Competition, Integration and Innovation

Tapas Kundu[*] Seongwuk Moon[†]

May 27, 2014

Please do not cite or circulate without authors' persmission

Abstract

We study how innovation contest affects organization of R&D activities in a model of vertical integration. In our framework, there are two upstream research units and two downstream customers. The research units have skill necessary to make an innovation but the customers have ability to commercialize an innovation. We adopt an incomplete contract approach in which neither effort nor the value of an innovation is contractible. A customer either (i) integrates with a research unit by buying ownership rights of an innovation before an innovation is realized, or (ii) remains non-integrated and bargains with research units over licensing fee after an innovation is realized. We model upstream competition in the form of an innovation contest. We find that the integrated and non-integrated R&D may coexist (semi integration) in equilibrium. Integration dampens integrated research unit's effort but can create positive externality on industry-wide innovative effort. When the effect of this positive externality is sufficiently strong, semi integration is more likely. Interestingly, an equilibrium market arrangement is not always efficient as those who gain from the positive externality cannot commit to compensate those who lose at the pre-innovation contracting stage.

JEL Code: L22; O31; O32

Keywords: R&D contest; Vertical integration; Innovation

[*]School of Business and Economics, University of Tromsø. Email: tapas.kundu@uit.no. Tapas Kundu is also associated with the ESOP Centre at the University of Oslo. ESOP is supported by the Research Council of Norway through its Centres of Excellence funding scheme.

[†]Sogang University Graduate School of Management of Technology. Email: seongwuk@sogang.ac.kr.

1 Introduction

What determines the boundary of a firm in the market for innovation? The innovative product market firms often obtain R&D based inputs from external sources or internal sources or both (Arora and Gambardella [2010]). At an industry level, we see firms doing in-house R&D coexist with firms buying R&D results from specialized entities. The pharmaceutical sector, in which the contract R&D has been most active, exhibits coexistence of external and internal R&D. In 2002, the top twenty largest pharmaceutical companies bought 43 percent of their drug development candidate from outside sources such as research oriented biotech companies, universities and other pharmaceutical companies and the remaining 57 percent based on internal research (Pisano [2006]).[1] How do firms decide whether to *make or buy* innovation?

The literature on firm boundary mostly explains integration or separation decisions based on transaction cost (TC hereafter) argument or property rights (PR hereafter) arguments (Lafontaine and Slade [2007]). The findings from TC models suggest that integration is more likely in situations in which transactions are complex and involve asset specific investments on assets with non verifiable quality, and in environments with high uncertainty (Mowery [1995]).[2] On the other hand, the PR models shows how ex post bargaining may adversely affect ex ante investment in non contractible asset. Integration is more likely if the manufacturer's marginal productivity of investment is stronger than the input supplier's marginal productivity of investment (Aghion and Tirole [1994]).[3]

While these models provide many important insights to understand reasons for integration or separation, the arguments are often restricted to factors that are within the decision-making domain of the firm and the supplier in context. Specifically, a firm's and its input supplier's decisions to integrate do not depend on the existence or non-existence of other firms or suppliers sharing a common environment. Unlike these theories, we, in this article, argue that the nature of R&D competition among input suppliers can explain why we may see coexistence of integrated and non integrated R&D in research based industries.

Specifically, we model R&D competition in the form of an innovation contest. As a successful innovation typically provides the innovator with a significant rent in research based industries, contest provides an ideal framework to model R&D competition.[4] By modeling competition for innovation as a contest we show that integration can have a

[1] In-house R&D also generated highly innovative drugs as collaboration with external R&D entities did: Examining 4,057 pharmaceutical projects by forty largest pharmaceutical companies, Guedj [2005] show that the novelty of drugs from in-house R&D was not statistically different from alliance.

[2] Transaction costs are in general any cost of establishing business relationship between agents, specifically associated with opportunistic behavior. The TC theory can be traced back to Coase [1937], but developed substantively later by Williamson [1971, 1975, 1979], Klein, Crawford and Alchian [1978] and others.

[3] Property rights confer the rights to make decisions concerning the use of an asset in situations not necessarily specified in contract. The PR theory can also be traced back to Coase [1937], but more formally developed by Grossman and Hart [1986], Hart and Moore [1990]; Hart [1995] and others. Whinston [2003] shows that the findings from PR approach can be substantially different from the findings of TC theories. Lafontaine and Slade [2007] provide a comprehensive survey of the empirical literature on both theories.

[4] See Konrad [2009] for a comprehensive analysis of contest frameworks.

positive externality on the rent seeking effort level of a non-integrated research unit: A non-integrated research unit exerts higher effort to win the innovation contest when its competitor is integrated. The effect of this positive externality is also partially extended to all the customers as they benefit from the higher effort. If the benefit is sufficiently high, an integrated arrangement survives in equilibrium.

In our model, two research based input suppliers ("research units") compete to generate and sell a new useful idea (innovation) and two firms want to buy the idea in order to commercialize the innovation in market. Before those research units make an innovation, firms try to buy the ownership of any forthcoming innovation (integration).[5] If a research unit refuses to sell ownership right of a forthcoming innovation, it works independently. If a research unit agrees to integrate, it works as an in-house department of the owner firm. In either case a research unit competes with the other research unit to make innovation in an innovation contest. If an independent research unit wins the contest, it bargains with the firms over the innovation price. If an integrated research unit wins, the integrating firm has two choices − It commercializes the innovation by itself or it can bargain with the other firm over the licensing fee. Although the integrating firm can elicit inefficient incentive of internal research unit, integration gives it an option to be a seller of innovation in the post innovation stage.

In equilibrium we can see either coexistence of integrated and non integrated R&D (semi integration), or only the non-integrated R&D activities (no integration). Integration reduces an integrated research unit's motivation to incur non-verifiable effort cost. Thus an integrated customer is adversely affected as the integrated research unit puts less effort. Integration however has a positive externality on the rent seeking effort level of a non-integrated research unit. An increase in non-integrated research unit's effort level also increases the payoff of all customers when the aggregate innovation probability is higher in semi integration than in no integration. If the increase in customer's payoff is sufficiently high, an integrated customer is able to compensate its loss due to the reduced effort of its own integrated research unit. Thus a semi integration arrangement survives in equilibrium. As all agents are assumed risk neutral, it is worth noting that a research unit's decision to integrate is not to avoid uncertainty of innovation,

We find that a full integration and a semi integration arrangement are always socially inefficient. The efficiency of a non-integrated arrangement can however be ambiguous. Specifically, a non-integrated industry structure produces socially wasteful effort if innovations have high expected value or if the customers have low bargaining power. An equilibrium R&D structure may not necessarily be more socially valuable compared to an alternate R&D structure. In our framework, those who benefit from an integration cannot necessarily commit to compensate those who lose, in any credible way. Thus the aggregate social value of an innovation is not necessarily maximized in equilibrium.

We also make a technical contribution in the literature on innovation contests. The contest success function in our model has a game theoretic foundation. The function is derived from an underlying environment in which an innovation is an uncertain event and

[5] Precisely, our notion of integration resembles what is called backward integration in the vertical integration literature (Lafontaine and Slade [2007]). In backward integration, the manufacturers decide whether to "make or buy" the input .

players exert effort strategically. The derived contest success function is multiplicatively separable in efforts, which makes the derivation of marginal effect of effort on contest success probabilities and payoffs easy and tractable. The framework is particularly useful in modeling contests in which efforts are productive and the value of the contest prize is uncertain.

1.1 Related Literature

[To be added]

The paper is organized as follows. In Section 2, we develop our model in three stages: Pre-innovation contract, innovation contest and post-innovation bargaining. Section 3 analyzes the basic model. In Section 4, we discuss various implications of our analysis. Section 5 concludes. The Appendix contains proofs that are omitted in the main text.

2 The model

We consider a game with four players - Two upstream research units, RU_1 and RU_2, and two downstream customers, C_1 and C_2. Customers derive value from commercialization of an innovation. The research units perform research that is necessary to realize an innovation. The customers, on the other hand, are only able to commercialize an innovation. The game proceeds in the following three stages - Pre-innovation contracting of ownership right, innovation contest and post-innovation bargaining.

2.1 Pre-innovation contracting

We consider an incomplete contract framework, similar to Grossman and Hart [1986]. In particular, we assume that the exact nature of innovation is unknown at the contracting stage, and therefore the realized value of an innovation is non-contractible. In addition, we also assume that research effort is non-contractible. The contract can only specify allocation of the ownership right of any forthcoming innovation.

At the beginning of the game, C_1 and C_2 simultaneously offer prices p_1 and p_2, to buy ownership right of any forthcoming innovation. The two research units observe prices and decide whether or not to sell ownership rights. When a research unit sells the ownership right of its potential innovation, it receives no further reward when its research may result in realization of an actual innovation. We call this case as a case of integration, and the corresponding customer-research unit pair is referred to as integrated. We assume that a customer (or a research unit) can only be integrated with one research unit (or one customer). If a customer (or a research unit) is not integrated, we will call it a non-integrated customer (or a non-integrated research unit).

A research unit's decision to integrate depends on the price that it would receive in return. As the customers offer prices simultaneously, we need to be specific about the mechanism by which a research unit is matched with a customer in an integrated arrangement. We assume the following. For a given price profile, if both research units are willing to integrate at the highest offered price, then one of the research unit is randomly matched with the customer offering the highest price. If the other research unit is willing to sell the ownership right at the second highest price, it will only then be matched with the other customer offering the second highest price (the minimum price in our model with two customers). If no research unit is willing to integrate at the highest offered price, then there will be no integration. If only one research unit is willing to integrate at the highest offered price, then that research unit is integrated with the customer offering the highest price, and the other research unit remains non-integrated. If two customers offer the same price, we randomly choose one customer as the one offering the highest price, and follow the above mentioned matching procedure.[6]

2.2 Innovation contest

Next, we move to the stage when research units perform research by exerting costly effort. Effort determines the probability of making an innovation. The minimum effort is normalized to zero. The probability that a research unit makes an innovation given an effort $e \in [0,1]$ is $q(e)$, where $q(e) \in [0,1]$ is an increasing function of e. Effort is costly, and its cost is given by an increasing function $c(e)$ with $c(0) = 0$. Additional assumptions will be needed to support the first order approach in various scenarios. We defer discussion of those to Section 3.2.

To capture the competition aspect between two research units, we introduce an innovation contest. We assume that time is fixed at the interval $[0,1]$. At the beginning of time, both research units simultaneously incur effort cost. Effort cost is sunk, and cannot be altered once the contest begins. We can interpret effort as a fixed investment such as building of research environment, hiring of research employees etc., that a research unit incurs at the beginning of the contest. A research unit wins the contest if it comes up with an innovation ahead of its competitor in the fixed time interval $[0,1]$. Let x_i denote the time that RU_i takes to make an innovation. We assume that x_i follows a uniform distribution over the time interval $\left[0, \frac{1}{q(e_i)}\right]$, so that the probability that RU_i makes an innovation within the time interval $[0,1]$ is exactly $q(e_i)$.

The winning probability of a research unit however differs from the individual success probability. In particular, for a given effort profile $\underline{e} = (e_1, e_2)$ where e_i denotes the effort level of RU_i, the winning probability of RU_i is given by $\pi_i(\underline{e}) = Pr\left[x_i = min\left\{x_1, x_2\right\} \leq 1\right]$ where $x_1 \sim Uniform\left[0, \frac{1}{q(e_1)}\right]$, and $x_2 \sim Uniform\left[0, \frac{1}{q(e_2)}\right]$. We can rewrite the winning probabilities in the following simpler forms.

$$\pi_1(\underline{e}) = Pr\left[x_1 = min\left\{x_1, x_2\right\} \leq 1\right] = \int_0^1 q(e_1)(1 - tq(e_2))\,dt = q(e_1)(1 - \frac{q(e_2)}{2}), \quad (1)$$

[6] We assume that the customer offering the second highest price does not renegotiate its price offer after one of research unit is integrated with the customer offering the highest price. We make this assumption to keep our analysis simple. The assumption however does not affect our results in any significant way. This is because in our model, when the customer offering the highest price gets integrated, the other customer does not gain any additional advantage in dealing with the non integrated research unit as a non-integrated research unit has an option to sell its innovation to an integrated customer.

and,

$$\pi_2(\underline{e}) = q(e_2)\left(1 - \frac{q(e_1)}{2}\right). \tag{2}$$

Note that $\pi_1(e_1, e_2) = \pi_2(e_2, e_1)$. It is worth noting that the sum of the winning probabilities is the same as the probability of realizing an innovation. We denote the innovation probability for a given effort profile $\underline{e}$ by $\pi_{inv}(\underline{e})$. We have, $\pi_{inv}(\underline{e}) = 1 - (1 - q(e_1))(1 - q(e_2)) = \pi_1(\underline{e}) + \pi_2(\underline{e})$.

If no research unit innovates within the time interval $[0, 1]$, the contest ends at time 1 with no innovation. Otherwise, the contest ends at the time when a research unit comes up with an innovation ahead of its competitor.

2.3 Post-innovation bargaining

At the end of innovation contest, we move to the post-innovation bargaining stage. Customers are the only users of innovation, and are therefore indispensable for realization of the innovation value. Once a research unit wins the innovation contest, it gets an exclusive right to use that innovation for the current period. This implies that if a non-integrated research unit wins the contest, it has the ownership right of that innovation and can bargain with the customers over a licensing fee. On the other hand, if an integrated research unit wins the contest, the corresponding integrated customer has the ownership right of that innovation, and can either commercialize that innovation or can bargain with the other customer over a licensing fee.

We model the post innovation bargaining in reduced form. Two customers realize their innovation values, once the innovation contest ends with a specific innovation. The customer C_i's innovation value is denoted by v_i, which can either be h with probability θ or l with probability $1 - \theta$, with $h > l > 0$. Let $v_{\max}$ and $v_{\min}$ denote the maximum and the minimum of two valuations respectively. For simplicity, we consider symmetric Nash Bargaining payoff. Specifically, when a seller (either a non-integrated research unit or an integrated customer) whose reservation value of the innovation is r_s, bargains with a buyer (a customer) whose value of the innovation is r_b, the additional value $(r_b - r_s)$ will be equally split between the seller and the buyer. Therefore, the payoff of the buyer and that of the seller are $\frac{r_b - r_s}{2}$ and $r_s + \frac{r_b - r_s}{2} = \frac{r_b + r_s}{2}$ respectively.[7]

If an integrated customer sells the commercialization right to another customer (such a possibility may arise if the corresponding integrated research unit wins the innovation contest), the integrated customer's reservation value is its own valuation of the innovation. On the other hand, if a non-integrated research unit sells the commercialization right to one customer, its reservation value will be the innovation value realized by the other customer.

[7] Note that our assumption of equal split of the additional rent is common in literature (see Aghion and Tirole [1994]). The assumption does not affect the results in any significant way in the class of models where utility in the ex post bargaining game is transferable. See Rubinstein (1982) for a micro-foundation of an equal split outcome in bargaining games.

2.4 Payoffs

We assume that all players are risk neutral.

The ex post payoff of RU_i is given by

$$U_i^{RU} = \begin{cases} p - c(e_i) & \text{if } RU_i \text{ is integrated} \\ \frac{v_{max}+v_{min}}{2} - c(e_i) & \text{if } RU_i \text{ is non-integrated and wins the contest} \\ -c(e_i) & \text{if } RU_i \text{ is non-integrated and does not win the contest} \end{cases},$$

where p is the price at which RU_i sells the innovation property right to its customer if it is integrated and e_i is the effort level of RU_i in the contest.

The ex post payoff of the customer C_i is

$$U_i^{C} = \begin{cases} \frac{v_i-v_{min}}{2} & \text{if } C_i \text{ is non-integrated} \\ \frac{v_{max}+v_i}{2} - p & \text{if } C_i \text{ is integrated with some } RU_j \text{that wins the contest} \\ \frac{v_i-v_{min}}{2} - p & \text{if } C_i \text{ is integrated with some } RU_j \text{that does not win the contest} \end{cases},$$

where p is the price at which C_i buys the innovation property rights from the research unit with which it is integrated and v_i is C_i's realized innovation value.

2.5 Strategies and solution concept

We focus only on pure strategies due to their analytical tractability. Customers offer prices to buy ownership rights during the pre-innovation contracting stage. Prices are offered simultaneously, and C_i's strategy is to choose p_i. The research units take two types of action. They decide whether to integrate with customer and they exert effort at the beginning of the innovation contest. The research units decide effort simultaneously. A research unit's effort strategy is contingent on prices offered and the integration decisions made in the pre-innovation contracting stage. A research unit decides whether to integrate on two occasions. First, observing a price profile, a research unit decides whether to integrate with the customer offering the highest price. Therefore the first integration strategy is contingent on the highest offered price. Second, in the event when a research unit is willing to integrate at the highest price but is not integrated (as the other research unit gets integrated with the customer), it decides whether to integrate with the customer offering the second highest price. Therefore, the second integration strategy is contingent on the second highest price, but is conditional on the fact that an integration has already happened between the other research unit and the customer offering the highest price.

We consider the subgame perfect Nash Equilibrium in pure strategies as the solution concept.

3 Equilibrium analysis

We solve the game by backward induction.

3.1 Post innovation bargaining

We will study the expected payoff of the four players at the beginning of the post-innovation stage. At this point, it is worthwhile to introduce two simplifying notations. We denote $E\left(\frac{v_{max}+v_{min}}{2}\right)$ and $E\left(\frac{v_{max}-v_{min}}{2}\right)$ by $\overline{v}$ and $\underline{v}$ respectively. It can be shown (see equation (3)) that with two customers, $\overline{v}$ also represents the expected value of an innovation to a customer. We have

$$\bar{v} = E\left(\frac{v_{max}+v_{min}}{2}\right) = \theta^2 h + (1-\theta)^2 l + 2\theta(1-\theta)\left(\frac{h+l}{2}\right) = \theta h + (1-\theta) l, \quad (3)$$
$$\underline{v} = E\left(\frac{v_{max}-v_{min}}{2}\right) = \theta^2 \cdot 0 + (1-\theta)^2 \cdot 0 + 2\theta(1-\theta)\left(\frac{h-l}{2}\right) = \theta(1-\theta)(h-l).$$

and

$$E(v_{max}) = \overline{v} + \underline{v},\ E(v_{min}) = \overline{v} - \underline{v}.$$

Depending on the outcome of the innovation contest, we can have three different cases at the beginning of the post innovation bargaining stage: (i) a non-integrated research unit wins the innovation contest, (ii) an integrated research unit wins the innovation contest, and (iii) the innovation contest results in no successful innovation.

First, consider the case in which a non-integrated research unit wins the innovation contest. The winning research unit can bargain with the customer who has the maximum valuation, and its reservation value is the second highest realized valuation. As there are only two customers, the winning research unit's expected payoff is given by $E\left(\frac{v_{max}+v_{min}}{2}\right) = \bar{v}$. The losing research unit will have zero payoff at this stage. Each customer will have an expected payoff of $\frac{1}{2}E\left(\frac{v_{max}-v_{min}}{2}\right) = \frac{\underline{v}}{2}$. It is worth noting that the aggregate expected payoff of all the four players is the same as the expected value of v_{max}. Thus, the bargaining process does not generate any inefficiency at this stage.

Second, we consider the case in which an integrated research unit wins the innovation contest. The corresponding integrated customer owns the innovation. The customer can either commercialize the innovation or bargain with the other customer if the other customer has higher valuation. The expected payoff of the winning customer (who is integrated with the winning research unit) is therefore given by $E\left(\frac{v_{max}+v_i}{2}\right) = \overline{v} + \frac{\underline{v}}{2}$. The expected payoff of the losing customer (who is not integrated with the winning research unit) is given by $E\left(\frac{v_i-v_{min}}{2}\right) = \frac{\underline{v}}{2}$. Each of the two research units will have zero expected payoff. As before, we can see that the aggregate expected payoff is the same as the expected value of v_{max}.

It is worth noting that the bargaining process does not generate any inefficiency at the post innovation stage. This is because we allow trading of the commercialization right between customers. If we had not allowed trading, we could have come across an ex post inefficient situation in which an innovation is not commercialized at the maximum possible value.

Finally, in the third case in which an innovation contest leads to no successful innovation, the post innovation bargaining is trivially resolved with each player having zero expected payoff at the post innovation stage.

3.2 Innovation contest

We now solve for the optimal effort levels in the innovation contest.

We will assume that the success function $q(e)$ and the cost function $c(e)$ are such that there exists a unique solution of a research unit's payoff maximization problem and the solution lies in the open interval $(0,1)$. Our first assumption below is sufficient (though not necessary) to ensure that the unique solution of a research unit's payoff maximization problem in various scenarios can be found by solving the first order condition. Formally, we assume:

Assumption 1. $\overline{v}q(e) - c(e)$ is strictly concave in e.

Assumption 1 is typically satisfied as long as the cost function $c(e)$ is sufficiently convex compared the success function $q(e)$. Our second assumption is sufficient to ensure that the solution of a research unit's payoff maximization problem in various scenarios lies in the open interval $(0,1)$. Formally, we assume:

Assumption 2. $\overline{v}q'(1) - c'(1) < 0 < \frac{\overline{v}}{2}q'(0) - c'(0)$.

In the remainder of our paper, we assume that both Assumptions 1 and 2 hold true unless explicitly stated.[8]

Depending on the outcome of the pre-innovation contracting stage, we can have three different industry structures at the beginning of the innovation contest: (i) both research units are integrated, (ii) one of the research unit integrates while the other does not, and (iii) no research unit integrates. We call these three structures as *full integration*, *semi integration* and *no integration* respectively.

First, consider the case of full integration. As the integrated research units get zero payoff at the post innovation stage, they will exert no effort at the contest. By (1) and (2), the winning probability of the two research units are identical and it is given by $q(0)\left(1 - \frac{q(0)}{2}\right)$.

Next, consider the case of semi integration. At this stage, without loss of generality, we can assume that RU_2 is integrated to C_2, and RU_1 is not integrated. RU_2 will therefore exert no effort as it gets zero payoff at the post innovation stage. From (1) and (3), we see that RU_1's expected payoff, given an effort level e_1, is $\overline{v}q(e_1)\left(1 - \frac{q(0)}{2}\right) - c(e_1)$. The optimal effort of RU_1, denoted by e^{SI}, therefore satisfies the following first order condition:

$$\overline{v}\left(1 - \frac{q(0)}{2}\right)q'\left(e^{SI}\right) - c'\left(e^{SI}\right) = 0. \tag{4}$$

Finally, consider the case of no integration. In a symmetric Nash equilibrium, the

[8] Assumptions 1 and 2 do not affect our results in any significant way. If we relax Assumption 1, we will have to deal with multiple solutions and subsequently, with an equilibrium selection problem. If we relax Assumption 2, we will have boundary solution, which makes the solution insensitive to changes in the parameter values to some extent.

optimal effort levels of both research units, denoted by e^{NI}, solves the following condition:

$$e^{NI} = \underset{e\in[0,1]}{argmax}\ \overline{v}q\left(e\right)\left(1-\frac{q\left(e^{NI}\right)}{2}\right)-c\left(e\right).$$

From the first order condition, we get that e^{NI} satisfies

$$\overline{v}\left(1-\frac{q\left(e^{NI}\right)}{2}\right)q'\left(e^{NI}\right)-c'\left(e^{NI}\right) \quad = \quad 0. \tag{5}$$

Under Assumptions 1 and 2, a comparison of the effort levels shows that $e^{SI} > e^{NI} > 0$. Thus, integration though dampens the integrated research unit's incentive to exert effort, it creates a positive externality to the other non-integrated research unit's choice of effort. The following lemma formally proves this observation.

Lemma 1. $e^{SI} > e^{NI} > 0$.

Proof. First note that $e^{NI} > 0$ by Assumption 2. Denote $\left(1-\frac{q(0)}{2}\right)$ and $\left(1-\frac{q\left(e^{NI}\right)}{2}\right)$ by A and B respectively. We have $A > B$ as $e^{NI} > 0$. Note that e^{NI} solves $\overline{v}Bq'\left(e^{NI}\right)-c'\left(e^{NI}\right)=0$. As $A > B$ and $q'\left(e^{NI}\right) > 0$, we must have $\overline{v}Aq'\left(e^{NI}\right)-c'\left(e^{NI}\right) > 0$. Further note that e^{SI} solves $\overline{v}Aq'\left(e^{SI}\right)-c'\left(e^{SI}\right)=0$. By Assumption 1, $\overline{v}Aq\left(e\right)-c\left(e\right)$ is also strictly concave and therefore, we must have $e^{NI} < e^{SI}$. □

Below we provide an example with a specific form of linear success function $q\left(e\right)$.

Example 1. Consider the following parameter values: $h = 3$, $l = 1$, $\theta = 0.5$. We therefore have $\overline{v} = 2$ and $\underline{v} = 0.5$. We consider $c\left(e\right) = e^2$. We assume that $q\left(e\right) = \frac{1-\alpha}{4} + \frac{3+\alpha}{4}e$ for $\alpha \in [0,1]$. The parameter α is positively related to the level at which non-verifiable effort affects the possibility of realizing an innovation. In Figure 1, we consider $\alpha = 0.8$. The two straight lines plot the best response functions of the two research units in the (e_1, e_2) space (the flatter one corresponds to RU_2's best response for a given choice of e_1). The response functions intersect each other at the optimal effort level of a non-integrated research unit in no integration ($e^{NI} = 0.638$). The point of intersection of the response function of RU_2 (the response function of RU_1) and the vertical axis (the horizontal axis) is the optimal effort level of a non-integrated research unit in semi integration ($e^{SI} = 0.926$). The dotted curves present the choices of e_1 and e_2 at which the innovation probability $\pi_{inv}\left(e_1, e_2\right)$ is constant. For $\alpha = 0.8$, we have $\pi_{inv}\left(e^{NI}, e^{NI}\right) = 0.882$ and $\pi_{inv}\left(e^{SI}, 0\right) = 0.933$. In Figure 2, we consider $\alpha = 0.4$ and we plot the response functions. For $\alpha = 0.4$, we have $e^{NI} = 0.578$, $e^{SI} = 0.786$, $\pi_{inv}\left(e^{NI}, e^{NI}\right) = 0.871$ and $\pi_{inv}\left(e^{SI}, 0\right) = 0.846$.

As illustrated in Example 1, the effect of integration on the innovation probability can be ambiguous. Let us denote the innovation probability, computed at the optimal effort

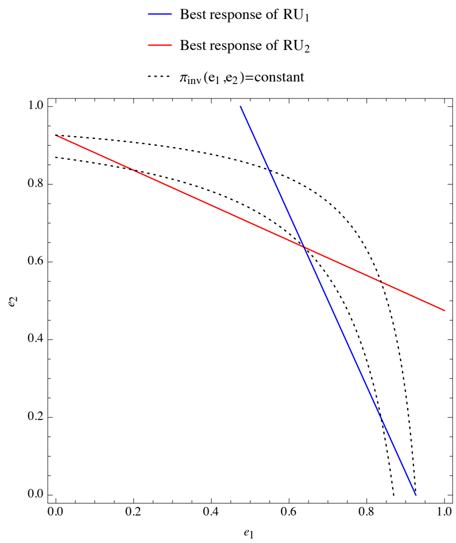

Figure 1: Response functions ($\alpha = 0.8$)

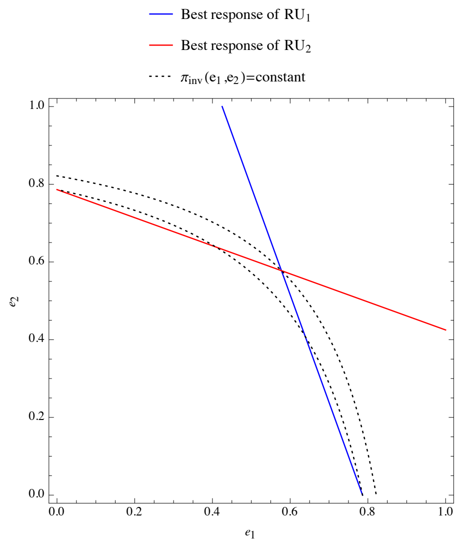

Figure 2: Response functions ($\alpha = 0.4$)

profile, in cases of no integration, semi integration and full integration by π_{inv}^{FI}, π_{inv}^{SI} and π_{inv}^{NI} respectively. We have

$$\begin{aligned}
\pi_{inv}^{FI} &= \pi_{inv}(0,0) = 1 - (1 - q(0))^2, \\
\pi_{inv}^{SI} &= \pi_{inv}\left(e^{SI}, 0\right) = 1 - (1 - q(0))\left(1 - q\left(e^{SI}\right)\right), \\
\pi_{inv}^{NI} &= \pi_{inv}\left(e^{NI}, e^{NI}\right) = 1 - \left(1 - q\left(e^{NI}\right)\right)^2.
\end{aligned} \tag{6}$$

It turns out that the innovation probability is the least in case of full integration, i.e., $\pi_{inv}^{FI} \le \min\left\{\pi_{inv}^{SI}, \pi_{inv}^{NI}\right\}$. The comparison between π_{inv}^{SI} and π_{inv}^{NI} is ambiguous. The following lemma documents this observation.

Lemma 2. *$\pi_{inv}^{NI} \le \pi_{inv}^{SI}$ if and only if $\frac{\left(1-q\left(e^{NI}\right)\right)^2}{(1-q(0))(1-q(e^{SI}))} \le 1$. Further, $\pi_{inv}^{FI} \le \min\left\{\pi_{inv}^{SI}, \pi_{inv}^{NI}\right\}$.*

The proof follows by comparing π_{inv}^{FI}, π_{inv}^{SI} and π_{inv}^{NI} and by the fact that $e^{SI} > e^{NI} > 0$. In the following example, we compare the innovation probabilities for a particular class of linear success function.

Example 2. We continue with the same parameter specification considered in Example 1. We assume that $q(e) = \frac{1-\alpha}{4} + \frac{3+\alpha}{4}e$ for $\alpha \in [0,1]$. Figure 3 plots the innovation probabilities π_{inv}^{FI}, π_{inv}^{SI} and π_{inv}^{NI} as functions of α. A non-integrated research unit's effort is always higher in semi integration. For high values of α, the innovation probability is higher in semi integration than in no integration.

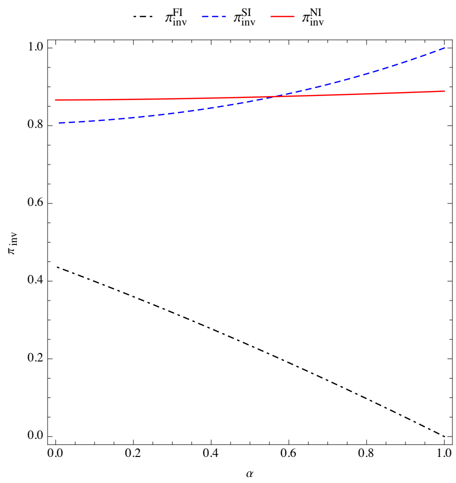

Figure 3: Innovation probability

3.3 Pre-innovation contracting

We now consider the pre-innovation contracting stage. At this stage, the customers simultaneously offer prices to buy ownership rights. After observing a price profile (p_1, p_2), the research units decide whether or not to integrate. Recall that integration occurs if at least one research unit is willing to integrate at the highest offered price. If both research units are willing to integrate at the highest offered price, then one of them is randomly assigned to the customer offering the highest price and the other research unit decides whether to integrate at the second highest price.

Depending on the integration decision, three different industry structures may arise in equilibrium. Table 3.3 presents payoffs of the research units and the customers in different structures. Without loss of generality, we consider that RU_2 and C_2 are integrated in semi integration. Note that as between-customers trading is allowed in the post innovation stage and customer's realized valuations are independently drawn, both customers would have an expected payoff of at least $\frac{\underline{v}}{2}$ times the innovation probability. However, by integrating, the integrated customer gets a premium, which equals $\overline{v}$ times the probability that its integrated research units wins the contest, and it pays a price. In the following subsections, we analyze equilibrium possibilities in various structures.

Table 3.3: Payoff in different market structures

	Full integration		No integration		Semi integration	
	1	2	1	2	1	2
RU	p	p	$(\overline{v}/2)\,\pi_{inv}^{NI} - c\left(e^{NI}\right)$	$(\overline{v}/2)\,\pi_{inv}^{NI} - c\left(e^{NI}\right)$	$\overline{v}\pi_1\left(e^{SI},0\right) - c\left(e^{SI}\right)$	p
C	$(\overline{v}/2)\,\pi_{inv}^{FI} + (\underline{v}/2)\,\pi_{inv}^{FI} - p$	$(\overline{v}/2)\,\pi_{inv}^{FI} + (\underline{v}/2)\,\pi_{inv}^{FI} - p$	$(\underline{v}/2)\,\pi_{inv}^{NI}$	$(\underline{v}/2)\,\pi_{inv}^{NI}$	$(\underline{v}/2)\,\pi_{inv}^{SI}$	$\overline{v}\pi_2\left(e^{SI},0\right) + (\underline{v}/2)\,\pi_{inv}^{SI} - p$

3.3.1 Equilibrium with full integration

First consider the case of full integration. Note that if an equilibrium with full integration exists, then both customers must offer the same price in equilibrium. It is because the customer offering the higher price will have a strict incentive to decrease the offered price, without affecting its chance to be integrated with any of the research units. Let us denote the common price, offered by C_1 and C_2, as p. In a full integration arrangement, a research unit has an incentive to integrate if the price p is above its opportunity cost of integration, which is given by $\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right)$. On the other hand, the customer will offer a price p (weakly) below the relative benefit from integration (derived in the proof of Lemma 3), which is given by $\overline{v}\pi_1\left(0,0\right) + \frac{v}{2}\pi^{FI}_{inv} - \frac{v}{2}\pi^{SI}_{inv}$. Therefore an equilibrium with full integration exists for some price p if and only if

$$\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right) \quad \leq \quad \overline{v}\pi_2\left(0,0\right) - \frac{v}{2}\left(\pi^{SI}_{inv} - \pi^{FI}_{inv}\right) \tag{7}$$

In this case, the optimal price p coincides with the lower bound of (7) as prices are offered before integration decisions are made. The following lemma documents the finding.

Lemma 3. *An equilibrium with full integration exists if and only if condition (7) holds true.*

A formal proof is given in the appendix. The following proposition shows that the condition required for the existence of an equilibrium with full integration cannot be satisfied for any parameter values. This is because when one of the research units is integrated, the other research unit's scope of rent-seeking from non-integration is high. Subsequently, its opportunity cost of integration is high. The customer's premium from integration in a fully integrated industry structure is too low to compensate the high opportunity cost.

Proposition 1. *There is no competitive equilibrium with full integration.*

Proof. The inequality (7) can be rewritten as

$$\frac{v}{2}\left(\pi^{SI}_{inv} - \pi^{FI}_{inv}\right) \leq \overline{v}\pi_2\left(0,0\right) - \left(\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right)\right).$$

The left hand side is always positive as $\pi^{SI}_{inv} > \pi^{FI}_{inv}$. But the right hand side is always negative as $\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right) > \overline{v}\pi_2\left(0,0\right) - c\left(0\right) = \overline{v}\pi_2\left(0,0\right)$. Hence, (7) cannot be satisfied. □

The mechanism behind this result is as follows. Note that the contracted price is simply a transfer between a research unit and a customer. Therefore, in any equilibrium, a customer-research unit pair must be able to maximize the joint payoff. In the full integration case, a customer-research unit pair gets a joint payoff of $\frac{\overline{v}+v}{2}\pi^{FI}_{inv}$. If they do not agree, they can deviate to a semi integration arrangement, which provides this pair a joint payoff of $\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right) + \frac{v}{2}\pi^{SI}_{inv}$. The joint payoff from deviation is always higher than the joint payoff in full integration as the non integrated research unit optimally chooses its effort to maximize payoff in semi integration.

3.3.2 Equilibrium with no integration

Next we consider the case of no integration. Note that if an equilibrium with no integration exists, then both research units are not willing to integrate at the maximum offered price. A research unit's payoff in this equilibrium is given by $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$. A research unit prefers not to integrate if the maximum price is less than this payoff. On the other hand, a customer will not offer a price above its relative benefit from integration when the other customer is not integrated. A customer's relative benefit from integration in this case (derived in the proof of Lemma 4) is given by $\overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI}$. Therefore if an equilibrium with no integration exists for some price profile (p_1, p_2), the following must be true:

$$\begin{aligned} \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI} &\leq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right) \\ \Leftrightarrow \frac{v}{2}\left(\pi_{inv}^{SI} - \pi_{inv}^{NI}\right) &\leq \delta\left(e^{NI}\right) \end{aligned} \tag{8}$$

where

$$\delta(e) \triangleq \overline{v}\pi_2(e, e) - c(e) - \overline{v}\pi_2\left(e^{SI}, 0\right).$$

It is easy to see that $\delta\left(e^{NI}\right)$ is always positive.[9] The expression in the left hand side of (8) can be positive or negative, as we see in Lemma 2. The condition (8) will be violated only if π_{inv}^{SI} is sufficiently greater than π_{inv}^{NI}. The following lemma formally proves that condition (8) is indeed a necessary and sufficient condition to have an equilibrium with no integration.

Lemma 4. *An equilibrium with no integration exists if and only if condition (8) holds true.*

A formal proof is given in the appendix.

3.3.3 Equilibrium with semi integration

Finally, we consider the case of semi integration in equilibrium. Without loss of generality, we assume that in a typical semi integration arrangement RU_2 is integrated to C_2 and RU_1 and C_1 are not integrated. If an equilibrium with semi integration exists, then p_2 must be above RU_2's opportunity cost of integration, which is $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$. Similarly, p_2 must be below C_2's relative benefit of integration in a semi integration arrangement, which is given by $\overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI}$. Thus in order to sustain semi integration in equilibrium, we must have

$$\begin{aligned} \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI} &\geq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right) \\ \Leftrightarrow \frac{v}{2}\left(\pi_{inv}^{SI} - \pi_{inv}^{NI}\right) &\geq \delta\left(e^{NI}\right) \end{aligned} \tag{9}$$

[9] We have $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right) \geq \overline{v}\pi_2\left(e^{NI}, 0\right) \geq \overline{v}\pi_2\left(e^{SI}, 0\right)$, where the first inequality follows from the fact that e^{NI} is RU_2's best response given that RU_1 exerts e^{NI} levels of effort and the second inequality follows from the fact that $e^{SI} > e^{NI}$.

The following lemma formally proves that condition (9) is also a necessary and sufficient condition to have an equilibrium with no integration. In such an equilibrium, we will have $p_2 = \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$ and the optimal response of C_1 would be to offer any price strictly below p_2. In response, RU_2 integrates with C_2 while others remain non-integrated.

Lemma 5. *An equilibrium with no integration exists if condition (9) holds true.*

A formal proof is given in the appendix. The following proposition characterizes all the competitive equilibria in pure strategies.

Proposition 2. *There always exists a competitive equilibrium. The equilibrium exhibits semi-integration if and only if condition (9) holds true. Otherwise, we have no integration in equilibrium.*

The proof directly follows from the preceding discussion, and is therefore skipped. The proposition shows that there are possibilities of observing either semi integration or no integration in equilibrium. Further, semi integration is likely to occur if the innovation probability is sufficiently higher in semi integration than in no integration.

In order to understand this result, let us consider a non integrated market structure to begin with. A research unit (assume RU_2) will be willing to integrate at an asking price, which is as high as its payoff under no integration. RU_2's decision to integrate however, increases RU_1's rent-seeking motivation and subsequently, RU_1's effort level in contest. An increase in RU_1's effort not only increases RU_1's expected payoff, but also increases all customers' expected payoff, if the aggregate innovation probability is higher in semi integration than in no integration. If the increase in innovation probability is sufficiently high, the customer C_2 can improve its payoff from integration, even after paying RU_2 its asking price.[10] Thus we see semi integration in our framework because of the nature of innovation contest $-$ A research unit's integration decision can have positive effect on other non-integrated unit's effort in innovation contest, which in turn can benefit all customers by increasing the aggregate innovation probability.

4 Discussion

4.1 Innovation contest

Semi integration occurs in equilibrium only if the innovation probability in semi integration is sufficiently higher than that in no integration. In our model, it is possible to have higher innovation probability in semi integration (compared to the case of no integration) as when one firm integrates and reduces its effort (to zero), the other non-integrated firm responds by increasing its rent seeking effort. Technically, the response curve (best action of a firm for a given action of the other firm) can be downward sloping (as illustrated in Figure 1 in example 1). In the discussion below, we address the question whether or not

[10] Note that the customer also gets a premium from integration, which is given by $\overline{v}\pi_2\left(e^{SI}, 0\right)$. But the premium is never sufficient to compensate the research unit's opportunity cost of integration.

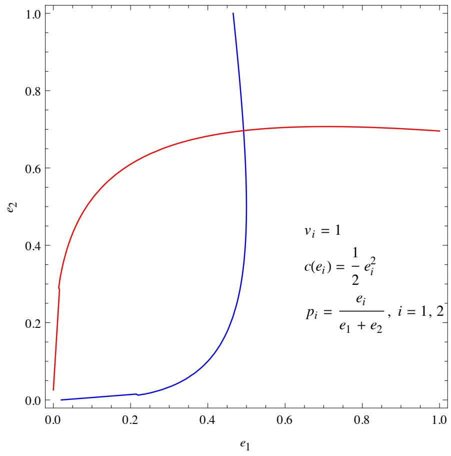

Figure 4: Response curves in Tullock contest when efforts are non-productive

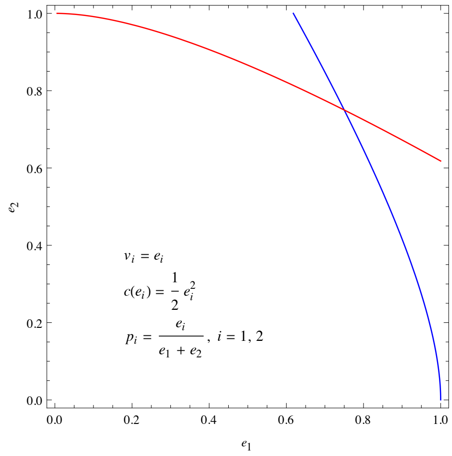

Figure 5: Response curves in Tullock contest when efforts are productive

the downward sloping response curves are specific features of our model or if they can be generalized to any reasonable model of innovation game.

We model competition in the form of a contest, in which players compete by expanding effort to increase the probability of winning a prize. In a standard $n-$player contest, we can write player i's payoff as $v_i p_i - c_i(e_i)$ where v_i is the prize value, π_i is the contest success probability and c_i is the cost of effort. Two critical components of modeling a contest are the contest success function (CSF) and the prize structure.[11] The standard models of a rent seeking contest typically assume a prize of fixed valuation and use specific classes of functional forms of CSF, of which the most commonly used one is the Tullock contest success function (Tullock 1980) of the following form:

$$p_i(e_1, \ldots, e_n) = \begin{cases} \frac{e_i^r}{\sum_{j=i}^n e_j^r} & \text{if } \max\{e_1, \ldots, e_n\} > 0 \\ \frac{1}{n} & \text{otherwise .} \end{cases} \tag{10}$$

Example 3. Consider a two-player contest with the Tullock contest success function (10) and assume $r = 1$. We assume a quadratic effort cost function $c(e) = \frac{1}{2}e^2$. In Figure 4, we plot the response curves when the prize has a fixed valuation, which is given by $v_i = 1$. The response curves are concave and are increasing at effort levels close to zero. It implies that if a player reduces it effort to zero, the other player will have an incentive to reduce her optimal effort. In Figure 5, we plot the response curves when the value of prize is given by $v_i = e_i$. The response curves are decreasing. In this case, by expanding effort, a player increases not only her winning probability, but also the value of her prize.

[11]For a comprehensive review of the contest literature, see Corchón [2007] and Konrad [2009].

In Example 3, we show that in a standard rent seeking contest framework with Tullock contest success function and with prize of fixed valuation (where v_i does not depend on effort), the response curves can be increasing when the competitor's effort is close to zero. The shape of the response curve is however, driven by the fact that the contest prize has a fixed value. Therefore, a player only expands its effort to defeat the competitor. In case of an innovation contest, the assumption of a prize with fixed valuation can be questionable. Specifically, in an innovation game, the event of making a successful innovation is an uncertain event and its probability depends on how much effort that the players exert. Thus a player's incentive to expand effort comes from two objectives: to defeat the competitor, and to make a successful innovation.

It is worth noting that the prevalence of Tullock success function in modeling contest is due to its strong axiomatic foundation. Skaperdas [1996] shows that any CSF with the following axiomatic properties − imperfect discrimination, monotonicity, anonymity, consistency and independence − must be of the additive form:

$$p_i\left(e_1,\ldots,e_n\right)=\begin{cases}\frac{f(e_i)}{\sum_{j=i}^{n} f(e_j)} & \text{if } \max\left\{f\left(e_1\right),\ldots,f\left(e_n\right)\right\}>0\\ \frac{1}{n} & \text{otherwise .}\end{cases} \tag{11}$$

where f is a positive increasing function of its argument. In addition, Skaperdas [1996] shows that the above axioms along with the assumption that CSF satisfies homogeneity of degree zero imply that the CSF must be of the form given in (10).[12]

In our framework, instead of assuming any specific CSF, we derive the winning probabilities from an underlying environment in which we treat innovation as a random event. In order to compare the winning probabilities with other forms of CSF, we first need to separate out the contest probability from the innovation probability. In our framework, a non-integrated research unit R_i chooses effort e_i to maximize her payoff $\overline{v}\pi_i\left(\underline{e}\right)-c\left(e_i\right)$, where $\pi_i\left(\underline{e}\right)$ is R_i's winning probability. We can rewrite her payoff as follows:

$$\overline{v}\pi_i\left(\underline{e}\right)-c\left(e_i\right)=\overline{v}\pi_{inv}\left(\underline{e}\right)\frac{\pi_i\left(\underline{e}\right)}{\pi_{inv}\left(\underline{e}\right)}-c\left(e_i\right),$$

where $\overline{v}\pi_{inv}\left(\underline{e}\right)$ is the expected value of an innovation and $\frac{\pi_i(\underline{e})}{\pi_{inv}(\underline{e})}$ is R_i's contest success probability given an innovation is realized. The contest success probability $\frac{\pi_i(\underline{e})}{\pi_{inv}(\underline{e})}$ satisfies all the desired axiomatic properties, and therefore it can indeed be rewritten in the form given in (11). To see this,

$$\begin{aligned}\frac{\pi_i\left(e_i,e_j\right)}{\pi_{inv}\left(e_i,e_j\right)}=&\frac{q\left(e_i\right)\left(1-\frac{q(e_j)}{2}\right)}{q\left(e_i\right)\left(1-\frac{q(e_j)}{2}\right)+q\left(e_j\right)\left(1-\frac{q(e_i)}{2}\right)}\\ =&\frac{f\left(e_i\right)}{\sum_{j=i}^{n} f\left(e_j\right)}\end{aligned}$$

[12] An extension of Skaperdas' result to non-anonymous CSF is given by Clark and Riis [1998].

where $f(e_i) = \frac{q(e_i)}{\left(1 - \frac{q(e_i)}{2}\right)}$. Further, with suitable choice of $q(e_i)$ (consider $q(e_i) = \frac{e_i^r}{e_i^r + 2}$), the contest probability coincides with the Tullock CSF in (10). In this sense, our framework generalizes the Tullock framework. We also incorporate the possibility of uncertain innovation as the contest prize does not have a deterministic value. Thus our derived winning probabilities are founded in a game theoretic framework and they are particularly suitable in modeling contest with uncertain prizes. A critical advantage of our model is that the winning probabilities $\pi_i(e_i, e_j) = q(e_i)\left(1 - \frac{q(e_j)}{2}\right)$ are multiplicatively separable. Therefore, the marginal effect of effort on probabilities and payoffs are relatively easy to derive and work with.

The patent race models are also related to our innovation contest model. In the classic model of patent race as pioneered by Loury [1979] and Dasgupta and Stiglitz [1980] (referred as LDS model hereafter), multiple firms compete for a patent. The patent has a fixed value and the probability of making a discovery in the infinite time horizon is always one. However, the model assumes that time is costly in the sense that an early discovery is better than a late discovery. The winner's payoff decreases with expected time of discovery. An implication of positive time-discount factor is that if a firm expands its effort (keeping others effort at a fixed level), it increases not only its chance of winning the patent, but also its payoff from winning as the expected time of discovery reduces. Baye and Hoppe [2003] show that LDS models are strategically equivalent to Tullock rent seeking contest model with fixed value prize when the time discount factor approaches to zero. It implies that the response curves can be increasing at effort levels close to zero when the discount factor approaches zero. However, if the discount factor is close to zero, we are effectively dealing with a situation when players are contesting for a prize with a deterministic value. It can be shown that if the discount factor is sufficiently higher than zero, the LDS models can also exhibit decreasing response curves $-$ when one firm reduces effort, the other firm responds by increasing effort.[13]

4.2 Efficiency

We allow customers to license innovation to each other. Therefore, the customer, who has the maximum valuation, always commercializes an innovation. The social value of an innovation based on an effort profile (e_1, e_2) is therefore given by

$$W(e_1, e_2) = E(v_{max})\,\pi_{inv}(e_1, e_2) - c(e_1) - c(e_2)\,.$$

Let W^* denote the maximum possible value of innovation, i.e., $W^* \triangleq \max_{(e_1,e_2)\in[0,1]^2} W(e_1, e_2)$. Since utility can be costlessly transferred among players (through contracted prices between research units and customers and through licensing fees between the customers), an outcome is efficient only if the aggregate payoff is W^*.[14] In order to tractably char-

[13] The examples are available with the authors.

[14] In other words, for any outcome (let us call it A) with aggregate payoff less than W^*, we can always construct another payoff profile, supported by some effort profile yielding higher social value of innovation, such that the new payoff profile Pareto dominates the payoff profile associated with A.

acterize the social value maximizing effort profile, we assume that the social value of innovation is concave in efforts. Formally, we assume:

Assumption 3. *$W(\underline{e})$ is strictly concave in $\underline{e}$.*

Assumption 3 is typically satisfied as long as the cost function $c(e)$ is sufficiently convex. Given Assumption 3, from the first order condition, we can uniquely characterize the social value maximizing effort profile, which is given by (e^w, e^w) and e^w satisfies the following condition:

$$(\overline{v} + \underline{v})(1 - q(e^w))\, q'(e^w) - c'(e^w) \quad = \quad 0. \tag{12}$$

Further, Assumption 2 ensures that $e^w > 0$, and so full integration is not an efficient outcome. By Assumption 3, we can also conclude that semi integration is not efficient in our framework. However, the efficiency of no-integration outcome is not unambiguous. On the one hand, there is an incentive to under-supply effort as non integrated research units receive only a part of the innovation's maximum value. On the other hand, the contest can lead to over-supply of effort as both units exert effort to increase the size of the rent. The following lemma shows that a no integration outcome can be socially wasteful if the success probability at the optimal effort level in no integration is sufficiently high.

Lemma 6. *$e^w \leq e^{NI}$ if and only if $q^{-1}\left(\frac{2\underline{v}}{\overline{v}+2\underline{v}}\right) \leq e^{NI}$.*

Proof. Note that e^{NI} satisfies (5) and e^w satisfies (12). Denote $(\overline{v} + \underline{v})\left(1 - q\left(e^{NI}\right)\right)$ and $\overline{v}\left(1 - \frac{q\left(e^{NI}\right)}{2}\right)$ by A and B respectively. A direct comparison of A and B shows that $A \leq B$ if and only if $\frac{2\underline{v}}{\overline{v}+2\underline{v}} \leq q\left(e^{NI}\right)$. By (5), $B = \frac{c'\left(e^{NI}\right)}{q'\left(e^{NI}\right)}$. Therefore,

$$\begin{aligned} A \leq B \quad &\Leftrightarrow \quad (\overline{v} + \underline{v})\left(1 - q\left(e^{NI}\right)\right) \leq \frac{c'\left(e^{NI}\right)}{q'\left(e^{NI}\right)} \\ &\Leftrightarrow \quad (\overline{v} + \underline{v})\left(1 - q\left(e^{NI}\right)\right) q'\left(e^{NI}\right) - c' \leq 0 \\ &\Leftrightarrow \quad e^w \leq e^{NI} \text{ (by Assumption 3).} \end{aligned}$$

Finally, note that as $q(e) \in [0, 1]$ is an increasing function, $q^{-1}\left(\frac{2\underline{v}}{\overline{v}+2\underline{v}}\right)$ is well defined. □

Lemma 6 shows that if individual success probability in no integration is sufficiently high, no integration produces wasteful effort. In a sense, the result is not so surprising − the research units' incentive to expand effort increases with their individual success probability and so innovation contest in no integration can be socially wasteful.[15]

[15] Chung [1996] shows that a contest in which efforts are productive can be socially wasteful. Our framework shares common features with Chung's framework in the sense efforts are also productive in our model. Chung assumes that the rent is a function of aggregate effort. In our framework, the rent is not a straight forward function of the aggregate effort, rather a function of the composite effort profile. Our result is, however, different from that in Chung's framework − the non integrated research units can also exert sub-optimal effort in equilibrium in our model. The difference in the results is driven by the

Recall that the optimal effort e^{NI} is endogenous and satisfies (5). It is easy to see that e^{NI} is increasing in $\overline{v}$ and it does not depend on $\underline{v}$. On the other hand, $\frac{2\underline{v}}{\overline{v}+2\underline{v}}$ is decreasing in $\overline{v}$ and increasing in $\underline{v}$. Together, we can conclude that research units over-supply effort in no integration if $\overline{v}$ is sufficiently high or if $\underline{v}$ is sufficiently low.

Corollary 1. *$e^w \leq e^{NI}$ if and only if $\overline{v}$ is sufficiently high or if $\underline{v}$ is sufficiently low.*

Note that $\overline{v}$ measures the expected value of an innovation where as $\underline{v}$ (which is $E\left(\frac{v_{max}-v_{min}}{2}\right)$) is an imperfect measure of the highest valued customer's bargaining power. A customer's barganining power is low when valuations are positively correlated so that v_{max} is likely to be close to v_{min}. The model predicts that when an innovation is highly valuable or if the customers' valuations are positively correlated, the non-integrated arrangement is likely to be socially wasteful.

While all arrangements can be potentially inefficient, it is interesting to see welfare comparison among different arrangements. In particular, we are interested to see whether an equilibrium arrangement can be dominated by another arrangement in terms of social value of innovation. The social value of an innovation, computed at the optimal effort in contest, in cases of full integration, semi integration and no integration are respectively given by

$$\begin{aligned} W^{FI} = && W(0,0) = && (\overline{v}+\underline{v})\,\pi_{inv}^{FI} \\ W^{SI} = && W\left(e^{SI},0\right) = && (\overline{v}+\underline{v})\,\pi_{inv}^{SI} - c\left(e^{SI}\right) \\ W^{NI} = && W\left(e^{NI},e^{NI}\right) = && (\overline{v}+\underline{v})\,\pi_{inv}^{NI} - 2c\left(e^{NI}\right). \end{aligned} \tag{13}$$

To compare different structures in terms of the social value of an innovation, we introduce a notion of inefficiency here. We call an industry structure *inefficient* if there exists an alternative structure with higher social value of an innovation, computed at the optimal effort profile in the innovation contest. It can be easily shown that the full integration structure is always inefficient as the social value of an innovation in full integration is dominated by the social value of an innovation in semi integration. At the same time, we see that full integration is never realized in equilibrium as both customer-research unit pairs receive higher payoff in semi integration than in no integration.

The comparison between the cases of semi integration and no integration, we see that the comparison is ambiguous. The social value of an innovation in semi integration is higher than the social value of an innovation under no integration if the following holds true:

$$\begin{aligned} W\left(e^{SI},0\right) &\geq W\left(e^{NI},e^{NI}\right) \\ \Leftrightarrow \pi_{inv}^{SI} - \pi_{inv}^{NI} &\geq \frac{c\left(e^{SI}\right) - 2c\left(e^{NI}\right)}{\overline{v}+\underline{v}}. \end{aligned} \tag{14}$$

fact that the research units, in our framework, can only realize a part of the total innovation value ($\overline{v}$ out of $E(v_{\max}) = \overline{v}+\underline{v}$) because of ex post bargaining. If we assume $\underline{v} = 0$ (which implies that v_is are positively correlated and consequently the customers have little bargaining power), then we will also find that non-integrated arrangement always produces socially wasteful effort.

The right hand side expression in (14) measures the difference in total effort between the two cases of semi integration and no integration, per unit of commercial valuation (recall that the innovation is commercialized at the maximum customer valuation, which has an expected value of $\overline{v} + \underline{v}$). The following proposition shows that the industry structure in competitive equilibrium may not necessarily be efficient. We specify the conditions under which an equilibrium with semi integration or an equilibrium with no integration can be inefficient.

Proposition 3. *The competitive equilibrium exhibits inefficient non-integration if the following is true*

$$\frac{\underline{v}}{2}\frac{\left(c\left(e^{SI}\right)-2c\left(e^{NI}\right)\right)}{\overline{v}+\underline{v}} \leq \frac{\underline{v}}{2}\left(\pi_{inv}^{SI}-\pi_{inv}^{NI}\right) \leq \delta\left(e^{NI}\right).$$

The competitive equilibrium exhibits inefficient semi integration if the following is true

$$\delta\left(e^{NI}\right) \leq \frac{\underline{v}}{2}\left(\pi_{inv}^{SI}-\pi_{inv}^{NI}\right) \leq \frac{\underline{v}}{2}\frac{\left(c\left(e^{SI}\right)-2c\left(e^{NI}\right)\right)}{\overline{v}+\underline{v}}.$$

The proof is trivial, and it follows from combining (14) with the equilibrium conditions derived in Proposition 2.

To understand why we may see inefficient structures in equilibrium, note that an outcome can be sustained in equilibrium as long as each pair of a research unit and a customer cannot make themselves better off by deviating to an alternate structure. Consider, for example, a situation in which no integration is observed in equilibrium. It implies that in this situation if a non-integrated pair of research unit and customer (we call it pair A) decides to integrate, their joint payoff will be less. However, such a move can have positive externality on the rent seeking effort level of the other non-integrated research unit in the contest. Thereby, the integration decision by pair A may increase the joint payoff of the other pair (we call it pair B) in a semi integration arrangement compared to what pair B is currently getting in the no integration equilibrium. If the increase in pair B's joint payoff exceeds the loss in pair A's joint payoff, the no integration arrangement is inefficient. However, as pair A is not compensated for the loss it would be making by integration, we observe no integration in equilibrium. The customers end up restricting themselves from offering higher price as the non integrated research unit cannot commit to compensate the integrated customer at the contracting stage.

Similarly, we see inefficient semi integration in equilibrium when an integrated pair makes loss from non-integration but such a move can increase the payoff of the other non-integrated pair by a high margin. Specifically, there will be a reduction in the rent seeking effort of the other non integrated research unit in the contest (from e^{SI} to e^{NI}). As in the previous scenario, the non integrated research unit cannot commit to compensate the integrated customer's loss from no integration, and thus we see inefficient semi integration in equilibrium.

4.3 Resource constraints

In our model, all players have positive expected payoff ex ante, but they can incur a positive loss ex post. Our analysis assumes away any resource constraint on the customer's side and on the research unit's side. Such constraints can have implications in our framework. Though we do not introduce these constraints in a formal manner here, but it is easy to see their implication in our model.

First consider the possibility that a research unit has resource constraints such that it may not necessarily finance its full effort cost through borrowing. In this case, a non-integrated research unit's effort level will be adversely affected. Let us assume a research unit can borrow only up to an amount $S > 0$. If S is above $c\left(e^{SI}\right)$, then the constraint has no impact on the equilibrium arrangement. If $S \in \left(c\left(e^{NI}\right), c\left(e^{SI}\right)\right)$, then the optimal effort in no integration arrangement will be unaffected, but the optimal effort of a non integrated research unit in semi integration arrangement will reduce to $c^{-1}(S)$, which is lower than e^{SI}. It reduces a research unit's payoff from not integrating in a semi integration arrangement, which in turn adversely affects the possibility semi integration in equilibrium. When the resource constraint is severe such that S is less than $c\left(e^{NI}\right)$, a non-integrated research unit's effort reduces to $c^{-1}(S) < e^{NI} < e^{SI}$ in both semi integration and no integration arrangements. In an extreme case, only full integration arrangement will be observed in equilibrium.

A resource constraint on the customer's side can be introduced in a similar way, such that there is an exogenous upper bound on the price that it can offer at the pre-innovation contracting stage. Unlike the previous case, the resource constraint of the customer does not affect the possibility of no integration arrangement. This is because the customers do not pay any price upfront in a no integration arrangement. Hence, the resource constraint of the customer is stronger, only the equilibrium with no integration survives.

5 Conclusion

We examine a simple model of vertical integration in the context of innovation. The analysis shows that contest affect the organization of R&D activities in a non-trivial way. In equilibrium we can see either coexistence of integrated and non integrated R&D activities (semi integration), or only the non-integrated R&D activities (no integration). Integration dampens a research unit's incentive to exert non-verifiable effort which is socially costly. The integration can, however, have a positive externality on the rent seeking effort level of a non-integrated research unit: A non-integrated research unit exerts higher effort to win the innovation contest when its competitor is integrated. An increase in non-integrated research unit's effort level benefits all the customers when the aggregate innovation probability is higher in semi integration than in no integration. When the benefit to the customers are sufficiently high, we observe integration in equilibrium. It is interesting to note that an equilibrium R&D structure may not necessarily be more socially valuable compared to an alternate R&D structure. This is because those who benefit from an integration cannot necessarily commit to compensate those who lose in any credible way.

Appendix

Our analysis focuses on the pure strategies only. At the pre-innovation contracting stage, C_i offers a price $p_i \in [0, \infty)$. Let $\mathbf{p} = (p_1, p_2)$ denote a price profile. At the pre-innovation contracting stage, RU_i decides whether to integrate with a customer. Its integration strategy is given by a tuple $\mathbf{int}_i = \left(int_i^f, int_i^s\right)$. The first component $int_i^f(\mathbf{p})$ is RU_i's integration decision at $p_{\max}$, the maximum price of the price profile $\mathbf{p}$. The second component $int_i^s(\mathbf{p})$ is RU_i's integration decision at $p_{\min}$, the second highest price of the price profile $\mathbf{p}$, given that the other research unit is integrated with the customer offering the highest price $p_{\max}$. For simplicity, we assume that $int_i^f(\mathbf{p})$ and $int_i^s(\mathbf{p})$ take binary values, 0 and 1, such that the value 1 corresponds to a decision to integrate. Let $\mathbf{int} = (\mathbf{int}_1, \mathbf{int}_2)$ denote a profile of integration strategies. The research units simultaneously decide the effort level in the innovation contest. RU_i's effort strategy is to choose $e_i \in [0, 1]$, given a price-integration strategy profile $(\mathbf{p}, \mathbf{int})$. A pure strategy of RU_i is given by $\sigma_i = (\mathbf{int}_i, e_i)$. Below we present the proofs that are omitted in the main text.

Proof of Lemma 3:

Proof. If an equilibrium with full integration exists, then it must be the case that the research units are willing to integrate at both prices p_1 and p_2. It is easy to see that in this case, both customers will offer the same price in equilibrium, as otherwise the customer offering the higher price can increase her payoff by decreasing price. We denote the common price by p. As RU_2 integrates at p when RU_1 is already integrated, we must have

$$p \geq \overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right). \tag{15}$$

C_2's expected payoff in this equilibrium is $\left(\overline{v} + \frac{v}{2}\right)\pi_2(0,0) + \frac{v}{2}\pi_1(0,0) - p$. The first component is C_2's expected payoff when RU_2 wins the contest times the probability that RU_2 wins the contest. Recall that (from our discussion in section 3.1) the expected payoff of the customer integrated with the winning research unit is $E\left(\frac{v_{\max}+v_i}{2}\right) = \overline{v} + \frac{v}{2}$. The second component is C_2's expected payoff when RU_1 wins the contest times the probability that RU_1 wins the contest. After simplifying, we can rewrite C_2's expected payoff as $\overline{v}\pi_2(0,0) + \frac{v}{2}\pi_{inv}^{FI} - p$.

On the other hand, if C_2 deviates by lowering its price, its expected payoff will be $\frac{v}{2}\left(\pi_1\left(0, e^{SI}\right) + \pi_2\left(0, e^{SI}\right)\right) = \frac{v}{2}\pi_{inv}^{SI}$. Comparing the above expressions, the no deviation condition for C_2 is given by

$$p \quad \leq \quad \overline{v}\pi_2(0,0) - \frac{v}{2}\left(\pi_{inv}^{SI} - \pi_{inv}^{FI}\right). \tag{16}$$

From (15) and (16), we see that a necessary condition to have an equilibrium with full integration is that

$$\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right) \quad \leq \quad \overline{v}\pi_2(0,0) - \frac{v}{2}\left(\pi_{inv}^{SI} - \pi_{inv}^{FI}\right). \tag{17}$$

The above condition is also a sufficient condition to have an equilibrium with full integration. To see this, we construct an equilibrium as follows. Let us denote $\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right)$ by A and $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$ by B. We have $B \leq A$ as $\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right) \geq \overline{v}\pi_2\left(0, e^{NI}\right) - c\left(e^{NI}\right) \geq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$. Consider the following strategies. C_i chooses $p_i = A$. The integration strategies $\left(\left(int_1^f(\mathbf{p}), int_1^s(\mathbf{p})\right), \left(int_2^f(\mathbf{p}), int_2^s(\mathbf{p})\right)\right)$ by the research units are as follows:

$$\begin{aligned} int_1^f(\mathbf{p}) &= \begin{cases} 1 & \text{if } p_{\max} \geq A \\ 0 & \text{otherwise} \end{cases} \\ int_2^f(\mathbf{p}) &= \begin{cases} 1 & \text{if } p_{\max} \geq B \\ 0 & \text{otherwise} \end{cases} \\ int_1^s(\mathbf{p}) = int_2^s(\mathbf{p}) &= \begin{cases} 1 & \text{if } p_{\min} \geq A \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{18}$$

And RU_i's effort strategy e_i is as follows:

$$e_i = \begin{cases} 0 & \text{if } RU_i \text{ is integrated} \\ e^{NI} & \text{if both research units are not integrated} \\ e^{SI} & \text{otherwise} \end{cases} \tag{19}$$

Claim 1: The integration strategies and the effort strategies given by (18) and (19) are Nash equilibrium strategies in the subgame induced by the price profile $\mathbf{p}$.

We have already shown in section 3.2 that that e_is are the Nash equilibrium effort strategies in the innovation contest. We will now show that for a given price profile $\mathbf{p}$, $\mathbf{int}_1 = \left(int_1^f(\mathbf{p}), int_1^s(\mathbf{p})\right)$ is RU_1's optimal integration strategy given RU_2 follows $\mathbf{int}_2 = \left(int_2^f(\mathbf{p}), int_2^s(\mathbf{p})\right)$ and vice versa. Note that RU_i's expected

payoff from no integration when the other research unit is integrated is given by A. Therefore, it prefers to integrate if and only if $p_{\min} \geq A$. We next show that int_1^f is RU_1's best response against RU_2's first stage integration strategy int_2^f and vice versa. To see this, note that for any price profile with $p_{\max} \geq A$, both research unit's dominant strategy is to integrate, as a research unit's maximum payoff from non-integration can never exceed A in any situation. Similarly, for any profile with $p_{\max} < B$, both research unit's dominant strategy is not to integrate, as it can always a payoff as high as B by non-integration. Finally, if $p_{\max}$ lies in the interval $[B, A]$, and if one of the research unit integrates, the other research unit's optimal strategy is not to integrate and vice versa. This completes the proof of Claim 1.

Further, note that the customers by offering $p_i = A$, can induce both firms to integrate, and given condition (17), none of the customer can improve the payoff by lowering its offered price when the other customer offers a price equal to A. Hence the above strategies constitute a subgame perfect Nash equilibrium. These strategies will lead to an outcome of full integration as both research unit integrate at $p_1 = p_2 = A$. □

Proof of Lemma 4:

Proof. If an equilibrium with no integration exists, then it must be the case that both research units are not willing to integrate at the maximum price. Assume that the research units face a price profile (p_1, p_2). We compare RU_2's payoff from integration and that from no integration. When RU_1 does not integrate, RU_2's payoff from integrating is $max\{p_1, p_2\}$ and from not integrating is $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$. If an equilibrium with no integration exists, we must have

$$max\{p_1, p_2\} \leq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right). \tag{20}$$

We next consider the customers' incentive to offer low prices. Suppose that C_1 offers a price $p_1 \leq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$. If C_2 also offers a price $p_2 \leq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$, its expected payoff is $\frac{v}{2}\pi_{inv}\left(e^{NI}, e^{NI}\right) = \frac{v}{2}\pi_{inv}^{NI}$. If C_2 deviates by increasing its price above (weakly) $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$, then one of the two research units (without loss of generality, assume RU_2) will choose to accept the offer. In such a case, C_2's expected payoff will be given by $\left(\overline{v} + \frac{v}{2}\right)\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_1\left(e^{SI}, 0\right) - p_2 = \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - p_2$. Comparing the above expressions, the no deviation condition for C_2 is given by

$$p_2 \quad \geq \quad \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI}. \tag{21}$$

Hence, a necessary condition to have an equilibrium with no integration is

$$\begin{aligned} \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI} \quad &\leq \quad \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right) \\ \Leftrightarrow \frac{v}{2}\left(\pi_{inv}^{SI} - \pi_{inv}^{NI}\right) \quad &\leq \quad \delta\left(e^{NI}\right). \end{aligned} \tag{22}$$

The above condition is also a sufficient condition to have an equilibrium with full integration. To see this, assume that condition (22) holds true and we consider the following strategies. C_i chooses $p_i = 0$. As before, we denote $\overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right)$ by A and $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$ by B. And, we consider the integration strategies $\left(\left(int_1^f(\mathbf{p}), int_1^s(\mathbf{p})\right), \left(int_2^f(\mathbf{p}), int_2^s(\mathbf{p})\right)\right)$ and the effort strategies given by (18) and (19) respectively.

As shown in the proof of lemma 3 (see Claim 1 in the proof), the integration strategies and the effort strategies are the Nash equilibrium strategies in the subgame induced by the price profile $\mathbf{p}$. We will have to show that $p_1 = 0$ and $p_2 = 0$ are Nash equilibrium price strategies by the customer. To see this, let us suppose that C_1 offers $p_1 = 0$. By increasing $p_2 \geq B$, C_2 can get a payoff of $\overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - p_2$, which can never be higher than its current payoff from no integration $\frac{v}{2}\pi_{inv}^{NI}$ as the condition (22) holds true. Hence, the above mentioned strategies are indeed Nash equilibrium strategies. In this equilibrium, no research unit integrates. □

Proof of Lemma 5:

Proof. Without loss of generality, we assume that RU_2 is integrated to C_2, and RU_1 and C_1 are not integrated in semi integration. If an equilibrium with semi integration exists, then it must be the case that RU_1 does not integrate at $p_{\min}$. Notice that the payoff of a non integrated research unit is $A = \overline{v}\pi_2\left(0, e^{SI}\right) - c\left(e^{SI}\right)$ when the other research unit is integrated. Hence, we must have $p_{\min} \leq A$. However, there are two possibilities in which we can see semi integration. First, both research units are willing to integrate at $p_{\max}$, and second, RU_2 integrates at $p_{\max}$ but RU_1 does not integrate at $p_{\max}$. We assume that $p_{\min} \leq A$ and analyze the two cases below.

Case 1: Both research units are willing to integrate at $p_{\max}$. When RU_2 is integrated, RU_1 gets $\frac{1}{2}p_{\max} + \frac{1}{2}A$ by integrating (RU_1 is matched with the customer offering $p_{\max}$ with $\frac{1}{2}$ probability) and it gets A by not integrating. Hence, in this case we must have $p_{\max} \geq A$.

Case 2: RU_2 is willing to integrate at $p_{\max}$, but RU_1 is not. Comparing RU_1's payoff from integration and no integration (when RU_2 is integrated), we see that $p_{\max} \leq A$. Similarly, comparing RU_2's payoff from integration and no integration (when RU_1 is not integrated) we see that $p_{\max} \geq B = \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$. Hence, in this case we must have $p_{\max} \in [B, A]$.

Next, we look at the customers' optimal price responses. For given p_1, we consider the optimal response of RU_2.

If $p_1 < B$, C_2 gets $\frac{v}{2}\pi_{inv}^{NI}$ by offering $p_2 < B$. And, if it offers $p_2 \geq B$, one of the research unit integrates while the other is not. Therefore, by offering $p_2 \geq B$, C_2 gets $\overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - p_2$, which is decreasing in p_2. Hence, when $p_1 < B$, the optimal response of C_2 is B if $B \leq \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI}$, and any $p_2 < B$ if $B > \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI}$.

If $p_1 \in [B, A]$, C_2 gets $\frac{v}{2}\pi_{inv}^{SI}$ by offering $p_2 < p_1$ (as only one research unit integrates with C_1 in that case). By offering $p_2 \geq p_1$, C_2 gets $\overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - p_2$, which is always less than $\frac{v}{2}\pi_{inv}^{SI}$ for all $p_2 \geq B$. This is because $\overline{v}\pi_2\left(e^{SI}, 0\right) \leq \overline{v}\pi_2\left(e^{NI}, 0\right) \leq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right) = B$. Hence, when $p_1 \in [B, A]$, the optimal response of C_2 is any $p_2 < p_1$.

Finally, if $p_1 > A$, C_2 gets $\frac{v}{2}\pi_{inv}^{SI}$ by offering $p_2 < A$ (as only one research unit integrates with C_1 in that case). By offering $p_2 \geq A$, C_2 gets $\overline{v}\pi_2(0, 0) + \frac{v}{2}\pi_{inv}^{FI} - p_2$, which is always less than $\frac{v}{2}\pi_{inv}^{SI}$ for all $p_2 \geq A$. Hence, when $p_1 > A$, the optimal response of C_2 is any $p_2 < A$.

The optimal response of C_1 for a given price p_2 would also be symmetric. It is evident in no circumstances, any customer would offer a price as high as A. Thus the case 1 depicted above, in which $p_{\max} \geq A$, will never be realized in equilibrium. Therefore, if we see semi integration in equilibrium, it must be that case 2 holds true, in which we have $p_{\max} \in [B, A]$. From the optimal response functions, we see that such a possibility can occur only if $B \leq \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI}$, in which case $p_{\max} = B$ and $p_{\min} < p_{\max}$. Hence a necessary condition to have an equilibrium with semi integration is

$$\begin{aligned} \overline{v}\pi_2\left(e^{SI}, 0\right) + \frac{v}{2}\pi_{inv}^{SI} - \frac{v}{2}\pi_{inv}^{NI} &\geq \overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right) \\ \Leftrightarrow \frac{v}{2}\left(\pi_{inv}^{SI} - \pi_{inv}^{NI}\right) &\geq \delta\left(e^{NI}\right). \end{aligned} \tag{23}$$

The above condition is also a sufficient condition to have an equilibrium with full integration. To see this, assume that condition (23) holds true and we denote $\overline{v}\pi_2\left(0, e^{SI}\right) -$

$c\left(e^{SI}\right)$ by A and $\overline{v}\pi_2\left(e^{NI}, e^{NI}\right) - c\left(e^{NI}\right)$ by B. Consider the following strategies. C_1 chooses $p_1 = 0$ and C_2 chooses $p_2 = B$. And, we consider the integration strategies $\left(\left(int_1^f(\mathbf{p}), int_1^s(\mathbf{p})\right), \left(int_2^f(\mathbf{p}), int_2^s(\mathbf{p})\right)\right)$ and the effort strategies given by (18) and (19) respectively.

As shown in the proof of lemma 3 (see Claim 1 in the proof), the integration strategies and the effort strategies are the Nash equilibrium strategies in the subgame induced by the price profile $\mathbf{p}$. From our derivation of the optimal response functions above, we see that $p_1 = 0$ and $p_2 = B$ are Nash equilibrium price strategies when the condition (23) holds true. Hence, the above mentioned strategies are indeed Nash equilibrium strategies. In this equilibrium, RU_2 integrates with C_2 while RU_1 and C_1 are not integrated. $\square$

References

[1994] Aghion, P., and J. Tirole. 1994. "The Management of Innovation." *Quarterly Journal of Economics* 109(4): 1185-209.

[2010] Arora, A., and A. Gambardella. 2010. "The Market for Technology," In *Handbook of the Economics of Innovation*. Elsevier B.V.

[2003] Baye, M. R., and Hoppe, H. C. 2003. "The strategic equivalence of rent-seeking, innovation, and patent-race games." *Games and Economic Behavior*, 44(2): 217-226.

[1996] Chung, T. Y. 1996. "Rent-seeking contest when the prize increases with aggregate efforts." *Public Choice*, 87(1-2): 55-66.

[1998] Clark, D. J., and& Riis, C. 1998). "Contest success functions: an extension." *Economic Theory*, 11(1): 201-204.

[1937] Coase, Ronald H. 1937. "The Nature of the Firm." *Economica*, 4(16): 386–405

[2007] Corchón, Luis C. 2007. "The theory of contests: a survey." *Review of Economic Design* 11(2): 69-100.

[1980] Dasgupta, P., and Stiglitz, J. 1980. "Uncertainty, industrial structure, and the speed of R&D." *The Bell Journal of Economics*, 11: 1-28.

[1986] Grossman, S., and O. Hart. 1986. "The Costs and Benefits of Ownership: A Theory of Lateral and Vertical Integration," *Journal of Political Economy*, LXXXXIV: 691-719.

[2005] Guedj, I. 2005. "Ownership Vs. Contract: How Vertical Integration Affects Investment Decisions in Pharmaceutical R&D." *McCombs Research Paper Series No. FIN-01-06.*

[1995] Hart, Oliver. 1995. *Firms, Contracts, and Financial Structure*. Clarendon Lectures in Economics. Oxford and New York: Oxford University Press, Clarendon Press.

[1990] Hart, Oliver, and John Moore. 1990. "Property Rights and the Nature of the Firm." Journal of Political Economy, 98(6): 1119–58.

[1978] Klein, Benjamin, Robert G. Crawford, and Armen A. Alchian. 1978. "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process." *Journal of Law and Economics*, 21(2): 297–326.

[2009] Konrad, Kai A. 2009. *Strategy and dynamics in contests*. Oxford University Press.

[2007] Lafontaine, Francine, and Margaret Slade. 2007. "Vertical Integration and Firm Boundaries: The Evidence." *Journal of Economic Literature*, 45(3): 629-685.

[1979] Loury, G. C. 1979. "Market structure and innovation." *The Quarterly Journal of Economics*, August 1979, 395-410.

[1995] Mowery, D., "The Boundaries of the U.S. Firm in R&D." In *Coordination and Information: Historical Perspectives on the Organization of Enterprise*, edited by N. Lamoreaux and D. Raff. University of Chicago Press, 1995.

[2006] Pisano, G., *Science Business: The Promise, the Reality, and the Future of Biotech.* Harvard Business Press, 2006.

[1996] Skaperdas, S. 1996. "Contest success functions." *Economic Theory*, 7(2), 283-290.

[2003] Whinston, Michael D. 2003. "On the Transaction Cost Determinants of Vertical Integration." *Journal of Law, Economics, and Organization*, 19(1): 1–23.

[1971] Williamson, Oliver E. 1971. "The Vertical Integration of Production: Market Failure Considerations." *American Economic Review*, 61(2): 112–23.

[1975] Williamson, Oliver E. 1975. *Markets and Hierarchies: Analysis and Antitrust Implications*. New York: Free Press.

[1979] Williamson, Oliver E. 1979. "Transaction-Cost Economics: The Governance of Contractual Relations." *Journal of Law and Economics*, 22(2): 233–61.