and to be allowed to produce a lower quality level q_C, in exchange for a bribe. Celentani and Ganuza (2002), in their framework, assume that the agent can manipulate the bid of the bribing firm only up to a certain extent so as to avoid detection. We follow the similar line of argument and assume that the replaced bid is the equilibrium bid of a hypothetical firm whose type is higher than that of the firm other than the bribing one. Along with that, the expected payment made by the corrupt agent under the auction mechanism is equal to the expected payment made by a honest agent or when there is no intermediary agent involved.

4 First-Score Auction: Bribe extortion

In the First-Score Auction format, the firm whose price-quality combination leads to the highest score is awarded the contract. As shown in Chandel and Sarkar (2014), a solution to the below system of differential equations with relevant boundary conditions constitutes an equilibrium of the first-score auction.

$$(\phi_i(b) - b) f_j(S_o^{-1}(\phi_j(b)))(S_o^{-1}(\phi_j(b)))' = F_j(S_o^{-1}(\phi_j(b))) \qquad \forall\, i, j = s, w$$

These are nothing but the first-order conditions for both the firms to be maximizing their expected profits simultaneously. It is difficult to obtain a general solution to the system of differential equations due to the unspecified functional form $S_o(\theta)$. We would therefore consider an example where $S_o(\theta)$ takes a specific form.

4.1 When the agent is honest and/or the firm decide to be honest

We define $S_o(\theta) = \max_q s(q) - c(q, \theta)$ and $q_o(\theta) = argmax_q s(q) - c(q, \theta)$ to convert the two-dimensional First-Score Auction into a unidimensional exercise (Chandel and Sarkar, 2014). When either or both of the procurement agent and the firm partaking in corrupt transaction choose to remain honest, the auction process is similar to that when there is no agent involved in the procurement. Let $V(q) = 2\sqrt{q}$ and $c(q, \theta) = \dfrac{q}{\theta}$. The proposition 1 from Chandel and Sarkar (2014) defines the equilibrium strategies of the firms as below:

Proposition 1 **:**

(a) The bidding strategies of the firms are

$$\beta_j^{FS}(\theta) = \underline{\eta} + \frac{1}{k_j(\theta - \underline{\eta})}\left(-1 + \sqrt{1 + k_j(\theta - \underline{\eta})^2}\right) \qquad \forall j = s, w \tag{1}$$

(b) In this first-score auction, each firm in equilibrium offers

$$q_s^{FS}(\theta) = q_w^{FS}(\theta) = q_o(\theta) = \theta^2 \tag{2}$$

$$p_i^{FS}(\theta) = 2\theta - \underline{\eta} - \frac{1}{k_i(\theta - \underline{\eta})}\left(-1 + \sqrt{1 + k_i(\theta - \underline{\eta})^2}\right) \tag{3}$$

where $k_j = \frac{1}{(\eta_i - \underline{\eta})^2} - \frac{1}{(\eta_j - \underline{\eta})^2} \qquad \forall i, j = s, w$

Hence, the expected winning offer in this first-score auction is

$$E(q)^{FS} = \sum_{i \neq j}^{s,w} \int_{\underline{\eta}}^{\eta_i} q_i^{FS}(\theta) \frac{\phi_j^{FS}(\beta_i^{FS}(\theta)) - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta \tag{4}$$

$$E(p)^{FS} = \sum_{i \neq j}^{s,w} \int_{\underline{\eta}}^{\eta_i} p_i^{FS}(\theta) \frac{\phi_j^{FS}(\beta_i^{FS}(\theta)) - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta \tag{5}$$

Under the Naive Scoring Rule, the expected utility in the first-score auction turns out to be

$$EU^{FS} = \bar{b} - \frac{(\eta_s - \underline{\eta})^2(\eta_w - \underline{\eta})^2}{(\eta_s - \eta_w)^{3/2}(\eta_w + \eta_s - 2\underline{\eta})^{3/2}} \left[\log\left(\frac{\sqrt{\eta_w + \eta_s - 2\underline{\eta}} + \sqrt{\eta_s - \eta_w}}{\sqrt{\eta_w + \eta_s - 2\underline{\eta}} - \sqrt{\eta_s - \eta_w}} \right) - 2\arctan\sqrt{\frac{\eta_s - \eta_w}{\eta_w + \eta_s - 2\underline{\eta}}} \right] \tag{6}$$

4.2 When the agent is dishonest and the firm decide to be corrupt

We solve for the equilibrium strategies in this case by backward induction. One important question to ask is that if the firms know that the agent is corrupt with some probability, will the bidding functions of the firms be different from what they were before? The answer is no. Since the bidding takes place after the agent and the chosen firm have colluded, the firm that was not made the offer can in no way alter its bidding behaviour so as to increase the probability of it being declared the winner. As for the firm that is chosen for corrupt arrangement, it can decide to bid in whichever manner for the agent has guaranteed its win even before the bidding process has taken place. One such way would be to adhere to bid according to equilibrium bidding function in the honest mechanism.

The Expected payoff of firm i when it chooses to accept the agent's offer to bribe, denoted by π_i^C, is given as

$$\pi_i^C = E(p)^{FS} - B_i - c(q_C, \theta_i) - \gamma P^B \tag{7}$$

Firm i chooses to accept the bribe offer when the expected profits earned by the firm under such offer are atleast equal or higher than those earned by the firm when it decides to be honest. That is to say, $\pi_i^C \geq \pi_i^H$ in case of the First-Score auction. Hence, the bribe paid by the firm at equilibrium would be

$$B_i = E(p)^{FS} - \pi_i^H - c(q_C, \theta_i) - \gamma P^B \tag{8}$$

The expected payoff to the procurement agent when he decides to be corrupt and ask for a bribe from firm i ($i = s$ or w) is given by

$$\pi_i^A = x.(B_i - \gamma P^A) + (1 - x).0 \tag{9}$$

Likewise, the expected payoff to the agent when the firm rejects his offer is just 0. Similar to the argument made for the Second-Score Auction, the equilibrium bribe as in equation (8) is admissible if and only if $E(p)^{FS} - \pi_i^H - c(q_C, \theta_i) - \gamma(P^A + P^B) > 0$ for both $i = s, w$.

At this stage, the firm type θ is unknown to the agent so the expected payoff under corrupt agreement would be given by

$$\begin{aligned} E_\theta(\pi_i^A) &= E_\theta(B_i - \gamma P^A) && (10) \\ &= E(p)^{FS} - E_\theta(\pi_i^H + c(q_C, \theta_i)) - \gamma(P^A + P^B) && (11) \end{aligned}$$

The agent asks the weak firm for a bribe when,

$$\begin{aligned} & E_\theta(\pi_s^A) \leq E_\theta(\pi_w^A) \\ \implies & E_\theta((B_s - \gamma P^A)) \leq E_\theta((B_w - \gamma P^A)) \\ \implies & E_\theta(B_s) \leq E_\theta(B_w) \\ \implies & E(p)^{FS} - E_\theta(\pi_s^H + c(q_C, \theta_s)) - \gamma P^B \leq E(p)^{FS} - E_\theta(\pi_w^H + c(q_C, \theta_w)) - \gamma P^B \\ \implies & E_\theta(\pi_s^H + c(q_C, \theta_s)) \geq E_\theta(\pi_w^H + c(q_C, \theta_w)) \qquad (12) \end{aligned}$$

Equation (12) gives us the condition under which the agent will always ask the weak firm to bribe in exchange for allowing a lower level of quality good to be supplied.

For $V(q) = 2\sqrt{q}$ and $c(q, \theta) = \dfrac{q}{\theta}$,the condition becomes

$$E_\theta(\pi_s^H) + q_C E_\theta(\frac{1}{\theta_s}) \geq E_\theta(\pi_w^H) + q_C E_\theta(\frac{1}{\theta_w}) \tag{13}$$

where

$$E_\theta(\pi_i^H) = \int_{\underline{\eta}}^{\eta_i} (\theta - \beta_i(\theta)) \frac{\phi_j^{FS}(\beta_i^{FS}(\theta)) - \underline{\eta}}{(\eta_i - \underline{\eta})(\eta_j - \underline{\eta})} d\theta$$

and $E_\theta(\frac{1}{\theta_i}) = \frac{1}{(\eta_i - \underline{\eta})} \log\left(\frac{\eta_i}{\underline{\eta}}\right)$ for all $i = s, w$.

Using Taylor's expansion it is easy to show that for $\eta_w < \eta_s$, the function $f(x) = \frac{1}{(x - \underline{\eta})} \log\left(\frac{x}{\underline{\eta}}\right)$ is always decreasing in x. So, for an exogenously given q_C, it is always true that $E_\theta(\frac{1}{\theta_s}) \leq E_\theta(\frac{1}{\theta_w})$. However, it is not that straightforward when we compare the expected profit of firms under honest case.

$$E_\theta(\pi_i^H) = \frac{1}{k_i(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} \int_{\underline{\eta}}^{\eta_i} \left(-1 + \sqrt{1 + k_i(\theta - \underline{\eta})^2}\right) d\theta \qquad \forall\, i = s, w. \tag{14}$$

which when solved give us the expected profits under honest case to be

$$E_\theta(\pi_i^H) = \frac{1}{2k_i(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} \Bigg[(\eta_i - \underline{\eta})\left(2 + \sqrt{1 + k_i(\eta_i - \underline{\eta})^2}\right) - \frac{3}{\sqrt{k_i}} \log\left(\sqrt{k_i}(\eta_i - \underline{\eta}) + \sqrt{1 + k_i(\eta_i - \underline{\eta})^2}\right)\Bigg] \quad \forall\ i = s, w$$

In case of symmetry, $E_\theta(\pi_s^H) = E_\theta(\pi_w^H) = \frac{\eta - \underline{\eta}}{6}$ where $\eta_s = \eta_w = \eta$.

Define a variable Y^{FS} such that

$$Y^{FS} = E_\theta(\pi_s^H) + q_C E_\theta(\frac{1}{\theta_s}) - E_\theta(\pi_w^H) - q_C E_\theta(\frac{1}{\theta_w}) \tag{15}$$

From equation (13), we can see that the agent will approach the strong firm to bribe if $Y^{FS} \leq 0$ and correspondingly the agent will approach the weak firm if $Y^{FS} \geq 0$.

It is difficult to observe algebraically how the variable Y^{FS} behaves with respect to the level of asymmetry between the firms. Let $\alpha \in (0, 1)$ be such that $\eta_w = \underline{\eta} + \frac{1}{1+\alpha}$ and $\eta_s = \underline{\eta} + \frac{1}{1-\alpha}$. This $\alpha > 0$ denotes the level of asymmetry between the two firms. For

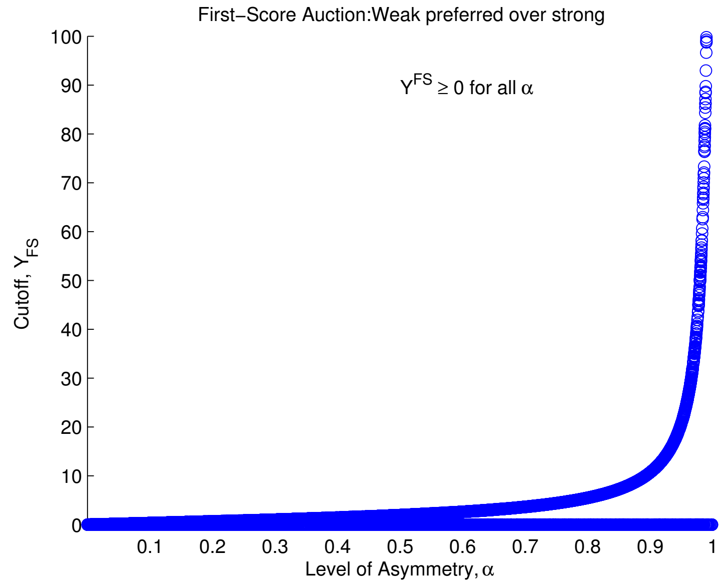

Figure 1: First Score Auction under Corruption

$\alpha = 0$ and $\eta_s = \eta_w = \eta$, the firms are symmetric and the expected profits for both the firms are same and equal to $\frac{\eta - \underline{\eta}}{6}$ which is 1/6 in the previous case which makes $Y^{FS} = 0$.

Using simulations, we find that $Y^{FS}(\alpha)$ is non-negative and increasing in α as shown in fig 1. This suggests that the agent will approach the weak firm to bribe under the First-Score Auction Mechanism. Now, when the firms are symmetric, the agent is indifferent between the firms for approaching them for a bribe. However, with asymmetry in their costs of production, the agent approaches the weak firm for a bribe, declaring that firm the winner. We can argue that since the bribe offered in equilibrium is nothing but the net surplus generated for the firm when it engages in corruption, the stronger firm will offer a lower bribe than the weaker firm in equilibrium as even though the strong firm's expected cost of production is less than that of the weak firm, the expected profits earned by the strong firm under honest mechanism, $E_\theta(\pi_s^H)$, are much higher than the expected profits earned by the weak firm under honest mechanism, $E_\theta(\pi_w^H)$.

Result: Let $\alpha \in (0, 1)$ be such that $\eta_w = \underline{\eta} + \frac{1}{1+\alpha}$ and $\eta_s = \underline{\eta} + \frac{1}{1-\alpha}$. Then, for randomly selected levels of α, it can be seen that the agent approaches the weak firm for a bribe under first-score auction mechanism.

5 Second-Score Auction:Bribe extortion

In a *Second-score auction*, the winning firm is asked to match the second highest score in the auction. However, it is not essential to offer the exact price and quality combination that the firm with the second highest score did.

5.1 When the agent is honest and/or the firm decide to be honest

As shown in Chandel and Sarkar (2014), we define $S_o(\theta) = \max_q s(q) - c(q, \theta)$ and $q_o(\theta) = argmax_q s(q) - c(q, \theta)$ to convert the two-dimensional Second-Score Auction into a unidimensional exercise.

Each firm would bid a score equal to $S_o(\theta_i)$ and the most efficient firm (largest θ) would win the auction. The expected profit earned, π_i^H, would be zero and as shown in Proposition 3 (Chandel and Sarkar, 2014), each firm in equilibrium would offer

$$
\begin{aligned}
q_i^{SS}(\theta) &= q_o(\theta) \\
p_i^{SS}(\theta) &= c(q_o(\theta), \theta) \qquad \forall i = s, w
\end{aligned}
$$

Therefore, the expected winning offer in this second-score auction is

$$E(q)^{SS} = E\{q_o(\theta_1)\} \tag{16}$$
$$E(p)^{SS} = E\{s(q_o(\theta_1)) - s(q_o(\theta_2)) + c(q_o(\theta_2), \theta_2)\} \tag{17}$$

where $\theta_1 = Max(\theta_s, \theta_w)$ and $\theta_2 = Min(\theta_s, \theta_w)$.

And under Naive Scoring Rule, the buyer's expected utility under the second-score auction becomes

$$
\begin{aligned}
EU^{SS} &= E\{s(q_o(\theta_2)) - c(q_o(\theta_2)\theta_2) \\
&= E\{S_o(\theta_2)\} \qquad (18)
\end{aligned}
$$

5.2 When the agent is dishonest and the firm decide to be corrupt

We solve for the equilibrium strategies in this case by backward induction. The Expected payoff of firm i when it chooses to accept the agent's offer to bribe, denoted by π_i^C, is given as

$$\pi_i^C = E(p)^{SS} - B_i - c(q_C, \theta_i) - \gamma P^B \tag{19}$$

Firm i chooses to accept the bribe offer when the expected profits earned by the firm under such offer are atleast equal or higher than those earned by the firm when it decides to be honest. That is to say, $\pi_i^C \geq \pi_i^H = 0$ in case of the Second-Score auction. Hence, the bribe paid by the firm at equilibrium would be

$$B_i = E(p)^{SS} - c(q_C, \theta_i) - \gamma P^B \tag{20}$$

The expected payoff to the procurement agent, if dishonest, from firm i ($i = s$ or w) is given by

$$\pi_i^A = x.(B_i - \gamma P^A) + (1 - x).0 \tag{21}$$

Likewise, the expected payoff to the agent when the firm rejects his offer is just 0. Before the type of the other firm is realized, the equilibrium bribe as in equation (20) is admissible if and only if $\pi_i^A > 0$ or $B_i - \gamma P^A > 0$ for all types of θ_i. This implies that the bribe offer under equilibrium is valid if and only if $E(p)^{SS} - c(q_C, \theta_i) - \gamma(P^A + P^B) > 0$ for all $i = s, w$.

It is important to note that the agent's decision to choose between a strong and a weak firm is rendered moot when the above condition does not hold for either or both the firms. For instance, when $E(p)^{FS} - c(q_C, \theta_i) - \gamma(P^A + P^B) > 0$ for just one of the firms, then for the other firm, the bribe offered would not be enough to surpass the penalty that the agent would have to incur in order to make a corrupt offer. In that case, it would be obvious for the agent to approach the firm that can actually bribe.

At this stage, the firm type θ is unknown to the agent so the expected payoff under corrupt agreement would be given by

$$\begin{aligned} E_\theta(\pi_i^A) &= E_\theta(B_i - \gamma P^A) & (22)\\ &= E(p)^{SS} - E_\theta(\pi_i^H + c(q_C, \theta_i)) - \gamma(P^A + P^B) & (23)\\ &= E(p)^{SS} - E_\theta(c(q_C, \theta_i)) - \gamma(P^A + P^B) & (24) \end{aligned}$$

An important question to ask is that who does the agent approach for a bribe, the strong or the weak firm. To answer this, we compare the expected payoff to the agent when he approaches the strong firm to that when he approaches the weak firm. Therefore, the agent asks the strong firm for a bribe when,

$$\begin{aligned} E_\theta(\pi_s^A) &\geq E_\theta(\pi_w^A) \\ \implies E(c(q_C, \theta_s)) &\leq E(c(q_C, \theta_w)) & (25) \end{aligned}$$

This constitutes as our first proposition. Formally,

Proposition 1: In the Second-Score Auction with corruption, the procurement agent always asks the stronger firm to bribe when $E(c(q_C, \theta_s)) \leq E(c(q_C, \theta_w))$.

An Example

Let $V(q) = 2\sqrt{q}$ and $c(q,\theta) = \dfrac{q}{\theta}$, the bidding strategies of the firms are $\beta_i^{SS}(\theta) = \theta$ for all $i = s, w$ and each firm in equilibrium offers

$$q_i^{SS}(\theta) = \theta^2$$
$$p_i^{SS}(\theta) = \theta \qquad \forall i = s, w$$

The expected winning offer in the second-score auction is

$$E(q)^{SS} = E\{\theta_1^2\} \tag{26}$$
$$E(p)^{SS} = E\{2\theta_1 - \theta_2\} \tag{27}$$

where $\theta_1 = Max(\theta_s, \theta_w)$ and $\theta_2 = Min(\theta_s, \theta_w)$.

And the buyer's expected utility in this case is

$$EU^{SS} = E\{\theta_2\} \tag{28}$$

As shown in the appendix,

$$E(\theta_1) = \frac{(2\eta_s + \underline{\eta})(\eta_s - \underline{\eta})}{3(\eta_w - \underline{\eta})} \tag{29}$$
$$E(\theta_2) = \frac{1}{6(\eta_s - \underline{\eta})}\left[(3\eta_s - \eta_w)(\eta_w + \underline{\eta}) - 4\underline{\eta}^2\right] \tag{30}$$
$$E(\theta_1^2) = \frac{(\eta_s - \underline{\eta})(3\eta_s^2 + 2\eta_s\underline{\eta} + \underline{\eta}^2)}{6(\eta_w - \underline{\eta})} \tag{31}$$

In this case, the agent asks the strong firm to bribe when,

$$q_C E(\frac{1}{\theta_s}) \leq q_C E(\frac{1}{\theta_w})$$
$$\frac{1}{(\eta_s - \underline{\eta})}\log\left(\frac{\eta_s}{\underline{\eta}}\right) \leq \frac{1}{(\eta_w - \underline{\eta})}\log\left(\frac{\eta_w}{\underline{\eta}}\right) \tag{32}$$

This is always true for $\eta_w < \eta_s$ as using Taylor's expansion it is easy to show that the function $f(x) = \dfrac{1}{(x - \underline{\eta})}\log\left(\dfrac{x}{\underline{\eta}}\right)$ is always decreasing in x. Hence, in the Second-Score Auction with corruption, the procurement agent always asks the stronger firm to bribe.

This makes sense since asymmetry amongst bidders does not effect the bidding behaviour of the firms and the agent's decision solely depends upon the expected cost incurred by the firms in producing the good of inferior quality q_C. As the strong firm's expected cost of production is less than that of the weak firm, the agent would be able to extort a higher level of bribe from the strong firm than that from its weaker opponent.

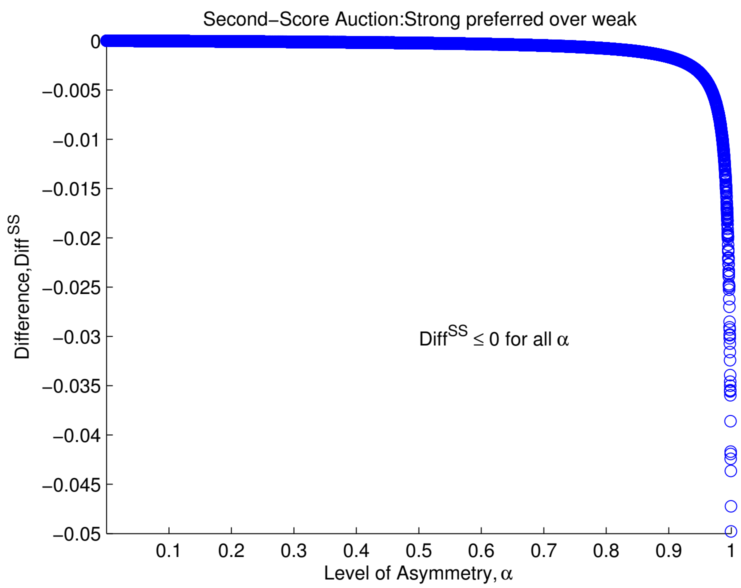

Figure 2: Second Score Auction under Corruption

Corollary 1: In the Second-Score Auction with corruption, the procurement agent always asks the stronger firm to bribe when the cost function of the firms is of functional form $c(q, \theta) = \dfrac{q}{\theta}$.

Figure 2 depicts the agent's decision to approach the strong firm for a bribe in the second-score auction by constructing a variable $\text{Diff}^{SS} = \dfrac{1}{(\eta_s - \underline{\eta})} \log\left(\dfrac{\eta_s}{\underline{\eta}}\right) - \dfrac{1}{(\eta_w - \underline{\eta})} \log\left(\dfrac{\eta_w}{\underline{\eta}}\right)$. It is shown that Diff^{SS} is always non-positive for all randomly selected levels of asymmetry.

Therefore, the procurement agent will always approach the strong firm to bribe under Second-Score Auction mechanism while the agent is most likely to ask the weak firm to bribe under First-Score Auction mechanism.

6 Conclusion

In the case of government contracts, a bureaucrat or a government official is delegated the task to procure a non-homogeneous good that can be produced at different quality levels. In a special form of auctions known as the 'Scoring Auctions', the bidders are requested to submit both a price and a quality bid which are then aggregated by means of a pre-announced Scoring Rule. The firm whose price and quality combination generates a maximum score wins the contract. We examine corruption under two kinds of scoring auctions, 'first-score' and 'second-score' when the suppliers are heterogeneous in their cost of production. If the agent who is in charge of the auction is also in charge of the verification of the quality of the good, she can enter into an agreement with one of the suppliers before the bids are placed that allows the supplier to produce the good at a cheaper quality in exchange for a bribe. We find that in our two-bidder setup, the form of auction matters. The agent will always ask the stronger firm to bribe under the second-score auction mechanism. However, while simulating the problem in case of the first-score auction mechanism, we find that the agent will approach the weaker firm to enter into a corrupt agreement.

In the above model, the bid of the bribing firm is manipulated such that the buyer is unaware of the presence of corruption in the procurement process. Then the buyer's preference between the first-score or the second-score auction under corruption would not alter and remain the same as that when the agent is honest. In other words, if the buyer prefers first-score over second-score auction when the agent is honest or absent, then the buyer would continue to do the same even when the agent is known to be corrupt with some probability.

In our model, if we allow for the agent to ask for a bribe after the bids have been submitted, the equilibrium bidding functions and the bribing mechanism is expected to no longer remain the same. The firms would then want to bid in such a way so as to increase their probability of being selected by the agent for corrupt arrangement. As for the agent, the decision to chose either strong or weak would disolve into when he or she then can either grant a *loser's bid revision*(Type II Corruption) or a *winner's bid revision*(Type I Corruption). In what we describe as a *loser's bid revision*, the corrupt auctioneer can allow the losing firm to win the auction and provide inferior quality, in exchange of a bribe. Under *winner's bid revision*, the winning firm can be allowed to provide inferior quality in exchange of a bribe. Another extension of the model would be to consider a buyer who knows that the procurement agent assigned to manage the auction is corrupt with some probability.

A Appendix

Proof of Lemma 1 :
Suppose for firm j with $\theta_j < \eta_s$, the equilibrium bid is (q_j, p_j) where $q_j \neq q_o(\theta_j)$. Then we show that this bid-tuple (q_j, p_j) is strictly dominated by another tuple (Q_j, P_j) where $Q_j = q_o(\theta_j)$ and $P_j = p_j + s(q_o(\theta_j)) - s(q_j)$. Note that $S(q_j, p_j) = S(Q_j, P_j)$ as $S(Q_j, P_j) = s(Q_j) - P_j = s(q_o(\theta_j)) - [p_j + s(q_o(\theta_j)) - s(q_j)] = s(q_j) - p_j = S(q_j, p_j)$

To show the strict dominance, we must show that $\pi(Q_j, P_j|\theta_j) > \pi(q_j, p_j|\theta_j)$.
Now,

$$\begin{aligned}\pi(Q_j, P_j|\theta_j) &= \{P_j - c(Q_j, \theta_j)\}.Prob\,[\,win\,|S(Q_j, P_j)]\\ &= \{p_j + s(q_o(\theta_j)) - s(q_j) - c(q_o(\theta_j), \theta_j)\}.Prob\,[\,win\,|S(q_j, p_j)]\\ &= \{p_j - c(q_j, \theta_j) + (s(q_o(\theta_j)) - c(q_o(\theta_j), \theta_j)) - (s(q_j) - c(q_j, \theta_j))\}.Prob[win|S(q_j, p_j)]\end{aligned}$$

Since $q_o(\theta_j) = argmax\, s(q_j) - c(q_j, \theta_j)$ and $q_j \neq q_o(\theta_j)$, $\quad s(q_o(\theta_j)) - c(q_o(\theta_j), \theta_j) > s(q_j) - c(q_j, \theta_j)$.

$\therefore\ \pi(Q_j, P_j|\theta_j) > \{p_j - c(q_j, \theta_j)\}.Prob\,[\,win\,|S(q_j, p_j)]$, which is nothing but equal to the $\pi(q_j, p_j|\theta_j)$,provided $Prob\,[\,win\,|S(q_j, p_j)] > 0$.

Claim: $Prob\,[\,win\,|S(q_j, p_j)] > 0$

To prove this claim, define $\underline{S} = inf\{S|Prob\,[\,win\,|S(q_j, p_j)] > 0\}$. Then $S_o(.)$ is a increasing function in θ_j. Also, $\underline{S} \leq S_o(\underline{\eta})$ for the trade to always take place. We shall prove the claim by contradiction. Suppose $\exists$ a $\theta_m > \underline{\eta}$ such that $Prob\,[\,win\,|S(q_m, p_m)] = 0$. Then the chosen score $S_m = S(q_m, p_m)$ must be such that $S_m \leq \underline{S}$. However, since $\underline{S} \leq S_o(\underline{\eta}) \leq S_o(\theta_m)$, firm with cost type θ_m can bid a score $S'_m \in (\underline{S}, S_o(\theta_m))$ which allows positive profits for that firm, thereby contradicting the optimality of the score choice S_m. Hence the claim.

So, with the proof of this claim, we have shown that $\pi(Q_j, P_j|\theta_j) > \pi(q_j, p_j|\theta_j)$ which contradicts the fact the bid-tuple (q_j, p_j) is the optimal choice for the firm j.

Proof of Claim 1 :
We prove the above claim by contradiction. Suppose $\beta_i(S_o(\underline{\eta})) \neq S_o(\underline{\eta})\ \forall\ i = s, w$. Then $\beta_i(S_o(\underline{\eta}))$ cannot be greater than $S_o(\underline{\eta})$, for the firm then will incur a loss if it wins the auction. Also, it is not a dominant strategy for the firm of the lowest type $\underline{\eta}$ to bid less than $S_o(\underline{\eta})$ since the probability of winning the auction reduces with the bid moving further away from the valuation. To elaborate further, suppose both the realized types are $\underline{\eta}$ and while $\beta_i(S_o(\underline{\eta})) < S_o(\underline{\eta})$, let us suppose $\beta_j(S_o(\underline{\eta})) = S_o(\underline{\eta}) \quad \forall i \neq j$. So, firm i will lose the auction even when it could have won, by bidding $\beta_i(S_o(\underline{\eta})) = S_o(\underline{\eta})$.
Moreover, if $\beta_s(S_o(\eta_s)) > \beta_w(S_o(\eta_w))$, then the strong bidder of type η_s would win with probability 1. However, it can increase its payoff by bidding slightly less than $\beta_s(S_o(\eta_s))$ and likewise, will get maximum benefit by bidding equal to $\beta_w(S_o(\eta_w))$.

Proof of Proposition 1 :
Substituting $S_o(\theta) = \theta$ and $\mathcal{H}(.) = F(.)$ in equation (1), we get

$$\begin{aligned} (\phi_i(b) - b) f_j(\phi_j(b))\phi_j'(b) &= F_j(\phi_j(b)) \qquad \forall\, i, j = s, w \\ \implies (\phi_i(b) - b)\frac{1}{\eta_j - \underline{\eta}}\phi_j'(b) &= \frac{\phi_j(b) - \underline{\eta}}{\eta_j - \underline{\eta}} \\ \implies (\phi_i(b) - b)\phi_j'(b) &= \phi_j(b) - \underline{\eta} \end{aligned} \tag{33}$$

which is equivalent to

$$(\phi_i(b) - b)(\phi_j'(b) - 1) = \phi_j(b) - \underline{\eta} - \phi_i(b) + b$$

Adding the two equations for $i, j = s, w$, we get

$$\frac{d}{db}(\phi_s(b) - b)(\phi_w(b) - b) = 2b - 2\underline{\eta} \tag{34}$$

Integrating this, we obtain

$$(\phi_s(b) - b)(\phi_w(b) - b) = b^2 - 2\underline{\eta}b + K \tag{35}$$

where K is the constant of integration. Substituting $\phi_s(\underline{\eta}) = \phi_w(\underline{\eta}) = \underline{\eta}$, we get $0 = \underline{\eta}^2 - 2\underline{\eta}^2 + K$ which implies that $K = \underline{\eta}^2$.
Therefore the above equation becomes,

$$(\phi_s(b) - b)(\phi_w(b) - b) = (b - \underline{\eta})^2 \tag{36}$$

Using Claim 1, we can calculate $\bar{b}$ by substituting $\phi_i(\bar{b}) = \eta_i \qquad \forall i = s, w$.

$$(\eta_s - \bar{b})(\eta_w - \bar{b}) = (\bar{b} - \underline{\eta})^2$$

which implies that,

$$\bar{b} = \frac{\eta_s\eta_w - \underline{\eta}^2}{\eta_s + \eta_w - 2\underline{\eta}} \tag{37}$$

Now, we can rewrite equation (33) as,

$$\phi_j'(b) = \frac{(\phi_j(b) - \underline{\eta})(\phi_j(b) - b)}{(b - \underline{\eta})^2} \tag{38}$$

We apply a change of variables by defining $(\phi_j(b) - \underline{\eta}) = \Phi_j(b)$ and $(b - \underline{\eta}) = B$. Therefore, the above equation reduces to,

$$\Phi_j'(B) = \frac{\Phi_j(B)(\Phi_j(B) - B)}{B^2} \tag{39}$$

Let $\Phi_j(B) - B = B\Gamma_j(B)$. Then

$$\Phi_j'(B) - 1 = \Gamma_j(B) + B\Gamma_j'(B)$$

Using this, equation (39) becomes,

$$\Gamma_j(B) + B\Gamma_j'(B) + 1 = \Gamma_j(B)(\Gamma_j(B) + 1) \tag{40}$$

or

$$\frac{\Gamma_j'(B)}{(\Gamma_j^2(B) - 1)} = \frac{1}{B}$$

Using integration by partial fractions, we obtain,

$$\Gamma_j(B) = \frac{1 + k_j B^2}{1 - k_j B^2} \tag{41}$$

where k_j is a constant of integration $\forall j = s, w$

Reverting back to the original variables,

$$\frac{\Phi_j(B)}{B} - 1 = \frac{1 + k_j B^2}{1 - k_j B^2}$$
$$\text{or} \qquad \Phi_j(B) = \frac{2B}{1 - k_j B^2} \tag{42}$$

$\therefore \qquad \forall j = s, w,$

$$\phi_j^{FS}(b) = \underline{\eta} + \frac{2(b - \underline{\eta})}{1 - k_j(b - \underline{\eta})^2} \tag{43}$$

Since $\phi_j(\bar{b}) = \eta_j$, where $\bar{b}$ is defined in (37), we obtain the constants of integration as

$$k_j = \frac{1}{(\eta_i - \underline{\eta})^2} - \frac{1}{(\eta_j - \underline{\eta})^2} \qquad \forall j = s, w \tag{44}$$

The bidding strategies, obtained by inverting (43) are,

$$\beta_j^{FS}(\theta) = \underline{\eta} + \frac{1}{k_j(\theta - \underline{\eta})}\left(-1 + \sqrt{1 + k_j(\theta - \underline{\eta})^2}\right) \qquad \forall j = s, w \tag{45}$$

Proof of Proposition 3:
To prove that, let $\beta_j = v_j$ be the equilibrium strategy for the firm j. What is the optimal response for firm i ?

Equation , then, can be rewritten as

$$\pi(\beta_i, v_i) = E\left[(v_i - \tilde{v_j}) I_{\beta_i > \tilde{v_j}}\right] \qquad \forall\ i, j = s, w \tag{46}$$

where $\tilde{v_j}$ is the observable v_j, a random variable ,since the actual pseudo-valuation of the firm i is unknown. Therefore,

$$\begin{aligned} \pi(\beta_i, v_i) &= \int_{S_o(\underline{\eta})}^{\beta_i} (v_i - x)\, h_j(x) dx \\ &= \int_{S_o(\underline{\eta})}^{\beta_i} (v_i - x)\, f_j(S_o^{-1}(x))(S_o^{-1}(x))' dx \end{aligned} \tag{47}$$

Firm i's problem is choose β_i so as to maximize equation (47). We show that β_i is neither greater than nor lesser than v_i.

Let $\boldsymbol{\beta_i < v_i}$
If β_i is increased to v_i, the change in the integral in equation (47) is

$$\Delta\pi = \int_{\beta_i}^{v_i} (v_i - x)\, f_j(S_o^{-1}(x))(S_o^{-1}(x))' dx$$

Since, $f_j(.)$ and $S_o^{-1}(.)$ are both increasing and $\beta_i < x < v_i$, we see that $\Delta\pi > 0$. Therefore, β_i , cannot be less than v_i.

Let $\boldsymbol{\beta_i > v_i}$
The difference in profit in that case is

$$\Delta\pi = \int_{v_i}^{\beta_i} (v_i - x)\, f_j(S_o^{-1}(x))(S_o^{-1}(x))' dx$$

which is negative for $v_i < x < \beta_i$. Firm i would deviate from its previous strategy $\beta_i > v_i$ to $\beta_i = v_i$.

Hence, the expected profit maximizing bids are

$$\beta_i^{SS} = v_i \qquad \forall i = s, w \tag{48}$$

This implies that,

$$s(q_o(\theta)) - p(\theta) = S_o(\theta) = s(q_o(\theta)) - c(q_o(\theta), \theta)$$
$$\therefore p(\theta) = c(q_o(\theta), \theta)$$

Therefore, in the second-score auction, each firm in equilibrium offers

$$\begin{aligned} q_i^{SS}(\theta) &= q_o(\theta) \\ p_i^{SS}(\theta) &= c(q_o(\theta), \theta) \qquad \forall i = s, w \end{aligned} \tag{49}$$

The expected winning offer in this second-score auction is

$$E(q)^{SS} = E\{q_o(\theta_1)\} \tag{50}$$
$$E(p)^{SS} = E\{s(q_o(\theta_1)) - s(q_o(\theta_2)) + c(q_o(\theta_2), \theta_2)\} \tag{51}$$

where $\theta_1 = Max(\theta_s, \theta_w)$ and $\theta_2 = Min(\theta_s, \theta_w)$.

Also, under Naive Scoring Rule, the buyer's expected utility in the second-score auction becomes

$$\begin{aligned} EU^{SS} &= E\{s(q_o(\theta_2)) - c(q_o(\theta_2)\theta_2) \\ &= E\{S_o(\theta_2)\} \end{aligned} \tag{52}$$

Let $s(q) = V(q) = 2\sqrt{q}$ and $c(q, \theta) = \dfrac{q}{\theta}$. Using these functional forms, we have $q_o(\theta) = \theta^2$, $S_o(\theta) = \theta$, $\mathcal{H}(.) = F(.)$ and therefore the bidding functions are

$$\beta_i^{SS}(\theta) = \theta \qquad \forall i = s, w \tag{53}$$

The expected winning offer is

$$E(q)^{SS} = E\{\theta_1^2\} \tag{54}$$
$$E(p)^{SS} = E\{2\theta_1 - \theta_2\} \tag{55}$$

where $\theta_1 = Max(\theta_s, \theta_w)$ and $\theta_2 = Min(\theta_s, \theta_w)$.

And the buyer's expected utility in this case is

$$EU^{SS} = E\{\theta_2\} \tag{56}$$

To find out the expected values, we first derive the probability distribution functions of θ_1 and θ_2. Let $G_1(.)$, $G_2(.)$ be the cumulative distribution functions of θ_1 and θ_2 respectively with $g_1(.)$, $g_2(.)$ being the corresponding density functions.

$$\begin{aligned} G_1(x) &= Prob[Max(\theta_s, \theta_w) < x] \\ &= Prob[\theta_s < x]Prob[\theta_w < x] \\ &= F_s(x)F_w(x) \\ &= \frac{(x-\underline{\eta})^2}{(\eta_s-\underline{\eta})(\eta_w-\underline{\eta})} \end{aligned}$$

$\therefore g_1(x) = \dfrac{2(x-\underline{\eta})}{(\eta_s-\underline{\eta})(\eta_w-\underline{\eta})}$. Since, $\theta_s \in [\underline{\eta}, \eta_s]$ and $\theta_w \in [\underline{\eta}, \eta_w]$, $\theta_1 = Max(\theta_s, \theta_w) \in [\underline{\eta}, \eta_s]$. Thus,

$$\begin{aligned} E(\theta_1) &= \int_{\underline{\eta}}^{\eta_s} x \frac{2(x-\underline{\eta})}{(\eta_s-\underline{\eta})(\eta_w-\underline{\eta})} dx \\ E(\theta_1^2) &= \int_{\underline{\eta}}^{\eta_s} x^2 \frac{2(x-\underline{\eta})}{(\eta_s-\underline{\eta})(\eta_w-\underline{\eta})} dx \end{aligned}$$

Upon calculating the integrals, we get

$$E(\theta_1) = \frac{(2\eta_s+\underline{\eta})(\eta_s-\underline{\eta})}{3(\eta_w-\underline{\eta})} \tag{57}$$

and

$$E(\theta_1^2) = \frac{(\eta_s-\underline{\eta})(3\eta_s^2+2\eta_s\underline{\eta}+\underline{\eta}^2)}{6(\eta_w-\underline{\eta})} \tag{58}$$

Similarly,

$$\begin{aligned} G_2(x) &= Prob[Min(\theta_s, \theta_w) < x] \\ &= Prob[\theta_s < x]Prob[\theta_w > x]Prob[\theta_s > x]Prob[\theta_w < x] \\ &= F_s(x) + F_w(x) - F_s(x)F_w(x) \\ &= \frac{(x-\underline{\eta})}{(\eta_s-\underline{\eta})} + \frac{(x-\underline{\eta})}{(\eta_w-\underline{\eta})} - \frac{(x-\underline{\eta})^2}{(\eta_s-\underline{\eta})(\eta_w-\underline{\eta})} \end{aligned}$$

So, $g_2(x) = \dfrac{1}{(\eta_s - \underline{\eta})} + \dfrac{1}{(\eta_w - \underline{\eta})} - \dfrac{2(x-\underline{\eta})}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})}$ and $\theta_2 = Min(\theta_s, \theta_w) \in [\underline{\eta}, \eta_w]$. Thus,

$$E(\theta_2) = \int_{\underline{\eta}}^{\eta_w} x \left[\frac{1}{(\eta_s - \underline{\eta})} + \frac{1}{(\eta_w - \underline{\eta})} - \frac{2(x-\underline{\eta})}{(\eta_s - \underline{\eta})(\eta_w - \underline{\eta})} \right] dx$$

which when solved gives,

$$E(\theta_2) = \frac{1}{6(\eta_s - \underline{\eta})} \left[(3\eta_s - \eta_w)(\eta_w + \underline{\eta}) - 4\underline{\eta}^2 \right] \tag{59}$$

Notes

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