Dynamic Incentives and Compulsory Savings in Microfinance

Dyotona Dasgupta, Prabal Roy Chowdhury

Indian Statistical Institute, Delhi Centre

Abstract

It has been argued, in literature, that providing only credit to the poor without any other service might not be welfare improving. We have examined a dynamic model with continuous time framework where a micro-finance institution (MFI) provides credit along with compulsory savings service to its clients. By providing savings service, the MFI enables the borrowers to build collateral which is required to get loan from the formal sector. Eligibility of getting loan from the formal sector can be thought of as getting out of the poverty trap. In the benchmark case a certificate from MFI along with collateral is required to get loan from the formal sector, later this assumption has been relaxed. We have observed that, under some restrictions, the optimum loan size progressively increases over time in both the cases. Hence, this paper addresses a nearly universal feature of micro-finance loan scheme, namely progressive lending, which has largely been ignored in literature. More precisely, when certification from the MFI is required, initially loan size increases over time and after a certain point of time it hits the efficient amount and then onwards loan size becomes constant over time, given that the rate of interest on deposit is not too small or the borrower is patient enough. When the certificate is not required, then also loan size increases over time, but restriction required for that is stronger. However in this case whether the borrower would at all be able to get the efficient amount or not is not unambiguous. Lastly, we found that borrower's lifetime utility is increasing in fraction of forced savings and decreasing with increase in competition among MFI-s captured via increase in cost of capital.

Keywords: Dynamic Contracts, Collateral, Compulsory Savings, Ex post Moral Hazard, Progressive Lending
JEL: G21, C61, O12

1. Introduction

Informational problems like adverse selection, ex ante moral hazard and enforcement problem or altenatively the problem of ex post moral hazard get aggravated with lack of collateral. Hence, earlier poor people, who by definition are unable to provide any collateral, were thought of as unbankable. Over the last few decades micro-finance has challenged this traditional thinking. It has been shown that micro-finance institutions (henceforth MFIs) have reached millions of poor all over the world(Armendariz and Morduch 2005).

MFI-s use many innovative mechanisms and successfully provide loans to the poorer section of the population. As mentioned in Morduch (1999) MFI-s use i) group lending with or without joint liability, ii) regular repayment schedule, iii) dynamic incentives and iv) collateral substitutes as mechanisms to ensure repayment. The literature has devoted a lot of attention to group lending with or without joint liability and sequential lending among others (see subsection 1.1 for a discussion). However, the most widely used mechanism by MFI-s, dynamic incentives particularly increasing loan amount over time (Morduch 1999, Armendariz and Morduch 2005, Rutherford 2001), has largely remain unexamined.[1]. In this paper we seek

Email addresses: `dyotona11r@isid.ac.in` (Dyotona Dasgupta), `prabalrc1@gmail.com` (Prabal Roy Chowdhury)

[1]Grameen, BancoSol, Bolivia, Bank Rakyat Indonesia Unit Desa, Badan Kredit Desa, Indonesia, FINCA Village banks, Bandhan, SKS India, Annapurna Microfinance (P) Ltd., Spandan India, BRAC in Bangladesh (source: Morduch (1999) and respective websites) are only some examples where loan size increases over time.

to provide a theoretical explanation of two mechanisms used by MFI-s viz. progressive lending where MFI-s increase loan amount contingent on repayment and collateral substitutes (and their interactions).

We have assumed that there is no problem of adverse selection (all borrowers are identical) or ex ante moral hazard (exerting effort or project choice is not a problem). The problem we deal here is of ex post moral hazard in that the borrower will not repay if she has incentive to do so. Thus the contract has to be self-enforcing in that the borrower does not want to default. The MFI[2] stop future loans in case of default and since the borrowers are in a dynamic relationship with the MFI, it works as a credible threat. Incentives are further enhanced when the MFI increases the loan size over time. So, progressive lending coupled with the threat of denying the access to credit in case of default gives the borrowers incentive to repay. However, observe that if the lending relationship has a clear end, borrowers have incentives to default in the final period. Anticipating that, the lender will not lend in the final period, giving borrowers incentives to default in the penultimate period - and so forth until the entire mechanism unravels (Morduch 1999). So, either this relationship needs to go on forever or there should be substantial amount of uncertainty about the end date. Alternatively, collateral substitutes can play an important role here. We have assumed that the borrowers have to save a part of their income with the MFI[3]. A part of their savings is seized[4] by the MFI in case of immature ending of their relationship which happens when the borrowers default. This further prevents the borrowers from defaulting.

Here, we want to make some comments about compulsory savings in general and that taken by MFI-s in particular. Earlier it was perceived that poor people are too poor to save, so the provision of savings service to them was not even considered by policy makers. However, volatile nature of their income forces them to save (Rutherford 2009 among many others). Persistence of different informal savings arrangements like ROSCA, ASCA establishes the claim. And, existence of deposit collector like Jyothi in India (Rutherford 2009), Susu men in Africa (Besley 1995), where savings earn a negative rate of interest, reiterates it. Hence, of late policy makers are considering provision of savings, insurance to the poor people more seriously. In fact, though the main focus of MFI-s was to provide cheap credit to its clients sustainably, recently it has been argued that MFI-s should provide other services like savings, insurance etc. as well. Also, it has been argued that due to little income and many immediate demands (Rutherford 2009), intra-household bargaining problem (Banerjee, Duflo (2011)), hyperbolic discounting (Banerjee, Duflo (2011),Karlan et al., Basu (2015)), and the fact that it is very difficult to say no to the demands of neighbours or relatives in a close knitted community (Rutherdord 2009) it is hard for them to maintain the discipline of saving. So, they in general prefer commitment savings[5]. In this paper we have provided a theoretical model which establishes how provision of compulsory of savings service can be welfare improving.

Specifically, we have addressed compulsory savings in microfinance. Contingent on repayment poor borrowers get future loans only if they save with the MFI and ultimately those savings enable them to get out of the poverty trap. In this paper getting out of the poverty trap is equivalent to getting loan from a formal bank which requires a minimum amount of collateral. Alternatively, we can think that savings give the poor people a financial support and the MFI's job is to help the poor people to acheive that financial support[6]. Here we must mention we are getting this positive result because of the benevolent nature of the MFI. Though results of these paper will still hold if we assume partially motivated MFI-s, we are not sure what would happen with profit maximising monopolistic MFI. Hence this paper, along with other recent papers (reference), argues that welfare of the poor borrowers might not improve if they are served by some profit seeking monopolistic MFI, policy makers should ensure that MFI-s are (partially) motivated.

Turning to the formal framework, we examine a dynamic model where time is continuous. The framework comprises villagers, an MFI, a savings institution and a Bank which lies in the formal sector. Villagers have

[2] We have assumed that there is only one benevolent MFI. We seek to relax this assumption in our future work.

[3] Grameen I and following them many other MFI-s actually take this kind of compulsory savings (Armendariz and Morduch 2005).

[4] In our paper seizing a part of the savings is not necessary. Sufficient delay in returning that amount, such that borrower utility decreases, will also give the same result.

[5] Again emergence of ROSCA, ASCA, Jyothi, Susu men establishes this claim. Also, mention the experiment by Karlan et al.

[6] Compulsory savings/ savings mandate of MFI-s has been criticised (Armendariz and Morduch 2005). Here we have shown the positive effects of it. Given the growing evidences of positive effects of compuslory savings it might actually be the case. However, it needs further empirical study.

access to a production technology, but do not have any wealth so that they have to borrow to finance the project. Given that they also lack any collateral, they are not eligible for formal sector loans. So the MFI is the only source of loans for them in this framework. The MFI is benevolent and provides a sequence of dynamic loan contracts which includes loan amount of each instant and a termination date. As mentioned earlier, the central problem here is one of ex post moral hazard in that the borrowers can always decide not to repay and appropriate the loan extended in that instant. Thus the contract has to be self-enforcing in that the borrower does not want to default. Provision of future loans is conditional on the repayment history. The MFI also provides savings service to its clinets - in case of repayment a borrower has to save a part of her income with the MFI. The borrower saves a part of her net income with the savings institution as well. This provision of savings helps them to reach a target level of savings required for getting connected to the formal sector which yields a larger lifetime utility. Alternatively, we can interpret this lifetime utility as the financial stability achievable once your savings reach a certain level. However, in case of immature ending of the contract, which happens when the borrower defaults, the MFI confiscates a part of her savings. The borrower can withdraw her savings with the outside savings institution whenever she wants to.

At first we have analysed the case where certification from the MFI along with collateral is required to get loan from the formal sector. We have found that under some mild restrictions, loan size weakly increases over time - initially loan amount increases over time and then it hits the efficient amount. The borrower keeps on getting efficient amount from that instant onwards till the contract ends. After that we have considered the case where certification from the MFI is not required, the borrower gets loan from the formal sector if she has the minimum amount of collateral. We got similar results in this case as well, but the restrictions required are stronger here. And also whether the borrower at all gets the efficient amount or not is not ambiguous. However, if she starts getting the efficient amount at any instant, in this case also, she keeps on getting efficient amount from that instant onwards till the contract ends.

Lastly, we have done some comparative static analyses. We have shown that in both the cases that is when certification from the MFI is required and when it is not required, borrower's utility increases with increase in the fraction of forced savings with the MFI. And, it decreases with increase in fraction of forced savings they get back from the MFI in case of default. Utility of the borrower also decreases with increase in competition among MFI-s captured via increase in cost of capital. However, the effect of increase in fraction of savings with the outside savings institution on the borrower's utility is interesting - borrower's utility increases with this change in the former case whereas this effect is ambiguous when certification from the MFI is not required[7].

1.1. Literature Review

Traditionally the literature has focused on group lending and joint liability. Group lending and joint liability has been extensively analysed, among others, Ghatak (1999), Guinanne and Ghatak (1999), etc. The subsequent literature has focused more on dynamic schemes, e.g. immediate and frequent repayment, sequential financing, etc. While sequential lending has been analysed by Roy Chowdhury (2005, 2007) and Aniket (2004), immediate and frequent repayment has been analysed by Chowdhury et al. (2014), Jain and Mansuri (2003) and Fischer and Ghatak (2010).

This paper continues with this theme of dynamic incentives. One of the earliest papers which addresses the role of dynamic incentives in microfinance is Morduch (1999), who reports that the most practised dynamic incentive is increasing loan size over time. Tedeschi (2006) has shown how the welfare of the borrowers will increase if the punishment phase is decreased from infinity to a finite time period. We have followed traditional approach where punishment phase is infinite, however since there is no uncertainty in production so in equilibrium it does not affect the borrower's utility. Also we have allowed (forced) savings hence MFI lends for only finite period of time.

One of the very first paper which deals with progressive lending is "Microfinance Beyond Group Lending"by Aghion and Morduch (2000). In a simple two period model they have shown that dynamic incentives boil down to the threat not to refinance a borrower who defaults on her debt obligations. The threat is

[7]This gives support to the observation of Morduch (1999) "where borrowers have better alternatives, they are likely to value the programs less, and this drives up default rates."Also, they cite Manohar Sharma and Manfred Zeller (1996):"repayment rates are higher in remote communities - i.e., those with fewer alternative credit programs".

enhanced by increasing the loan size over time. However, since the borrowers do not have anything to lose in the last period, they always default in the second period.

Progressive lending in microfinance has also been addressed by Shapiro (2012). In his paper the uncertainty comes from the borrowers' patience level and lenders are unaware of that. He found that the loan size increase over time only in the unique efficient equlibrium. However, there exists inefficient equlibria where all borrowers default and it cannot be guaranteed that efficient equilibrium will always be chosen.

Ghosh and Ray (2001) analysed a credit market consisting of two types of borrowers - namely good and bad, where good borrowers always repay, and bad borrowers never do. In their mechanism, progressivity helps in screening good types from the bad. The optimal contract has two phases - new and old. The profit maximising lender offers the former contract to new borrowers whereas the latter is offered to those borrowers who have repaid loans in the previous period(s) successfully. The loan amount of new phase is lower than that of old phase. However, since bad borrowers never repay so new phase consists of only one period, the defaulters do not get any future loan and those who repay enter into old phase. So, progressivity in loan size for more than one period cannot be explained via this model.

Egli (2004) develops model, where there are two type of borrowers - one type (good) always repay and the other type (bad) is strategic. Due to the divisibility of a project progressive lending is observed in that model.

Above mentioned all the papers explain progressivity in loan size via some kind of informational asymmetry. In our paper there is no informational asymmetry, everything is perfectly observable. The only problem here is the borrower is strategic in that she defaults whenever she has incentive to do so.

In a very generalised infinite horizon model Ray (2002) has addressed *expost moral hazard* and has analysed the time structure of "Efficient Self Enforcing(ESE)"sequences. He has shown that a relationship can be divided into an "initial phase"and a "mature phase". In the initial phase loan size increases over time and in mature phase when borrower's payoff is maximum loan size remains constant over time and the length of the initial phase depends on the lender's bargaining power. He has shown that every "Efficient Self Enforcing(ESE)"sequence, must after a finite set of dates, exhibit a continuation that maximises the agent's payoff from the class of all ESE sequences. That is every relationship must end with a mature phase, whether there will be any initial phase or not depends on the starting point of the relationship. So, whether loan size will be increasing over time or not depends on the starting point of the relationship. However, loan size remains constant when borrower's utility is maximum.

In our model we have considered benevolent MFI. So, the borrower's utility is maximised from the very first instant of time. Even in this set up we have observed progressivity in loan size. The result is driven by the fact that unlike Ray's paper we have allowed compulsory savings which allows the borrower to get out of the poverty trap, so continuation payoff keeps on increasing with time which in turn increases the borrower's incentive to repay. Also, to ensure that no-deviation constraint does bind Ray has assumed that discount factors are not close to unity. Hence, unconstrained maximiser amount of loan is not incentive compatible. However, in our model since continuation payoff increases over time unconstrained maximiser amount of loan becomes incentive compatible after a certain instant of time. So, the borrower gets that amount of loan from that period onwards which was not acheivable in Ray's model.

Rutherford (2009), in his book, has extensively studied the lives of poor people and he has found that they do need "basic personal financial intermediation[8]". In one randomized study of microsavings Dupas and Robinson(2009) found that it reduces average poverty, among female vendors in a rural Kenyan market. Ashraf, Karlan and Yin in their series of papers have shown that access to commitment savings product increases the welfare and discipline of savings though it diminishes over time.

However there is hardly any theoretical literature which addresses savings in Microfinance. Aniket(2011) has addressed savings in microfinance in a theoretical model. In that setting two individuals form a group endogenously, one individual(the borrower) gets loan from the MFI and the other individual(the saver) saves with MFI and that saving, along with the loan from MFI, is used to finance the borrower's project. The wealth threshold for being a borrower is greater than that for being a saver. In equilibrium, their will be negative assortative matching within the group along wealth lines. Also, the paper shows that under some assumptions the MFI-s offering both borrowing and saving opportunities are able to reach poorer individuals

[8] services which enable poor people to convert their small savings into usefully large lump sums

than MFI-s which offer only borrowing opportunities. Our paper addresses individual lending and saving. And also shows that the utility of the poor is more when the savings technology is offered by MFI, in compare to any other outside savings opportunity.

Basu (2015) has proposed a model which addresses the puzzle of simultaneous borrowing and saving. He has done that by assuming that borrowers are present biased. While in our paper we have assumed that all borrowers are time consistent we have explicitly shown that how provision of compulsory savings can actually help poor borrowers to acheive financial stability.

The rest of the paper is organized as follows. Section 2 describes the benchmark case where certification from MFI is required to get loan from the formal sector, in Section 3 we will relax the assumption, only collateral is required to get loan from the bank. In Section 4 we have compared these two cases. Some comparative statics analyses have been done in section 5. Section 6 concludes.

2. Benchmark Case: Certification from MFI is required

We will start with the benchmark case where a certificate from the MFI along with S amount of collateral is required to get loan from the bank. We will argue that the optimum loan size progressively increases over time.

2.1. Economic Environment

The economy consists of many risk neutral villagers, an informal sector and a formal sector. An MFI and a savings institution lie in the informal sector whereas the formal sector comprises of a bank. The villagers can potentially be either in the informal or in the formal sector. Collateral is required to get loan from the bank. Hence, the villagers who do not have any money or assets are not eligible for formal sector loans. However, they can obtain collateral free loans from the MFI. Production function $f(\cdot)$ in the informal sector is deterministic in nature. And, the following assumption on $f(\cdot)$ will be maintained throughout:

A 1. $f(\cdot) > 0;\ f'(\cdot) > 0;\ f''(\cdot) < 0;\ f'(0) = \infty$ *and* $f'(\infty) = 0$.

Thus the production function is increasing, strictly concave, smooth and satisfies the Inada conditions.

Definition 1. *Given A1, the efficient scale of investment (k^*), where*

$$k^* = \underset{k}{argmax}[f(k) - k],$$

is well defined.

The villagers save with the savings institution and the MFI, that savings enable them to accumulate collateral required for formal sector loan. Turning to the formal sector, minimum S amount of collateral is required to get loan from the bank. Once a borrower obtains a loan from the bank, she gets connected to the formal sector, with the possibility of future loans, etc.. Let V denote the present discounted value of the life time utility from getting connected to the formal sector (gross of S). Alternatively, S can be interpreted as the minimal amount required so as to achieve a measure of financial stability and consumption smoothing, with V denoting the lifetime discounted utility from such savings.[9] We assume that the net gain from moving to the formal sector that is, the difference in lifetime continuation payoff and collateral required to move to the formal sector is sufficiently high.

A 2. $V - S > \int_0^\infty e^{-rt} f(k^*)dt = \frac{1}{r} f(k^*)$.

We will observe later that k^* is the maximum amount a borrower gets from the MFI. So, the assumption says that the maximum possible lifetime utility of the borrower from staying in the informal sector, even when repayment is not required, is less than the net gain from moving to the formal sector. This is reasonable, in that, once a borrower is connected to the formal sector, she has access to more productive technologies, greater legal and state protection, etc..

[9] Roodman (2011) suggests that such consumption smoothing is an important motivation behind joining MFIs.

We consider an infinite horizon dynamic framework where time, denoted by t, is continuous. Let r denote the real rate of interest and future discount factor, where $r > 0$. At the beginning, i.e. at $t = 0$, the MFI announces a loan scheme $\langle \{k_t\}_{t=0}^{\infty}, T \rangle$, where k_t denotes the amount loaned at every t and T denotes the termination date of this loan scheme. This contract also includes that at every t the borrower has to deposit $\alpha(\in [0, 1])$ part of her net income. An individual either accepts or rejects this loan scheme with the game ending in case the borrower rejects. If she rejects then she gets her outside option which is normalised to zero.

If she accepts, then at every t, $0 < t \leq T$; there are two stages :

Stage 1: Subject to there having been no default till this point, the MFI makes a loan of k_t to the borrower, which yields an instantaneous output of $f(k_t)$ to the borrower.

Stage 2: The borrower then decides whether to repay or not:

- In case the borrower decides to repay, the MFI obtains k_t towards loan repayment, α part of the remaining amount, i.e. $f(k_t) - k_t$, is deposited with the MFI and β part is deposited with the savings institution. Any amount deposited attracts an interest at the rate r. And the rest i.e. $(1 - \alpha - \beta)[f(k_t) - k_t]$ is consumed by the borrower instantaneously.
- In case the borrower defaults, she enjoys the entire current income $f(k_t)$. But the MFI terminates the contract, and the borrower does not get any loan from that point onwards. Moreover, $\alpha - \alpha'$ part of her savings with the MFI till date, is confiscated by the MFI. The borrower consumes $1 - \beta$ part and saves the rest with the savings institution. She also decides when to withdraw her savings from the savings institution. Let $T_D(t)$ be that instant of time. Observe, in this benchmark case a certificate from MFI, alongwith S amount of collateral, is required to get loan from the formal bank. So, she can never get connected to the formal sector in case of default.

At this point note that the framework involves compulsory savings[10]. The borrower has to deposit α portion of her net income with the MFI. This serves two purposes. First, of course, the amount deposited helps to incentivise repayment, and thus is beneficial to the MFI (and ultimately to the borrower as well). Second, such deposits also provide a direct service to the borrower, in that it serves as a savings technology to the borrower over and above savings technology available to her. Our analysis go through when outside savings technology is not available which is often the case in rural economy, so in that sense our model is robust. For simplicity, we have assumed that α and β i.e. the proportion of the net income to be deposited with MFI and other institution, respectively, are *exogenously* given.

The MFI is assumed to be benevolent, and maximises the lifetime utility of the borrowers.

2.2. Equilibrium

Consider a scheme $\langle \{k_t\}_{t=0}^{\infty}, T \rangle$. The objective of the MFI is to maximise the borrower's utility, subject to (i) the feasibility condition (FC) that the amount of savings accumulated by the borrower exceeds S by the end of the scheme (so that she can move to the formal sector), and (ii) the dynamic incentive compatibility constraints that is at no $t \leq T$, the borrower has an incentive to default.

Therefore the problem of the MFI is to:

[10] Bolivian village banking MFI Crédito com Educacíon Rural, Finca Nicaragua take forced savings.

$$\underset{<\{k_t\}_{t=0}^{\infty},T>}{\text{Maximize}} \qquad \int_0^T e^{-rt}\Big[(1-\alpha-\beta)[f(k_t)-k_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-k_t]dt - S + V\right]$$

Subject to:

$$\text{FC:} \quad \int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-k_t]dt \geqslant S,$$

$$\text{DIC:} \quad \int_t^T e^{-r(t^{'}-t)}\Big[(1-\alpha-\beta)[f(k_{t'})-k_{t'}]\Big]dt^{'} + e^{-r(T-t)}\left[\int_0^T e^{r(T-t^{'})}(\alpha+\beta)[f(k_{t'})-k_{t'}]dt^{'} - S + V\right]$$

$$\geqslant (1-\beta)\left[f(k_t) + \int_0^t e^{r(t-t^{'})}\alpha^{'}[f(k_{t'})-k_{t'}]dt^{'}\right]$$

$$+ e^{-r(T_D(t)-t)}\left[e^{r(T_D(t)-t)}\left[\beta\Big[f(k_t) + \int_0^t e^{r(t-t^{'})}\alpha^{'}[f(k_{t'})-k_{t'}]dt^{'}\Big] + \beta\int_0^t e^{r(t-t^{'})}[f(k_{t'})-k_{t'}]dt^{'}\right]\right] \qquad \forall t \leqslant T.$$

The left side of FC is the total savings till T^{th} instant which needs to be at least as high as S. Turning to the DIC, at any arbitrary instant of time t utility from repayment must be weakly greater than that from default. The left side of DIC is the utility from repayment where the first term is the dicounted utility from consumption. And the second term is the present discounted utility she enjoys at T^{th} instant. Observe, by the feasibility constraint, her savings becomes at least S by that T^{th} instant. She uses S part of her savings as collateral and gets connected to the formal sector. She derives utility by consuming the rest. Now, the right side of DIC is the utility from default. The borrower gets $f(k_t)$ and $\alpha^{'}$ part of her savings with the MFI till date, she consumes β part of that, the first term is the utility she derives from that consumption. Now, in case of default at t she withdraws the entire saved amount at $T_D(t)$ instant of time, second term is the discounted utility from that. Observe, DIC can be simplified to:

$$\int_t^T e^{-r(t^{'}-t)}\Big[(1-\alpha-\beta)[f(k_{t'})-k_{t'}]\Big]dt^{'} + e^{-r(T-t)}\left[\int_0^T e^{r(T-t^{'})}(\alpha+\beta)[f(k_{t'})-k_{t'}]dt^{'} - S + V\right]$$

$$\geqslant f(k_t) + \int_0^t e^{r(t-t^{'})}(\alpha^{'}+\beta)[f(k_{t'})-k_{t'}]dt^{'}; \qquad \forall t \leqslant T.$$

Therefore the problem of the MFI becomes:

$$\underset{<\{k_t\}_{t=0}^{\infty},T>}{\text{Maximize}} \qquad \int_0^T e^{-rt}\Big[(1-\alpha-\beta)[f(k_t)-k_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-k_t]dt - S + V\right]$$

Subject to:

$$\text{FC:} \quad \int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-k_t]dt \geqslant S, \tag{1}$$

$$\text{DIC:} \int_t^T e^{-r(t^{'}-t)}\Big[(1-\alpha-\beta)[f(k_{t'})-k_{t'}]\Big]dt^{'} + e^{-r(T-t)}\left[\int_0^T e^{r(T-t^{'})}(\alpha+\beta)[f(k_{t'})-k_{t'}]dt^{'} - S + V\right]$$

$$\geqslant f(k_t) + \int_0^t e^{r(t-t^{'})}(\alpha^{'}+\beta)[f(k_{t'})-k_{t'}]dt^{'}; \qquad \forall t \leqslant T. \tag{2}$$

2.3. Optimal Contract

Now we can state our first proposition.

Proposition 1. *Let A1 and A2 hold. Then the feasibility constraint (FC) binds at the optimal contract.*

Proof: Firstly we will prove that the borrower's lifetime utility decreases with T that is the borrower's lifetime utility decreases as the time, she remains in the informal sector, increases. Given a loan sequence we will increase T marginally, without changing the loan amount and will observe its effect. In Step 2 we have shown that from Step 1 it follows that if FC does not bind then that cannot be an optimum contract.

Step 1. Here, we prove that the borrower's utility decreases with T or alternatively, partial differentiation of the borrower's lifetime utility is negative, that is

$$\frac{\partial}{\partial T}\left[\int_0^T e^{-rt}\Big[(1-\alpha-\beta)[f(k_t)-k_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-k_t]dt - S + V\right]\right] < 0.$$

Note that, $e^{-rT}\Big[\int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-k_t]dt - S + V\Big]$
$= \int_0^T e^{-rt}(\alpha+\beta)[f(k_t)-k_t]dt + e^{-rT}\Big[V-S\Big].$

Thus, $\int_0^T e^{-rt}\Big[(1-\alpha-\beta)[f(k_t)-k_t]\Big]dt + e^{-rT}\Big[\int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-k_t]dt - S + V\Big]$
$= \int_0^T e^{-rt}[f(k_t)-k_t]dt + e^{-rT}\Big[V-S\Big]$

$$\begin{aligned}
&\frac{\partial}{\partial T}\left[\int_0^T e^{-rt}\Big[(1-\alpha-\beta)[f(k_t)-k_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-k_t]dt - S + V\right]\right] \\
= \ & \frac{\partial}{\partial T}\left[\int_0^T e^{-rt}[f(k_t)-k_t]dt + e^{-rT}\Big[V-S\Big]\right] \\
= \ & e^{-rT}[f(k_T)-k_T] - re^{-rT}(V-S) \\
= \ & e^{-rT}\Big[[f(k_T)-k_T] - r(V-S)\Big]. \qquad (3)
\end{aligned}$$

Clearly, given A2, it must be that $f(k_T) - k_T - r(V-S) < 0$, so that (3) is negative.[11]

Step 2. Now suppose that the FC does not bind. Consider another scheme that is identical to the original scheme, but ends at $T' < T$. For T' close to T, FC will be satisfied, and the borrower's utility increases by the preceding argument. Finally, DIC holds for all $t \le T'$, as the original scheme satisfied the DIC and $T' < T$ (we mimic the argument in step 1 to prove this). ■

Intuitively, if the borrower continues with the MFI after building the required collateral, his savings will go up but at the cost of delay in joining the formal sector. Due to this delay, present value of continuation payoff decreases and this waiting cost becomes more than the gain from extra savings.

Hence, the problem of the MFI becomes:

$$\underset{\langle\{k_t\}_{t=0}^{\infty},T\rangle}{\text{Maximize}} \quad \int_0^T e^{-rt}\Big[(1-\alpha-\beta)[f(k_t)-k_t]\Big]dt + e^{-rT}V$$

Subject to:

$$\text{FC:} \quad \int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-k_t]dt = S, \qquad (4)$$

$$\text{DIC:} \int_t^T e^{-r(t'-t)}\Big[(1-\alpha-\beta)[f(k_{t'})-k_{t'}]\Big]dt' + e^{-r(T-t)}V \geqslant f(k_t) + \int_0^t e^{r(t-t')}(\alpha'+\beta)[f(k_{t'})-k_{t'}]dt'; \quad \forall t \leqslant T. \qquad (5)$$

Let the optimum scheme be denoted by $<\{k_t^*\}_{t=0}^{\infty}, T^*>$, where T^* is the time required to save S under this scheme, i.e.

$$\int_0^{T^*} e^{r(T^*-t)}(\alpha+\beta)[f(k_t^*)-k_t^*]dt = S.$$

[11] $f(k^*) - k^* = \text{Max}[f(k)-k]$ i.e. $f(k_T) - k_T \leqslant f(k^*) - k^* < f(k^*)$. Therefore, when A2 holds $f(k_T) - k_T < r(V-S)$.

Before proceeding further let us introduce

Definition 2. $\mathbf{k_{It}}$ *is the amount of loan for which the DIC at t binds.*

Now, we are in a position to introduce our next proposition.

Proposition 2. *Consider the optimal scheme* $< \{k_t^*\}, T^* >$. *Then* $k_t^* = \min\{k_{It}, k^*\}$ *for almost all t.*

Proof: Observe that due to DIC $k_t^* \leq k_{It}$ $\forall t \leq T^*$.

Now, consider the set $\mathcal{M} = \{t \leqslant T^* : \text{Either } k_t^* < \min\{k_{It}, k^*\} \text{ Or } k_t^* \in (k^*, k_{It}]\}$. The claim is that the measure of the set $\mathcal{M}$ is zero[12]. Suppose not. Then $\exists M (\subsetneq \mathcal{M}$ and $M \ni t \leq T^{'} < T^*)$such that the measure of $M > 0$.

We then construct another scheme $< \{k_t'\}, T^* >$ such that:

$$k_t' = \begin{cases} k_t^* & \text{when } t \notin M \text{ and } t \leq T^* \\ (k_t^*, \min\{k_{It}, k^*\}] & \text{when } t \in M \text{ and } k_t^* < \min\{k_{It}, k^*\} \\ [k^*, k_t^*) & \text{when } t \in M \text{ and } k_t^* \in (k^*, k_{It}]. \end{cases}$$

By construction, $< \{k_t^{'}\}, T^* >$ satisfies DIC[13].

Also observe that, $\forall t \in M$, $[f(k_t^{'}) - k_t^{'}] > [f(k_t^*) - k_t^*]$[14]. Further, $\forall t \leqslant T^*$and $t \notin M$, we have that $k_t^{'} = k_t^*$, and therefore $[f(k_t^{'}) - k_t^{'}] = [f(k_t^*) - k_t^*]$. Therefore,

$$\int_0^{T^*} e^{r(T^*-t)}(\alpha+\beta)[f(k_t^{'}) - k_t^{'}]dt > \int_0^{T^*} e^{r(T^*-t)}(\alpha+\beta)[f(k_t) - k_t]dt = S.$$

Since time is continuous,
$\exists \Delta^{'} > 0$ such that $\int_0^{T^*-\Delta^{'}} e^{r(T^*-\Delta^{'}-t)}(\alpha+\beta)[f(k_t^{'}) - k_t^{'}]dt \geqslant S$ and $\Delta^{'} < T^* - T^{'}$.
Thus, for $\Delta^{'}$ small enough, we have constructed another scheme $< \{k_t^{'}\}, T >$[15] that satisfies DIC and FC, and ends earlier than T^*. Thus, by the argument in Proposition 1, T^* can't be the optimum.

So, we can conclude that $k_t^* = \min\{k_{It}, k^*\}$ for almost all t. ■

Intuitively, the MFI wants to design the scheme in such a way that the borrower can connect with the formal sector as soon as possible. Thus ideally the MFI would like to lend an amount of k^* at every instant as this maximises the amount saved. However, due to ex post moral hazard problem, the MFI cannot lend more than k_{It} at any $t \leqslant T^*$.

So, at optimum $k_t^* = \min\{k_{It}, k^*\}$ for almost all $t \leqslant T^*$ and $\int_0^{T^*} e^{r(T^*-t)}(\alpha+\beta)[f(k_t^*)-k_t^*]dt = S$. Observe, the sequnce $\langle k_t^* \rangle$ is determined recursively, in Appenix B we have discussed the procedure of determining the optimum loan sequence and corresponding time to reach a given S in details.

2.4. Dynamics of loan size

Now we are in a position to address our main finding of this paper. We will see that under some restrictions loan size will be non-decreasing over time.

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[12]Note that any $k_t^* > k_{It}$ is not incentive compatible, so to prove the proposition it is sufficient to prove that the measure of the set $\mathcal{M}$ is zero

[13]any $k_t \leq k_{It}$ where $t \leq T^*$ satisfies DIC and by construction $k_t^{'} \leq k_{It}$

[14]Observe that $f(k) - k$ is maximum at k^* Now, suppose
A. $t \in M$ and $k_t^* < \min\{k_{It}, k^*\}$ then if we increase k_t then $f(k_t) - k_t$ increases
B. $t \in M$ and $k_t^* \in (k^*, k_{It}]$ then if we decrease k_t then $f(k_t) - k_t$ increases.

[15]where $T = T^* - \Delta^{'}$

From proposition 2 we know that $k_t^* = \min\{k_{It}, k^*\}$. Now, k^* is a constant, so we can say that k_t^* (weakly) increases over time whenever k_{It} increases with t. Before proving it formally let us try to understand it intuitively.

Observe, from Definition 2 we have

$$f(k_{It}) = \int_t^{T^*} e^{-r(t'-t)}\Big[(1-\alpha-\beta)[f(k_{t'}^*)-k_{t'}^*]\Big]dt' + e^{-r(T^*-t)}V - \int_0^t e^{r(t-t')}(\alpha'+\beta)[f(k_{t'}^*)-k_{t'}^*]dt'; \qquad \forall t \leqslant T^*$$

Differentiating R.H.S. of DIC with respect to t we have:

$$(1-\alpha-\beta)r\int_t^{T^*} e^{-r(t'-t)}[f(k_{t'}^*)-k_{t'}^*]dt' + re^{-r(T^*-t)}V - (1-\alpha+\alpha')[f(k_t^*)-k_t^*]$$
$$-r\int_0^t e^{r(t-t')}(\alpha'+\beta)[f(k_{t'}^*)-k_{t'}^*]dt' \tag{6}$$

Now observe that the first term is positive, so k_{It} will surely be increasing over time if
$re^{-r(T^*-t)}V > (1-\alpha+\alpha')[f(k_t^*)-k_t^*] + r\int_0^t e^{r(t-t')}(\alpha'+\beta)[f(k_{t'}^*)-k_{t'}^*]dt'$

Also, observe $r\int_0^t e^{r(t-t')}(\alpha'+\beta)[f(k_{t'}^*)-k_{t'}^*]dt' < e^{-r(T^*-t)}rS$
So, k_{It} increases with t whenever $e^{-r(T^*-t)}(V-S) > \frac{1}{r}[f(k_t^*)-k_t^*]$. So, by A2 we can say that when T^* is not so large, k_{It} increases over time. Intuition behind this is quite simple - by A2 we know that the net gain in lifetime utility from moving to the formal sector is sufficiently high. So, when time left to achieve that is small, borrower would repay higher and higher amount such that she can move to the formal sector sooner.

Now observe that T^* is endogenously determined, so let us now try to find out the sufficient condition on parameters for k_{It} to be increasing. It turns out that the required sufficient condition is

A 3. $1-(\alpha-\alpha') < r$

So, the condition is saying that -

- Either r must be sufficiently high which can be interpreted in two ways - high r means interest rate on deposit is high enough so time required to save S is small (ceteris paribus) or altenatively, higher r means the borrower is patient enough and hence repay higher loan.
- And for a given r maximum amount the borrower repays increases over time when $\alpha-\alpha'$ is high enough. Observe that we can think $\alpha-\alpha'$ as the bargaining power of the MFI; it confiscates $\alpha-\alpha'$ part of the borrower's savings in case of default so when it is high enough the borrower's loss becomes higher and higher with default so she repays higher amount as her savings with the MFI increases.

Hence we observe that k_{It} increases over time given A3. Hence the following lemma.

Lemma 1. *Given A1, A2 and A3 k_{It} is increasing over time* $\forall t \in [0, T^*]$.

Proof: Observe equation (6) can be written as:
$r[\int_t^{T^*} e^{-r(t'-t)}\Big[(1-\alpha-\beta)[f(k_{t'}^*)-k_{t'}^*]\Big]dt' + e^{-r(T^*-t)}V - \int_0^t e^{r(t-t')}(\alpha'+\beta)[f(k_{t'}^*)-k_{t'}^*]dt']$
$-(1-\alpha+\alpha')[f(k_t^*)-k_t^*]$
$= rf(k_{It}) - (1-(\alpha-\alpha'))[f(k_t^*)-k_t^*]$
where the last equality comes from DIC.

Given A3 $rf(k_t)-(1-\alpha)[f(k_t)-k_t] > 0$[16]. Hence the result. ■

Before stating our main result we need to introduce another definition and lemma.

[16]From Proposition 2 we know that if $k_{It} < k^*$ then $k_t^* = k_{It}$, so it comes immediately. When $k_{It} > k^*$ $k_t^* = k^*$ and since $f'(\cdot) > 0$, $f(k_{It}) > f(k_t^*) > f(k_t^*) - k_t^*$

Definition 3. *Let $\hat{t}(T^*)$ solve*

$$k_{I\hat{t}} = k^*,$$

i.e. $\hat{t}(T^*)$ denotes the t such that the maximum permissible loan according to the DIC at t is equal to the efficient amount of loan.

Lemma 2. *Given A1, A2 and A3:*

1. *$\hat{t}(T^*)$ is unique and $\hat{t}(T^*) < T^*$, $\forall S$.*
2. *$k_{It} > k^*$, $\forall t > \hat{t}(T^*)$.*

Proof:

1. As we have observed in the earlier Lemma k_{It} increases monotonically with t given A3. Also, k_{It} is continuous. Hence, $\hat{t}(T^*)$ is unique, if it exists.
   Observe that, $f(k_{IT}) = (1-\alpha-\beta)[f(k_T) - k_T] + V - \int_0^T e^{r(T-t)}(\alpha' + \beta)[f(k_t^*) - k_t^*]dt > V - S$ [17] $> \frac{f(k^*)}{r}$ where the last inequality comes from A2. Hence, $\hat{t}(T^*) < T^*$, $\forall S$.
2. It is straight forward from the previous argument. ■

Observation. The above lemma proves that k_{It} will ultimately exceed k^*, however it might happen that $k_{It} > k^*$ $\forall t \in [0, T^*]$. This will not happen when
$f(k_{I0}) = \int_0^{T^*} e^{-rt}(1-\alpha-\beta)[f(k_t^*) - k_t^*]dt + e^{-rT^*}V = e^{-rT^*}\left[V - S + \frac{1}{\alpha+\beta}S\right] < f(k^*)$.
It is saying that DIC will bind initially when time required to achieve S or alternatively S is sufficiently high. It is quite intuitive in that when S is not very high, the borrower can move to the formal sector very soon, so the borrower repays even the efficient amount from the very beginning.

Now, we are equipped to state the main result result of this paper.

Proposition 3. Characterisation of Loan Size:
Given A1, A2, A3 there will be weak progressive lending for almost all $t \in [0, T^*]$.
Precisely, loan size will increase for almost all $t \in [0, \hat{t}(T^*))$ and then it will become constant over time.

Proof: From Lemma 2 we know $k_{It} < k^*$, $\forall t < \hat{t}(T^*)$, hence $k_t^* = k_{It}$ for almost all $t < \hat{t}(T^*)$ and from Lemma 1 we know that k_{It} is increasing $\forall t \leqslant \hat{t}(T^*)(< T^*)$. Hence, loan size will increase for almost all $t < \hat{t}(T^*)$.
And from $\hat{t}(T^*)$ onwards $k^* = \min\{k^*, k_{It}\}$ hence $k_t^* = k^*$ for almost all $t \in [\hat{t}(T^*), T^*]$.
Hence, the result. ■

3. Certification from MFI is not required to get loan from the formal sector

3.1. Economic Environment

In this section we will relax the assumption that a certificate from the MFI is required to get loan from the formal sector. Like before, minimum S amount of collateral is required to get loan from the bank and V is the present discounted value of the lifetime utility from getting connected to the formal sector (gross of S). At $t = 0$ the MFI announces a loan scheme $\langle \{k_{st}\}_{t=0}^{\infty}, T_R \rangle$ where k_{st} and T_R are corresponding loan amount of every t and termination date of the loan scheme respectively. If the borrower accepts the contract then at every t, $0 < t \leq T$; there are two stages :

Stage 1: Subject to there having been no default till this point, the MFI makes a loan of k_{st} to the borrower, which yields an instantaneous output of $f(k_{st})$ to the borrower.

Stage 2: The borrower then decides whether to repay or not:

[17] $S = \int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t^*) - k_t^*]dt > \int_0^T e^{r(T-t)}(\alpha' + \beta)[f(k_t^*) - k_t^*]dt$, since $\alpha > \alpha'$

- In case of repayment, the MFI obtains k_{st} towards loan repayment. α_s and β_s part of the net income are deposited with the MFI and the savings institution respectively, r being the interest rate. The rest i.e. $(1-\alpha_s-\beta_s)[f(k_{st})-k_{st}]$ is consumed by the borrower instantaneously.
- In case of default,

  The MFI terminates the contract, and the borrower does not get any loan from that point onwards.

  The borrower enjoys the entire current income $f(k_{st})$. Also she gets back $\alpha_s^{'}$ part of her savings with the MFI till date. The borrower consumes β_s part and saves the rest with the outside savings institution.

  Now, since she can get loan from the formal bank in case she has at least S amount of collateral. She decides when to withdraw her savings from the savings intitution accordingly. Let $T_D(t)$ be the time of withdrawal (corresponding to default at t).

3.2. Equilibrium

So, after accepting the contract at every instant t, the borrower's problem is to decide whether to repay or not and in case of default she chooses $T_D(t)$ such that her utility is maximum. Observe, while choosing $T_D(t)$ there can be two cases: (*i*) she withdraws her saving before it reaches S, (*ii*) she waits till it becomes at least S, so that she can use that as collateral and move to the formal sector. Maintaining similarity in notation let us call the constraint as feasibility condition in case of default (FC$_D$):

$$\mathrm{FC}_D : e^{r(T_D(t)-t)}\left[\beta_s\left[f(k_{st})+\int_0^t e^{r(t-t^{'})}\alpha_s^{'}[f(k_{st^{'}})-k_{st^{'}}]dt^{'}\right]+\left[\int_0^t e^{r(t-t^{'})}\beta_s[f(k_{st^{'}})-k_{st^{'}}]dt^{'}\right]\right]\geq S$$

where the first term is the amount she saves at t^{th} instant with the savings institution and the second term is the savings she already had with the institution. She withdraws at $T_D(t)$ instant, so feasibility condition requires her total saving at that time be at least S.

Simplifying the condition we can write it as:

$$\mathrm{FC}_D : e^{r(T_D(t)-t)}\beta_s\left[f(k_{st})+(1+\alpha_s^{'})\int_0^t e^{r(t-t^{'})}[f(k_{st^{'}})-k_{st^{'}}]dt^{'}\right]\geq S$$

So, her utility from default is:

$$\begin{aligned}
&(1-\beta_s)\left[f(k_{st})+\alpha_s^{'}\int_0^t e^{r(t-t^{'})}[f(k_{st^{'}})-k_{st^{'}}]dt^{'}\right]\\
&+e^{-r(T_D(t)-t)}\left[e^{r(T_D(t)-t)}\beta_s\left[f(k_{st})+(1+\alpha_s^{'})\int_0^t e^{r(t-t^{'})}[f(k_{st^{'}})-k_{st^{'}}]dt^{'}\right]\right] \quad \text{when FC}_D \text{ does not hold,}\\
&(1-\beta_s)\left[f(k_{st})+\alpha_s^{'}\int_0^t e^{r(t-t^{'})}[f(k_{st^{'}})-k_{st^{'}}]dt^{'}\right]\\
&+e^{-r(T_D(t)-t)}\left[e^{r(T_D(t)-t)}\beta_s\left[f(k_{st})+(1+\alpha_s^{'})\int_0^t e^{r(t-t^{'})}[f(k_{st^{'}})-k_{st^{'}}]dt^{'}\right]\quad -S+V\right] \quad \text{when FC}_D \text{ holds.}
\end{aligned}$$

where the first term of each expression corresponds to the utility she derives from consumption at t^{th} instant and the second term is the utility she enjoys at $T_D(t)^{\text{th}}$ instant.

Now, given that moving to the formal sector is welfare improving, captured by A2, the borrower would always wait till her saving reaches S. However, after that she has no incentive to wait since her utilty would increase with increase in saving, but this increase in utility is less than the waiting cost of moving to the formal sector. So, the borrower withdraws the saving as soon as it becomes S. Hence, the following proposition.

Proposition 4. *FC_D binds at optimum $\forall t \leq T_R$.*

Proof: First we will prove that the borrower would not choose any $T_D(t)$ such that FC_D does not hold. Then we will show that FC_D binds at optimum.

Step 1: Observe that, for any $T_D(t)$ utility from default, when FC_D does not hold, is

$$f(k_{st}) + e^{rt}(\alpha_s^{'} + \beta_s)\Big[\int_0^t e^{-rt^{'}}[f(k_{st'}) - k_{st'}]dt^{'}\Big]$$

which is independent of $T_D(t)$[18].
Now, utility from default, when FC_D holds, is

$$f(k_{st}) + e^{rt}(\alpha_s^{'} + \beta_s)\Big[\int_0^t e^{-rt^{'}}[f(k_{st'}) - k_{st'}]dt^{'}\Big] + e^{-r(T_D(t)-t)}\Big[V - S\Big] \tag{7}$$

Given A2, $V - S > 0$ Hence, $f(k_{st}) + e^{rt}(\alpha_s^{'} + \beta_s)\Big[\int_0^t e^{-rt^{'}}[f(k_{st'}) - k_{st'}]dt^{'}\Big] + e^{-r(T_D(t)-t)}\Big[V - S\Big]$
$> f(k_{st}) + e^{rt}(\alpha_s^{'} + \beta_s)\Big[\int_0^t e^{-rt^{'}}[f(k_{st'}) - k_{st'}]dt^{'}\Big]$
So, at optimum, the borrower would choose $T_D(t)$ such that FC_D holds.

Step 2: Differentiating (7) with respect to T_D we get $-re^{-r(T_D(t)-t)}[V - S] < 0$.
So, the borrower's utility decreases with increase in T_D, so she will withdraw the savings as soon as it becomes S. ■

Let, $T_D^*(t)$ be the time, corresponding to t such that

$$\int_0^t e^{r(T_D^*(t)-t^{'})}\beta_s(1 + \alpha_s^{'})[f(k_{st'}) - k_{st'}]dt^{'} + \beta_s e^{r(T_D^*(t)-t)} f(k_{st}) = S$$

So, given a contract, the borrower's utility from default at t is:
$(1 - \beta_s)\Bigg[f(k_{st}) + \alpha_s^{'}\int_0^t e^{r(t-t^{'})}[f(k_{st'}) - k_{st'}]dt^{'}\Bigg] + e^{-r(T_D^*(t)-t)}V.$

3.3. Optimal Contract

The MFI knows the borrower's problem, so it designs a contract such that the borrower does not have any incentive to default at any instant of time. So, the problem of the MFI is to choose $< \{k_{st}\}_{t=0}^{\infty}, T_R >$ such that the borrower's utility is maximised subject to (i) the feasibility condition in case of repayment (FC_R) and (ii) the dynamic incentive compatibility constraints such that at no $t \leq T_R$ the borrower has an incentive to default.
Therefore, the problem of the MFI is to

$$\underset{<\{k_{st}\}_{t=0}^{\infty}, T_R>}{\text{Maximize}} \int_0^{T_R} e^{-rt}\Big[(1 - \alpha_s - \beta_s)[f(k_{st}) - k_{st}]\Big]dt + e^{-rT_R}\Bigg[\int_0^{T_R} e^{r(T_R-t)}(\alpha_s + \beta_s)[f(k_{st}) - k_{st}]dt - S + V\Bigg]$$

Subject to:

$$\text{FC}_R : \int_0^{T_R} e^{r(T_R-t)}(\alpha_s + \beta_s)[f(k_{st}) - k_{st}]dt \geqslant S, \tag{8}$$

$$\begin{aligned}\text{DIC:} \int_t^{T_R} e^{-r(t^{'}-t)}\Big[(1 - \alpha_s - \beta_s)[f(k_{st'}) - k_{st'}]\Big]dt^{'} + e^{-r(T_R-t)}\Bigg[\int_0^{T_R} e^{r(T_R-t^{''})}(\alpha_s + \beta_s)[f(k_{st''}) - k_{st''}]dt^{''} - S + V\Bigg] \\ \geqslant (1 - \beta_s)\Bigg[f(k_{st}) + \alpha_s^{'}\int_0^t e^{r(t-t^{'})}[f(k_{st'}) - k_{st'}]dt^{'}\Bigg] + e^{-r(T_D^*(t)-t)}V; \quad \forall t \leqslant T_R.\end{aligned} \tag{9}$$

[18]Observe that we are getting this result because we have assumed that discount rate and interest rate on savings are equal.

The left side of FC is the total savings till T_R which needs to be at least S. Left hand side of DIC is the utility borrower gets in case of repayment at any t whereas right hand side is the utility she derives from default at that instant. DIC says that utility from repayment must be at least as high as utility from default at any arbitrary $t \in [0, T_R]$.

Like before it can be shown that at optimum feasibility condition (FC$_R$) binds.

Proposition 5. *Let A1 and A2 hold. Then the feasibility constraint (FC$_R$) binds at the optimal contract.*

Proof: The proof of this proposition is same as the proof of proposition 1, so it is omitted in the main text and can be found in Appendix A.

The intuition of the proof is simple. Since moving to the formal sector gives the borrower more utility, the MFI would design a contract in such a way that the borrower can move to the formal sector as soon as possible, that is as soon as her total savings becomes exactly S.

Hence, the problem of the MFI now becomes

$$\underset{<\{k_{st}\}_{t=0}^{\infty}, T_R>}{\text{Maximize}} \int_0^{T_R} e^{-rt}\Big[(1-\alpha_s-\beta_s)[f(k_{st})-k_{st}]\Big]dt + e^{-rT_R}V$$

Subject to:

$$\text{FC}_R: \int_0^{T_R} e^{r(T_R-t)}(\alpha_s+\beta_s)[f(k_{st})-k_{st}]dt = S, \tag{10}$$

$$\text{DIC:} \int_t^{T_R} e^{-r(t'-t)}\Big[(1-\alpha_s-\beta_s)[f(k_{st'})-k_{st'}]\Big]dt' + e^{-r(T_R-t)}V$$
$$\geqslant (1-\beta_s)\left[f(k_{st}) + \alpha_s' \int_0^t e^{r(t-t')}[f(k_{st'})-k_{st'}]dt'\right] + e^{-r(T_D^*(t)-t)}V; \qquad \forall t \leqslant T_R. \tag{11}$$

Let the optimum scheme be denoted by $<\{k_{st}^*\}_{t=0}^{\infty}, T_R^*>$, where T_R^* is the time required to save S under this scheme, i.e. $\int_0^{T_R^*} e^{r(T_R^*-t)}(\alpha_s+\beta_s)[f(k_{st}^*)-k_{st}^*]dt = S$.

Now let us, introduce the following definition:

Definition 4. $\mathbf{k_{sIt}}$ *is the amount of loan for which the DIC at t binds.*

Our next proposition characterizes the optimal loan size. Since, the MFI is benevolent, the borrower gets efficient amount of loan (k^*) whenever incentive compatibility constraint allows that. When k^* is not incentive compatible the borrower gets the maximum amount which is incentive compatible i.e. k_{sIt}.

Proposition 6. *Consider the optimal scheme $<\{k_{st}^*\}, T_R^*>$. Then $k_{st}^* = \min\{k_{sIt}, k^*\}$ for almost all t.*

Proof: The proof is very similar to the proof of proposition 2 and can be found in Appendix A. ■

3.4. Dynamics of loan size

In this section we will observe how loan size changes over time. In particular, we will find a parametric condition under which loan size increases over time. However, since here the borrower has less to lose from default, sufficient condition for progressive lending is stronger than the benchmark case.

By proposition 6 we know that $k_{st}^* = \min\{k_{sIt}, k^*\}$. So, k_{st}^* will be (weakly) increasing over time, if k_{sIt} is increasing over time whenever $k_{sIt} \leq k^*$. Before proving it formally, let us introduce another assumption.

A 4. $r > 1 - (\alpha_s - \alpha_s') + \frac{\alpha_s\beta_s(V-S)}{S+\beta_s(V-S)}$

Now, we are in a position to state our next lemma.

Lemma 3. *Given A4, k_{sIt} increases over time $\forall t$ such that $k_{sIt} \leq k^*$.*

Proof: From definition 4 we know that

$$f(k_{sIt}) = \frac{1}{1-\beta_s}\left[\int_t^{T_R^*} e^{-r(t^{'}-t)}(1-\alpha_s-\beta_s)[f(k^*_{st^{'}})-k^*_{st^{'}}]dt^{'} + e^{-r(T_R^*-t)}V - e^{-r(T_D^*(t)-t)}V\right]$$
$$-\alpha_s^{'}\int_0^t e^{r(t-t^{'})}[f(k^*_{st^{'}})-k^*_{st^{'}}]dt^{'} \tag{12}$$

So, given A4 if we can prove that R.H.S. of (12) is increasing $\forall t$ such that $k_{sIt} \leq k^*$ then we are done.

Now, observe that $T_D^*(t)$ changes with t. So, first we need to write $T_D^*(t)$ or $e^{-r(T_D^*(t)-t)}$ as a function of t.

From FC$_D$: $e^{r(T_D^*(t)-t)}\beta_s f(k^*_{st}) + \int_0^t e^{r(T_D^*(t)-t^{'})}\beta_s(1+\alpha_s^{'})[f(k^*_{st^{'}})-k^*_{st^{'}}]dt^{'} = S$

Or, $e^{r(T_D^*(t)-t)}\left[\beta_s f(k^*_{st}) + \beta_s(1+\alpha_s^{'})\int_0^t e^{r(t-t^{'})}[f(k^*_{st^{'}})-k^*_{st^{'}}]dt^{'}\right] = S$

Or, $e^{-r(T_D^*(t)-t)} = \frac{\beta_s f(k^*_{st})+\beta_s(1+\alpha_s^{'})\int_0^t e^{r(t-t^{'})}[f(k^*_{st^{'}})-k^*_{st^{'}}]dt^{'}}{S}$

So, equation (12) can be written as:

$$f(k_{sIt}) = \frac{1}{1-\beta_s}\Bigg[\int_t^{T_R^*} e^{-r(t^{'}-t)}(1-\alpha_s-\beta_s)[f(k^*_{st^{'}})-k^*_{st^{'}}]dt^{'} + e^{-r(T^*-t)}V$$
$$-\frac{\beta_s f(k^*_{st})+\beta_s(1+\alpha_s^{'})\int_0^t e^{r(t-t^{'})}[f(k^*_{st^{'}})-k^*_{st^{'}}]dt^{'}}{S}V\Bigg] - \alpha_s^{'}\int_0^t e^{r(t-t^{'})}[f(k^*_{st^{'}})-k^*_{st^{'}}]dt^{'} \tag{13}$$

Differentiating R.H.S. of (13) with respect to t we get:

$$f^{'}(k_{sIt})\frac{\partial f(k_{sIt})}{\partial t} = rf(k_{sIt}) - [\frac{1-\alpha_s-\beta_s}{1-\beta_s}+\alpha_s^{'}][f(k^*_{st})-k^*_{st}] + \frac{\beta_s V}{(1-\beta_s)S}[rf(k^*_{st}) - (1+\alpha_s^{'})[f(k^*_{st})-k_{st}*]$$
$$-\frac{\beta_s V}{(1-\beta_s)S}f^{'}(k^*_{st})\frac{\partial f(k^*_{st})}{\partial t}$$

Since, we are considering only those t such that $k_{sIt} < k^*$, hence by our previous proposition we have $k^*_{st} = k_{sIt}$.

So, the last expression becomes:

$$[1+\frac{\beta_s V}{(1-\beta_s)S}]f^{'}(k_{sIt})\frac{\partial f(k_{sIt})}{\partial t} = r[1+\frac{\beta_s V}{(1-\beta_s)S}]f(k_{sIt}) - [\frac{1-\alpha_s-\beta_s}{1-\beta_s}+\alpha_s^{'}+\frac{\beta_s(1+\alpha_s^{'})V}{(1-\beta_s)S}][f(k_{sIt})-k_{sIt}]$$
$$= r[1+\frac{\beta_s V}{(1-\beta_s)S}]f(k_{sIt}) - \frac{S-\alpha_s S-\beta_s S+\alpha_s^{'}S-\alpha_s^{'}\beta_s S+\beta_s V+\beta_s\alpha_s^{'}V}{(1-\beta_s)S}[f(k_{sIt})-k_{sIt}]$$
$$= r[1+\frac{\beta_s V}{(1-\beta_s)S}]f(k_{sIt}) - \frac{S(1-\beta_s)+\beta_s V-(\alpha_s-\alpha_s^{'})S-(\alpha_s-\alpha_s^{'})\beta_s(V-S)+\alpha_s\beta_s(V-S)}{(1-\beta_s)S}[f(k_{sIt})-k_{sIt}]$$
$$= [1+\frac{\beta_s V}{(1-\beta_s)S}]\left[rf(k_{sIt}) - [1-\frac{(\alpha_s-\alpha_s^{'})[S-\beta_s(V-S)]+\alpha_s\beta_S(V-S)}{S+\beta_s(V-S)}][f(k_{sIt})-k_{sIt}]\right]$$
$$\Rightarrow f^{'}(k_{sIt})\frac{\partial f(k_{sIt})}{\partial t} = rf(k_{sIt}) - [1-(\alpha_s-\alpha_s^{'})+\frac{\alpha_s\beta_s(V-S)}{S+\beta_s(V-S)}][f(k_{sIt})-k_{sIt}]$$

So, given A4, k_{sIt} will be increasing $\forall t$ such that $k_{sIt} < k^*$. ■

Like the benchmark case, here also we need to introduce a definition.

Definition 5. *Let $\hat{t}_s(T_R^*)$ solve $k_{sI\hat{t}_s(T_R^*)} = k^*$ i.e. $\hat{t}_s(T_R^*)$ denotes the t such that the maximum permissible loan according to the DIC at t is equal to the efficient amount of loan.*

Observe that, k_{sIt} can potentially be always greater than k^*, to rule that out we will make another assumption.

A 5. $f(k^*) > \frac{e^{-rT_R^*}(V+\frac{1-\alpha_s-\beta_s}{\alpha_s+\beta_s}S)S}{S+\beta_s(V-S)}$.

In the next lemma, we will show that given A 4. $\hat{t}_s(T_R^*)$ is unique in that k_{It} becomes equal to k^* at most once.

Lemma 4.
1. *Given A 4, $\hat{t}_s(T_R^*)$ is unique.*
2. *Given A 5, $\hat{t}_s(T_R^*) > 0$*
3. *Given A 4 and A 5, $k_{sIt} < k^*$ $\forall t \in [0, \hat{t}_s(T_R^*))$.*

Proof: 1. Observe that we have proved in Lemma 3 that k_{sIt} is increasing $\forall t$ such that $k_{sIt} \leq k^*$, so k_{sIt} can intersect k^* only from below. So, $\hat{t}_s(T_R^*)$ is unique.
2. Given A 5., it is easy to show that $k_{I0} < k^*$ hence, $\hat{t}_s(T_R^*) > 0$.
3. It is straight forward from the previous arguments. ■

However, unlike the benchmark case we cannot say that $\hat{t}_s(T_R^*) < T_R*$. In this case $k_{IT_R^*}$ can be less than k^*.
The next proposition states that given A4, optimum loan size increases over time in this case also.

Proposition 7. *Given A1, A2, A4, A5 there will be weak progressive lending for almost all $t \in [0, T_R^*]$. Precisely, if $\hat{t}_s(T_R^*) < T_R^*$ then, loan size increases for almost all $t \in [0, \hat{t}_s(T_R^*))$ and after that it becomes constant over time. And if $k_{sIt} < k^*$ $\forall t \in [0, T_R^*]$ then loan size increases over time for almost all $t \in [0, T_R^*]$.*

Proof: When $\hat{t}_s(T_R^*) < T_R^*$ then from lemma 4 we have $k_{sIt} < k^*$ $\forall t < \hat{t}_s(T_R^*)$ and by lemma 3 we know that k_{sIt} is increasing. Also by proposition 6 $k_{st}^* = k_{sIt}$ for almost all $t \in [0, \hat{t}_s(T_R^*)]$ and from Lemma 2 we know that k_{sIt} is increasing $\forall t \leqslant \hat{t}_s(T_R^*)$. Hence, loan size will increase for almost all $t < \hat{t}_s(T_R^*)$.
And from $\hat{t}_s(T_R^*)$ onwards $k^* = \min\{k^*, k_{sIt}\}$ hence $k_{st}^* = k^*$ for almost all $t \in [\hat{t}_s(T_R^*), T_R^*]$.

When $k_{sIt} < k^*$ $\forall t \in [0, T_R^*]$ then $k_{st}^* = k_{sIt}$ so by lemma 3 k_{st}^* is increasing over time for almost all $t \in [0, T_R^*]$.
Hence, the result. ■

4. Comparison

In this section we will compare the optimum loan sequences and time required to save S of the two cases. To observe the effect of requirement of certificate from the MFI on optimum loan sequence and ultimately on borrower's welfare, we will assume $\alpha_s = \alpha$ and $\beta_s = \beta$. Though they can be potentially different.

Observe that the DIC when certification from the MFI is required can be written as[19]:

$$f(k_t^*) + [1-(\alpha-\alpha')]\int_0^t e^{r(t-t')}[f(k_{t'}^*) - k_{t'}^*]dt' \leq e^{-r(T^*-t)}[V - S + \frac{S}{\alpha+\beta}]; \qquad \forall t \leqslant T$$

And, the the DIC when certification from the MFI is not required can be written as

$$f(k_{st}^*) + [1-(\alpha-\alpha')]\int_0^t e^{r(t-t')}[f(k_{st'}^*) - k_{st'}^*]dt' \leq e^{-r(T_R^*-t)}[V - S + \frac{S}{\alpha+\beta}] - e^{-r(T_D^*(t)-t)}(V-S); \quad \forall t \leqslant T_R$$

[19]Derivation can be found in Appendix A

Since $e^{-r(T_D^*(t)-t)}(V-S)$, at any $t \leq T^*$ optimum loan amount in the benchmark case is strictly greater than that when certification is not required. As a result optimum time required to save S is less in the former case than in the later case. Hence, even if it seems that requirement of certification from the MFI would make the borrower worse off, actually it is welfare improving. It is quite intuitive in that when certification is not required the borrower can use the saved amount to get out of the poverty trap, utility from which is much higher than that from consuming the saved amount which happens when certificate from the MFI is required. Hence, borrower's incentive to default decreases when certification is required. Both borrower and the MFI know this and hence MFI chooses the optimum contract accordingly. Hence, the loan size is higher when certification is required than that when certification is not required, which in turn decreases the duration for which the borrower remains in the poverty trap.

Observation: Sufficient condition for progressive lending when certification is not required captured via $r > 1-(\alpha-\alpha^{'})+\frac{\alpha\beta(V-S)}{S+\beta(V-S)}$ is stronger than the condition when certification is required captured via $r > 1-(\alpha-\alpha^{'})$.

5. Comparative Statics

In this section we will explore the effect of changes in the values of different parameters on borrower's welfare. We will start with the case where the fraction of forced savings with the MFI (namely α) changes, then we will change the fraction of forced savings with the savings institution (namely β) and then we will explore the case where the part of savings with the MFI the borrower gets back ($\alpha^{'}$) changes. Lastly, we will observe the effect of increase in competition via increase in cost of capital which in turn increases the amount to be repaid.

We will start the case where certification from the MFI is required.

5.1. Cerification from the MFI is required

5.1.1. Change in the fraction of forced savings with MFI (α)

In this section we will observe the effect of increase in the fraction of compulsory savings, that is α, on borrower's utility.

Proposition 8. *Given A2, utility of a borrower increases with α[20].*

Proof: *Step1* Firstly, we will prove that, for a given sequence of $\langle k_t^* \rangle$, as α increases optimum T, that is, time required to save S decreases.

So, we will show: $\left.\frac{dT}{d\alpha}\right|_S < 0$

Differentiating FC with respect to α we get

$$\frac{dT^*}{d\alpha} = -\frac{S}{(\alpha+\beta)\big[rS+(\alpha+\beta)[f(k^*)-k^*]\big]} < 0$$

So, as α increases T decreases, for a given sequence of $\langle k_t^* \rangle$. However, that sequence might not remain available with change in α.

[20]However, note that we are getting this result due to the particular form of utility function. In more realistic situation minimum amount of consumption is required for subsistence. So, the borrower would have valued consumption more when α is very high, in that case borrower's utility will decrease with increase in α. More, precisely there would exist a threshold value of α such that utility of the borrower will decrease if α is increased beyond that threshold. We are not using that here, as it will complicate the analysis without adding anything to the qualitative result of this paper.

Step 2. Now, using $\frac{dT^*}{d\alpha}$ we found in Step 1, we will show that dynamic incentive compatibility constraint(DIC) will be relaxed with increase in α. For that we will show that L.H.S. of DIC will increase. Differentiating DIC with respect to α we get

$$e^{-r(T-t)}\frac{S}{(\alpha+\beta)\big[rS+(\alpha+\beta)[f(k_T)-k_T]\big]}\Big[r(V-S)-[f(k^*)-k^*]\Big]+(\alpha-\alpha^{'})\int_0^t e^{r(t-t^{'})}[f(k_{t^{'}})-k_{t^{'}}]dt^{'}$$

>0 where the last inequality comes from *A2* [21].

So k_{It} will increase $\forall t$. Now, observe that in Step 1, we evaluated $\frac{dT}{d\alpha}<0$ for original sequence of $\langle k_t^*\rangle$, now after change in α optimum loan amount (weakly) increases (from Step 2), which will further decrease T^*, hence $\frac{dT^*}{d\alpha}<0$ for the new optimum sequence as well.

Step 3. Since k_{It} increases $\hat{t_\alpha}(T^*_\alpha)$ will decrease, hence, the borrower will start getting k^* faster. So, that will further decrease T and so on. And from *Step1* of *Proposition1* we know that the utility of the borrower increases as T decreases. Hence the result. ■

Intuitively, this result is quite plausible. As the fraction of forced savings increases the borrower's instantaneous consumption decreases but his savings increases and she can acheive V faster, so if V is sufficiently large (captured through A2), the instantaneous loss will be much lesser than the gain from obtaining V faster. So, the utility at every instant increases which in turn relaxes the dynamic incentive compatibility constraint, hence per period loan amount (weakly) increases. Hence, the borrower will be able to acheive the required savings sooner, which will increase her utility and so on. Hence, the result.

5.1.2. Change in the fraction of forced savings with other savings institution (β)

As expected borrower's utility also increases with β however it's effect is lesser than increase in α.

Proposition 9. *Given A2, utility of a borrower increases with β. The increase in utility due to increase in β is less than that due to increase in α.*

Proof: *Step1* Like earlier, we will prove that as β increases optimum T, that is, time required to save S decreases.

So, we will show: $\left.\frac{dT}{d\beta}\right|_S<0$

Differentiating FC with respect to β we get

$$\frac{dT}{d\beta}=-\frac{S}{(\alpha+\beta)\big[rS+(\alpha+\beta)[f(k_T)-k_T]\big]}<0$$

Step 2 Now like before by differentiating R.H.S. of $f(k_{It})$ we get:

$$e^{-r(T-t)}\frac{S}{(\alpha+\beta)\big[rS+(\alpha+\beta)[f(k_T)-k_T]\big]}\Big[r(V-S)-[f(k_T)-k_T]\Big]>0.$$

Using the argument we used in the proof of proposition 8 we can argue that borrower's utility increases with increase in β. ■

Intuition behind the fact that increase in utility due to increase in β is less than that with increase in α is with increase in α or β, saved amount increases, hence the time required to save S decreases which also motivates borrower to repay more, hence permissible loan amount increases which again decreases the time required to build collateral and so on. However, as β increases borrower's incentive to default increases, hence, increase in permissible loan amount is less in the former case hence, decrease in time required to save S is smaller. Hence, the result.

[21]By A2 $r(V-S)>f(k^*)>f(k^*)-k^*\geq f(k_T)-k_T$. And, $e^{rt}\int_0^t e^{-rt^{'}}[f(k_{t^{'}})-k_{t^{'}}]dt^{'}>0$.

5.1.3. Change in $\alpha^{'}$

Proposition 10. *Borrower's utility decreases with increase in $\alpha^{'}$.*

Proof: Observe, FC does not change with $\alpha^{'}$, we will first see what happens to $f(k_{It})$ when $\alpha^{'}$ increases but T^* remains constant.

So by differentiating R.H.S. of $f(k_{It})$ we get:
$-\int_0^t e^{r(t-t^{'})}[f(k_{t'})-k_{t'}]dt^{'}<0$

Now, observe if k_{It} decreases the optimum loan amount decreases which increases T^* which further decreases k_{It}. And hence the borrower gets weakly lesser amount of loan with increase in $\alpha^{'}$, hence time required to save S increases hence, borrower's utility decreases with increase in $\alpha^{'}$.

5.1.4. Competitive MFI

Now, suppose the MFI faces competition, such that the cost of capital increases, so even a benevolent MFI will not be able to charge zero interest rate in order to sustain. Let for k amount of capital the borrower has to repay ck where $c>1$. However, we are assuming that double dipping is not possible. We are assuming this just to focus exclusively on the effect of change in cost of capital, that is even if it is possible to regulate the villagers to borrow from more than one MFI-s, the increase in competition among MFI-s will affect the borrowers adversely.

In this set up we will define the efficient scale of investment k_c^* as

$$k_c^* = \underset{k}{\operatorname{argmax}}[f(k)-ck]^{22}$$

So, in this setup the problem of the MFI-s becomes:

$$\underset{<\{k_t\}_{t=0}^{\infty},T>}{\text{Maximize}} \qquad \int_0^T e^{-rt}\Big[(1-\alpha-\beta)[f(k_t)-ck_t]\Big]dt + e^{-rT}\left[\int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-ck_t]dt - S + V\right]$$

Subject to:

$$\text{FC}^{'}: \quad \int_0^T e^{r(T-t)}(\alpha+\beta)[f(k_t)-ck_t]dt \geqslant S, \tag{14}$$

$$\begin{aligned}\text{DIC}^{'}: \quad &\int_t^T e^{-r(t^{'}-t)}\Big[(1-\alpha-\beta)[f(k_{t'})-ck_{t'}]\Big]dt^{'} + e^{-r(T-t)}\left[\int_0^T e^{r(T-t^{''})}(\alpha+\beta)[f(k_{t''})-ck_{t''}]dt - S + V\right] \\ &\geqslant f(k_t) + \int_0^t e^{r(t-t^{'})}\beta[f(k_{t'})-ck_{t'}]dt^{'}; \quad \forall t \leqslant T.\end{aligned} \tag{15}$$

Clearly, when A2 holds the inequality $f(k_c^*) < r(V-S)$ also holds.

So, our next proposition is:

Proposition 11. *Let A1 and A2 hold. Then the modified feasibility constraint (FC$^{'}$) binds at the optimal contract.*

Proof: Same as proof of Proposition 1. ■

Let the optimum scheme be denoted by $<\{k_{ct}^*\}_{t=0}^{\infty}, T_c^*>$, where T_c^* is the time required to save S under this scheme, i.e.

$$\int_0^{T_c^*} e^{r(T_c^*-t)}(\alpha+\beta)[f(k_{ct}^*)-ck_{ct}^*]dt = S.$$

Now we define the maximum permissible loan amount by DIC.

[22] Observe , $f^{'}(k_c) = c > 1 = f^{'}(k)$ and since $f(\cdot)$ is a concave function $k_c^* < k$

Definition 6. $\mathbf{k_{cIt}}$ *is the amount of loan for which the modified dynamic incentive compatibility constraint (DIC$^{'}$) at t binds.*

Our next proposition characterizes optimum loan sequence.

Proposition 12. *Consider the optimal scheme* $< \{k^*_{ct}\}, T^*_c >$. *Then* $k^*_{ct} = \min\{k_{cIt}, k^*_c\}$ *for almost all t.*

Proof: Same as proof of Proposition 2. ■

Now, we need to introduce another definition:

Definition 7. *Let* $\hat{t}_c(T^*_c)$ *solve*

$$k_{cI\hat{t_c}} = k^*_c,$$

i.e. $\hat{t}_c(T^*_c)$ *denotes the t such that the maximum permissible loan according to the DIC$^{'}$ at t is equal to the efficient amount of loan.*

Proposition 13. *Given A3 there will be weak progressive lending for almost all* $t \in [0, T^*_c]$.
Precisely, loan size will increase for almost all $t \in [0, \hat{t}_c(T^*))$ *and then it will become constant over time.*

Proof: Same as proof of Proposition 3. ■

Welfare Analysis:

Proposition 14. *Utility of the borrower decreases as competition among MFI-s increases.*

Proof: Observe, $k^*_{ct} < k^*_t \; \forall t$. Hence, $T^*_c < T^*$ And we have already observed that borrower's utility decreases with T. Hence, welfare of the borrower will decrease as competition increases, which is captured by c.

5.2. Certification from the MFI is not required

It can be shown that the effect of change in α_s and $\alpha^{'}_s$ and introduction of competition among MFIs are same as the former case. However, change in β_s has interesting effect on utility of the borrower, so we will address only that in this section. We observe this because as β_s increases time required to save S decreases ceteris paribus in case of repayment as well as in case of default. While the first effect would increase maximum amount the borrower repays the second effect decreases it, so at optimum which effect would be dominant is ambiguous.

Proposition 15. *Effect of change in* β_s *on utility is ambiguous.*

Proof. *Step 1.* Like before, we can show that T^*_R decreases with increase in β_s which is captured by $\frac{dT^*_R}{d\beta_s} = -\frac{S}{(\alpha_s+\beta_s)\left[rS+(\alpha_s+\beta_s)[f(k^*_{sT_R})-k^*_{sT_R}]\right]} < 0.$

But also observe $T^*_D(t)$ also decreases with increase in β_s which is captured by
$\frac{dT^*_D(t)}{d\beta_s} = -\frac{f(k^*_{st})+(1+\alpha^{'}_s)\left[\int_0^t e^{r(t-t^{'})}[f(k^*_{st^{'}})-k^*_{st^{'}}]dt^{'}\right]}{S} < 0.$

However, like before that sequence might not remain available with change in β_s.

Step 2. $f(k_{sIt}) = \int_t^{T^*_R} e^{r(t-t^{'})}[f(k^*_{st^{'}}) - k^*_{st^{'}}]dt^{'} + e^{-r(T^*_R-t)}V + \int_0^t e^{r(t-t^{'})}(\alpha - \alpha^{'}_s)[f(k^*_{st^{'}}) - k^*_{st^{'}}]dt^{'}$
$+ e^{-r(T^*_D(t)-t)}V$

Step 2. Differentiating DIC with respect to β_s we get

$$e^{-r(T^*_R-t)}\frac{S}{(\alpha_s+\beta_s)\left[rS+(\alpha_s+\beta_s)[f(k^*_{sT_R})-k^*_{sT_R}]\right]}\left[r(V-S) - [f(k^*_{sT_R}) - k^*_{sT_R}]\right]$$
$$+ (\alpha_s - \alpha^{'}_s)\int_0^t e^{r(t-t^{'})}[f(k^*_{st^{'}}) - k^*_{t^{'}}]dt^{'} - \frac{f(k^*_{st})+(1+\alpha^{'}_s)\left[\int_0^t e^{r(t-t^{'})}[f(k^*_{st^{'}})-k^*_{st^{'}}]dt^{'}\right]}{S}(V-S)$$

Now observe we cannot say anything about the sign of the last expression $\forall t \leq T^*_R$. Hence the result. ■

6. Conclusion

Though there are many empirical/experimental evidences that providing savings service makes poor people better off. There is hardly any literature which gives a theoretical explanation to it. In this paper we have tried to provide a simple model where savings service helps poor people to get out of the poverty trap. Also, we have addressed a near universal feature of microfinance, namely progressive lending - in our model optimum loan size (weakly) increases over time when fraction of forced savings is not too small. Also, we have found that increase in fraction of forced savings is welfare improving and competition among MFI-s negatively affects the borrowers even when double dipping is not a problem. It has been observed that some kind of commitment towards MFI, here captured via requirement of a certificate from MFI, actually makes borrower better off.

Provision of compulsory savings has been explained via hyperbolic discounting of clients in many papers. It will be an interesting exercise to solve our model where borrowers are time inconsistent.

What will happen if we allow competition among MFI-s, where they do not share information? It is observed that borrowers actually take loans from multiple MFI-s, what will happen in that case? Is it possible to provide savings service in that case as well? Will that be welfare improving?

Also, MFI-s can use this savings to provide insurance to its clients. Poor people are more vulnerable to shocks so providing insurance to them is unambiguously welfare improving. Another interesting exercise is to observe how a credit-constrained MFI chooses number of clients and amount of loan to be provided to a particular client. While providing loan to a particular borrower affects other prospective clients adversely, providing loan to her not only increases utilty of that borrower but also increases her savings which increases the available fund of the MFI - the problem of the MFI is to solve these two tensions optimally. Hence, provision of savings via MFI-s is welfare improving. The analysis here suggests that micro-finance and micro-savings need to be studied in unison, and provides a starting point of this analysis.

Appendix A

Proof of proposition 5: Firstly we will prove that the borrower's lifetime utility decreases with T_R that is the borrower's lifetime utility decreases as the time, she remains in the informal sector, increases, keeping loan amount constant. In Step 2 we have shown that from Step 1 it follows that if FC_R does not bind then that cannot be an optimum contract.

Step 1. Here, we prove that partial differentiation of the borrower's utility with respect to T_R is negative, that is

$$\frac{\partial}{\partial T_R}\left[\int_0^{T_R} e^{-rt}\Big[(1-\alpha_s-\beta_s)[f(k_{st})-k_{st}]\Big]dt + e^{-rT_R}\left[\int_0^{T_R} e^{r(T_R-t)}(\alpha_s+\beta_s)[f(k_{st})-k_{st}]dt - S + V\right]\right] < 0.$$

Observe, $\int_0^{T_R} e^{-rt}\Big[(1-\alpha_s-\beta_s)[f(k_{st})-k_{st}]\Big]dt + e^{-rT_R}\left[\int_0^{T_R} e^{r(T_R-t)}(\alpha_s+\beta_s)[f(k_{st})-k_{st}]dt - S + V\right]$

$= \int_0^{T_R} e^{-rt}[f(k_{st})-k_{st}]dt + e^{-rT_R}\Big[V-S\Big]$

$$\begin{aligned}
&\frac{\partial}{\partial T}\left[\int_0^{T_R} e^{-rt}\Big[(1-\alpha_s-\beta_s)[f(k_{st})-k_{st}]\Big]dt + e^{-rT_R}\left[\int_0^{T_R} e^{r(T_R-t)}(\alpha_s+\beta_s)[f(k_{st})-k_{st}]dt - S + V\right]\right]\\
=\ &\frac{\delta}{\delta T}\left[\int_0^{T_R} e^{-rt}[f(k_{st})-k_{st}]dt + e^{-rT_R}\Big[V-S\Big]\right]\\
=\ &e^{-rT_R}\Big[[f(k_{sT_R})-k_{sT_R}] - r(V-S)\Big]. \qquad (16)
\end{aligned}$$

Clearly, given A2, it must be that $f(k_{sT_R}) - k_{sT_R} - r(V-S) < 0$, so that (20) is negative.[23]

[23] $f(k^*)-k^* = \mathrm{Max}[f(k)-k]$ i.e. $f(k_{sT_R})-k_{sT_R} \leqslant f(k^*)-k^* < f(k^*)$. Therefore, when A2 holds $f(k_{sT_R})-k_{sT_R} < r(V-S)$.

Step 2. Now suppose that the FC_R does not bind. Consider another scheme that is identical to the original scheme, but ends at $T_R^{'} < T_R$. For T_R' close to T_R, FC_R will be satisfied, and the borrower's utility increases by the preceding argument. Finally, DIC holds for all $t \leq T_R^{'}$, as the original scheme satisfied the DIC and $T_R^{'} < T_R$ (we mimic the argument in step 1 to prove this). ∎

Proof of Proposition 6: Observe that due to DIC $k_{st}^* \leq k_{sIt}$ $\forall t \leq T_R^*$.

Now, consider the set $\mathcal{N} = \{t \leqslant T_R^* : \text{Either } k_{st}^* < \min\{k_{sIt}, k^*\} \text{ Or } k_{st}^* \in (k^*, k_{sIt}]\}$. The claim is that the measure of the set $\mathcal{N}$ is zero[24]. Suppose not. Then $\exists N (\subsetneq \mathcal{N}$ and $N \ni t \leq T_R^{'} < T_R^*)$such that the measure of $N > 0$.

We then construct another scheme $< \{k_{st}^{'}\}, T_R^* >$ such that:

$$k_{st}^{'} = \begin{cases} k_{st}^* & \text{when } t \notin N \text{ and } t \leq T_R^* \\ (k_{st}^*, \min\{k_{sIt}, k^*\}] & \text{when } t \in N \text{ and } k_{st}^* < \min\{k_{sIt}, k^*\} \\ [k^*, k_{st}^*) & \text{when } t \in N \text{ and } k_{st}^* \in (k^*, k_{sIt}]. \end{cases}$$

By construction, $< \{k_{st}^{'}\}, T_R^* >$ satisfies DIC[25].

Also observe that, $\forall t \in N$, $[f(k_{st}^{'}) - k_{st}^{'}] > [f(k_{st}^*) - k_{st}^*]$[26]. Further, $\forall t \leqslant T_R^*$ and $t \notin N$, we have that $k_{st}^{'} = k_{st}^*$, and therefore $[f(k_{st}^{'}) - k_{st}^{'}] = [f(k_{st}^*) - k_{st}^*]$. Therefore,

$$\int_0^{T_R^*} e^{r(T_R^*-t)}(\alpha_s + \beta_s)[f(k_{st}^{'}) - k_{st}^{'}]dt > \int_0^{T_R^*} e^{r(T_R^*-t)}(\alpha_s + \beta_s)[f(k_{st}) - k_{st}]dt = S.$$

Since time is continuous,
$\exists \Delta_R^{'} > 0$ such that $\int_0^{T_R^*-\Delta_R^{'}} e^{r(T_R^*-\Delta_R^{'}-t)}(\alpha_s + \beta_s)[f(k_{st}^{'}) - k_{st}^{'}]dt \geqslant S$ and $\Delta_R^{'} < T_R^* - T_R^{'}$.
Thus, for $\Delta_R^{'}$ small enough, we have constructed another scheme $< \{k_{st}^{'}\}, T_R >$[27] that satisfies DIC and FC_R, and ends earlier than T_R^*. Thus, T_R^* can't be the optimum.

So, we can conclude that $k_{st}^* = \min\{k_{sIt}, k^*\}$ for almost all t. ∎

When certification is required: Observe that from FC we get:

$$\begin{aligned}
&\int_0^{T^*} e^{r(T^*-t)}(\alpha + \beta)[f(k_t^*) - k_t^*]dt = S \\
\Rightarrow &\int_0^{t} e^{r(T^*-t^{'})}[f(k_{t^{'}}^*) - k_{t^{'}}^*]dt^{'} + \int_t^{T^*} e^{r(T^*-t^{'})}[f(k_{t^{'}}^*) - k_{t^{'}}^*]dt^{'} = \frac{S}{\alpha + \beta} \\
\Rightarrow &e^{-r(T^*-t)}\left[\int_0^{t} e^{r(T^*-t^{'})}[f(k_{t^{'}}^*) - k_{t^{'}}^*]dt^{'} + \int_t^{T^*} e^{r(T^*-t^{'})}[f(k_{t^{'}}^*) - k_{t^{'}}^*]dt^{'}\right] = e^{-r(T^*-t)}\frac{S}{\alpha + \beta} \\
\Rightarrow &\int_t^{T^*} e^{-r(t^{'}-t)}[f(k_{t^{'}}^*) - k_{t^{'}}^*]dt^{'} = e^{-r(T^*-t)}\frac{S}{\alpha + \beta} - \int_0^{t} e^{r(t-t^{'})}[f(k_{t^{'}}^*) - k_{t^{'}}^*]dt^{'}
\end{aligned}$$

---

[24] Note that any $k_{st}^* > k_{sIt}$ is not incentive compatible, so to prove the proposition it is sufficient to prove that the measure of the set $\mathcal{N}$ is zero

[25] any $k_{st} \leq k_{sIt}$ where $t \leq T_R^*$ satisfies DIC and by construction $k_{st}^{'} \leq k_{sIt}$

[26] Observe that $f(k) - k$ is maximum at k^* Now, suppose
A. $t \in N$ and $k_{st}^* < \min\{k_{sIt}, k^*\}$ then if we increase k_{st} then $f(k_{st}) - k_{st}$ increases
B. $t \in N$ and $k_{st}^* \in (k^*, k_{sIt}]$ then if we decrease k_{st} then $f(k_{st}) - k_{st}$ increases.

[27] $T_R = T_R^* - \Delta_R^{'}$

$$\text{DIC: } \int_t^{T^*} e^{-r(t'-t)}\Big[(1-\alpha-\beta)[f(k_{t'}^*)-k_{t'}^*]\Big]dt' + e^{-r(T^*-t)}V \geqslant f(k_t^*) + \int_0^t e^{r(t-t')}(\alpha'+\beta)[f(k_{t'}^*)-k_{t'}^*]dt'; \quad \forall t \leqslant T$$

$$\Rightarrow e^{-r(T^*-t)}\frac{(1-\alpha-\beta)S}{\alpha+\beta} - (1-\alpha-\beta)\int_0^t e^{r(t-t')}[f(k_{t'}^*)-k_{t'}^*]dt'$$

$$-\int_0^t e^{r(t-t')}(\alpha'+\beta)[f(k_{t'}^*)-k_{t'}^*]dt' + e^{-r(T^*-t)}V \geqslant f(k_t^*); \qquad \forall t \leqslant T$$

$$\Rightarrow f(k_t^*) + [1-(\alpha-\alpha')]\int_0^t e^{r(t-t')}[f(k_{t'}^*)-k_{t'}^*]dt' \leq e^{-r(T^*-t)}[V-S+\frac{S}{\alpha+\beta}]; \qquad \forall t \leqslant T$$

When certification is not required:
Observe like before from FC_R we can get:

$$\int_t^{T_R^*} e^{-r(t'-t)}[f(k_{st'}^*)-k_{st'}^*]dt' = e^{-r(T_R^*-t)}\frac{S}{\alpha+\beta} - \int_0^t e^{r(t-t')}[f(k_{st'}^*)-k_{st'}^*]dt'$$

$$\text{DIC: } \int_t^{T_R^*} e^{-r(t'-t)}\Big[(1-\alpha-\beta)[f(k_{st'}^*)-k_{st'}^*]\Big]dt' + e^{-r(T_R^*-t)}V \geqslant (1-\beta)\big[f(k_{st}^*) + \alpha'\int_0^t e^{r(t-t')}[f(k_{st'}^*)-k_{st'}^*]dt'\big]$$

$$+ e^{-r(T_D^*(t)-t)}\Big[e^{r(T_D^*(t)-t)}\beta\big[f(k_{st}^*) + (1+\alpha')\int_0^t e^{r(t-t')}[f(k_{st'}^*)-k_{st'}^*]dt'\big] - S + V\Big]; \qquad \forall t \leqslant T_R$$

$$\Rightarrow \int_t^{T_R^*} e^{-r(t'-t)}\Big[(1-\alpha-\beta)[f(k_{st'}^*)-k_{st'}^*]\Big]dt' + e^{-r(T_R^*-t)}V$$

$$\geqslant f(k_{st}^*) + \int_0^t e^{r(t-t')}(\alpha'+\beta)[f(k_{st'}^*)-k_{st'}^*]dt' + e^{-r(T_D^*(t)-t)}(V-S); \qquad \forall t \leqslant T_R$$

$$\Rightarrow e^{-r(T_R^*-t)}\frac{(1-\alpha-\beta)S}{\alpha+\beta} - (1-\alpha-\beta)\int_0^t e^{r(t-t')}[f(k_{st'}^*)-k_{t'}^*]dt' - \int_0^t e^{r(t-t')}(\alpha'+\beta)[f(k_{st'}^*)-k_{st'}^*]dt'$$

$$+ e^{-r(T_R^*-t)}V \geqslant f(k_{st}^*); \qquad \forall t \leqslant T_R$$

$$\Rightarrow f(k_{st}^*) + [1-(\alpha-\alpha')]\int_0^t e^{r(t-t')}[f(k_{st'}^*)-k_{st'}^*]dt' \leq e^{-r(T_R^*-t)}[V-S+\frac{S}{\alpha+\beta}] - e^{-r(T_D^*(t)-t)}(V-S); \quad \forall t \leqslant T_R$$

Appnedix B

Algorithm for determining optimum loan sequence

When Certification from the MFI is required:

Observe from the feasibility constraint (FC) we have:

$$S = \int_0^{T^*} e^{r(T^*-t)}(\alpha+\beta)[f(k_t^*)-k_t^*]dt$$

$$\frac{S}{\alpha+\beta} = \int_0^t e^{r(T^*-t+t-t')}[f(k_{t'}^*)-k_{t'}^*]dt' + \int_t^{T^*} e^{r(T^*-t')}[f(k_{t'}^*)-k_{t'}^*]dt' \text{ for any } t \in [0, T^*]$$

$$\int_0^t e^{r(t-t')}[f(k_{t'}^*)-k_{t'}^*]dt' = e^{-r(T^*-t)}\Big[\frac{S}{\alpha+\beta} - \int_t^{T^*} e^{(T^*-t')}[f(k_{t'}^*)-k_{t'}^*]dt'\Big]$$

So, we can write $f(k_{It}) = \int_t^{T^*} e^{-r(t^{'}-t)}\Big[(1-\alpha-\beta)[f(k^*_{t^{'}}) - k^*_{t^{'}}]\Big]dt^{'} + e^{-r(T^*-t)}V$
$- \beta e^{-r(T^*-t)}\Big[\frac{S}{\alpha+\beta} - \int_t^{T^*} e^{r(T^*-t^{'})}[f(k^*_{t^{'}}) - k^*_{t^{'}}]dt^{'}\Big]; \qquad \forall t \leqslant T^*$

Then we can start with any arbitrarily large T and determine the optimum k^*_t. Now, suppose it ends at some $\hat{t}$, then we will shift the entire thing to the origin. Observe here we need only diferences in time, so shifting the origin will not change anything. So, optimum time (T^*_R) is $T-\hat{t}$, $k^*_T = k^*_{T-\hat{t}} = k^*_{T^*}$, $k^*_{\hat{t}} = k^*_0$ and so on.

When Certification from the MFI is not required:
From FC$_R$ we have:

$$
\begin{aligned}
S &= \int_0^{T^*_R} e^{r(T^*_R-t)}(\alpha_s+\beta_s)[f(k^*_{st}) - k^*_{st}]dt \\
\frac{S}{\alpha_s+\beta_s} &= \int_0^{t} e^{r(T^*_R-t+t-t^{'})}[f(k^*_{st^{'}}) - k^*_{st^{'}}]dt^{'} + \int_t^{T^*_R} e^{r(T^*_R-t^{'})}[f(k^*_{st^{'}}) - k^*_{st^{'}}]dt^{'} \text{ for any } t \in [0, T^*_R] \\
\int_0^{t} e^{r(t-t^{'})}[f(k^*_{st^{'}}) - k^*_{st^{'}}]dt^{'} &= e^{-r(T^*_R-t)}\Big[\frac{S}{\alpha_s+\beta_s} - \int_t^{T^*_R} e^{r(T^*_R-t^{'})}[f(k^*_{st^{'}}) - k^*_{st^{'}}]dt^{'}\Big]
\end{aligned}
$$

Now, observe from FC$_D$ we have:

$$
\begin{aligned}
S &= \int_0^{t} e^{r(T^*_D(t)-t^{'})}\beta_s[f(k^*_{st^{'}}) - k^*_{st^{'}}]dt^{'} \\
&= e^{r(T^*_D(t)-t)}\beta_s \int_0^{t} e^{r(t-t^{'})}[f(k^*_{st^{'}}) - k^*_{st^{'}}]dt^{'} \\
e^{-r(T^*_D(t)-t)} &= \frac{\beta_s}{S}\left[e^{-r(T^*_R-t)}\Big[\frac{S}{\alpha_s+\beta_s} - \int_t^{T^*_R} e^{r(T^*_R-t^{'})}[f(k^*_{st^{'}}) - k^*_{st^{'}}]dt^{'}\Big]\right]
\end{aligned}
$$

So, we can write $f(k_{sIt}) = \int_t^{T^*_R} e^{-r(t^{'}-t)}\Big[(1-\alpha_s-\beta_s)[f(k^*_{st^{'}}) - k^*_{st^{'}}]\Big]dt^{'} + e^{-r(T^*_R-t)}V$
$- \frac{\beta_s}{S}\left[e^{-r(T^*_R-t)}\Big[\frac{S}{\alpha_s+\beta_s} - \int_t^{T^*_R} e^{r(T^*_R-t^{'})}[f(k^*_{st^{'}}) - k^*_{st^{'}}]dt^{'}\Big]\right]V; \qquad \forall t \leqslant T^*_R$

Now, we can determine optimum loan sequence and time following the above method.

References

[1] Aniket, K., (2004). *Sequential group lending with moral hazard,* Working Paper.

[2] Aniket, K., (2011). *Beyond Microcredit: Giving the Poor a Way to Save Their Way Out of Poverty,* Working Paper.

[3] Armendariz, B., and Morduch, J. (2005). *The Economics of Microfinance,* Cambridge, MA: MIT Press.

[4] Armendariz, B., and Morduch, J. (2000). *Microfinance Beyond Group Lending,* The Economics of Transition, 8 (2), 401 - 420.

[5] Ashraf, N., Karlan, D. and Yin, W. (2006). *Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines,* Quarterly Journal of Economics, 121(2), pp. 635-672.

[6] Banerjee, A., Duflo, E. (2011). *Poor Economics,* Random House Publishers.

[7] Banerjee, A., Duflo, E., Glennerster R., and Kinnan, C. (2013). *The Miracle of Microfinance? Evidence From a Randomized Evaluation,*

[8] Banerjee, A. (2013) *Microcredit Under the Microscope: What Have We Learned in the Past Two Decades, and What Do We Need to Know?,* The Annual Review Economics 2013. 5:487519

[9] Banerjee, A., Karlan, D. and Zinman, J. (2015). *Six Randomized Evaluations of Microcredit: Introduction and Further Steps* American Economic Journal: Applied Economics, 7(1): 121

[10] Banerjee, A., Duflo, E., Goldberg, N., Karlan, D., Osei, R., Pariente, W., Shapiro, J., Thuysbaert, B., Udry C. (May 2015) *A Multi-faceted Program Causes Lasting Progress for the Very Poor: Evidence from Six Countries* Science 348(6236): pp. 772.

[11] Basu, K.(2015). *A Behavioral Model of Simultaneous Borrowing and Saving* Working Paper.

[12] Besley, T. (1995). *Savings, Credit, and Insurance* Handbook of Development Economics, vol. 3A. T.N. Srinivasan and Jere Behrman, eds. Amsterdam: Elsevier. pp. 2125207.

[13] Chowdhury, S., Roy Chowdhury, P. and Sengupta, K. (2014). *Gradual Repayment and sequential lending in micro-finance,* Working Paper.

[14] de Quidt, J., Fetzer, T. and Ghatak, M. (2013). *Market Structure and Borrower Welfare in Microfinance,* Working Paper.

[15] Dupas, P. and Jonathan, R (2009). *Savings Constraints and Microenterprise Development: Evidence from a Field Experiment in Kenya.* Working Paper 146983. NBER.

[16] Dupas, P. and Jonathan, R (2013). *Why don't the Poor Save More? Evidence from Health Savings Experiments.* American Economic Review , 103(4): 11381171.

[17] Egli. D. (2004). *Progressive Lending as an Enforcement Mechanism in Microfinance Programs,* Review of Development Economics, 8(4), 505-520.

[18] Fischer, G. (2010) and Ghatak, M. *Repayment Frequency and Lending Contracts with Present-Biased Borrowers,* Working Paper.

[19] Ghatak, M. (1999). *Group Lending, Local Information and Peer Selection,* Journal of Development Economics, Vol.60, No.1.

[20] Ghatak, M. and Guinnane, T. W. (1999). *The Economics of Lending with Joint Liability : Theory and Practice,* Journal of Development Economics, Vol.60, No.1.

[21] Ghosh, P. and Ray, D (2015). *Information and Enforcement in Informal Credit Markets,* Economica (forthcoming).

[22] Jain, S. and Mansuri, G., 2003. *A little at a time: the use of regularly scheduled repayments in microfinance.,* Journal of Development Economics 72 (1), 253 279.

[23] Morduch, J., 1999. *The microfinance promise.* Journal of Economic Literature 37 (4), 1569 1614.

[24] Ray, D. 2002. *The Time Structure of Self-Enforcing Agreements,* Econometrica, Vol. 70, No. 2, pp. 547-582

[25] Roodman, D. and Morduch, J. 2009. *The impact of Microcredit on the Poor in Bangladesh:Revisiting the Evidence.* Working Paper 174. Center for Global Development.

[26] Roodman, D. 2011. *Due Diligence: An Impertinent Inquiry into Microfinance,* Center for Global Development.

[27] Roy Chowdhury, P. (2005). *Group-lending: Sequential Financing, Lender Monitoring and Joint Liability,* Journal of Development Economics 77, 415-439.

[28] Roy Chowdhury, P. (2007). *Group-lending with Sequential Financing, Contingent Renewal and Social Capital,* Journal of Development Economics 84, 487-507.

[29] Rutherford, S. (2009). *The Poor and Their Money: Microfinance from a Twenty-first Century Consumer's Perspective.*

[30] Shapiro, D. A. (2012). *Microfinance and Dynamic Incentives,* Working Paper.

[31] Tedeschi, G. A. (2006). *Here today, gone tomorrow: Can dynamic incentives make microfinance more flexible?,* Journal of Development Economics, 80(1), 84-105.

[32] Westley, G. 2004. *A Tale of Four Village Bankings Programs: Best Practices in Latin America,* Inter-American Development Bank.