The Role of Small and Medium Enterprises in Structural Transformation and Economic Development

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Abstract

Developing countries are often faced with the difficult task of balancing rapid economic growth with equitable growth. The inequalities that arise during the course of rapid economic growth are a cause of serious concern as they are not temporary but are structural in nature. This study directs attention to the role of Small and Medium Enterprises (SMEs) in overcoming these structural rigidities. The model presented in this study shows that even though SMEs have the potential for being a bridge for economies facing structural poverty and inequalities, they may not always be effective in bringing about the desired increase in incomes and reduction of inequalities in income and opportunities. It is shown that a country's history plays an important role in determining the effectiveness of SMEs in bringing about structural changes and under certain conditions can render promoting these enterprises as a completely ineffective policy prescription.

Keywords: SME, Structural Transformation, Intergenerational Growth, Entrepreneurship, Education, Intermediate Inputs, Neighbourhood Effects, History Dependence.

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1 Introduction

The promotion of Small and Medium Enterprises (SMEs) has always been considered an important policy instrument for dealing with persistence of poverty and inequality in developing countries. For example, targeted support for SMEs constitutes a core component of the development strategy followed by the Word Bank Group, with a gross expenditure on this account of around $3 billion per year over the period 2006 - 12.[1] In India, the foundations for policies for SMEs were laid as early as in the second five year plan in 1956 (Kashyap, 1988), a programme that still finds favour among the policy-makers today.[2] The justification for such support comes from the recognition of the role of SMEs in creating job opportunities, alleviating poverty, and in fostering competition and entrepreneurship which enhance growth (Beck et al., 2005). Moreover, in emerging economies like India, where agriculture still constitutes the primary source of employment for majority of the population[3], SMEs are viewed as the vehicle of structural transformation, facilitating the transition of the labour force from less productive traditional occupations to more productive manufacturing production.

Despite the perceived importance of SMEs in the development process, there are very few studies in the literature which explore the theoretical pathways through which such linkages work. This paper is an attempt in this direction. In this paper, we explore the mechanisms through which SMEs can usher in structural transformation in an economy and in the process act as an effective measure in pulling households out of long run poverty traps.

The production activities of SMEs in an economy are often quite diverse and can range from production of final goods to provision of intermediate inputs. This study however, focuses on the specific role that they play as ancillary firms in providing intermediate inputs to large firms. While large firms are highly skill-intensive (Idson and Oi, 1999), these small ancillary units are run by innovating entrepreneurs who operate with little formal training or specialized skills but are able to develop inputs that are useful to the large firms. Limiting ourselves to this narrow definition of SMEs allows us to discern the relationship between SMEs and entrepreneurship. The structure of the modern sector (consisting of large firms), on the other hand, draws from a product variety model of endogenous growth (a la Romer (1987)), which ties the entrepreneurial abilities in the SMEs with the growth process of the economy.

To explore the issue of structural transformation and poverty alleviation, we posit a dual economy framework, where a less productive traditional or cottage sector coexists with a highly productive and skill-intensive modern sector. Skill formation entails a lumpy investment and in the presence of credit market imperfections, and in the absence of an intermediate inputs sector, the economy falls into a long run poverty trap (Galor and Zeira, 1993). In this context, the appeal of SMEs lies in their role as a middle sector which provides alternative employment opportunities to low skill workers. In this paper, we examine the scope of the intermediate inputs sector in facilitating transition of workers

[1] World Bank Report on "The Big Business of Small Enterprises", http://ieg.worldbank.org.

[2] For example, the stated objective of the Micro, Small and Medium Enterprises Development Act (2006) is to "provide for facilitating the promotion and development and enhancing the competitiveness of micro, small and medium enterprises and for matters connected therewith or incidental thereto."

[3] One can get a sense of this by looking at the Employment and Unemployment Survey, NSS 68^{th} round, $2011-12$, which shows that in India, 67.15% of workers were engaged in the agriculture according to usual status in the rural areas and 37.7% of the overall population was employed in agriculture according to usual status.

out of the cottage sector even when credit constraints are binding.

The main finding of this paper is that the role of SMEs in its dual role of ushering in structural transformation as well as eradicating poverty is limited by the size of the modern manufacturing sector, and in particular, by the size of the skilled labour force employed in the modern sector. The intuition behind this result is quite straight forward. Since the SMEs provide intermediate inputs to the modern sector, the profits from producing these ancillary goods and services depend on the level of demand coming from the modern sector. Unless the demand for each input is sufficiently high, it does not pay to switch from traditional occupation to the intermediate production. At the same time, skilled labour and the ancillary goods and services being complementary inputs in the modern production process, the non-availability of the ancillary inputs lowers the productivity of skilled labour, thereby affecting growth. Thus, our model argues that focusing on promotion of SMEs alone may not be sufficient to initiate structural change and alleviate poverty. One needs a holistic approach towards designing a policy which puts equal emphasis on education and skill formation.

To the extent that education decisions are undertaken by the parents and their choices are influenced by the people they interact with, one could argue that skill formation itself could be influenced by the presence of an intermediate goods sector, which allows for a closer interaction between the skilled labour force and the intermediate goods producers. The interaction at their workplace might allow them to expand their social network, which may positively influence the schooling decisions regarding their children. The gain from such social networks may range from direct benefits like better information and guidance to indirect benefits of providing role models. In contrast, households working in the cottage sector might be entirely left out of this loop, given their limited proximity to the skilled workers. The growth model presented in this study adds this additional dimension by attempting to incorporate this shift in the education investment decision of parents working in different sectors. We show that the presence of such neighbourhood effects may propagate the structural transformation process once it has been initiated. But inadequate size of the modern sector remains a structural bottleneck that hinders the initiation of such a process in the first place. Thus, the argument about a holistic policy approach stays.

While there are very few theoretical works in the literature which have explored the role of SMEs in the process of structural transformation, two studies (Dias and McDermott, 2006; Gries and Naudé, 2010) come close to the idea discussed in our paper. Gries and Naudé (2010) develop a dual economy model to explore the role of entrepreneurship in the process of structural transformation. However, their study differs crucially from ours in terms of the interpretation and the role envisaged for the intermediate goods producers in the process of structural transformation. The driving force of structural transformation in Gries and Naudé (2010) is essentially the start up entrepreneurs or the intermediate goods producers, and whether an economy can attain structural transformation or not depends on the size of these innovative business enterprises and their innate entrepreneurial abilities. Skill formation and education do not play any role in their model. In contrast, in our model, the small scale manufacturers do not come from any specially talented group. In fact they come from the rather disadvantageous section of the population who, due to lack of enough education, cannot take part in modern production as skilled labour. Small scale manufacturing therefore becomes a substitute for low skill traditional cottage production, but its demand is limited by the size of the modern sector. This key difference allows us to examine the effectiveness of policies that promote SMEs vis-a-vis policies that facilitate skill formation in ushering in structural transformation. In Gries and Naudé (2010) on the other hand, entrepreneurial talents are innate and hence there is limited role of policy in the structural transformation process.

The study by Dias and McDermott (2006) examines a problem different to ours, but the mechanism is complementary to our paper. Dias and McDermott (2006) emphasize the need of institutional polcies that promote entrepreneurship to complement strides in human capital formation, while we argue that policies to promote education and human capital formation are essential for structural transformation over and above promoting SMEs. In their study, entrepreneurs are described as owners of modern manufacturing firms and the study describes their role in pulling people from the traditional sector to work in their firms and in the process encouraging them to acquire higher human capital. However, if the number of entrepreneurs is below a minimum critical number then the economy does not grow and migration does not happen. Our model broadly differs in the structure of the economy and the pathway through which human capital drives structural transformation. In our model, the role of entrepreneurs is not as owners of the modern sector firms but as suppliers of efficiency enhancing intermediate inputs. The entrepreneurial sector acts as a bridge for workers in the traditional sector who will never be able to impart the minimum level of human capital that will allow their children to work in the modern sector, given their low wages and credit market imperfections. The study by Dias and McDermott (2006) on the other hand assumes that workers in the traditional sector, though lowly paid, can always make lumpy investments in human capital by borrowing using their future labour wages as collateral, which seems very unlikely in a developing country setting. Further, in our model, choosing entrepreneurship as an occupation comes with a possibility of failure. Human capital formation facilitates transformation by increasing the probability of success by bridging information gaps required for coming up with a product that is demanded by the modern sector and this rise in probability pulls more people into the entrepreneurial sector. However, whether an economy will stagnate or will transform either completely or partially into a modern economy critically depends on the size of the skilled labour force.

The rest of the paper is organized into five sections. The second section describes the structure of the economy, and explains the household side and the production side of the economy. The third section describes the occupation choice of the households. The fourth section uses the structure of the economy from the preceding two sections to describe the dynamics of structural transformation in the economy. The fifth section presents the comparative statics of the model and tries to explain the impact of the shift in two critical parameter of the model on the outcomes of structural transformation. The sixth and final section concludes with a discussion of possible policy implications.

2 Basic Structure of the Model

2.1 The Household Side of the Economy

We consider a closed economy with overlapping-generations, comprising of a continuum of heterogeneous households of measure one. Each agent in any household lives for exactly two periods: first period as a child and second period as an adult. A single child is born to the agent at the beginning of the second period of her life. Thus at any point of time, each household has a single adult and a single child. The population in each generation remains constant at unity.

The life cycle of an agent belonging to any generation is as follows. In the first period of her life, an individual consumes nothing and only acquires education, the extent of which depends on parental investment in education. All individuals are identical with regards to their innate abilities, but they differ only by the level of education that is bestowed on them by their parents. Although the level of education received by the child is decided by the parent, the occupational choices are made upon adulthood, when she enters the labour market.

In the second period of her life, the agent as an adult chooses an occupation based on the level of education received as a child and the income associated with various occupations. The occupational choice of the agent determines the level of income that she earns upon adulthood, which she spends on her own consumption as well as in educating her own child. The agent dies at the end of this period.

2.1.1 Utility Function

We assume that an agent derives utility from her current consumption as well as from the level of education attained by her child. The assumption that parents care for children's education is fairly standard in the literature [4] and can be explained by the warm glow felt from educating their children which comes from knowing that education has its virtues along with the gains from higher potential earning for children when they enter the work force. The utility function of the parent is assumed to be a simple Cobb-Douglas form described as follows-

$$U_t(c_t, e_t) = (c_t)^{1-\beta} \cdot (e_{t+1})^{\beta}$$

Here, U_t is the utility of an agent belonging to generation t, c_t is the second period (adulthood) consumption of the agent and e_{t+1} is the education level bestowed on the child. The coefficient, β, which lies between 0 and 1, represents the weight attached or importance which the parent places on the child's education. In other words, the parameter β captures the importance of education as perceived by the parents.

Parents choose a level of education e for their child which lies between 0 and 1. As a simplification, we assume that the level of education also captures the investment required to acquire that level of education. Investing an amount $e \in [0, 1)$ in education generates a semi-skilled worker with a corresponding skill level of $e \in [0, 1)$. The semi-skilled workers can either work in a self-sustained cottage sector or in small-scale manufacturing, producing intermediate inputs for modern production. On the other hand, an investment of $e = 1$ (i.e, completing the highest possible level of education) generates a fully skilled worker, who has the option of working as a skilled worker in the large scale enterprises.

One of the factors which influences the importance that the parent places on the child's education is the environment at their work place. We postulate that working in a more educated environment enhances the weightage that parents place on their children's education. This positive linkage between average level of education at the workplace and children's education can be explained by several factors like existence of positive role models, face to face transfer of information and experiences in educating children, and better networks (Sheldon, 2002). To capture this effect we assume that the parameter β takes different values for parents working in different sectors. In particular it takes a value $\overline{\beta}$ when parents are engaged in modern production either as skilled workers or as

[4] See for example Tamura (2010).

intermediate inputs providers, while it takes a value $\underline{\beta}$ when parents are engaged in the cottage sector, such that $0 < \underline{\beta} < \overline{\beta} < 1$.

Preferences aside, parents are constrained by their income and they need to optimally allocate resources between their current consumption and their children's education. Their budget constraint is given as follows-

$$c_t + e_{t+1} = y_t$$

Here c_t and e_{t+1} are in terms of money spent on current consumption and education of the child, and y_t is the income of the parent. Solving the maximization problem of the parent gives us the optimal level of education, given as follows-

$$c_t = (1 - \beta) \cdot y_t$$

$$e_{t+1} = \beta \cdot y_t$$

The corresponding indirect utility of an agent:

$$U_t^* = (1 - \beta)^{1-\beta} \cdot (\beta)^{\beta} \cdot y_t$$

Since the indirect utility is monotonic in y_t, the agent will choose the occupation that is expected to generate higher income (given her education level). We shall come back to the optimal occupational choice after we discuss the production structure, which pins down the incomes from various occupations.

2.2 The Production Side of the Economy

A single final good is produced in the economy using two different technologies: a high skill-intensive modern technology as well as a low skill-intensive cottage technology. In particular, we assume that production in the cottage sector requires only unskilled labour; thus having some degree of skill does not make any difference in terms of productivity of workers engaged in cottage production. The modern sector, on the other hand, employs the services of skilled workers. In addition, the modern sector may also employ various specialized intermediate inputs which augment the productivity of skilled workers employed in modern firms. Thus, in addition to the two sectors producing the final goods, there may exist an intermediate goods sector producing a variety of inputs for the modern sector and are operated by semi-skilled workers with varying degrees of skills. These enterprises can range from ancillary units providing intermediate goods for production, to enterprises which provide services that assist in the production process of the larger units. The following sections provide a more detailed description of the three sectors.

2.2.1 The Cottage Sector

The cottage sector produces the final good using rudimentary technology, characterised by low productivity. Examples of enterprises which have features of the cottage sector can include small village crafts (pottery, blacksmith, handloom, blacksmiths), urban cottage industry (gold-smithy, matches) and urban seasonal industries (brick and pottery) (Kashyap, 1988). The sector can employ workers of all education levels but there are no returns to education in this sector. The production function of the cottage sector can be described as follows-

$$Y_{ct} = w_c L_t$$

Here, Y_{ct} is the output of cottage sector and w_c is the marginal product of labour. Irrespective of the level of education, every worker in this sector gets the same wage w_c, equal to the marginal product in this sector. The fraction of the total working population engaged in this sector is denoted by L_t.

2.2.2 The Modern Sector

The structure of the modern or large scale sector of the economy draws from the product differentiation model by Romer (1987). This sector comprises of large skill intensive enterprises operating at the technology frontier and having high productivity. The modern sector produces the same final good as the cottage sector. But unlike the cottage sector, it only employs workers with the highest level of skill i.e. $e = 1$. In addition, it may also employ a collection of intermediate inputs as a complementary factor to skilled labour. The production technology that incorporates this complementarity between skilled labour and intermediate inputs is described as follows:

$$Y_{st} = AH_t + H_t^{1-\alpha}(\sum_{i=1}^{M_t} x_{it}^{\alpha})$$

This sector only employs highly educated skilled workers, which is denoted by H_t. It also employs M_t varieties of intermediate inputs. The quantity of the i^{th} intermediate input used, at a particular time t, is given by x_{it}. An interesting feature of this technology is that while skilled workers constitute an essential component of the production process, the sector can operate even in the absence of the intermediate inputs. The use of intermediate inputs however, raises the output as well as the productivity of skilled workers.

The production technology in the modern sector exhibits constant returns to scale and is operated by perfectly competitive firms, such that each input is paid its marginal product.

The wages paid to the workers in any time period t is given by:

$$w_{st} = \frac{\delta Y_{st}}{\delta H_t} = A + (1-\alpha)H_t^{-\alpha} \sum_{i=1}^{M_t} x_{it}^{\alpha} \tag{1}$$

The price paid for each intermediate good i is again given by its marginal product:

$$p_{it} = \alpha x_i^{\alpha-1} H_t^{1-\alpha} \tag{2}$$

Having described the modern sector, we now move on to the characterization of the intermediate inputs sector.

2.2.3 Intermediate Inputs Sector

The intermediate inputs sector works very closely with the modern sector and generates a positive externality on the skilled workers as described in the previous section. The role of this sector is to provide inputs in the form of intermediate goods and services to the modern sector. There are M_t varieties of intermediate inputs supplied at any point of time t and each variety is monopolized by the entrepreneur who creates it. Thus, M_t also captures the number of agents who are engaged in intermediate input production (who are the small-scale entrepreneurs/manufacturers and monopoly owners of the respective intermediate inputs). In our model, at each time t, M_t is determined endogenously - emerging out of the occupational choices of agents.

The production technology for intermediate inputs is symmetric across all inputs: producing 1 unit of the intermediate input requires 1 unit of the final good as the input. Since the final good is the numeraire, the price of the final good is 1. This gives a cost function for each intermediate good produced as $C(x_{it})$, which is linear in the intermediate output i.e. $C(x_{it}) = x_{it}$. Notice that equation (2) denotes the inverse demand function for each intermediate input i. The monopoly producer of the i-th intermediate input knows his demand function and decides on the profit-maximizing level of output i (x_i) accordingly. The profit function of a intermediate goods producer is described as follows:

$$w_{it} = \max_{x_{it}} [\ p_{it} x_{it} - x_{it}\ ]$$

Replacing p_{it} in the above equation by equation (2), described in the previous section, and then solving for the first order conditions gives us:

$$x_{it}^{*} = \alpha^{\frac{2}{1-\alpha}} H_t$$

Facing symmetric demand and symmetric cost conditions, each entrepreneur in equilibrium will produce the same level of output i.e. $x_{it} = x_t^{*}$ for all i. Using the symmetric solution in the inverse demand function of the intermediate inputs given in equation 2, we find that the prices are also symmetric and are a mark-up over the marginal cost. Since the marginal cost is the price of the final good, the price of all the intermediate inputs is as follows:

$$p_t = \frac{1}{\alpha}$$

Using the optimal solutions for the production of intermediate inputs and the corresponding prices, the income of each entrepreneur can be shown to be -

$$w_{It} = \Pi H_t \quad , \qquad where\ \Pi \ = \ (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}$$

Now using the symmetric solution of intermediate inputs in the production function of the modern sector, we get -

$$Y_{st} = [A + \alpha^{\frac{2\alpha}{1-\alpha}} M_t] H_t$$

Moreover, the wage equation of the workers in the modern sector can now be written as -

$$w_{st} = A + \frac{\Pi}{\alpha} M_t \tag{3}$$

One can now clearly see the complementarity between the modern sector and the intermediate inputs sector. If output in the modern sector rises due to employment of a higher number of skilled workers, then the corresponding demand, and therefore profit, of the intermediate inputs also rises. Further, employing more of the intermediate inputs also raises the productivity of the modern sector workers and this raises their wages.

Having described the production structure of the economy, and the corresponding incomes from various occupations, we can move on to describing the occupational choice of workers.

3 Occupational Choice

There are two factors affecting the occupation choice of workers: the first is their education level, which is determined by their parents, and the second is the wage rate in each sector, which is determined endogenously in the model. If a worker has education level equal to 1 then she can work in the modern sector provided that the wages are favourable. However, the workers who do not have the highest level of education, i.e. $e = 1$, cannot work in the modern sector and have to choose between being engaged in intermediate input production or working in the cottage sector.

We assume that each agent in the economy has identical entrepreneurial ability. However, working in the intermediate inputs sector requires an initial investment of τ fraction of the total labour hours of the worker. This time investment is required for the agents to develop a particular variety of intermediate inputs that will aid the skilled workers in the modern production process. However, this time investment is a sunk cost and does not guarantee that the worker will be able to find a marketable intermediate input. We assume that even after incurring the fixed time cost in R&D, the potential entrepreneur-producer successfully develops an intermediate good with probability p - which is same across all agents in the economy. However, this probability of success of an entrepreneur is endogenous - it depends on the ease of accessing knowledge available in the economy.

Knowledge and information can thought of as being embodied in education and the ease of accessing it can be thought to be captured by the average level of education. A higher average level of education would mean that the education level of all the workers is high or even if the education level of a worker is not high, then it is more likely that she can easily seek information from another worker who is better educated. The probability of success can hence be thought to be a function of average level of education in the economy. Higher is the average level of education, the easier it is for the worker to gather information and this increases their chance to come up with a marketable innovation. For simplicity we assume that the probability of success (p) is described by the average level of education itself. Since the education level of the worker $e_t \in [0, 1]$, the average level of education also lies in the same range. The probability of success can now be described as -

$$p(\bar{e}_t) = \bar{e}_t \in [0, 1]$$

If everyone in the economy is educated to the highest level, i.e. $\bar{e}_t = 1$, then every entrepreneur in pursuit of innovative intermediate good will always find enough information to ensure that she is always successful i.e. $p(1) = 1$. On the other hand, if the average level of education is zero, then it is impossible to innovate i.e, $p(0) = 0$.

Since there is a sunk cost as well as uncertainty in terms of whether one will be successful in finding a marketable intermediate input, it implies when an individual enters the intermediate goods production, she compares the expected income from intermediate goods production with the wages she would earn in the cottage sector. The workers who fail as entrepreneurs are absorbed into the cottage sector. Since the failed workers have lost a fraction τ of their total labour time while attempting to come up with an innovative intermediate input, they only have a fraction $(1 - \tau)$ of their total labour time available for working in the cottage sector. This means that the expected wages from working in the intermediate inputs sector is,

$$E[intermediate\ goods\ sector\ wage] = p(\bar{e}_t)w_{It} + [1 - p(\bar{e}_t)](1 - \tau)w_c$$

The workers who do not have an education level of 1 will choose to work in the intermediate inputs sector if only if the expected wages from being an entrepreneur exceed the full wages in the cottage sector i.e.

$$E[Intermediate\ Goods\ Sector\ Wage] \geq w_c$$

Even though having an education level of 1 makes a worker eligible for a modern sector job, she will choose to work in this sector only if the wages are higher than the wages she expects to earn from being an entrepreneur i.e workers choose to work in the modern sector iff-

$$w_{st} \geq E[Intermediate\ Goods\ Sector\ Wage]$$

Now that we know the structure of the economy as well as how workers choose their respective occupation, we can now proceed to the analysis of how education, occupation choices and incomes of individuals evolve over time.

4 Dynamics of Education, Occupation Choices and Income

This section will describe the dynamics of education, occupation choices and income in the economy by analysing the levels of average education and the number of modern sector workers. We begin our analysis by looking at the education choices that parents make for their children which will help us describe the average level of education in the economy. The education level of the child, which comes from the utility maximizing exercise of the parent, is a function of the parent's income and their belief on how important their children's education is vis a vis their current consumption i.e. $e_{t+1} = \beta \cdot y_t$. But parents working in all sectors of the economy do not value education in the same way. We had mentioned earlier that parents in the modern sector place a higher weightage $\overline{\beta}$ on their children's education compared to a lower weightage $\underline{\beta}$ placed by parents in the cottage sector. Depending on the sector in which parents choose to work in, the education level of the children can be described as follows-

$$e_{t+1} = \begin{cases} \overline{\beta} w_{st}, & \text{If } e_t = 1 \text{ and parents work in the modern sector.} \\ \overline{\beta}\ w_{It}, & \text{If parents are entrepreneurs in the intermediate inputs sector.} \\ \underline{\beta}\ w_{ct}, & \text{If the parents work in the cottage sector.} \end{cases}$$

The children use their endowment of education and choose to work in one of the three sectors in the economy and the total labour-force at any point of time is normalized to 1. In this model, we assume that the economy has full employment and each worker is employed in either of the three sectors such that,

$$H_t + M_t + L_t = 1$$

We begin our description of the dynamics by assuming that the economy at time t has a given distribution of education such that h_t[5] fraction of adult agents have education

[5]It is important to point out the difference between h_t and H_t here. Note that h_t is the fraction of workers who are *eligible* to work in the modern sector, while the *actual* number of workers in the modern sector is give by H_t. Being eligible does not mean that they actually choose to work in the modern sector. This choice is determined entirely by wage incentives.

level 1 and the remaining $(1-h_t)$ fraction of adult agents have some education level lying between zero and unity. The corresponding average level of education of adult agents in any time period t will be given by,

$$\overline{e}_t = h_t \cdot 1 + \int_0^{1-h_t} e_{it} di.$$

Since by assumption, h_t and all the e_{it}s lie between zero and unity, so will be $\overline{e}_t$. In other words, the possible combination of $(\overline{e}_t, h_t)$ will lie on or below the forty five degree line passing through the $(\overline{e}_t, h_t)$ plane.

Based on the optimal educational investment choices that parents make for their children in any period t, the average level of education in the next period can be determined by considering the wages and the number of parents working in each sector. This is given as follows-

$$\overline{e}_{t+1} = \overline{\beta} w_{st} \cdot H_t + \overline{\beta} w_{It} \cdot M_t + \underline{\beta} w_{ct} \cdot L_t \tag{4}$$

The numbers of workers in each sector (H_t, M_t, L_t) and the corresponding wages (w_{st}, w_{It}, w_{ct}) are determined endogenously in the model.

Describing the number of modern sector workers in each period of time requires a careful analysis of how agents choose their occupations. We make here three parametric assumptions which make the occupational choices interesting and non-trivial. These are specified below.

Assumptions:

1. $A > w_c$
2. $\overline{\beta} A \geq 1$ and $\underline{\beta} w_c < 1$
3. $A < \Pi$

The first assumption means that the skilled workers employed in the modern sector always earn more than the unskilled workers employed in the cottage sector. The reason for using A and w_c as benchmarks for comparison is because, A is the lowest possible wage for a worker in the modern sector [6] and w_c is the constant wage rate for a cottage sector worker. Thus assumption 1 ensures that the skilled workers have no incentive to join the cottage production; they will always seek employment in modern production.

The second assumption means that parents engaged as skilled worker in the modern sector will always educate their children to the maximum extent[7] and this is never true for the parents working in the cottage sector. The cause of low levels of education in the cottage sector can be thought to be the result of low wages as well as low importance that parents place on their children's education.

The third assumption means that if all agents were educated to the maximum level and all of them were working in the modern sector then the potential income in the

[6]Recall from equation 3 that the modern sector wage rate is given by $w_{st} = A + \frac{\Pi}{\alpha} M_t$. If there is no intermediate inputs sector i.e. $M_t = 0$ then $w_{st} = A$.

[7]Although $\overline{\beta} A \geq 1$, parents will spend till the point where $\overline{\beta} A = 1$ since there is no point investing more than the maximum level.

intermediate inputs sector would be higher than the skilled wage rate in the modern sector. This would indirectly imply that we will never observe a situation where every agent in the economy is employed as skilled labour in the modern sector. Even if everyone in the economy was educated to the full extent, there will be incentives for agents to move between the modern sector and the intermediate inputs sector till the wage differentials between the sectors are evened out.

We now take a closer look at the conditions determining the agent's choice of occupation. The first condition that describes the occupation choice of an agent with an education level less than unity is denoted by condition A, which is specified below:

<u>Condition A</u>: Workers with education level less than 1 ($e_t < 1$) will work in the intermediate inputs sector if and only if the expected income from being an entrepreneur exceeds the wages in the cottage sector, or,

$$\overline{e}_t \cdot \Pi H_t + (1 - \overline{e}_t) \cdot (1 - \tau) w_c \geq w_c$$

The above equation can be rearranged to find the possible combination of H_t and $\overline{e}_t$ that satisfy Condition A. The rearranged equation is as follows-

$$H_t \geq \frac{w_c}{\Pi} \left(\frac{\tau + \overline{e}_t(1 - \tau)}{\overline{e}_t} \right) \tag{5}$$

The combinations of $(\overline{e}_t, H_t)$ that satisfy the above expression can be represented in the $\overline{e}_t$-H_t plane. However, we also need to remember that $H_t \leq h_t$ and all combinations of $(\overline{e}_t, H_t)$ will always lie below the forty five degree line. The set representing all points where Condition A is satisfied is arrived at by making the following observations about equation 5, if it holds with equality,-

$$When\ \overline{e}_t \to 0\ then\ H_t \to \infty$$

$$When\ \overline{e}_t \to 1\ then\ H_t \to \frac{w_c}{\Pi}$$

$$When\ H_t \to 1\ then\ \overline{e}_t \to \frac{w_c - (1 - \tau) w_c}{\Pi - (1 - \tau) w_c}$$

Now, we take a look at the first and second derivatives of H_t with respect to $\overline{e}_t$ -

$$\frac{dH_t}{d\overline{e}_t} = -\frac{\tau w_c}{\Pi\ \overline{e}_t^2} < 0$$

$$\frac{d^2(H_t)}{(d\overline{e}_t)^2} = \frac{\tau w_c}{\Pi\ \overline{e}_t^3} > 0 \qquad (\text{Since } 0 \leq \overline{e}_t \leq 1)$$

This gives all the possible values of $\overline{e}_t$ and H_t, which satisfy Condition A and also fall in the admissible set. This is the upper contour set of the equation of Condition A and is indicated by the shaded area in figure 1.

The second condition that describes the occupation choice of a worker with education level equal to unity is denoted by condition B, which is specified below:

<u>Condition B</u>: Workers with education level equal to 1 ($e_t = 1$) will choose to work

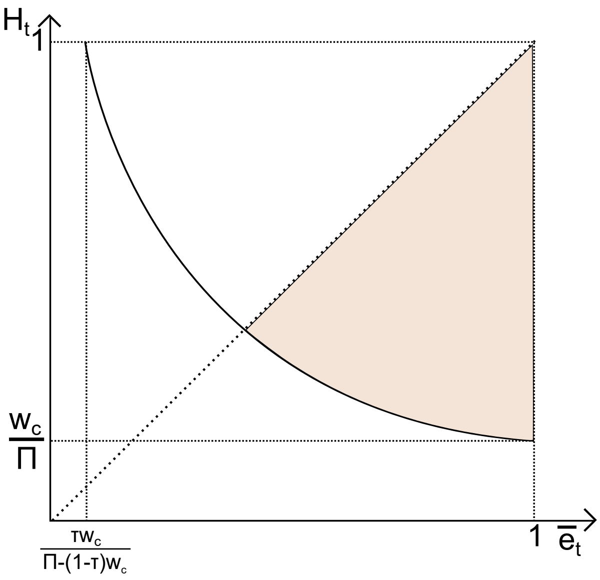

Figure 1: Condition A - Combinations of $(\overline{e}_t, H_t)$ for which the workers with education level less than 1 will always choose to work as entrepreneurs.

in the modern sector if and only if the expected wages from being a modern sector worker exceed the expected wages from being an entrepreneur, or,

$$A + \frac{\Pi}{\alpha} M_t \geq \overline{e}_t \cdot \Pi H_t + (1 - \overline{e}_t) \cdot (1 - \tau) w_c$$

The above expression is rearranged to get the possible combinations of $\overline{e}_t$ and H_t that satisfy Condition B. This is as follows-

$$H_t \leq \left[\frac{A + (1-\tau)w_c}{\Pi(1+\alpha)} \right] \frac{\alpha}{\overline{e}_t} + \left[\frac{1}{1+\alpha} - \frac{(1-\tau)\alpha w_c}{\Pi(1+\alpha)} \right] \tag{6}$$

The following observations can be made about the above equation if it holds with equality-

$$When\ \overline{e}_t \to 0\ then\ H_t \to \infty$$

$$When\ \overline{e}_t \to 1\ then\ H_t \to \frac{A + \frac{\Pi}{\alpha}}{\Pi + \frac{\Pi}{\alpha}}$$

$$When\ H_t \to 1\ then\ \overline{e}_t \to \frac{A - (1-\tau)w_c}{\Pi - (1-\tau)w_c}$$

Now, we take a look at the first and second derivatives of H_t with respect to $\overline{e}_t$ -

$$\frac{dH_t}{d\overline{e}_t} = - \left[\frac{A - (1-\tau)w_c}{\Pi + \frac{\Pi}{\alpha}} \right] \frac{1}{\overline{e}_t^2} < 0$$

$$\frac{d^2(H_t)}{(d\overline{e}_t)^2} = \left[\frac{A - (1-\tau)w_c}{\Pi + \frac{\Pi}{\alpha}} \right] \frac{1}{\overline{e}_t^3} > 0$$

The combinations of $\overline{e}_t$ and H_t that satisfy Condition B and also lie in the admissible set can be represented by the lower contour set of the equation of Condition A. This is

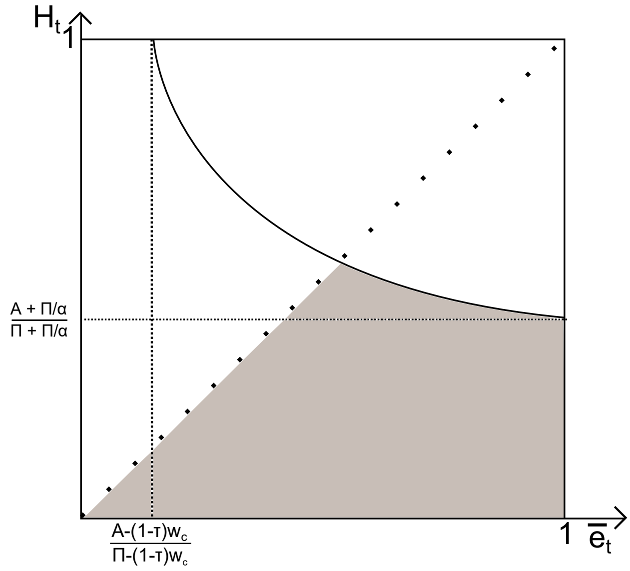

Figure 2: Condition B - Combinations of $(\overline{e}_t, H_t)$ for which the workers with education level 1 will always choose to work in the modern sector.

indicated by the shaded region in figure 2.

The combinations of $\overline{e}_t$ and H_t which satisfy Conditions A or B are shown in same graph in figure 3 . It is useful to simultaneously analyse both the conditions because there is a certain portion of overlap i.e. a set of combinations where both conditions are simultaneously satisfied [8]. Region A represents the combinations where only Condition A is satisfied. Region B represents the region where only Condition B is satisfied. Region AB represent the region where Conditions A and B are simultaneously satisfied. Now the stage has been set for the analysis of the dynamics of education, occupation choices and income. The rest of this section will deal with the dynamics in each of the three regions shown in figure 3, beginning with region B, then moving to regions AB and A.

We now analyse the evolution of an economy, starting from an initial combination of $(\overline{e}_t, h_t)$. Recall that, although h_t is historically given, H_t in this model is endogenously determined by the optimal occupational choices of the households, such that at any point of time t, $H_t \leq h_t$. Thus given $(\overline{e}_0, h_0)$, we have to first determine the corresponding occupational choices of agents, which, through the children's education decision, will generate the next period's combination of $(\overline{e}, h)$. In other words, to trace the dynamics we first have to optimally solve the occupational choices of various agents. Since these optimal occupational choices vary depending on whether the economy is in Region B, or Region AB or Region A, the corresponding dynamics for each of these regions will differ as well. Accordingly, we analyse below three mutually exclusive cases.

4.1 CASE 1: Only Condition B is satisfied (Region B)

Let us first look at economies which start with an initial combination of $(\overline{e}, h)$ which lies in region B. Economies starting in this region will have all the highly educated agents joining the modern sector. On the other hand, since Condition A is not satisfied in

[8]Using the parametric assumptions, it can be easily shown that $\frac{A+\frac{\Pi}{\alpha}}{\Pi+\frac{\Pi}{\alpha}} > \frac{w_c}{\Pi}$ and $\frac{A-(1-\tau)w_c}{\Pi-(1-\tau)w_c} > \frac{w_c-(1-\tau)w_c}{\Pi-(1-\tau)w_c}$.

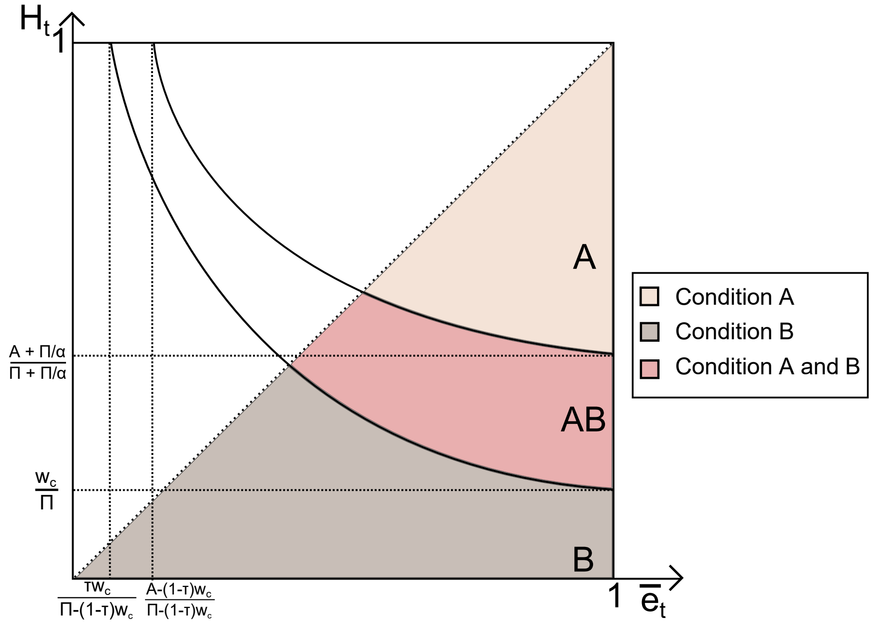

Figure 3: Combinations of $(\overline{e}_t, H_t)$ for which the Conditions A or B are satisfied.

this region, agents with education level less than 1 will not find it worthwhile being entrepreneurs. These two features together will generate the following pattern of occupational choices in Region B. First, every agent with education level 1 will work in the modern sector, implying $H_t = h_t$. Secondly, agents with education level less than 1 will work full-time in the cottage sector, implying, $M_t = 0$; $L_t = (1 - H_t)$. Finally, given these values of H_t, M_t and L_t, the corresponding incomes are given by $w_{ct} = w_c$ and $w_{st} = A$.

Since all agents with education level 1 always work in the modern sector and the agents with education level less than 1 will always choose to remain in the cottage sector, there is no mobility of workers between sectors. The lack of mobility of the workers implies that $H_t = h_t = h_0$ and $L_t = (1 - h_0)$, or, $\Delta H_t = \Delta h_t = 0$

Next, in analysing the evolution of average level of education $(\overline{e}_t)$, notice that since all semi-skilled agents working in the cottage sector earn the same income (irrespective of their education level), they will bestow the same level of education on their children $(= \underline{\beta} w_c)$. Likewise, since all skilled agents working in the modern sector earn the same income, they will bestow the same level of education on their children $(= 1)$. Hence we can easily calculate the average education level of the next generation of agents as follows:

$$\overline{e}_{t+1} = 1 \cdot H_t + \underline{\beta} w_c \cdot (1 - H_t)$$

Noting that in this region, $H_t = h_{0,}$

$$\overline{e}_{t+1} = 1 \cdot h_0 + \underline{\beta} w_c \cdot (1 - h_0) \tag{7}$$

Since H_t is fixed at h_0 for all subsequent time periods, the average level of education of the children also gets fixed at a constant value such that $\overline{e}_{t+1} = \overline{e}_{t+2} = \ldots = (1 - \underline{\beta} w_c) \cdot h_0 + \underline{\beta} w_c$. In other words, $(1 - \underline{\beta} w_c) \cdot h_0 + \underline{\beta} w_c$ represents a steady state level of average education in the economy when the economy starts at region B. Moreover, there are multiple such steady state points depending on the exact value of h_0. These multiple steady states can be expressed as a function of the initial level of workers with education level 1, as follows-

$$\overline{e}^B(h_0) = (1 - \underline{\beta} w_c) \cdot h_0 + \underline{\beta} w_c. \tag{8}$$

The above function describing the locus of steady states is linear in the initial level of high education workers (h_0) and the line describing the above equation can be plotted in the $\overline{e}_t$ - H_t plane. This is shown by the dashed lined in figure 4. The above figure

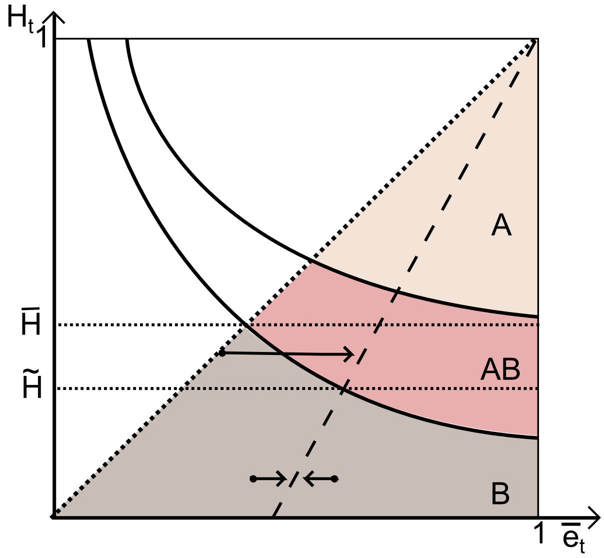

Figure 4: Steady State Level of Average Education and Modern Sector Workers when only Condition B holds.

also shows the paths of economies as they move towards their steady states. Starting from any $(\overline{e}_0, h_0)$ combination lying in region B, the economy will immediately jump to a point $(\overline{e}^B(h_0), h_0)$ in the very next period, and will stay there forever as long as the point $(\overline{e}^B(h_0), h_0)$ also lies in Region B. In terms of figure 4, an economy with the initial size of highly educated agents less than level $\tilde{H}$ (as shown in the figure 4) will continue to be in region B. On the other hand an economy with $h_0 = H_0 \in [\tilde{H}, \overline{H}]$ will move to region AB.

4.2 CASE 2: Conditions A and B are both satisfied (Region AB)

Next, let us consider economies which start with an initial combination of $(\overline{e}, h)$ which lies in the region AB. Economies starting in this region will have all highly educated agents joining the modern sector. Along with this, all the workers with education level less than 1 will always attempt to be entrepreneurs despite the associated uncertainty in incomes. These two features together will generate the following pattern of occupational choices in Region AB. First, every agent with education level 1 will always choose to work in the modern sector, implying $H_t = h_t$. Secondly, all agents with education level less than 1 will always try to be entrepreneurs. However, not all of them are successful and the probability of success is linked to their ability to access and gather information, which in this model is captured by the average level of education. Only a fraction $\overline{e}_t$ of the $1 - H_t$ workers attempting to be entrepreneurs will be successful, implying $M_t = (1 - H_t) \cdot \overline{e}_t$. The rest of the unsuccessful agents will return to work in the cottage sector, implying $L_t = (1 - H_t) \cdot (1 - \overline{e}_t)$. Finally, notice that in this region, the intermediate inputs sector is functional and it complements the modern sector. The concomitant increase in productivity of the workers translates into higher wages for the skilled workers, which at any point of time t is given by $w_{st} = A + \frac{\Pi}{\alpha} M_t$. On the other hand, income of an intermediate goods producer is given by $w_{It} = \Pi H_t$. Finally, the unsuccessful entrepreneurs work in the cottage sector, but they loose out on a fraction τ of their total labour time and have to settle with a lower income of $(1 - \tau)w_c$.

Using equation 4, the average level of education at any point of time $t + 1$ given the average education level and the number of workers in each of the three sectors, in period t, is given as-

$$\overline{e}_{t+1} = 1 \cdot H_t + \overline{e}_t(1 - H_t) \cdot \overline{\beta} \cdot w_{It} + (1 - \overline{e}_t)(1 - H_t) \cdot \underline{\beta} \cdot w_{ct}$$

Replacing values of w_{It} and w_{ct},

$$\overline{e}_{t+1} = 1 \cdot H_t + \overline{e}_t(1 - H_t) \cdot \overline{\beta} \cdot \Pi H_t + (1 - \overline{e}_t)(1 - H_t) \cdot \underline{\beta} \cdot (1 - \tau)w_c \tag{9}$$

A crucial condition determining the education dynamics is the level of investment that parents working as entrepreneurs make for their children. Depending on the investment level we get the following two sub-cases.

Sub-Case 1 : The investment in education, $\overline{\beta} \cdot \Pi H_t$, is not sufficient to ensure that their children have an education level of 1.

This means that despite there being some migration of workers out of the cottage sector, all households that historically started with an initial education level less than 1 will perpetually be restricted to working as entrepreneurs or as cottage sector workers. The reason being that the investments in education by parents in either sector is not sufficient enough to ensure that their children can to move to the modern sector. The households which historically started with education level at 1 are the only ones who work in the the modern sector i.e. $H_t = h_0$. Now, moving on to analysing the dynamics of average education and the number of modern sector workers. The following observation is made regarding the number of workers in the modern sector at any point of time t -

$$\Delta\ H_t = \Delta\ h_t = 0$$

Further, the change in average education at any point of time t is given by-

$$\Delta\ \overline{e}_t = \overline{e}_{t+1} - \overline{e}_t = 1 \cdot H_t - [1 - (1 - H)_t \cdot \overline{\beta}\ \Pi H_t]\overline{e}_t + (1 - \overline{e}_t)(1 - H_t) \cdot \underline{\beta}\ (1 - \tau)w_c \tag{10}$$

Steady state is achieved when both, the number of modern sector workers and the average education of workers stop changing over time. Since the number of modern sector workers is already at its steady state, it is only the average education that needs to adjust to its steady state. This happens when $\Delta\ \overline{e}_t$ is zero. Solving for this using the above equation we get the steady state level of average education $\overline{e}^{AB}$ given as follows-

$$\overline{e}^{AB}(h_0) = \frac{h_0 + (1 - h_0)\underline{\beta}(1 - \tau)w_c}{1 - (1 - h_0)[\overline{\beta}\ \Pi h_0 - \underline{\beta}(1 - \tau)w_c]}$$

Given the initial number of workers with education level 1 (h_0) and the parameters of the model, we can uniquely determine the steady states of these economies. We can further check for the stability of the steady state by re-arranging equation 10 in terms of the steady state as follows -

$$\Delta\ \overline{e}_t = [H_t + (1 - H_t)\overline{B}(1-\tau)w_c]\left(1 - \frac{\overline{e}_t}{\overline{e}^{AB}}\right)$$

From the above equation it can be inferred that the change in average education will be positive if the average education at any time period t is less than the steady state or $\overline{e}_t < \overline{e}^{AB}$. The change will be negative if average education at time t exceeds the steady state level. Since deviations from the steady state are self correcting, the steady state given by $\overline{e}^{AB}$ is a stable one. The locus of steady states for different values of h_0 is shown in figure 5. The economies having average education of its workers and the number of high education workers in region AB will converge to the dashed line, as long as $\overline{\beta}\cdot\Pi H_t \leq 1$ or $H_t = h_t \leq \frac{1}{\overline{\beta}\cdot\Pi}$. Derivation of the locus of steady state for different initial h_0 is obtained by making the following observations about the steady state equation-

$$\frac{d\ \overline{e}(0)}{d\ H_t} > 0\ \ and\ \ \frac{d\ \overline{e}(1)}{d\ H_t} < 0$$

$$\frac{d\ \overline{e}(\frac{1}{\beta\Pi})}{d\ H_t} > 0,\ \ The\ slope\ of\ the\ locus\ is\ positive\ when\ \ H_t = \frac{1}{\beta\Pi}$$

$$\overline{e} = 1\ \ when\ H_t = 1\ or\ H_t = \frac{1}{\beta\Pi}$$

It is observed that the locus of steady states is positively sloped in the region below $H_t = \frac{1}{\overline{\beta}\cdot\Pi}$. Further, an economy in region AB with H_t above $\hat{H}$ and below $H_t = \frac{1}{\overline{\beta}\cdot\Pi}$ will converge to the dashed line representing the steady state $\overline{e}^{AB}$. In this portion of region AB an economy will have a functioning intermediate input sector but these countries will be unable to make a complete transition away from cottage production. The economies with H_t below $\hat{H}$ will move into region B and settle at the steady stage given by $\overline{e}^B$ described in Case 1.

Sub-case 2: Investment in education is sufficient to ensure that the next generation has education level at 1 i.e. $\overline{\beta}\cdot\Pi H_t \geq 1$.

When this condition is satisfied, the children of all the successful entrepreneurs will be highly educated and since Condition B is satisfied in this region, they will choose to work in the modern sector when they join the workforce. If h_t is the number of workers with education level 1 in period t then in the next period,

$$h_{t+1} = H_t + M_t = H_t + (1 - H_t)\cdot\overline{e}_t$$

Since Condition B is satisfied, all these workers will join the modern sector ($H_t = h_t$ for all time periods in region AB). This means that-

$$H_{t+1} = H_t + (1 - H_t)\cdot\overline{e}_t$$

The change in the number of modern sector workers / workers with full education is given by-

$$\triangle H_t = \overline{e}_t(1 - H_t)$$

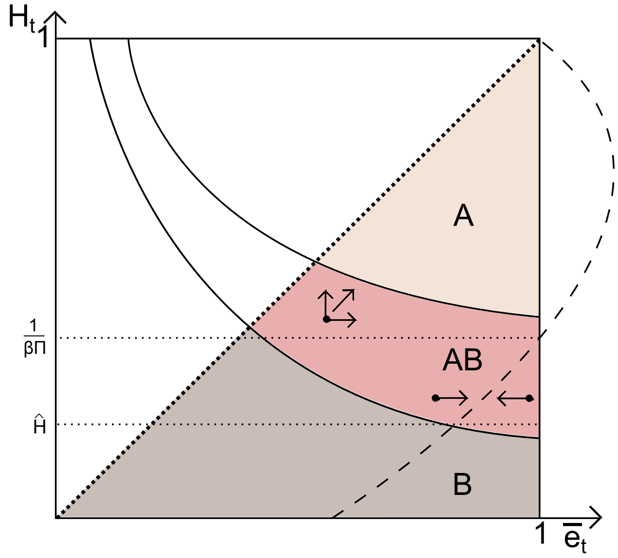

Figure 5: Dynamics and Steady State Levels of Average Education and modern Sector Workers when Conditions A and B hold Simultaneously.

The number of highly educated workers will stop changing only when either $\overline{e}_t = 0$ or $H_t = 1$, but neither of the two happens in regions AB. Hence, as long as the economy is in region AB and $H_t \geq \frac{1}{\beta \cdot \Pi}$, h_t or H_t keeps increasing. Now, coming to the dynamics of education, we revert back to equation 9 and update education investments by entrepreneur parents to 1, this is given below-

$$\overline{e}_{t+1} = 1 \cdot [H_t + (1 - H_t)\overline{e}_t] + (1 - H_t)(1 - \overline{e}_t) \cdot \underline{\beta} \cdot (1 - \tau)w_c$$

and from equation 10 we get,

$$\Delta\ \overline{e}_t = (1 - \overline{e}_t)[H_t + (1 - H_t) \cdot \underline{\beta}\ (1 - \tau)w_c]$$

The above equation is always positive since $\overline{e}_t$ and H_t lie in the range of 0 and 1. Further, the equation tells us that average education will stop adjusting only when everyone is educated to the highest level or $h_t = H_t = 1$ and hence $\overline{e}_t = 1$, but this does not happen in region AB. Hence, as long as the economy is in region AB and $H_t \geq \frac{1}{\beta \cdot \Pi}$, $\overline{e}_t$ keeps increasing.

So we see that both average education and the number of highly educated agents keep increasing till the economy exits this region. Figure 5 shows the direction of motion of the variables in the portion of region AB above $H_t \geq \frac{1}{\beta \cdot \Pi}$.

4.3 CASE 3: Only Condition A is satisfied (Region A)

Finally, let us look at economies which start with an initial combination of $(\overline{e}, h)$ which lies in region A. Economies in region A will find that the cottage sector workers find it profitable to be entrepreneurs. At the same time the highly educated workers do not find it profitable to work as skilled worker in the modern sector. The complementarity [9] between the modern sector and the intermediate inputs sector means that the highly

[9] The complementarity arises because $w_{st} = A + \frac{\Pi}{\alpha} M_t$ and $w_{It} = \Pi H_t$, as workers move out of the modern sector H_t falls and M_t rises and this helps bridge the difference between the wages in the two sectors.

educated workers will continue to switch from the modern sector to the intermediate input sector until the wages in the two sectors are equalised and the marginal agent with education level 1 is indifferent between the two sectors. These two features together will generate the following pattern of occupational choices in Region A. First, note that since not all workers with education level 1 find it profitable to be modern sector workers, we no longer have $h_t = H_t$. However, what continues to hold is that $H_t \leq h_t$, where H_t represents the measure of the highly educated agents who join the modern sector. In fact in every period, H_t and M_t adjust such that -

$$A + \frac{\Pi}{\alpha} M_t = \overline{e}_t \cdot \Pi H_t + (1 - \overline{e}_t) \cdot (1 - \tau) w_c$$

Notice that M_t now consists two sets of agents: $\overline{e}_t$ fraction of the highly educated ones who did not join the modern sector and $\overline{e}_t$ fraction of all the semi-skilled ones, who chose the intermediate input sector over modern sector. Thus

$$M_t = \overline{e}_t \left(h_t - H_t \right) + \overline{e}_t (1 - h_t).$$

Using this relationship in the above equation, we get the optimal occupational choice in this economy as

$$A + \frac{\Pi}{\alpha} \left[\overline{e}_t \left(h_t - H_t \right) + \overline{e}_t (1 - h_t) \right] = \overline{e}_t \cdot \Pi H_t + (1 - \overline{e}_t) \cdot (1 - \tau) w_c$$

Solving the above equation, we get the precise value of H_t, given h_t. Notice that if we write this condition in terms of H_t and M_t then, it describes precisely the boundary points defining region A. This boundary represents the locus of points where Condition B is exactly satisfied. Finally, the unsuccessful entrepreneurs will return to work in the cottage sector and the number of these workers is given by $L_t = (1 - H_t) \cdot (1 - \overline{e_t})$. These workers loose out on a fraction τ of their total labour time and have to settle with a lower income of $(1 - \tau) w_c$. We make an assumption that the failed entrepreneurs with education level 1 are unable to go back to the modern sector because of the time lost during product discovery and hence are forced to join the cottage sector. Since the entire of region A has combinations of $(\overline{e}_t, H_t)$ where $H_t \geq \frac{1}{\beta \cdot \Pi}$ [10], in such economies, all parents who are successful as entrepreneurs invest enough in their children's education to ensure that they have the option of being modern sector workers.

Using the above information we can update the equation of the average level of education in period $t + 1$ as follows-

$$\overline{e}_{t+1} = 1 \cdot H_t + 1 \cdot (1 - H_t) \overline{e}_t + (1 - H_t)(1 - \overline{e}_t) \cdot \underline{\beta} \cdot (1 -_\tau) w_c$$

and the change in average education is given by-

$$\Delta \ \overline{e}_t = (1 - \overline{e}_t) [H_t + (1 - H_t) \cdot \underline{\beta} \ (1 - \tau) w_c]$$

The above equation is always positive and average education keeps growing till everyone in the economy has the highest level of education. On the other hand, not all workers who have the highest level of education want to work in the modern sector and they will

[10] It can be shown that $\frac{A + \frac{\Pi}{\alpha}}{\Pi + \frac{\Pi}{\alpha}} > \frac{1}{\beta \cdot \Pi}$.

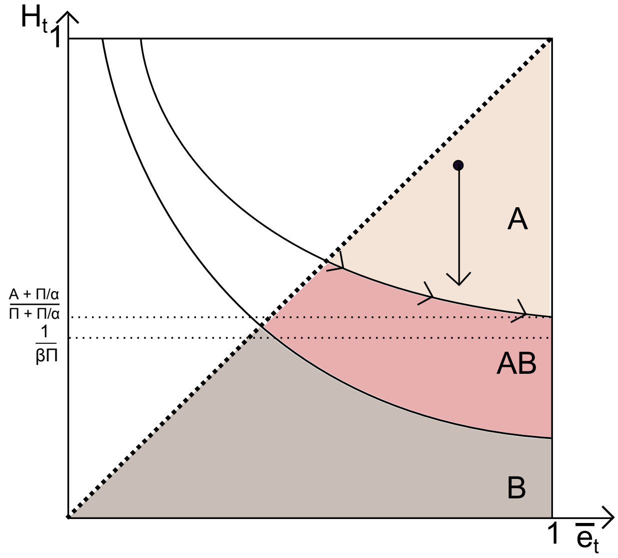

Figure 6: Dynamics and Steady State Levels of Average Education and Modern Sector Workers when only Conditions A holds.

choose to become entrepreneurs till the expected wages in the two sectors equalize and this happens when Condition B is exactly satisfied or,

$$H_t = \left[\frac{A + (1-\tau)w_c}{\Pi(1+\alpha)}\right]\frac{\alpha}{\bar{e}_t} + \left[\frac{1}{1+\alpha} - \frac{(1-\tau)\alpha w_c}{\Pi(1+\alpha)}\right] \tag{11}$$

The change in the number of modern sector workers in each period is given by-

$$\triangle H_t = H_{t+1} - H_t = \left[\frac{A + (1-\tau)w_c}{\Pi(1+\alpha)}\right]\frac{-\alpha \triangle \bar{e}_t}{\bar{e}_{t+1}\bar{e}_t}$$

Since the change in average education is always positive, the number of modern sector workers will fall every period while maintaining that equation 11 is satisfied. The steady state level of average education in this region is $\bar{e}_t = 1$ and the corresponding number of modern sector workers in steady state is $\frac{A+\frac{\Pi}{\alpha}}{\Pi+\frac{\Pi}{\alpha}}$. This adjustment in average education and the number of modern sector workers is shown in figure 6.

A country which finds itself in this region will move to a steady state level where all workers are educated to the maximum extent and the easy availability of knowledge makes the choice of entrepreneurship risk free. Further, incomes across the modern sector and the intermediate inputs sector are equalized, and the overall income of the economy is at its maximum level. Notice that in this region the production structure becomes completely modernised: the low paying cottage sector closes down entirely and production of the final good happens only in the modern sector. In other words, the process of structural transformation is complete for the economy in this region. Given that all workers have full education and they all work either in the modern sector or in the intermediate inputs sector, the wages adjust to the highest level while ensuring that there are no higher paying opportunities in either of the two sectors. The combined dynamics of all the three previous cases is given in figure 7.

This diagram allows us to analyse the long run position of an economy with a given initial size of highly educated agents (h_0) and an initial average level of education ($\bar{e}_0$)

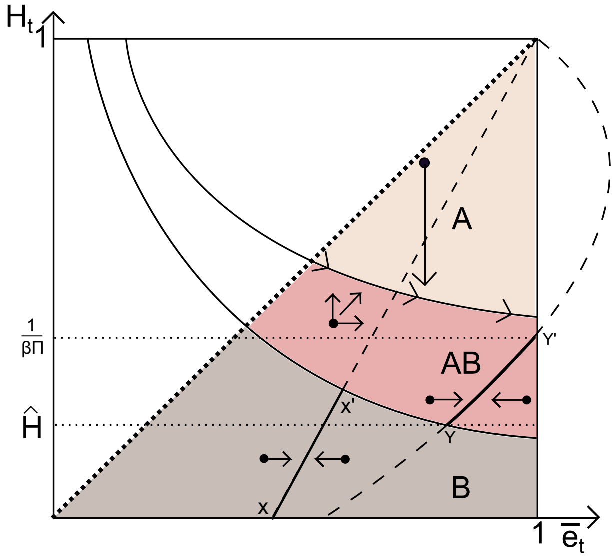

Figure 7: The Dynamics of Education, Occupation and Income.

and its implications for the process of structural transformation. It is easy to see from figure 7 that all economies having the initial size of educated agents below level $\hat{H}$ will immdiately converge to a steady state along line XX', and would be characterized by the presence of a large cottage sector and a small modern sector with no mobility of agents across sectors and therefore no scope for structural transformation. Economies where the initial size of educated agents is above $\hat{H}$ but below $\frac{1}{\beta \cdot \Pi}$ will converge either to a steady state along line XX' or to a steady state along curve YY', depending on the corresponding initial level of average education ($\bar{e}_0$). In particular, for the same size of the highly educated population, the lower is the average level of education, the higher is the probability that the economy will reach a steady state along XX' rather than YY'. Even if it reaches a steady state along YY', the process of structural transformation would not be complete and the economy in the long run would still be characterized by the presence of a cottage sector (albeit smaller). Finally, all economies with size of the highly educated population above $\frac{1}{\beta \cdot \Pi}$ converge to a steady state where $\bar{e}_t = 1$ and $H_t = \frac{A+\frac{\Pi}{\alpha}}{\Pi+\frac{\Pi}{\alpha}}$. These economies are characterized by complete modernization with the productivity of the modern sector being at its highest level.

The inferences made from this model are summarized in the following proposition:

Proposition 1: *Presence of an intermediate inputs sector does not bring about complete structural transformation in economies where the initial size of the educated agents and the concommitant scale of the modern sector is below a minimum level. This minimum scale will vary from economy to economy depending on the importance that parents place on their children's education and the returns to entrepreneurship. Such economies will witness perpetuation of a low productivity cottage sector as well as persistence of inequalities in education, occupation choices and incomes.*

Proposition 2: *The economies where the initial size of the educated agents and the concommitant scale of the modern sector exceeds the minimum scale will undergo com-*

plete structural transformation with an increase in the income generated in the economy and complete equality in education, occupation choices and incomes.

5 Comparative Statics

Not all economies will share the same values of parameters used in the description of the model. The focus of this discussion is on two particular parameters: the parameter measuring the importance that parents place on their children's education (β) and the returns to entrepreneurship in the intermediate inputs sector (Π). The previous section on the dynamics of structural transformations shows that there is a minimum scale of the modern sector for the transformation to be complete. The minimum scale which is given by $H_t = \frac{1}{\overline{\beta} \cdot \Pi}$, depends on the two parameters $\overline{\beta}$ and Π. This minimum scale is essential to ensure that the parents choosing to be entrepreneurs end up investing sufficiently in their children's education so as to ensure that they have the options of moving to the modern sector if it provides them with a wage advantage. This is the exact mechanism which helps bridge the gap between the modern sector and the cottage sector.

The higher is the importance that parents place on their children's education, captured by the $\overline{\beta}$, the higher is the chance of the economy being able to structurally transform. One of the assumptions made in the model is that parents in the cottage sector have low incomes which does not allow them to educate their children to the maximum extent. This is where the intermediate input sector acts as a channel where the entrepreneurs working in this sector can afford to educate their children to the full extent. However, the actual investment depends on how much importance they place on their children's education. A higher importance will lead them to invest more in their children's education which increases the chances of the economy being able to complete its transformation into a modern production economy. We also conjecture that the interactions of the entrepreneurs with the highly educated workers in the modern sector during the process of production influences the importance that they place on their children's education. Even though such shifts in the perceptions about education are not critical in explaining the mechanism of the model, it adds an interesting dimension to the study which warrants further investigation. The effect of the parameter determining the importance on education placed by parents in the intermediate inputs sector can be summarized in the following proposition-

Proposition 3: *Economies are more likely to undergo the structural transformation in the presence of the intermediate inputs sector if parents place a lot of importance on their children's education or if associating with the large scale enterprises helps increase the importance that parents place on the education of their children vis a vis current consumption.*

The changes in the returns to the entrepreneurs, captured by Π, has a broader role to play in the structural transformation. Not only does it bring down the minimum threshold of the modern sector, but it also shrinks the region where the economy can perpetuate in inequality and poverty traps. This is because an increase in Π also causes the the locus of points where Conditions A and B hold with strict equality, to shift towards the origin. This is shown as follows-

The equation representing strict satisfaction of Condition A is,

$$H_t = \frac{w_c}{\Pi}\left(\frac{\tau + \overline{e}_t(1-\tau)}{\overline{e}_t}\right)$$

This locus shifts towards the origin because both the limits given by $\lim_{\overline{e}_t \to 1} H_t = \frac{w_c}{\Pi}$ and $\lim_{H_t \to 1} \overline{e}_t = \frac{w_c - (1-\tau)w_c}{\Pi - (1-\tau)w_c}$ fall when Π increases. Similarly for the equation representing Condition B is given by-

$$H_t = \left[\frac{A + (1-\tau)w_c}{\Pi(1+\alpha)}\right]\frac{\alpha}{\overline{e}_t} + \left[\frac{1}{1+\alpha} - \frac{(1-\tau)\alpha w_c}{\Pi(1+\alpha)}\right]$$

The locus again shifts towards the origin when Π increases and this is because the limits given by $\lim_{\overline{e}_t \to 1} H_t = \frac{A + \frac{\Pi}{\alpha}}{\Pi + \frac{\Pi}{\alpha}}$ and $\lim_{H_t \to 1} \overline{e}_t = \frac{A - (1-\tau)w_c}{\Pi - (1-\tau)w_c}$ fall when Π rises. Refer to figure 3 for a graphical representation of the loci of the two conditions. This clearly indicates that higher returns to working in the intermediate input sector will facilitate the transformation in several ways and reduce the region of failure for an economy. This is summarized in the following proposition-

Proposition 4: *An economy where the intermediate inputs sector is more profitable, keeping all other things the same, is more likely to undergo structural transformation.*

6 Conclusion

This study highlights the possible roadblocks that policy makers might face when using SMEs as a tool for transforming the economy. This is because the history of an economy, in terms of the initial distribution of education of its workforce, has a major role in determining whether it is possible for the SMEs to facilitate structural transformation and in the process overcome the persistence of inequalities in education, work opportunities and incomes. The paper, at the same time, also highlights the importance of educational policy in assisting the transformation of the economy. This is because, as soon as educational inequalities are removed, all the other inequalities will follow their way out. The study also continues to highlight the role of SMEs in promoting entrepreneurship and in acting as a middle sector providing alternatives to cottage sector jobs. Further, the SMEs while acting as a bridge also play an important role in raising the overall income in the large scale sector as well. An economy which undergoes structural transformation in the presence of the intermediate inputs sector will have a higher steady state wage in the large scale sector compared to an economy which has everyone working in the large scale sector with no intermediate inputs sector. This is because the intermediate inputs provided by the SMEs to the large scale enterprises raises their productivity which translates into higher wages for its workers. However, such process of structural transformation gets throttled if the initial size of the highly educated agents is too small, which in turns limits the size of the modern sector. Thus this study underscores the need for a holistic approach in terms of development policies which focuses not just on the SMEs as a vehicle of structural transformation but also emphasizes on the need for promoting higher education for the process of structural transformation to be effective.

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