Translation-Consistent Subgroup Decomposable Inequality Indices

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Abstract

The paper suggests a two-parameter extension of the family of subgroup decomposable absolute inequality indices identified in Chakravarty and Tyagarupananda (1998). Maintaining similarity with Zheng (2007), we replace the notion of 'translation invariance' by 'translation consistency' and hence characterize the relevant class of subgroup decomposable inequality indices.

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1. Introduction

Over several decades of the last century, the study of economic inequality has drawn a considerable attention of welfare economists all over the globe. Of particular interest is the notion of 'subgroup decomposability' of an inequality index, which deals with the contribution of different groups in the measurement of inequality in a multi-group context. This type of decomposition helps the policy-maker identifying the subgroup responsible for the enhancement of inequality. In this situation, the subgroup-decomposability of the inequality index under consideration requires that the overall inequality can be expressed as a sum of a between-group term and a within-group term. The between-group term measures the inequality among the subgroups obtained by replacing the individual incomes by the mean incomes of the subgroups while the within group term is a weighted sum of the inequality measures of the subgroups. The notion of subgroup-decomposability of an inequality index came to the fore on the eighties of the last century. Bourguigon (1979), Shorrocks (1980, 1984), Blackorby et al (1981) and Foster (1983) were among those who contributed to the area.

Shorrocks (1980) identified the class $\left(I_c\right)$ of all symmetric (that is, invariant under all permutations of incomes), scale-invariant (homogeneous of degree zero), twice continuously differentiable and subgroup decomposable inequality indices satisfying Population Principle (invariance under replications of the population) and non-negativity (the index is non-negative valued and it vanishes if only if all the incomes are equal). The family I_c is popularly known as the generalized entropy family.

Replacing the notion of 'scale invariance' by 'translation invariance', Chakravarty and Tyagarupananda (1998) characterized the class of all absolute subgroup decomposable indices (satisfying other conditions as in Shorrocks (1980)). The isolated class includes variance and the Kolm absolute inequality index I_θ, where θ is a positive real number.

Bosmans and Cowell (2010) demonstrated how 'translation invariance' can be used as a tool for identification of the family of 'absolutely decomposable' inequality indices. In particular, an inequality measure satisfying anonymity, the transfer principle, population replication invariance, absolute decomposability and translation invariance if and only if there exists a real number θ and a continuous and strictly increasing transform of the inequality index equals either variance or Kolm absolute inequality index.

In a remarkable contribution, Zheng (2007) contended that the notion of invariance (scale/ translation/ intermediate) depends on the value judgment of the policy maker. To get rid of this problem, he suggested the use of 'unit consistency' condition (a natural generalization of 'scale invariance') and then located the relevant class of decomposable measures. It has been shown that the latter is a two-parameter extension of the one-parameter generalized entropy family.

Following the argument put forward by Zheng (2007), one may wonder if it is possible to find an ordinal counterpart of 'translation invariance' and axiomatize the corresponding class of decomposable inequality indices. This paper makes an attempt to answer this question. We first define a 'translation consistent' inequality index. To illustrate the notion, consider two income distributions D_1 and D_2 and let the inequality index I rank D_1 higher than D_2. Now, if all the incomes in both the distributions be increased/decreased by a constant amount, then translation consistency demands that I should rank the former higher than the latter. Obviously, a translation invariant inequality index is 'translation consistent', but the converse is not true. Thus, the class of all 'translation consistent' inequality indices includes I_θ and the variance. In other words, the class characterized in this paper can be viewed as a generalization of the class mentioned in Chakravarty and Tyagarupananda (1998).

The scheme of the paper is as follows. After discussing the background material in the following section, we present the characterization theorems in section 3. Finally, section 4 concludes the treatise.

2. The Background

For a population of size *n*, the vector $x = (x_1, x_2, .., x_n)$ represents the distribution of income, where each x_i is assumed to be drawn from the non-degenerate interval $[\upsilon, \infty)$ in the positive part R^1_{++} of the real line R^1. Here x_i stands for the income of person i of the population. For any

$i, x_i \in [\upsilon, \infty)$ and so, $x \in D^n = [\upsilon, \infty)^n$, the n-fold Cartesian product of $[\upsilon, \infty)$. The set of all possible income distributions is $D = \bigcup_{n \in N} D^n$, where N is the set of natural numbers. For all $n \in N$, for all $x = (x_1, x_2, ..., x_n) \in D^n$, $\sum_{i=1}^{n} (x_i / n)$, the mean of x, is denoted by $\lambda(x)$ (or simply by λ). For all $n \in N$, 1^n denotes the n-coordinated vector of ones. The non-negative orthant of the n-dimensional Euclidean space R^n is denoted by R_+^n. An inequality index is a function $I : D \to R_+^1$.

An index of inequality can satisfy invariance of two types viz. scale invariance and translation invariance. A scale invariant (relative) index remains constant under equi-proportionate changes in all incomes. In contrast, a translation invariant (absolute) index remains unaltered under equal absolute changes in all incomes. These two concepts of relative and absolute inequality express two different notions of value judgments about inequality equivalence. As labeled by Kolm (1976), the former refers to the 'rightist' view and the latter to the 'leftist' view. Apart from these two extreme modes of invariance, there is a notion of intermediate invariance which corresponds to the so-called 'centralist' viewpoint (see Chakravarty and Tyagarupananda, 2009).

To be precise, an inequality index $I_R : D \to R_+^1$ is a relative or scale invariant index if proportional changes in all incomes do not change inequality, that is, for all $n \in N$, $x \in D^n$,

$$I_R(cx) = I_R(x), \tag{1}$$

where $c > 0$ is any scalar. Similarly, an inequality index $I_A : D \to R_+^1$ is absolute / translation invariant if for all $n \in N$, $x \in D^n$,

$$I_A(x + c1^n) = I_A(x), \tag{2}$$

where c is a scalar such that $x + c1^n \in D^n$.

The following postulates are considered to be standard regularity conditions of an arbitrary inequality index $I : D \to R_+^1$ (whether relative or absolute).

Symmetry (SYM): For an arbitrary $n \in N$, if $x \in D^n$, then $I(x) = I(y)$, where y is any permutation of x.

Principle of Transfers (POT): For an arbitrary $n \in N$ and $x \in D^n$, suppose that y is obtained from x by the following transformation

$$y_i = x_i + c \leq y_j,$$

$$y_j = x_j - c,$$

and $$y_k = x_k \text{ for all } k \neq i, j, \tag{3}$$

where $c > 0$. Then $I(y) < I(x)$.

Principle of Population (POP): For all $n \in N$, $x \in D^n$, $I(x) = I(y)$, where $y = (x^1, x^2, ..., x^l)$, each $x^i = x$ and $l \geq 2$ is arbitrary.

Normalization (NOM): For all $n \in N$, $I\left(c1^n\right) = 0$ for all $c > 0$.

Non-negativity (NON): For all $n \in N$, $x \in D^n$, $I(x) = 0$ if and only if $x = \lambda\left(x\right)1^n$.

According to **SYM**, a condition of anonymity, I remains invariant under reordering of all incomes. Thus, **SYM** implies that any characteristic other than income has no relevance in the measurement of inequality. **POT** (popularly known as the Pigou-Dalton principle; also referred to as strict Schur-concavity) says that a transfer of income from a rich person j to a poor person i that does not change their relative positions reduces inequality while the reverse happens in case of a transfer from a poor to a rich. According to **POP,** inequality remains unaltered under replications of the population. Thus, **POP** plays significant role in cross population comparisons of inequality. **NOM** stipulates that inequality vanishes if there is perfect equality in the underlying distribution. Finally, **NON** imposes a typical restriction on **NOM**. It demands that the inequality index can never vanish unless there is perfect equality.

We now fix our attention to the notion of decomposability of an inequality index. An inequality index is said to be population subgroup decomposable if it satisfies the following axiom:

Subgroup Decomposability (SUD): For all $k \geq 2$ and for all $x^1, x^2, ..., x^k \in D$,

$$I\left(x\right) = I\left(\lambda_1 1^{n_1}, \lambda_2 1^{n_2}, ..., \lambda_k 1^{n_k}\right) + \sum_{i=1}^{k} \omega_i\left(\underline{n}, \underline{\lambda}\right) I\left(x^i\right), \tag{4}$$

where n_i is the population size associated with the distribution x^i, $n = \sum_{i=1}^{k} n_i$, $\lambda_i = \lambda(x^i)$ = mean of the distribution x^i, $\underline{\lambda} = (\lambda_1, \lambda_2, \ldots\ldots, \lambda_k)$, $\underline{n} = (n_1, n_2, \ldots\ldots, n_k)$, $\omega_i(\underline{n}, \underline{\lambda})$ is the positive weight attached to inequality in x^i, assumed to depend on the vectors $\underline{n}$ and $\underline{\lambda}$, and $x = (x^1, x^2, \ldots\ldots, x^k)$.
SUD shows that for any partitioning of the population, total inequality can be broken down into its between-group and within-group components. The between-group term (BI) gives the level of inequality that would arise if each income in a subgroup were replaced by the mean income of the subgroup and the within- group term (WI) is the weighted sum of inequalities in different subgroups (see Foster, 1985 and Chakravarty, 2009).

Shorrocks (1980) demonstrated that a twice continuously differentiable inequality index $I : D \rightarrow R_+^1$ satisfying scale invariance, **SUD, POP, SYM** and **NON** must be a positive multiple of the following form:

$$I_e(x) = \begin{cases} \dfrac{1}{ne(e-1)} \displaystyle\sum_{i=1}^{n} \left[\left(\dfrac{x_i}{\lambda(x)} \right)^e - 1 \right], e \neq 0,1, \\ \dfrac{1}{n} \displaystyle\sum_{i=1}^{n} \log \dfrac{\lambda(x)}{x_i}, e = 0, \\ \dfrac{1}{n} \displaystyle\sum_{i=1}^{n} \dfrac{x_i}{\lambda(x)} \log \dfrac{x_i}{\lambda(x)}, e = 1. \end{cases} \tag{5}$$

The family I_e, which is popularly known as the generalized entropy family, satisfies **POT**. A transfer from a rich person to a poor decreases I_c by a larger amount the lower is the value of c.

Proceeding analogously, Chakravarty and Tyagarupananda (2009) characterized the SUD-family of absolute inequality indices. To be specific, the class of twice continuous differentiable inequality indices satisfying **SUD, POP, SYM** and **NON** that remains invariant under equal translation of all incomes comprises:

$$I_\theta(x) = \frac{1}{n} \sum_{i=1}^{n} \left[e^{\theta(x_i - \lambda)} - 1 \right], \ \theta \neq 0,$$

$$I_V(x) = \frac{1}{n} \sum_{i=1}^{n} x_i^2 - \lambda^2. \tag{6}$$

The variance I_V and the index I_θ(commonly known as the Kolm absolute measure of inequality), satisfy POT for all real non-zero values of θ (see Chakravarty and Tyagarupananda, 2009).

Bosmans and Cowell (2010) introduced the notion of 'absolute decomposability' of an inequality measure. According to their definition, an inequality measure $I : D \to R_+^1$ is decomposable if there exists a function A such that for all $x, y \epsilon D$,

$$I(x,y) = A\big(I(x), I(y), \lambda(x), \lambda(y), n(x), n(y)\big), \tag{7}$$

where A is continuous and strictly increasing in its first two arguments and $n(x)$ denotes the dimension of x.

They then demonstrated that an inequality measure $I: D \to R$ satisfies **SYM, POT, POP,** absolute decomposability and translation invariance if and only if there exists a real number θ and a continuous and strictly increasing function $f: R \to R$ with $f(0) = 0$ and such that for all $x \epsilon D$,

$$f\big(I(x)\big) = I_\theta(x) \text{ or } I_V(x) \tag{8}$$

as defined in Chakravarty and Tyagarupananda (2009).

Zheng (2007) argued that the notion of scale/translation invariance of an inequality index should be replaced by that of 'unit consistency'. The latter demands that for any two distributions $x, y \in D$ if $I(x) < I(y)$, then $I(\kappa x) < I(\kappa y)$ for all $\kappa \in R_{++}^1$. The author then characterized all differentiable, unit-consistent **SUD** inequality indices satisfying **SYM, POP, POT** and **NOM**. The resulting index is a positive multiple of

$$I_{\alpha,\beta}(x) = \begin{cases} \dfrac{1}{n\alpha(\alpha-1)\big(\lambda(x)\big)^{\beta}} \displaystyle\sum_{i=1}^{n} \left[x_i^{\alpha} - \big(\lambda(x)\big)^{\alpha} \right], \alpha \neq 0,1, \\ \dfrac{1}{n\big(\lambda(x)\big)^{\beta-1}} \displaystyle\sum_{i=1}^{n} \dfrac{x_i}{\lambda(x)} \log \dfrac{x_i}{\lambda(x)}, (\alpha = 0), \\ \dfrac{1}{n\big(\lambda(x)\big)^{\beta}} \displaystyle\sum_{i=1}^{n} \log \dfrac{\lambda(x)}{x_i}, (\alpha = 1).. \end{cases} \tag{9}$$

for $\alpha, \beta \in R$.

Clearly, this family is a two parameter extension of the generalized entropy family; if $\alpha = \beta$, then $I_{\alpha,\beta}$ coincides with I_e.

3. The Characterization Theorem

We begin this section with the formal definition of translation consistency of an inequality measure.

Definition 1: An inequality index $I : D \to R^1_+$ is said to be translation consistent if for all $x, y \in D^n$, $I(x) < I(y)$ implies $I\left(x + c1^n\right) < I\left(y + c1^n\right)$ for all scalar c such that $x + c1^n$, $y + c1^n \in D^n$.

As already mentioned in the previous section, both variance and the Kolm measure, being translation invariant, are translation consistent as well. However, it can be demonstrated that no member of the generalized entropy class satisfies translation consistency. For example, with $x = (1,3,8)$, $y = (2,3,10)$ and $c = 10$ we have, $I_2(x) = 1.625$ and $I_2(y) = 1.52$ while $I_2\left(x + c1^3\right) = 0.1326$ and $I_2\left(y + c1^3\right) = 0.1689$. Thus, $I_2(x) > I_2(y)$ but $I_2\left(x + c1^3\right) < I_2\left(y + c1^3\right)$. In other words, I_2 fails to satisfy translation consistency.

The first result of this section, whose proof is similar to that of Proposition 1 in Zheng (2007), is on the necessary and sufficient condition of translation consistency.

Proposition 1: An inequality index $I : D \to R^1_+$ is translation consistent if and only if for all $x \in D$ and for all $c > 0$, there exists a continuous function $f : R^1_{++} \times R^1_+ \to R^1_+$, which is increasing in the second argument such that

$$I\left(x + c1^n\right) = f\left(c, I(x)\right) \tag{10}$$

We next mention a result borrowed from Shorrocks (1980).

Proposition 2: A differentiable inequality index I satisfies **SYM, POP, SUD** and **NOM** if and only if these exist functions $\xi : R^1_+ \to R^1_{++}$ and $\phi : R^1_+ \to R^1_+$ such that for any $x \in R^n_+$,

$$I(x) = \frac{1}{n\xi\left(\lambda(x)\right)} \sum_{i=1}^{n} \left[\phi(x_i) - \phi\left(\lambda(x)\right)\right] \tag{11}$$

where ξ is differentiable; ϕ is strictly convex and continuously differentiable.

The first major finding of this section is on the implication of translation consistency.

Proposition 3: If an inequality index I satisfies **SYM, NOM, SUD, POT** and translation consistency, then

$$I\left(x + c1^n\right) = \tau^c I(x) \tag{12}$$

for all $c > 0$ and some constant $\tau > 0$.

Proof: Proceeding as in Proposition 3 of Zheng (2007) and maintaining the same set of notations we arrive at

$$f(c,k) = a(c)k \tag{13}$$

whenever $c > 0$, for some constant $a(c) \neq 0$.

(13), along with (10) implies that for any $x \in D$,

$$I(x + c1^n) = a(c)I(x) \tag{14}$$

from which it follows that for arbitrary $c_1, c_2 > 0$,

$$I\left(x + \overline{c_1 + c_2}\, I^n\right) = a\left(c_1 + c_2\right) I(x) \tag{15}$$

and

$$\begin{aligned} I\left(x + \overline{c_1 + c_2}\, I^n\right) &= a\left(c_2\right) I(x + c_1 I^n) \\ &= a(c_1)a(c_2)I(x) \end{aligned} \tag{16}$$

(15) and (16) together yield:

$$a(c_1 + c_2) = a(c_1)a(c_2) \tag{17}$$

for all $c_1, c_2 > 0$.

The only continuous solution to this equation is given by

$$a(c) = \tau^c \tag{18}$$

where $c > 0$. Positivity of τ is a consequence of non-negativity of I. This completes the proof of the proposition. ■

We have now caught hold of all the machineries necessary for proving our main result.

Theorem 1: An inequality index $I : D \to R_{+}^{1}$ satisfies **SYM, NOM, POP, SUD, POT,** continuous differentiability and translation consistency if and only if it is a positive multiple of the form

$$I_{\gamma,\delta}(x) = \begin{cases} \dfrac{1}{\delta^{\lambda(x)}} I_V(x), \gamma = 0, \delta > 0 \\ \dfrac{1}{n(x)\delta^{\lambda(x)}} \sum_{i=1}^{n} \left[e^{\gamma x_i} - e^{\gamma\lambda(x)} \right], \gamma > 0, \delta > 0 \end{cases}, \tag{19}$$

Proof of the proposition uses the following lemma.

Lemma 1: Whenever I satisfies (12) we have,

$$\sum_{i=1}^{n(x)} I_i(x) = (\ln \tau) I(x). \tag{20}$$

Proof of Lemma 1: Fix $x \in R_{++}^{n}$ and define $g : R_{+}^{1} \to R_{+}^{1}$ by

$$g(c) = I(x + c1^n) \tag{21}$$

By differentiability of I it follows that

$$g'(c) = \nabla I(x + c1^n).1^n \tag{22}$$

where ∇v is the gradient of v.

By continuous differentiability of I we have,

$$g'(0) = \nabla I(x).1^n = \sum_{i=1}^{n} I_i(x) \tag{23}$$

But

$$g(c) = \tau^c I(x). \tag{24}$$

Differentiating both sides of (24) we get,

$$g'(c) = \tau^c \ln \tau I(x) \tag{25}$$

From (25) it readily follows that

$$g'(0) = \ln \tau I(x) \tag{26}$$

Comparing (23) and (26) we are done. ■

Proof of Theorem 1: Let $x \in R^n_{++}$. Differentiating (11) partially w.r.t. x_i we get,

$$n\,I_i(x) = \frac{\psi(\lambda)\left[\phi'(x_i) - \phi'(\lambda)\frac{1}{n}\right] - \sum_{i=1}^{n}\left[\phi(x_i) - \phi(\lambda)\right]\psi'(\lambda)\frac{1}{n}}{\left(\psi(\lambda)\right)^2} \tag{27}$$

Next, taking sum over all i,

$$\sum_{i=1}^{n} n\,I_i(x) = \frac{\psi(\lambda)\left[\phi'(x_i) - \phi'(\lambda)\right] - \psi'(\lambda)\sum_{i=1}^{n}\left[\phi(x_i) - \phi(\lambda)\right]}{\left(\psi(\lambda)\right)^2} \tag{28}$$

Differentiating both sides of (11) partially w.r.t. x_i and making use of (21) we get,

$$\begin{aligned}&\psi(\lambda)\left[\phi'(x_i) - \phi'(\lambda)\right] - \psi'(\lambda)\sum_{i=1}^{n}\left[\phi(x_i) - \phi(\lambda)\right] \\ &\qquad = (\ln\tau)\psi(\lambda)\sum_{i=1}^{n}\left[\phi(x_i) - \phi(\lambda)\right]\end{aligned} \tag{29}$$

Rearranging we get,

$$\psi(\lambda)\sum_{i=1}^{\mu}\left[\phi'(x_{i'}) - \phi'(\lambda)\right] - \left\{\psi'(\lambda) + (\ln\tau)\psi(\lambda)\right\}\sum_{i=1}^{n}\left\{\phi(x_{i'}) - \phi(\lambda)\right\} = 0 \tag{30}$$

Differentiating (23) partially w.r.t. x_i we get,

$$\begin{aligned}&\psi'(\lambda)\left[\phi''(x_i) - \phi''(\lambda)\frac{1}{n}\right] + \sum_{i=1}^{n}\left[\phi'(x_i) - \phi'(\lambda)\psi'(\lambda)\frac{1}{n}\right] \\ &\quad = \left\{\psi'(\lambda) + (\ln\tau)\tau(\lambda)\right\}\left[\phi'(x_i) - \phi'(\lambda)\frac{1}{n}\right] \\ &\quad + \left[\psi''(\lambda)\frac{1}{n} + (\ln\tau)\psi'(\lambda)\frac{1}{n}\right]\sum_{i=1}^{n}\left\{\phi(x_i) - \phi(\lambda)\right\}\end{aligned} \tag{31}$$

Replacing x_i and x_j in (24) and taking difference of both sides we get,

$$\psi(\lambda)\left\{\phi''(x_i) - \phi''(x_j)\right\} = \left\{\psi'(\lambda) + (\ln\tau)\psi(\lambda)\right\}\left[\phi'(x_i) - \phi'(x_j)\right] \tag{32}$$

Since this holds for all $x_i, x_j \in R^1_{++}$, it follows that

$$\frac{\psi'(\lambda)}{\psi(\lambda)} = \delta, \text{ a constant.} \tag{33}$$

Solution to (33) is given by:

$$\psi(\lambda) = K\delta^{\lambda} \tag{34}$$

for some constant K. Since ψ is positive-valued, it follows that $K > 0$.

Substituting (34) in (32) we get,

$$\phi''(x_i) - \phi''(x_j) = (\delta + \ln\tau)\left[\phi'(x_i) - \phi'(x_j)\right], \tag{35}$$

that is,

$$\phi''(x_i) - (\delta + \ln\tau)\phi'(x_i) = \phi''(x_j) - (\delta + \ln\tau)\phi'(x_j) \tag{36}$$

It follows that

$$\phi''(x_i) - (\delta + \ln\tau)\phi'(x_i) = b \tag{37}$$

for some constant b.

Thus, $y = \phi(x)$ satisfies the differential equation

$$\left(D^2 - \gamma D\right)y = b \tag{38}$$

where $Dy = \frac{dy}{dx}, D^2 y = \frac{d^2 y}{dx^2}$ and $\gamma = (\delta + \ln\tau)$.

The complete solution to (38) is given by

$$y = \phi(x) = \begin{cases} c_0 + c_1 x + \frac{b}{2}x^2\,;\ b, c_0, c, \in R\ (\gamma = 0) \\ c_1 - \frac{b}{\gamma}x + c_2\, e^{\gamma x}\,;\ c_1, c_2, b \in R\ (\gamma \neq 0) \end{cases} \tag{39}$$

Using strict convexity of ϕ it further follows that

$b > 0$ in the first case and $c_2 > 0$ in the second.

Simplified forms of I corresponding to the solutions in (39) can be described as follows.

Case I: $\gamma = 0$.

$$I(x) = \frac{1}{nK\delta^{\lambda(x)}} \sum_{i=1}^{n} \left\{ c_0 + c_1 x_i + \frac{b}{2} x_i^2 - c_0 - c_1 \lambda(x) - \frac{b}{2} \lambda^2 \right\} = \frac{b \ \mathrm{var}(x)}{2K\,\delta^{\lambda(x)}} \tag{40}$$

Case II: $\gamma \neq 0$.

$$I(x) = \frac{1}{Kn\delta^{\lambda(x)}} \sum_{i=1}^{n} \left[c_1 - \frac{b}{\alpha} x_i + c_2\, e^{\gamma x_i} - c_1 + \frac{b}{\alpha} \lambda(x) - c_2 e^{\gamma\lambda(x)} \right] = \frac{c_2}{Kn\delta^{\lambda(x)}} \left\{ e^{\gamma x_i} - c_2 e^{\gamma\lambda(x)} \right\} \tag{41}$$

(40) and (41) can be clubbed together to produce a positive multiple of (19). ■

Remark 1: The substitution $\delta = e^{\gamma}$ transforms $I_{\gamma,\delta}$ to I_e. Thus, $I_{\gamma,\delta}$ can be viewed as a 2-parameter extension of the family $\left(I_\theta, I_V\right)$ characterized by Chakravarty and Tyagarupananda (1998).

Remark 2: It can be easily verified that none of the measures $I_{\gamma,\delta}$ is scale invariant. However, the variance is the only member of this family which is unit consistent. Thus, there is a subgroup decomposable inequality index which is both unit consistent and translation consistent.

Zheng (2007) talks of extreme rightist and extreme leftist views of inequality measurement. An extreme rightist measure *I* is one which is reduce when all the incomes are increased by the same proportion, that is, if $I(ax) < I(x)$ for all $a > 1$. One can easily see that none of the $I_{\gamma,\delta}$ indices agrees with the extreme rightist view. Similarly, *I* is an extreme leftist measure if it is increased when all the incomes are increased by the same amount, that is, if $I\left(x + c1^n\right) > I(x)$ for all $c > 0$. A simple calculation shows that $I_{\gamma,\delta}$ conforms to the extreme leftist view if $\delta < e^{\gamma}$.

4. Conclusion

Following Zheng (2007), we have thus identified in this paper a generalization of the class of subgroup-decomposable absolute indices of inequality. A number of questions are yet to be answered. For example, one may be interested to know whether any member of the generalized family satisfies the principle of 'diminishing transfers' suggested by Kolm (1976). One may also feel inclined to enquire whether the axiom of 'intermediate invariance' mentioned in

Chakravarty and Tyagarupananda (2009) can be relaxed to a condition of 'intermediate consistency' to classify a new family of subgroup-decomposable indices. We leave these problems for future research programs.

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