Efficient Coalitional Bargaining with Noncontingent Offers [*†]

Rakesh Chaturvedi [‡§]

First Draft: October 2013
Current Draft: October 2015

Abstract

A new feature pertaining to proposer's ability to implement offers is introduced in the extensive form bargaining mechanism studied in Okada (1996). This mechanism is used to analyze two classes of coalitional games with transferable utility. One class is that of strictly supermodular games; the other has the property that per capita value is increasing as a coalition adds to its members. The new feature in the mechanism is that the proposer has a choice to implement his proposal with any subset of responders who have accepted it. Thus the institutional feature of 'every responder has veto power' is relaxed here. It is shown that for all sufficiently high discount factors δ , there exists an efficient subgame perfect equilibrium in pure stationary strategies (SSPE) whose limiting outcome is the core-constrained Nash Bargaining Solution. For strictly supermodular games, Core constraints are binding on Nash Bargaining Solution while for the other class they are not. Also, all efficient SSPE are payoff-equivalent in the limit as $\delta \to 1$.

JEL Classification: C70, C71, C72, C78, D71
Keywords: bargaining, coalitions, Nash Bargaining Solution, Core, veto power

[*]Earlier drafts were circulated under the title 'An Efficient Bargaining Mechanism for Strictly Supermodular Coalitional Games'

[†]I thank Kalyan Chatterjee for suggesting the problem to me and for invaluable suggestions and advice. It's a pleasure to acknowledge discussions with James Jordan, Vijay Krishna and Bhaskar Dutta. I owe in good measure to Neil Wallace for numerous suggestions for improving the exposition of the paper and for many conversations that improved my understanding. I also acknowledge comments and feedback received from IWGTS-2014 Conference at Sao Paulo, Economic Theory Conference at PennState, Midwest Theory Conference, Cornell-PennState Macro Conference and Indian Statistical Institute Delhi.

[‡]Indian Institute of Management Udaipur, Udaipur, Rajasthan, India-313001.
[§]Contact Email: rakesh.dse@gmail.com

Contents

1 Introduction

Bargaining models of multilateral exchange must contend with the possibility that a part of a proposed multilateral deal for a set of parties may still be a feasible and consensual deal for some of the parties. This possibility is nonexistent in a bilateral deal because a deal by its definition needs a minimum of two parties to consent.

To facilitate collective decision making, institutions have evolved that are sparse in terms of the criteria needed to conclude multilateral deals. The criteria is set in terms of the number of parties who consent to the deal. Two arrangements at the extreme ends of such a criteria are dictatorship which requires consent of just one party and veto power to everyone (unanimity) that requires consent of every party. In between such extremes are majority rules. A veto power given to parties in a bilateral deal is equivalent to individual consent. In a multilateral deal, however, a veto power given to any party is more than individual consent. It is the power to encroach over the consent of other parties, irrespective of their number. It is also the power to block voluntary and mutually beneficial deals that may be struck by other parties.

Often there are no institutionalized negotiation rules like majority voting that are legally enforceable on the parties. Sovereign debt renegotiation is a prominent example. As documented in Hornbeck (2010) and Alfaro (2014), Argentina, after defaulting in 2002, on its legally incurred sovereign debt, went in for negotiations with its private creditors for debt restructuring. The negotiation process for sovereign debt restructuring is not legally enforceable. After failing to agree on the terms, Argentina made a unilateral take-it-or-leave-it offer to settle in the 2005 Bond Exchange. 76% of the creditors accepted the offer. The 2010 Bond Exchange took the acceptance to 91.3%. This has created two coalitions of bondholders: the exchange bondholders who have consented to the restructured deal and the holdouts who have not and are litigating in an attempt to get their full face value. In 2012, US Court of Appeals for Second Circuit, interpreting *pari passu* clause, prohibited Argentina from paying one class of creditors while others receive nothing, effectively giving a huge leverage to holdouts. In 2014, US Supreme Court declined to hear Argentina's appeal against the ruling[1]. In theoretical terms, the court has ruled in favor of an extreme criteria among a spectrum of criteria possible to conclude a deal. It has effectively given veto power in the hands of every creditor and the ruling is being widely seen as an impediment to restructuring deals.

Our objective, in this paper, is not to study the specific setting of sovereign debt renegotiation, but rather to take existing models of multilateral bargaining and relax any enforceable institutional requirement like veto power or majority rule for conclusion of deals. Thus the bargaining power of various parties will be determined endogenously by their ability to create value and their threat positions. Some parties will endogenously get veto power, others will not. The upshot is that relative to existing models, strategic incentives are better aligned to support efficiency.

[1] http://www.nytimes.com/2014/06/20/business/economy/ruling-on-argentina-gives-investors-an-upper-hand.html?$r_r = 0$

Our environment is an n-person coalitional game with transferable utility for which a well developed solution concept is the Core. The stability requirements imposed on allocations in the definition of the Core is the primary reason for its theoretical appeal. The Core emerges exactly as the set of stationary equilibrium outcomes of the n-person bargaining model (without discounting) of Perry and Reny (1994) [2]. Moldovanu and Winter (1995) also find the Core emerging as the set of payoffs of their order-independent equilbria of a family of undiscounted bargaining games. [3]

The n-person pure bargaining game (where the only possible outcomes are complete cooperation of all players or complete breakdown of cooperation) has been studied independently for which a well developed solution concept is the Nash Bargaining Solution. It was axiomatically derived for bilateral case by Nash Jr (1950) and later shown by Binmore et al. (1986) to be the limiting (as $\delta \to 1$) equilibrium outcome of two-person bargaining model of Rubinstein (1982). A similar limiting result was shown by Krishna and Serrano (1996) for the n-person case. Britz et al. (2010) and Britz et al. (2014) are two recent contributions that offer noncooperative support to the weighted Nash Bargaining Solution for the n-person case.

The central question of our concern is: What payoff outcomes can we expect in n-person coalitional games when parties do not have recourse to legally enforceable rules, for instance, majority rule or universal veto power, for concluding multilateral deals in negotiations. In this paper, we only give a limited answer to this question. We find there is a class of n-person coalitional games which may be analyzed essentially as n-person pure bargaining games. For this class, the Nash Bargaining Solution remains a limiting equilibrium outcome. More interestingly, there is a class of n-person coalitional games which cannot be analyzed as their associated n-person pure bargaining games but the pure bargaining games play a critical role for payoff outcomes. Thus for this class, both Core and Nash Bargaining Solution are important for a limiting equilibrium outcome.

We focus on studying two settings that have been studied before. One class, **S**, is that of strictly supermodular games [4]. It has the property that players are complements for coalition formation. The other class of games, **G**, have the property that per capita value is increasing as a coalition adds to its members. For either class, an efficient outcome has immediate formation of grand coalition. The class of games **G** and **S** are unrelated in that neither is a subset of the other. However both have nonempty cores [5]. An example of a strictly supermodular setting is a production partnership game [6]. The problem to be studied in this environment is to determine the coalitional structure i.e. which coalitions form and

[2] The environment in Perry and Reny (1994) is a totally balanced TU game.

[3] The environment in Moldovanu and Winter (1995) is a strictly superadditive NTU game that has nonempty core for each of its component games.

[4] They are traditionally called strictly convex games.

[5] The necessity of nonempty core for existence of efficient stationary equilibria is a result that holds in a variety of mechanisms studied and holds in the mechanism studied here as well.

[6] Each player owns some factors of production (like land or labor). Players have access to a (convex) production technology that displays increasing marginal productivity. Players only make participation decisions. A coalition S by cooperating can pool their factors of production and generate a value that is just the production output. See Rosenmüller (1981) and Example 18.A.A.6 in Mas-Colell et al. (1995).

for each such coalition formed, how is the surplus that accrues to that coalition shared among its members.

We now describe our mechanism recursively. Suppose the state of the game is such that players in S are still negotiating while the rest have left the game with some agreements reached in some fashion. At this point, a player in S is randomly chosen to be the proposer with probability $1/|S|$ [7]. The proposer makes a proposal which is a coalition $T \subset S$ and a distribution x_T of surplus of that coalition. Responders in T then move sequentially in some order saying Yes or No. After everyone has responded, the proposer decides whether to partially implement his offer with all, some or none of the responders who have accepted the offer. When a proposer decides to implement his offer with T_I (which if not empty necessarily includes the proposer), he gives each responder j in T_I what he offered to him i.e. x_j and he gets the residual of the surplus. The state then changes to one in which $S \setminus T_I$ is the set of players still negotiating in the game [8]. There is discounting when a new proposer is chosen. Players are expected utility maximizers. The notion of equilibrium is stationary subgame perfect equilibrium (SSPE).

Our mechanism embodies a proposer's ability to make noncontingent offers- even if some responder in his proposed coalition has rejected it, he has a choice to implement it with a subset of responders who have accepted it. In strictly supermodular environments, this ability to walk away with a subcoalition makes the proposer more powerful in that it potentially gives him access to a threat. It turns out this is enough for getting efficiency with probability 1. We will elaborate it further after stating our results.

The main result in this paper is to show for all games in $\mathbf{G} \cup \mathbf{S}$, for all sufficiently high discount factors, there exists an efficient pure strategy SSPE whose limiting outcome is the core-constrained Nash Bargaining Solution [9]. For games in $\mathbf{G}$, the Core does not act as a binding constraint on the Nash Bargaining Solution. For games in $\mathbf{S}$, the Core is a binding constraint on the Nash Bargaining Solution. We give a constructive proof describing a recursive algorithm for computing the proposals made by the players to the grand coalition in this SSPE. Also, efficient SSPE are payoff-equivalent in the limit as $\delta \to 1$. This limit value is the core-constrained Nash Bargaining Solution.

The ideas behind our constructive existence proof are properties of strictly supermodular environments. These are- a result due to Compte and Jehiel (2010) about nested structure of coalitions for which the core constraints are binding at any core allocation, the algorithmic characterization of the core-constrained Nash Bargaining Solution for supermodular games shown by Dutta and Ray (1989) and a further monotonicity result about such allocation shown in Dutta (1990). In the equilibrium we construct, the set of coalitions for which the core constraints bind at the core-constrained Nash Bargaining Solution are precisely those that constitute credible coalitional threats. First we partition the players by using the result of Compte and Jehiel (2010). Our description of equilibrium proposals is a result of

[7] Chatterjee et. al.(1993) mechanism differs at this point in that it has a fixed player chosen.

[8] Compte and Jehiel (2010) do not allow the rest of the players to continue bargaining once a coalition has formed.

[9] Compte and Jehiel (2010) call it the Coalitional Nash Bargaining Solution.

two recursive algorithms. The first algorithm inductively describes what will be coalitional threats and veto-demands of players in the equilibrium. It turns out that the limit (as $\delta \to 1$) of the vector of veto-demands of players in our equilibrium is the core-constrained Nash Bargaining Solution. A responder who is not a member of a coalitional threat that a proposer uses must be willing to lower his demand relative to what he would have demanded if he had veto power over the offer. This responder can therefore be compensated with less than his veto-demand. An interesting feature of the equilibrium is that the proposer is forced to concede this responder more than what he demands. This is the cost he has to pay in order to maintain the credibility of his coalitional threat. This is the primary difference in strategic incentives owing to noncontingent offers that supports efficiency. The purpose of the second algorithm is to describe the how much more do players get as proposers [10] and how much less do players get as non-veto players relative to their veto-demand. This is done by inductively using the equilibrium condition and the feasibility condition on the offers.

The idea of the proof for uniqueness of the limit allocation in any SSPE efficient with probability 1 starts with the observation that limit value of any such equilibrium is a core allocation. It is then shown that coalitional threats must be the ones for which the core constraints are binding. Claim D of Compte and Jehiel (2010) yields the nested structure of the threats. This generates a partition of players. In the next step, we argue that all players in any block of partition get the same payoff in the limit. The last step characterizes the coalitional threats and the limit allocation based on individual optimization in equilibrium. This unravels the coalitional threats as well as the limit allocation inductively. This inductive characterization is the same that characterizes the core-constrained Nash Bargaining Solution of strictly supermodular games as showed in Dutta and Ray (1989).

The plan of the paper is as follows. After discussing the literature in Section 2, we describe the model in Section 3 and state the results in Section 4. The proof of the existence result is described in Section 5 and is exposited as follows. The candidate equilibrium is described in Sections 5.1, 5.2, 5.3 both for a restricted model in which only one coalition is permitted to form as well as the model without this restriction. The reason why we exhibit the equilibrium for a restricted model first is that we only have to deal with the game with all players in it. There are no subgames with a smaller population. Also, the acceptance-rejection strategies are simple for the restricted model. Optimality of the strategies is discussed in Section 5.4. A monotonicity property of core-constrained Nash Bargaining Solution for strictly supermodular games shown in Dutta (1990) then assures us that the strategies so constructed can be supported as an SSPE in our model for all sufficiently high discount factors. We discuss the equilibrium construction in a simple 3-player example in Section 5.5 and contrast the efficiency implication with the other mechanisms that have been studied. The uniqueness of limit allocation for any efficient (with probability 1) equilibrium is discussed in Section 6. We conclude in Section 7.

[10] proposer's advantage

2 Related Literature

The literature on noncooperative analysis of coalitional games was pioneered by Selten (1980) and Harsanyi (1974). Since then, the literature can be classified along several dimensions. Selten (1980), Moldovanu and Winter (1994) and Compte and Jehiel (2010) study bargaining protocols that terminate as soon as the first coalition is formed. Chatterjee et. al.(1993), Okada (1996) and Moldovanu and Winter (1995) study bargaining protocols that allow the possibility of multiple agreements.

The works of Chatterjee et. al.(1993), Okada (1996) and Compte and Jehiel (2010) are closest to ours. A common feature of all these papers is that they give veto power to every responder towards whom the offer is directed. This embodies a constraint on the proposer's ability to implement offers- he cannot implement his offer if some responder rejects it. Put in a different way, the offers made by the proposer are *contingent offers* - their implementation is contingent on acceptance by everyone to whom the offer is directed. Indeed, the reason for inefficiency in these models is that when you give veto power to every responder, it increases their demands and so it gets costly for some players to propose to the grand coalition. They would rather propose to a smaller coalition and satisfy their veto-demands than propose to the grand coalition and satisfy everyone's veto-demands.

These papers differ in the way they define efficiency of an equilibrium. Okada (1996) defines an equilibrium in pure strategies to be subgame efficient if in every subgame, every player proposes the full coalition of players who are still negotiating. He further defines limit subgame efficient equilibrium to be one that is subgame efficient along a sequence of δ going to 1. His main result is that there exists a limit subgame efficient equilibrium if and only if the coalitional game has increasing returns per capita (i.e. the game is in $\mathbf{G}$).

Chatterjee et.al.(1993) do not insist on efficiency in every subgame. They find that efficiency obtains for all order of proposers if and only if the game is in $\mathbf{G}$. For strictly supermodular games (games in $\mathbf{S}$) and for all sufficiently high δ, efficiency obtains only for a particular choice of the player who makes the first offer when the game is played.

Compte and Jehiel (2010) work with the notion of asymptotic efficiency which they define to be efficiency in the limit and not necessarily along a sequence of δ going to 1. In other words, efficiency is approximated better as players get more patient. They obtain asymptotic efficiency for a class of games that includes $\mathbf{G} \cup \mathbf{S}$. It is pertinent to emphasize that if we were to insist on efficiency with probability 1, the equilibrium in Compte and Jehiel (2010) for games in $\mathbf{S}$ would be inefficient except for $\delta = 1$.

In this paper, we take the bargaining mechanism studied by Okada (1996) but do away with the feature that every responder has veto power. A proposer now has the choice to implement his offer with a subset of responders who have accepted it. In this sense, the offer is noncontingent- it's implementation is not contingent on acceptance by everyone in the proposed coalition. In a companion paper Chaturvedi (2013b), we embed this feature in the mechanism studied by Chatterjee. et. al. (1993). We work with the notion of efficiency with probability 1 (the same notion that is used in Okada (1996)) and our result is that for all sufficiently high δ, there exists an efficient (wp 1) equilibrium in pure stationary strategies for games in $\mathbf{G} \cup \mathbf{S}$.

3 The Model

3.1 The Coalitional Game

Let $N = \{1, \ldots, n\}$ be the set of all players. Let (N, v) be a coalitional game with transferable utility. Any coalition, $S \subset N$ has a nonnegative worth, $v(S) \geq 0$. We will denote the set of all coalitions of N by $\mathscr{C}$. When a coalition agrees to a payoff allocation, it can fully commit to it and there are no enforcement problems in implementing that agreement. We now describe some coalitional environments that have been studied in the literature.

(N, v) is strictly superadditive if

$$\forall S, T \subset N, \quad S \cap T = \emptyset, \quad v(S \cup T) > v(S) + v(T)$$

(N, v) has increasing returns per capita as a coalition adds to its members if

$$\forall S, T \subset N, \quad S \supset T, \quad \frac{v(S)}{|S|} > \frac{v(T)}{|T|}$$

(N, v) is strictly supermodular if

$$\forall i \in N, \quad \forall S, T \subset N \setminus i, \quad S \supset T, \quad v(S \cup \{i\}) - v(S) > v(T \cup \{i\}) - v(T)$$

Let $\mathbf{G}$ denote the class of games that has increasing returns per capita. Let $\mathbf{S}$ denote the class of strictly supermodular games. For both these environments, the economy splitting up into coalitions is an inefficient coalitional structure. The only efficient structure is the formation of the grand coalition. Also supermodular environments are superadditive as well. The following definition will be useful to us.

For stating our results, we will need the following definition.

Definition 1. Core-constrained Nash Bargaining Solution is the payoff allocation that maximizes the Nash product among all payoff allocations in the core. For any (N, v) with a nonempty core, this is uniquely defined as the core is a convex set and the Nash product is a strictly quasiconcave function.

$$\max_{x \in \mathbb{R}^n} \prod_{i \in N} x_i$$

$$\text{subject to } x(N) = v(N)$$

$$\forall S \subsetneq N, \; x(S) \geq v(S)$$

3.2 The Bargaining Mechanism

In any period $t = 1, 2, \ldots$, let S be the set of active players still in the game. A player from S is randomly chosen to be the proposer. Draws are independent and each player has an equal chance $1/|S|$ of being chosen. A proposer makes an offer (T, x_T) where $i \in T \subset S$ and $\sum_{i \in T} x_i = v(T)$. Players in $T \setminus i$ then respond sequentially according to some given order ϕ. Suppose $T_A \subset T$ accept the offer. Then player i can either choose to implement his offer (T, x_T) with a coalition $T_I \subset T_A$, $i \in T_I$ or choose 'DELAY'. If i implements (T, x_T) with some T_I , then coalition T_I exits the game [11]. Every player j in $T_I \setminus i$ gets x_j and i gets the residual $v(T_I) - \sum_{j \in T_I \setminus i} x_j$. The game continues next period with the set of active players $S \setminus T_I$. If i chooses DELAY, the game continues next period with the set of active players unchanged at S. The offers made to each prospective coalition partner are *noncontingent* in the sense that they are not contingent on acceptance by everyone in the proposed coalition. Preferences are linear in the share and intertemporal preferences are just discounted utility preferences with a common discount factor δ. Players are expected utility maximizers.

The concept of equilibrium we will use is that of a stationary subgame perfect equilibrium (SSPE) which we now define [12] for the model.

Definition 2. A strategy profile is a stationary subgame perfect equilibrium (SSPE) of the bargaining model $G(N, v)$ described above if it is a subgame perfect equilibrium with the property that for every $t = 1, 2, \ldots$, the period-t strategy of every player depends only on the set of players still negotiating in the game and the history of the game within period-t.

For any SSPE σ of the extensive form game $G(N, v)$ described above, let $u(S, \sigma) \in \mathbb{R}^S$ be expected payoff vector at a chance node when S is the set of players still in the game. Let $u^\star(S, \sigma) = \lim_{\delta \to 1} u(S, \sigma)$. Let $b(S, \sigma) := \delta u(S, \sigma)$ be the discounted value vector of σ. The payoff $b_j(S, \sigma)$ has the following interpretation- it is the demand that player j would make if he had veto power over an offer to which he must respond. We'll often refer to $b_j(S, \sigma)$ as the *veto-demand* of player j in the SSPE σ in subgame $G(S, v)$ because it is informative in this sense.

We'll refer to the environment and the mechanism described above as the unrestricted model. This is the object of our study and our results pertain to the unrestricted model. However, for expositional purposes, we find it convenient to work with a version of the mechanism where the bargaining terminates as soon as one coalition forms. We refer to this version as the restricted model. As noted in the last paragraph of the introduction, we exposit the strategies for the restricted model only. However, we do point out, what will be the corresponding strategies in the unrestricted model. Again when we discuss optimality of strategies, we carry out the proof for the restricted model. But we do point out what ensures perfection in the unrestricted model.

[11] In the event that other responders reject i's offer, i has the choice to exit the game forming the singleton coalition $\{i\}$.

[12] cf. Okada (1996)

4 Results

Proposition 1 is an existence result whose proof is constructive and illuminates how threats work in the model. Proposition 2 is a uniqueness (in the limit) result that is weak in so far as it applies only to SSPE that are efficient with probability 1.

Proposition 1. For all games in $\mathbf{G} \cup \mathbf{S}$ and for all sufficiently high discount factors, there exists a pure strategy SSPE that is efficient with probability 1 and whose limiting outcome is the core-constrained Nash Bargaining Solution.

Proposition 2. For all games in $\mathbf{G} \cup \mathbf{S}$, all SSPE *efficient* with probability 1 are payoff-equivalent in the limit as $\delta \to 1$. This limit value is the core-constrained Nash Bargaining Solution.

The remainder of the paper is devoted to proving these two results.

5 Proof of Proposition 1

The equilibrium that we construct requires that we first develop a particular partition of the set of players.

Definition 3. Suppose $b^\star$ is a core allocation for a game (N, v). Then we say $S \subset N$ is a binding coalition with respect to $b^\star$ if $b^\star(S) := \sum_{i \in S} b_i^\star = v(S)$.

Given a strictly supermodular (N, v), let $\{N_1, \ldots, N_L\}$ be the partition of the set of players N induced by the core-constrained Nash Bargaining Solution. We know this partition can be provided because of the following known result.

Claim D. Compte and Jehiel (2010). For (N, v) strictly supermodular, the set of binding coalitions $\mathscr{S}$ with respect to a core allocation $u^\star$ is nested. That is $\mathscr{S}$ is of the form $\{S_1, S_1 \cup S_2, \ldots, S_1 \cup \ldots \cup S_L\}$ where $N = S_1 \cup \ldots \cup S_L$ [13]. This naturally induces a partition $\{S_1, S_2, \ldots, S_L\}$ of players.

Dutta and Ray (1989) characterize the core-constrained Nash Bargaining Solution for supermodular games as the unique egalitarian allocation in the core. They provide an algorithm that computes this solution by generating a partition of the set of players. We remark here that our partition may differ from theirs. An example that illustrates this is $v(N) = 1, v(12) = 0.7, v(1) = 0.35$ and $v(S) = 0$ for all other S. For this strictly supermodular game, Dutta and Ray (1989) would give the partition $\{\{12\}, \{3\}\}$ while our partition will be $\{\{1\}, \{2\}, \{3\}\}$. Also note that for games in $\mathbf{G}$, there is no partition of players i.e.

[13] Compte and Jehiel (2010) do not include the grand coalition N in the set of binding coalitions but in order to generate a partition we do.

$L = 1$. When we describe the equilibrium construction, the strategies for games in $\mathbf{G}$ can be read out accordingly keeping this in mind.

5.1 Proposal Strategy

Consider the game $G(N, v)$ with the full set N as the population of players and an equilibrium σ. If i could veto an offer made by some player j in which he is a prospective coalition partner, then j must compensate i with i's discounted value of the game $b_i(N, \sigma)$. Put differently, j must compensate i with his *veto-demand* $b_i(N, \sigma)$. Player i can get more than his veto-demand only as a proposer and we call that addition as *proposer's advantage*. Player i can get less than his veto-demand only as a responder when he does not hold a veto power over the offer made by some proposer. When this happens, we say that the proposer has taken i as a *hostage* and we term i's loss relative to his veto-demand as *hostage's disadvantage*.

We now describe the proposal strategies in terms of players' *veto-demands*, *proposer's advantage* and *hostage's disadvantage*. Whenever $i \in N$ makes a proposal, he offers $(N, u(N, i))$ where for $i \in N_k$, for $j \in N_{k+1}$ and for $l \in N \setminus (N_{k+1} \cup \{i\})$

$$\begin{aligned} u_i(N, i) &= b_i(N, \sigma) + a_i(N, \sigma) \\ u_j(N, i) &= b_j(N, \sigma) - h_j(N, \sigma) \\ u_l(N, i) &= b_l(N, \sigma) \end{aligned}$$

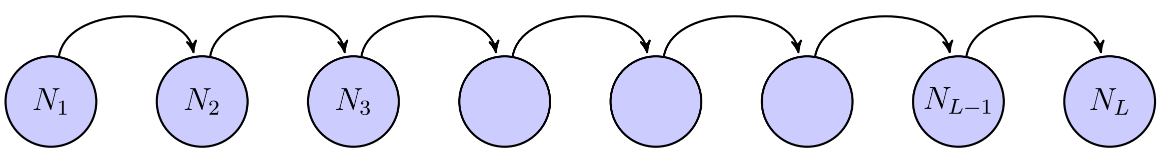

Figure 1: Equilibrium Structure of "Hostages"

Figure 1 depicts the structure of *hostages* in the equilibrium that we construct. For any $k < L$, any player in N_k takes all the players in N_{k+1} as his hostages. It's fruitful now to define what we mean by a coalitional threat of a player as a proposer.

As remarked in the introduction, the key feature that no responder has veto power and that the proposer has a choice to implement his offer with some players who have consented implies that inherent in his proposal and implementation strategy is a threat that he can walk away with a subcoalition. The concept of a coalitional threat formalizes this idea.

Definition 4. COALITIONAL THREAT. For an equilibrium σ of the bargaining game $G(N, v)$, let $(S, u(S, i))$ be i's offer in in the subgame $G(S, v)$. Then a coalition $T \subsetneq S$ is a coalitional threat for i if
(i) i compensates his coalition partners in T with their veto-demands i.e.

$$\forall j \in T \setminus i, \quad u_j(S, i) = b_j(S, \sigma)$$

(ii) i's implementation strategy is to implement his offer with T.
(iii) it is locally optimal for i to implement his offer with T. In other words, T solves the following constrained optimization problem

$$\max_{i \in T' \subset S} \quad \big[v(T') - \sum_{j \in T' \setminus i} b_j(S, \sigma)\big] \geq b_i(S, \sigma)$$

Our description of equilibrium offers is a result of two recursive algorithms. The first algorithm inductively describes what will be the coalitional threats and veto-demands of players in equilibrium. We will see that the limit (as $\delta \to 1$) of the vector of veto-demands of players in our equilibrium is the core-constrained Nash Bargaining Solution. A responder who is not part of a coalitional threat that a proposer uses must be willing to budge from his veto-demand. This responder can therefore be held *hostage* and suffer a *disadvantage* at the hands of this proposer. Care needs to be exercised at this point in deciding which players are held *hostages* by which players i.e. the assignment of hostages to proposers. We do this in a natural way suggested by the hierarchical nature of partition of players. The purpose of the second algorithm is to describe the proposer's advantage and the hostage's disadvantage. This is done by inductively using the equilibrium condition and the feasibility condition.

RECURSIVE ALGORITHM TO COMPUTE COALITIONAL THREATS AND VETO-DEMANDS

We now describe a simple recursive algorithm that gives coalitional threats [14] and computes veto-demands. The computation is based on three features. First, the set of coalitional threats is precisely $\{N_1, N_1 \cup N_2, \ldots, N_1 \cup \ldots \cup N_L\}$. Owing to the way we have partitioned the set of players, this means the set of coalitional threats is precisely the set of coalitions for which the core constraints are binding at the core-constrained Nash Bargaining Solution. Thus the only players who do not have a coalitional threat are those in the last block of partition, N_L. The second feature can be described as symmetry. Players in the same block of partition (because of the first feature, this means they have the same coalitional threat) have the same veto-demands. Lastly, a player who has a coalitional threat is indifferent between implementing this threat and choosing DELAY.

Step 1. For $i \in N_1$, his coalitional threat is N_1. The symmetry and indifference feature immediately give i's veto-demand $b_i(N, \sigma)$.

$$v(N_1) - (|N_1| - 1) b_i(N, \sigma) = b_i(N, \sigma)$$

which gives

$$b_i(N, \sigma) = \frac{v(N_1)}{|N_1|}$$

Suppose the coalitional threats and veto-demands have been computed for $i \in N_1, \ldots, i \in N_{k-1}$.

[14] Of course, this needs to be verified after we have presented the full equilibrium.

Step k. For $i \in N_k$, his coalitional threat is $\cup_{a=1}^{k} N_a$. Note for $j < k$ and $l \in N_j$, l's veto-demand $b_l(N, \sigma)$ has already been computed in Step j. The symmetry and indifference feature immediately give

$$v(\cup_{a=1}^{k} N_a) - \sum_{j<k} \sum_{l \in N_j} b_l(N, \sigma) - (|N_k| - 1) b_i(N, \sigma) = b_i(N, \sigma)$$

which gives

$$b_i(N, \sigma) = \frac{v(\cup_{a=1}^{k} N_a) - v(\cup_{a=1}^{k-1} N_a)}{|N_k|}$$

Step L. For $i \in N_L$, set

$$b_i(N, \sigma) = \frac{\delta v(N) - v(\cup_{a=1}^{L-1} N_a)}{|N_L|}$$

The full construction of proposals will justify the above assignment.

The equilibrium we construct has the feature that for $L \geq 2$, for $k < L$, for $i \in N_k$, if player i is the proposer, then players in N_{k+1} are *hostages* of i.

Table 1 below is a summary description of coalitional threats for various players.

Table 1: Coalitional Threats of Players in Equilibrium

Player in	Coalitional Threat
N_1	N_1
N_2	$N_1 \cup N_2$
...	...
N_{L-1}	$N_1 \cup \ldots \cup N_{L-1}$

RECURSIVE ALGORITHM TO COMPUTE PROPOSER'S ADVANTAGE AND HOSTAGE'S DISADVANTAGE

In what follows, for a player $i \in N_k$, a_i will denote his proposer's advantage when he is the proposer and h_i will denote his disadvantage when he is held a hostage (this happens when some player in N_{k-1} is a proposer). The computation has the feature that players in the same block of partition have the same disadvantage.

Step 1. For $i \in N_1$, set his proposer advantage as

$$a_i(N, \sigma) = \Big(\frac{|N|}{\delta} - |N|\Big) b_i(N, \sigma)$$

This is derived by writing the equilibrium condition. If $L = 1$, then there's nothing more to describe since there are no hostages in this case. Stop. If $L \geq 2$, move to the next Step.

Step k for $k \leq L$. This is the inductive step in the Algorithm. There are two substeps.

Step $k.1$. For $i \in N_k$, his disadvantage as a hostage, $h_i(N, \sigma)$ is determined from the feasibility condition of the proposal made by $j \in N_{k-1}$ and by noting that $b(N) = \delta v(N)$:

$$b_j + a_j + (|N_{k-1}| - 1)b_j + |N_k| (b_i - h_i) + \sum_{m \in N \setminus (N_k \cup N_{k-1})} b_m = v(N)$$
$$h_i(N, \sigma) = \frac{a_j(N, \sigma) - (1 - \delta)v(N)}{|N_k|}$$

Step $k.2$. For $i \in N_k$, his proposer's advantage, a_i is determined from equilibrium condition

$$\begin{aligned} a_i(N, \sigma) &= \Big(\frac{|N|}{\delta} - (|N| - |N_{k-1}|)\Big) b_i - |N_{k-1}| (b_i - h_i) \\ &= \Big(\frac{|N|}{\delta} - |N|\Big) b_i(N, \sigma) + |N_{k-1}| h_i(N, \sigma) \end{aligned}$$

where $h_i(N, \sigma)$ has already been computed in Step $k.1$.

The construction carried out above fully describes the proposal strategies in the restricted version of the model where only one coalition may form. This is because there are no subgames with a smaller population. For the model as we have described, in any subgame $G(S, v)$ with the set of players being S, the proposal strategies are computed as outlined above with the algorithms being carried out over (S, v). The reduced game (S, v) is also strictly supermodular and hence the same procedure carries over.

Remark. 1. For any player, his proposer's advantage and his hostage's disadvantage is continuously and monotonically decreasing in δ and vanishes in the limit as $\delta \to 1$. [15]

[15] For a player $j_k \in N_k$, it is easy to arrive at formulas for

$$\frac{\mathrm{d}a_{j_1}}{\mathrm{d}\delta} = -\frac{|N|}{\delta^2} \frac{v(N_1)}{|N_1|} < 0$$
$$\frac{\mathrm{d}h_{j_2}}{\mathrm{d}\delta} = -\frac{|N|}{|N_2|} \Big(\frac{v(N_1)}{\delta^2 |N_1|} - \frac{v(N)}{|N|}\Big) < 0$$

For k > 2

$$\frac{\mathrm{d}h_{j_k}}{\mathrm{d}\delta} = -\frac{|N| \sum_{a=1}^{k-1} |N_a|}{|N_{k-1}| |N_k|} \Big(\frac{v(\cup_{a=1}^{k-1} N_a)}{\delta^2 \sum_{a=1}^{k-1} |N_a|} - \frac{v(N)}{|N|}\Big) < 0$$

The inequalities follow because the expressions in the parentheses are always positive for strictly supermodular games by Claim 4.1 in Appendix A. For $k \geq 2$, from Step $k.2$ of Algorithm

$$\frac{\mathrm{d}a_{j_k}}{\mathrm{d}\delta} < 0$$

2. For any $i \in N$, the limiting (as $\delta \to 1$) payoff vector in i's offer is core-constrained Nash Bargaining Solution. We know this from the algorithmic characterization of the core-constrained Nash Bargaining Solution for supermodular games shown by Dutta and Ray (1989).

5.2 Implementation Strategy

Consider the game $G(N, v)$ with the full set N as the population of players. Fist consider the equilibrium-offers. Suppose $i \in N_k$, the k-th block of partition of players. If $k = L$, the last block of partition, then i has no coalitional threat. In this case i's implementation strategy is as follows

(a) If noone in N rejects the offer, then implement the offer with N.
(b) If someone in N rejects the offer, then choose DELAY.

If $k < L$, then i's implementation strategy is as follows.

(a) If noone in N rejects the offer, then implement the offer with N.
(b) If someone in i's coalitional threat $\cup_{a=1}^{k} N_a$ rejects the offer, then choose DELAY.
(c) If someone rejects the offer but everyone in i's coalitional threat $\cup_{a=1}^{k} N_a$ accepts, then i implements the offer with his coalitional threat.

For off-equilibrium offers, i implements a coalition which gives him the maximum payoff provided this payoff is at least as great as the payoff he gets by choosing DELAY. If no such coalition exists, he chooses DELAY.

This describes the implementation strategies of players in the restricted version of the model where only one coalition may form. For the unrestricted version as we have described, for any subgame $G(S, v)$ with S as the population of players, the corresponding partition is $(S_a)_a$. Players follow the implementation strategies as described above with this change.

5.3 Acceptance-Rejection Strategy

Consider j's response node in the game $G(N, v)$ where Q is the set of players who have accepted or proposed the standing offer (S, x_S) made by i so far. Since the responders move in a pre-determined order ϕ, anyone of them while contemplating accepting or rejecting the offer has to consider the effect of his decision on the decisions of the responders following him. The ARSs of responders are defined inductively, first for the last responder according to ϕ, then for the penultimate responder and so on backwards in order.

Let $Y_j \subset S \setminus (Q \cup \{j\})$ be the set of responders who will accept the proposal (S, x_S) according to their respective ARSs if j rejects it. For the last responder in S, $Y_j = \emptyset$. The ARS of j

is conditioned on whether in the event of j's rejection, i would implement his offer with a coalition $S_I \subset Q \cup Y_j$ or choose DELAY.

For equilibrium offers,
(a) If i's implementation strategy is to choose DELAY in the event of a rejection by j (i.e. j holds a veto-power over i's offer), then j accepts the offer if it gives him at least his veto-demand $b_j(N, \sigma)$ and rejects otherwise.
(b) If i's implementation strategy is to implement his offer with a coalition in the event of a rejection by j (i.e. j does not have a veto over i's offer), then j accepts the offer if it gives him at least $\delta v(j)$ and rejects otherwise.

For off-equilibrium offers,
(a) If i's implementation strategy is to choose DELAY in the event of a rejection by j, then j accepts the offer if it gives him more than his veto-demand $b_j(N, \sigma)$ and rejects otherwise.
(b) If i's implementation strategy is to implement his offer with a coalition in the event of a rejection by j, then j accepts the offer if it gives him more than $\delta v(j)$ and rejects otherwise.

Note for the equilibrium proposal, responders resolve any indifference by accepting while for any off-equilibrium proposal, they resolve it by rejecting. This describes the acceptance-rejection strategies of players in the restricted version of the model where only one coalition may form. For the unrestricted version as we have described, for any subgame $G(S, v)$ with S as the population of players, the corresponding partition is $(S_a)_a$. Players follow the acceptance-rejection strategies as described above with this change keeping in view that when a proposer threatens to form a subcoalition $T \subsetneq S$, the responders while making their decision look forward to their payoff in the continuation game $G(S \setminus T, v)$. We omit writing it here.

5.4 Optimality

Again for ease of exposition, we only show that the strategies described constitute an SSPE for the restricted model. The proof consists of verifying that the constructed strategies are unimprovable and can be found in Appendix A. The equilibrium is sustained by the following expectations. Any deviation by a proposer in which he tries to unilaterally gain at the expense of players outside his coalitional threat (outsiders) will be met by a rejection by players inside his coalitional threat (insiders). A rejection by insiders is a best response since they are indifferent between rejecting and accepting. Any deviation by a proposer in which he tries to gain at the expense of outsiders by bribing insiders will be met by a rejection by outsiders since the act of bribing insiders renders the threat incredible. Outsiders call it a bluff and anticipating this response, the insiders reject it as well since they realize the sweetened offer to them is just a mirage which is never going to materialize.

It is instructive to discuss the strategic incentives of players in our mechanism as they compare to incentives in Chatterjee et. al.(1993), Okada (1996) and Compte and Jehiel (2010).

In Chatterjee et. al.(1993) mechanism, a player who was not in the last block of partition when chosen as a proposer did not make an efficient offer. The reason was that since the responders had veto power over the offer, equilibrium offers must compensate the players in the proposed coalition with their veto-demands. The veto-demands of players were high enough that the proposer preferred to make an inefficient offer to a smaller coalition than an efficient offer to the grand coalition. Similar incentives are present in pure strategies in Okada (1996). Compte and Jehiel (2010) while maintaining the institutional feature of veto power consider equilibria in behavioral strategies. A player's veto-demand is calculated under the expectation that there is a probability of his exclusion. A chance of exclusion serves as a randomized threat that brings a reduction in veto-demands of players compared to the what they would be in a pure strategy efficient equilibrium (if it exists). Yet every player who is part of the proposed coalition must be compensated with his veto-demand. As a result, Compte and Jehiel (2010) are only able to approximate efficiency off the limit and get efficiency only at the limit $\delta = 1$. Since our mechanism gives a proposer the choice to implement his offer with a subcoalition of consenting responders, he can carry out his threat without incurring the cost of delay. Players inside the coalitional threat of a proposer are veto players while those outside are non-veto players for the proposer's equilibrium offer. Non-veto players must lower their demands relative to their veto-demands (i.e. discounted value of equilibrium). This incentivizes each player in that he prefers to make an offer to the grand coalition.

For the unrestricted model as we have described, the corresponding strategies described constitute an SSPE as well. This follows because the limit allocation, the core-constrained Nash Bargaining Solution satisfies a monotonicity property shown in Dutta (1990). For stating this property, let $u^\star(N, v)$ denote the core-constrained Nash Bargaining Solution for the coalitional game (N, v). For a vector $y \in R^N$, let y_S denote the projection of y along the axes of players in S.

Dutta (1990). Suppose (N, v) is strictly supermodular. For any $S \subsetneq N$, $u^\star_S(N, v) > u^\star(S, v)$ where the strict inequality is for all coordinates.

The result above ensures that in any subgame $G(S, v)$, for all sufficiently high δ, every responder agrees to the proposal no matter who proposes. Thus in every subgame $G(S, v)$, the entire coalition S is formed immediately. Thus we have shown Proposition 1.

5.5 Example

Consider a 3-player example that will illustrate the equilibrium construction and contrast it with the other mechanisms that have been studied.

Example 1. Consider the following 3-player strictly supermodular coalitional game.

S	v(S)	S	v(S)	S	v(S)
{1}	0	{2}	0	{3}	0
{1,2}	0.7	{1,3}	0.2	{2,3}	0.2
{1, 2, 3}	1	∅	0		

Remark. 1. The core-constrained NBS in this game is the allocation $u^\star = (0.35, 0.35, 0.3)$.
2. The set of coalitions for which the core constraints are binding is $\mathscr{S}_b = \{\{1,2\},\{1,2,3\}\}$.
3. For the Chatterjee et. al.(1993) mechanism, if Player 1(or 2) is the initial proposer, he proposes to $\{1,2\}$ in the unique SSPE and the limiting outcome is $(0.35, 0.35, 0)$.
4. For the Okada (1996) mechanism, a pure strategy SSPE does not exist.
5. For the Compte and Jehiel (2010) mechanism, there is an SSPE in behavior strategies for $\delta > \frac{3\sqrt{57}-9}{16}$ in which players 1 and 2 make an offer to the grand coalition N with probability $x = \frac{9-13\delta+8\delta^2}{2\delta(3-\delta)}$ and to coalition $\{1,2\}$ with probability $1-x$ while player 3 makes an offer to the grand coalition N with probability 1. Note that efficiency with probability 1 does not obtain along a sequence of δ going to 1. It only obtains in the limit when $\delta = 1$.

It may be verified that for $\delta \geq 0.7$, the following proposal strategies are supported as an SSPE with players 1 and 2 having coalition $\{1,2\}$ as a coalitional threat and player 3 having no coalitional threat.

Proposal Strategies. Player i makes an offer $u(N,i)$ to the grand coalition N. The offers are:

$$u(N,1) = (b_1 + a_1, b_2, b_3 - h_3)$$
$$u(N,2) = (b_1, b_2 + a_2, b_3 - h_3)$$
$$u(N,3) = (b_1, b_2, b_3 + a_3)$$

where

$$b_1 = 0.35 \qquad b_2 = 1.5 \qquad b_3 = \delta - 0.7$$
$$a_1 = \big(\frac{3}{\delta} - 3\big)b_1 \qquad a_2 = a_1 \qquad h_3 = a_1 - (1-\delta)$$
$$a_3 = \big(\frac{3}{\delta} - 3\big)b_3 + 2h_3$$

6 Proof of Proposition 2

The following result says that for strictly superadditive coalitional environments, no SSPE can exhibit delay in any subgame. The proof can be found in Appendix B.

Lemma 1. Suppose (N, v) is strictly superadditive. For an SSPE σ, for every $S \subset N$, every player $i \in S$ must make an acceptable offer in every subgame $G(S, v)$.

The proof of Lemma 1 does not involve any new ideas. It is reminiscent of Okada (1996) who gets this result for the mechanism he studies. We next turn to a conditional characterization of players' coalitional threats for a given vector of veto-demands.

Lemma 2. Given the vector of veto-demands $b(N,\sigma)$ in an SSPE σ of the game $G(N,v)$, players' coalitional threats $\{S^i\}_{i\in N}$ are solutions to the following constrained optimization problems

$$\forall i \in N, \quad \max_{i\in S^i \subset N} \Big[v(S^i) - \sum_{j\in S^i \setminus i} b_j(N.\sigma) \Big] \geq b_i(N,\sigma)$$

Proof. The constrained optimization problem for player i expresses the local optimality of his choice at his implementation node in the subgame $G(N,v,i)$ with full population N of players and when he is the proposer. By definition of a coalitional threat for player i, all members other than i must be compensated with their veto demands. The inequality denotes the credibility constraint for a coalitional threat: implementing the offer with the coalitional threat must be better for i than choosing 'DELAY'. The max operator ensures that i must choose a coalitional threat optimally. Q.E.D.

We need a definition that will be useful to state our next lemma.

Definition 5. Δ-core (N,v) [16] is the core of the coalitional game (N,v') where $v'(N) = v(N) - \Delta$ and for every subcoalition S, $v'(S) = v(S)$. Formally

$$\Delta\text{-core } (N,v) = \{x \in \mathbb{R}^n_+ : \sum_{j\in N} x_j = v(N) - \Delta; \forall S \subsetneq N, \sum_{j\in S} x_j \geq v(S)\}$$

Define $\Delta(\delta) := v(N) - \sum_{j\in N} b_j(N,\sigma)$. Our next result is that the vector of veto-demands $b(N,\sigma)$ in an efficient SSPE σ of a strictly superadditive coalitional game must lie in $\Delta(\delta)$-core of (N,v).

Lemma 3. Define $\Delta(\delta) := v(N) - \sum_{j\in N} b_j(N,\sigma)$. If (N,v) is strictly superadditive and σ is an SSPE that is efficient with probability 1, then $b(N,\sigma) \in \Delta(\delta)$-core (N,v).

Proof. See Appendix B. Q.E.D.

Corollary. If (N,v) is strictly superadditive and σ is an SSPE that is efficient with probability 1 for all sufficiently high δ, then $\lim_{\delta\to 1} b(N,\sigma) \in$ core (N,v).

Proof. Observe that in any such SSPE, $\lim_{\delta\to 1} \Delta(\delta) = 0$. Q.E.D.

[16] The definition is different from the definition that Compte and Jehiel (2010) work with. They define Δ-core (N,v) to be the core of the coalitional game (N,v') where $v'(N) = v(N) - \Delta$ and for every subcoalition S, $v'(S) = v(S) - \Delta$.

Lemma 3 also yields an immediate sharper characterization of coalitional threats in efficient SSPE of strictly superadditive games that is summarized in the next lemma.

Lemma 4. If (N, v) is strictly superadditive and σ is an SSPE that is efficient with probability 1, then players' coalitional threats $\{S^i\}_{i\in N}$ are solutions to the following constrained optimization problems

$$\forall i \in N, \quad \max_{i\in S^i\subset N}\left[v(S^i) - \sum_{j\in S^i\setminus i} b_j(N.\sigma)\right] = b_i(N,\sigma)$$

We know turn to the proof of Proposition 2 which is carried out in three steps. In the first step, Lemma 4 and Claim D of Compte and Jehiel (2010) yield a conditional characterization of coalitional threats given $b(N, \sigma)$. Lemma 4 tells us that the coalitional threats must be the ones for which the core constraints are binding and Claim D of Compte and Jehiel (2010) yields the nested structure of the threats. This generates a partition of players. In the next step, we argue that all players in any block of partition get the same payoff in the limit. The last step characterizes the coalitional threats and $\lim_{\delta\to 1} b(N, \sigma)$ based on individual optimization in equilibrium. This unravels the coalitional threats as well as the limit allocation inductively.

Proof of Proposition 2. Step 1. Fix an SSPE σ that is efficient with probability 1 for all sufficiently high δ. Let $b(N, \sigma)$ be the vector of veto-demands in σ. Lemma 4 gives the set of coalitional threats, $\mathscr{S}$, in σ. By Lemma 4 and Claim D of Compte and Jehiel (2010), we know that wlog $\mathscr{S}$ has the structure $\{S_1, S_1 \cup S_2, \ldots \cup_{a=1}^{L-1} S_a\}$. This naturally generates a partition of players $\{S_1, \ldots, S_{L-1}, S_L\}$ where $S_L = N \setminus \cup_{a=1}^{L-1} S_a$. Then $b(N, \sigma)$ must satisfy the system of equations:

$$\sum_{j\in S_1} b_j(N,\sigma) = v(S_1)$$
$$\sum_{j\in S_1} b_j(N,\sigma) + \sum_{j\in S_2} b_j(N,\sigma) = v(S_1 \cup S_2)$$
$$\cdots$$
$$\sum_{j\in S_1} b_j(N,\sigma) + \ldots + \sum_{j\in S_{L-1}} b_j(N,\sigma) = v(\cup_{a=1}^{L-1} S_a)$$
$$\sum_{j\in S_1} b_j(N,\sigma) + \ldots + \sum_{j\in S_L} b_j(N,\sigma) = \delta v(N)$$

where the last equation is by definition of $b(N, \sigma)$ as the vector of discounted values to players of the game $G(N, v)$.

The system of equations may be rewritten as linear constraints that the vector of veto-demands $(b_j(N,\sigma))_{j\in S_k}$ for every block S_k must satisfy.

$$\sum_{j\in S_1} b_j(N,\sigma) = v(S_1)$$
$$\sum_{j\in S_2} b_j(N,\sigma) = v(S_1\cup S_2) - v(S_1)$$
$$\dots$$
$$\sum_{j\in S_{L-1}} b_j(N,\sigma) = v(\cup_{a=1}^{L-2} S_a \cup S_{L-1}) - v(\cup_{a=1}^{L-2} S_a)$$
$$\sum_{j\in S_L} b_j(N,\sigma) = \delta v(N) - v(\cup_{a=1}^{L-1} S_a)$$

Step 2. In this step, we seek to characterize for every block of partition $k = 1,\dots,L$, the limit vector of veto-demands for that block $\lim_{\delta\to 1}(b_j(N,\sigma))_{j\in S_k}$. Let $a(N,\sigma)$ be the proposer's advantage that $i \in S_k$ earns as a proposer in subgame $G(N,v,i)$. Then $\lim_{\delta\to 1} a(N,\sigma) = 0$. For each block $k = 1,\dots,L$, define the coalitional game (S_k, v_k) by

$$v_k(T) = v(\cup_{a=1}^{k-1} S_a \cup T) - v(\cup_{a=1}^{k-1} S_a) + \sum_{i\in S_k} a(N,\sigma)\mathbb{1}_{\{T=S_k\}}$$

Then for every player $i \in S_k$, the sequence as $\delta \to 1$ of the restriction $(u_j(N,i))_{j\in S_k}$ of i's efficient offer $u(N,i)$ in σ to S_k is a sequence of efficient and contingent offers [17] that constitutes an equilibrium offer in $G(S_k, v_k, i)$. Put differently, the restriction $\sigma|_{S_k}$ of the SSPE σ of $G(N,v)$ constitutes an SSPE of $G(S_k, v_k)$ (with discounted value, say, $e(S_k,\sigma)$) that is efficient wp 1 for all sufficiently high δ and in which all offers are contingent offers. The problem is to determine $\lim_{\delta\to 1}(b_j(N,\sigma))_{j\in S_k}$.

We study the limit payoff vector of a different yet related game. Denote by O, Okada (1996)'s bargaining mechanism that has the institutional feature that every responder has veto-power over an offer directed to him. In other words, players can only make contingent offers. Fix an SSPE β of $O(S_k, v_k)$ that is efficient wp 1 for all sufficiently high δ. Then it can be shown [18] that β has a unique vector of veto-demands $c(S_k,\beta) \in \mathbf{R}_+^{S_k}$ given by

$$\forall j \in S_k, \quad c_j(S_k,\beta) = \frac{\delta}{|S_k|}\left[v(\cup_{a=1}^{k-1} S_a \cup S_k) - v(\cup_{a=1}^{k-1} S_a) + a(N,\sigma)\mathbb{1}_{\{k<L\}}\right]$$

It is now evident that the vectors $(b_j(N,\sigma))_{j\in S_k}$, $e(S_k,\sigma)$ and $c(S_k,\beta)$ must be equal in the limit. i.e.

$$\begin{aligned}\forall k = 1,\dots,L; \quad \forall j \in S_k; \quad \lim_{\delta\to 1} b_j(N,\sigma) &= \lim_{\delta\to 1} e_j(S_k,\sigma)\\ &= \lim_{\delta\to 1} c_j(S_k,\beta)\\ &= \frac{v(\cup_{a=1}^{k-1} S_a \cup S_k) - v(\cup_{a=1}^{k-1} S_a)}{|S_k|}\end{aligned}$$

[17] i compensates every player $j \in S_k \setminus i$ with his veto-demand $b_j(N,\sigma)$.

[18] See proof of Theorem 3 in Okada (1996)

Step 3. The characterization of coalitional threats and limit vector of veto-demands $\lim_{\delta\to 1} b_j(N,\sigma)$ can now be obtained by observing that in equilibrium σ, given other's choices of coalitional threats, every player i must choose his coalitional threat so as to maximize his payoff from playing the game or what amounts to the same thing, maximize his veto-demand $b_i(N,\sigma)$. Given our characterization of $\lim_{\delta\to 1} b_j(N,\sigma)$ in Step 2, this leads to a series of inductive optimization problems that unravel the coalitional threats $\{N_1, N_1 \cup N_2, \ldots \cup_{a=1}^{L-1} N_a\}$ and limit vector $\lim_{\delta\to 1}(b_j(N,\sigma))_{j\in N_a}$ inductively. By the inductive characterization of the core-constrained Nash Bargaining Solution of supermodular games in Dutta and Ray (1989), we know $\lim_{\delta\to 1} b_j(N,\sigma)$ is the core-constrained Nash Bargaining Solution of (N,v). Q.E.D.

7 Concluding Remarks

In the classical environment of coalitional game with transferable utility, the efficiency implications of relaxing veto power in a bargaining model was examined. An equilibrium efficient with probability 1 in pure strategies was displayed for strictly supermodular coalitional games for all sufficiently high discount factors, a result that does not obtain in the models of Chatterjee et al. (1993), Okada (1996) and Compte and Jehiel (2010). The limiting outcome of this equilibrium is found to be the core-constrained Nash Bargaining Solution. Stationary subgame perfect equilibria that are efficient with probability 1 for all sufficiently high discount factors are shown to be payoff equivalent in the limit. That limit is the limit of the efficient equilibrium displayed i.e. the core-constrained Nash Bargaining Solution. An attractive feature of the model is that players who get to have veto power over an offer made by a proposer are endogenous and they constitute a coalitional threat for a proposer which permits the demand reduction of non-veto players.

The efficiency implications of noncontingent offers are underscored by similar result we find when we embed this feature in Chatterjee et al. (1993) mechanism where the rule governing the selection of proposers is taken to be a fixed protocol and the first rejector becomes the new proposer. Such an exercise is undertaken in Chaturvedi (2013b). Studying the set of SSPE outcomes and the efficiency implications of the mechanism for coalitional games wider than those studied here are avenues for future work.

Appendices

Appendix A

Claim 4.1. Suppose (N, v) is strictly supermodular. Then $\forall k = 1, \ldots, L-1$

$$\frac{v(\cup_{a=1}^{k} N_a)}{\sum_{a=1}^{k} |N_a|} \geq \frac{v(N)}{|N|}$$

Proof. Step 1. Let $N_0 = \emptyset$. Given (N, v), for $k = 1, \ldots, L(v)$, define inductively the restricted game $(N \setminus \cup_{a=0}^{k-1} N_a, v_k)$ by $v_k(S) = v(\cup_{a=0}^{k-1} N_a \cup S) - v(\cup_{a=0}^{k-1} N_a)$. In this step, we show a property of supermodular games that may be called the 'principle of cascading averages'. It says that the average values of restricted (in the sense defined above) games of a supermodular game are ordered in a decreasing fashion. We show the following statement is true:

$$\forall k \in \{1, \ldots, L-1\}, \quad \frac{v(N) - v(\cup_{a=0}^{k-1} N_a)}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} \geq \frac{v(N) - v(\cup_{a=0}^{k} N_a)}{\left|N \setminus \cup_{a=0}^{k} N_a\right|}$$

Suppose to the contrary that for some k

$$\frac{v(N) - v(\cup_{a=0}^{k-1} N_a)}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} < \frac{v(N) - v(\cup_{a=0}^{k} N_a)}{\left|N \setminus \cup_{a=0}^{k} N_a\right|} \tag{1}$$

Since $k \neq L$, $N_k \subsetneq N \setminus \cup_{a=0}^{k-1} N_a$ is a maximizer at Step k of our algorithm for generating partitions. So

$$\frac{v(N) - v(\cup_{a=0}^{k-1} N_a)}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} \leq \frac{v(\cup_{a=0}^{k} N_a) - v(\cup_{a=0}^{k-1} N_a)}{|N_k|} \tag{2}$$

By strict supermodularity of (N, v), $v(N) - v(\cup_{a=0}^{k-1} N_a) > 0$. Adding (1) and (2),

$$\frac{\left|N \setminus \cup_{a=0}^{k} N_a\right| + |N_k|}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} < \frac{[v(N) - v(\cup_{a=0}^{k} N_a)] + [v(\cup_{a=0}^{k} N_a) - v(\cup_{a=0}^{k-1} N_a)]}{v(N) - v(\cup_{a=0}^{k-1} N_a)}$$

So we have

$$1 = \frac{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|}{\left|N \setminus \cup_{a=0}^{k-1} N_a\right|} < \frac{v(N) - v(\cup_{a=0}^{k-1} N_a)}{v(N) - v(\cup_{a=0}^{k-1} N_a)} = 1$$

a contradiction.

Step 2. By Step 1, we get $\forall k \in \{1, \ldots, L-1\}$

$$\frac{v(N)}{|N|} \geq \frac{v(N) - v(\cup_{a=1}^{k} N_a)}{\left|N \setminus \cup_{a=0}^{k} N_a\right|}$$
$$\frac{\left|N \setminus \cup_{a=0}^{k} N_a\right|}{|N|} \geq \frac{v(N) - v(\cup_{a=1}^{k} N_a)}{v(N)}$$
$$\frac{v(\cup_{a=1}^{k} N_a)}{\sum_{a=1}^{k} |N_a|} \geq \frac{v(N)}{|N|}$$

Q.E.D.

Claim 4.2. Suppose (N, v) is strictly supermodular. Let $u^\star$ be the core-constrained Nash Bargaining Solution for (N, v). Then for every $S \subset N$ such that $S \neq \cup_{a=1}^{k} N_a$ for some $k = 1, \ldots, L$, we have $\sum_{j \in S} u_j^\star > v(S)$.

Proof. This is because $u^\star$ is in the core of (N, v) and by definition the set of all coalitions that are binding at $u^\star$ is precisely $\{N_1, N_1 \cup N_2, \ldots, N_1 \cup \ldots \cup N_L\}$. Q.E.D.

Proof of Proposition 1 (ctd.) *Optimality of Implementation Strategy*

For off-equilibrium offers, the optimality of implementation strategy is clear from its definition. For equilibrium offers, it can be verified from the choice-payoff tables below that the strategies described are unimprovable. Suppose $i \in N_k$, the k-th block of partition of players. If $k = L$, the last block of partition, then i has no coalitional threat. In this case i's feasible choices and payoffs as follows

(a) If noone in N rejects the offer

i's choices	i's payoffs
implement with N	$b_i + a_i$
DELAY	b_i
implement with a subcoalition T	$v(T) - \sum_{j \in T \setminus i} b_j < b_i$

(b) If someone in N rejects the offer

i's choices	i's payoffs
DELAY	b_i
implement with a subcoalition T	$v(T) - \sum_{j \in T \setminus i} b_j < b_i$

If $k < L$, then i's feasible choices and payoffs as follows

(a) If noone in N rejects the offer

i's choices	i's payoffs
implement with N	$b_i + a_i$
DELAY	b_i
implement with	
(i) i's coalitional threat $\cup_{a=1}^k N_a$	b_i
(ii) any other subcoalition T	$v(T) - \sum_{j\in T\setminus i} u_j(N,i) \overset{\delta\to 1}{\to} v(T) - \sum_{j\in T\setminus i} b_j < b_i$

(b) If someone in i's coalitional threat $\cup_{a=1}^k N_a$ rejects the offer

i's choices	i's payoffs
DELAY	b_i
implement with a subcoalition $T \neq \cup_{a=1}^k N_a$	$v(T) - \sum_{j\in T\setminus i} u_j(N,i) \overset{\delta\to 1}{\to} v(T) - \sum_{j\in T\setminus i} b_j < b_i$

(c) If someone rejects the offer but everyone in i's coalitional threat $\cup_{a=1}^k N_a$ accepts

i's choices	i's payoffs
implement with $\cup_{a=1}^k N_a$	b_i
DELAY	b_i
implement with a subcoalition $T \neq \cup_{a=1}^k N_a$	$v(T) - \sum_{j\in T\setminus i} u_j(N,i) \overset{\delta\to 1}{\to} v(T) - \sum_{j\in T\setminus i} b_j < b_i$

In all the choice-payoff tables, the assertion that as $\delta \to 1$, $v(T) - \sum_{j\in T\setminus i} u_j(N,i) \to v(T) - \sum_{j\in T\setminus i} b_j$ is by construction while the assertion that for a subcoalition T that is not i's coalitional threat, $v(T) - \sum_{j\in T\setminus i} b_j < b_i$ is due to Claim 4.2.

Optimality of Acceptance-Rejection Strategy follows from its definition.

Optimality of Proposal Strategy

We first show that the equilibrium offer is accepted no matter who proposes. Suppose $i \in N_k$. Consider the last responder in $\cup_{a=1}^k N_a$ who finds himself in a situation where every other responder in $\cup_{a=1}^k N_a$ has already accepted the proposal. j knows that if he rejects, then i will choose DELAY as per his implementation strategy i.e. j has a veto power over i's offer. Since the offer compensates j with his veto-demand $b_j(N,\sigma)$, he accepts as per his acceptance-rejection strategy. By induction on the number of responders in $\cup_{a=1}^k N_a \setminus j$ who have not yet responded to the proposal, it follows for any responder $j \in \cup_{a=1}^k N_a$ that if every other responder in $\cup_{a=1}^k N_a \setminus j$ who has already responded to the proposal has accepted it, then it is optimal for j to accept it.

Consider a responder $j \in N \setminus \cup_{a=1}^k N_a$. If all players in $\cup_{a=1}^k N_a$ who have already responded before him have accepted the proposal, then j knows by the arguments of the preceding paragraph that the other players in $\cup_{a=1}^k N_a$ who will follow him will accept the proposal as well. Since i's implementation strategy is to implement the offer with $\cup_{a=1}^k N_a$, j does not have a veto-power over i's offer. Since the offer compensates him with $u_j(N,i) > \delta v(j)$, he accepts the offer as per his acceptance-rejection strategy.

Deviations from equilibrium offer. Suppose $i \in N_k$.

Case 1. $k < L$. Classify deviations as:

(1) (N,d) where $\forall j \in \cup_{a=1}^k N_a \setminus i, d_j = u_j(N,i), \forall j \in N \setminus \cup_{a=1}^k N_a, d_j \leq u_j(N,i)$ and $\exists j \in N \setminus \cup_{a=1}^k N_a$ such that $d_j < u_j(N,i)$. This is a deviation where the proposer i tries to further gain unilaterally at the expense of players outside $\cup_{a=1}^k N_a$. A unanimous rejection of this off equilibrium offer is obtained in what follows.
Consider the last responder j in $\cup_{a=1}^k N_a$ who finds himself in a situation where every other responder in $\cup_{a=1}^k N_a$ has already rejected the proposal. j knows that if he rejects, then i will choose DELAY as per his implementation strategy i.e. j has a veto power over i's offer. Since the offer compensates j with his veto-demand $b_j(N,\sigma)$, he rejects as per his acceptance-rejection strategy. The argument is easily extended now by induction on the number of responders in $\cup_{a=1}^k N_a \setminus j$ who have not yet responded to the proposal to say that for any responder $j \in \cup_{a=1}^k N_a$, if every other responder in $\cup_{a=1}^k N_a \setminus j$ who has already responded to the proposal has rejected it, then it is optimal for j to reject it.
Since all responders in $\cup_{a=1}^k N_a$ reject this off-equilibrium proposal, any responder $j \in N \setminus \cup_{a=1}^{k+1} N_a$ knows that i's threat to implement $\cup_{a=1}^k N_a$ is an empty threat. Since the offer does not compensate j with more than $b_j(N,\sigma)$, he rejects as per his acceptance-rejection strategy.

(2) (N,d) where $\forall j \in \cup_{a=1}^k N_a \setminus i, d_j \geq u_j(N,i), \exists j \in \cup_{a=1}^k N_a \setminus i, d_j > u_j(N,i), \forall j \in N \setminus \cup_{a=1}^k N_a, d_j \leq u_j(N,i)$ and $\exists j \in N \setminus \cup_{a=1}^k N_a$ such that $d_j < u_j(N,i)$. This is a deviation where the proposer i tries to gain at the expense of players outside his coalitional threat and offers to share the gains with some players inside $\cup_{a=1}^k N_a$. A unanimous rejection of this off-equilibrium offer is obtained in what follows.
Consider the last responder $j \in N \setminus \cup_{a=1}^k N_a$ who finds himself in a situation where every other responder in $N \setminus \cup_{a=1}^k N_a$ has already rejected the proposal. Now j knows that if he rejects, i would choose DELAY rather than implement the offer with $\cup_{a=1}^k N_a$. This is because

$$\begin{aligned} v(\cup_{a=1}^k N_a) - \sum_{j \in \cup_{a=1}^k N_a \setminus i} d_j &< v(\cup_{a=1}^k N_a) - \sum_{j \in \cup_{a=1}^k N_a \setminus i} u_j(N,i) \\ &= v(\cup_{a=1}^k N_a) - \sum_{j \in \cup_{a=1}^k N_a \setminus i} b_j(N,\sigma) \\ &= b_i(N,\sigma) \end{aligned}$$

Since the offer compensates j with $d_j \leq u_j(N,i) \leq b_j(N,\sigma)$, he rejects as per his acceptance-rejection strategy. Argue by induction on the number of responders in $N \setminus \cup_{a=1}^k N_a$ who have not yet responded to the proposal to say that for any responder $j \in N \setminus \cup_{a=1}^k N_a$, if every other responder in $(N \setminus \cup_{a=1}^k N_a) \setminus j$ who has already responded to the offer has rejected it, then it is optimal for j to reject it.

Since all responders in $N \backslash \cup_{a=1}^{k} N_a$ reject this off-equilibrium offer, any responder j in $\cup_{a=1}^{k} N_a$ knows that i's sweetened offer is just a "mirage" because even if everyone in $\cup_{a=1}^{k} N_a$ accepts, i will choose DELAY. So j rejects the offer as well.

(3) (S, d) where $S = \cup_{a=1}^{k} N_a$ and $\forall j \in S \setminus i, d_j = u_j(\delta, N, i) + \epsilon$ for $\epsilon > 0$. The payoff for i from such an offer is

$$\begin{aligned}
d_i &= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \backslash i} d_j \\
&= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \backslash i} u_j(N, i) - \Big(\sum_{a=1}^{k} |N_a| - 1 \Big) \epsilon \\
&= v(\cup_{a=1}^{k} N_a) - \sum_{j \in \cup_{a=1}^{k} N_a \backslash i} b_j - \Big(\sum_{a=1}^{k} |N_a| - 1 \Big) \epsilon \\
&= b_i - \Big(\sum_{a=1}^{k} |N_a| - 1 \Big) \epsilon \\
&< b_i + a_i = u_i(N, i)
\end{aligned}$$

Thus i cannot gain by this deviation.

(4) (S, d) where $S \neq \cup_{a=1}^{k} N_a$. The payoff for i from such an offer is arbitrarily close to $v(S) - \sum_{j \in S \backslash i} u_j^\star$ while the payoff from the equilibrium strategy is arbitrarily close to $u_i^\star$ as δ gets high enough where $u^\star$ is the core-constrained Nash Bargaining Solution. By Claim 4.2, i cannot gain by this deviation.

Case 2. $k = L$. It suffices to realize that in any deviation (S, d) there cannot be a subcoalition $T \subset S$ such that $i \in T$, $d_j \geq u_j(N, i)$ and $v(T) - \sum_{j \in T \backslash i} d_j = b_i$. In other words, no T can be coalitional threat for i. This is essentially because T was not a coalitional threat for the equilibrium offer as well. Formally $v(T) - \sum_{j \in T \backslash i} d_j \leq v(T) - \sum_{j \in T \backslash i} u_j(N, i) < b_i$. Q.E.D.

Appendix B

Proof of Lemma 1 Step 1. Suppose an SSPE σ involves a player $i \in S$ making an unacceptable offer in some subgame $G(S, v)$. We construct a profitable deviation (S, d) for i. Let d the payoff distribution defined by $\forall j \in S \setminus i$, $d_j = b_j(S, \sigma) + \epsilon$ where $0 < \epsilon < (v(S) - \sum_{j \in S} b_j(S, \sigma))/(|S| - 1)$. Then d will be unanimously accepted since any responder does worse by rejecting. i gets $d_i = v(S) - \sum_{j \in S \backslash i} (b_j(S, \sigma) + \epsilon)$. By making an unacceptable offer, i's payoff at his proposal node in $G(S, v)$ is $b_i(S, \sigma)$. It is then easy to see by the choice of ϵ that $d_i > b_i(S, \sigma)$ i.e. $v(S) > \sum_{j \in S} b_j(S, \sigma) + (|S| - 1)\epsilon$.

Step 2. We now show that we can always choose such an ϵ. In other words, $v(S) - \sum_{j \in S} b_j(S, \sigma) > 0$. Let $u_j(\delta, S, i)$ be the payoff to j in a unanimously acceptable offer

$(T^i, u(S,i))$ made by i where $T^i \subset S$. If either $j \notin T^i$ or i makes an unacceptable offer, then $u_j(S,i) = 0$. Now

$$\begin{aligned}\sum_{j\in S} b_j(S,\sigma) &= \sum_{j\in S} \frac{\delta}{|S|}\Big[\sum_{k\in S} u_j(S,k)\Big] \\ &= \frac{\delta}{|S|}\Big[\sum_{j\in S} u_j(S,i) + \sum_{j\in S}\sum_{k\in S\setminus i} u_j(S,k)\Big] \\ &= \frac{\delta}{|S|}\sum_{k\in S\setminus i}\sum_{j\in S} u_j(S,k)\end{aligned}$$

Now $\forall k \in S \setminus i$

$$\sum_{j\in S} u_j(S,k) = \begin{cases} 0 & \text{if } k \text{ makes an unacceptable offer} \\ v(T^k) & \text{if } k \text{ makes an acceptable offer to } T^k \subset S \\ v(S) & \text{if } k \text{ makes an acceptable offer to } S \end{cases}$$

By strict superadditivity,

$$\forall k \in S \setminus i, \quad \sum_{j\in S} u_j(S,k) \le v(S)$$

$$\begin{aligned}\sum_{j\in S} b_j(S,\sigma) &\le \frac{\delta}{|S|}(|S|-1)v(S) \\ &< v(S)\end{aligned}$$

Thus we have shown a profitable deviation by i. This contradicts the premise that $\sigma \in E$.

Q.E.D.

Proof of Lemma 3 Fix an SSPE σ that is efficient with probability 1. By definition of $\Delta(\delta)$,

$$\sum_{j\in N} b_j(N,\sigma) = v(N) - \Delta(\delta)$$

Suppose by way of contradiction that for some $S \subsetneq N$, $v(S) > \sum_{j\in S} b_j(N,\sigma)$. Fix $i \in S$. By Lemma 2, i has a coalitional threat, say S^i in σ such that

$$v(S^i) - \sum_{j\in S^i\setminus i} b_j(N,\sigma) > b_i(N,\sigma)$$

Step 1. We first show there exists a player $k \in N \setminus S^i$ such that i compensates k with more than $b_k(N \setminus S^i, \sigma)$ i.e. $u_k(N,i) > b_k(N \setminus S^i, \sigma)$. The following deductions about i's equilibrium offer in $G(N,v,i)$ are a direct consequence of subgame perfection.

$$\begin{aligned} \forall j \in S^i \setminus i, &\quad u_j(N,i) = b_j(N,\sigma) \\ \forall j \in N \setminus S^i, &\quad u_j(N,i) \geq b_j(N \setminus S^i, \sigma) \end{aligned}$$

Since S^i is a coalitional threat for player i, i must compensate players in $S^i \setminus i$ with their veto-demands in the game $G(N,v)$. Since any player $j \in N \setminus S^i$ does not have veto power over i's offer, he must be willing to accept any payoff at least as much as $b_j(N \setminus S^i, \sigma)$ (j's discounted equilibrium payoff in G(N $\setminus S^i, v$)).

Suppose to the contrary that $\forall j \in N \setminus S^i$, $u_j(N,i) = b_j(N \setminus S^i, \sigma)$. Then

$$\begin{aligned} \sum_{j \in N} u_j(N,i) &= \sum_{j \in S^i} u_j(N,i) + \sum_{j \in N \setminus S^i} u_j(N,i) \\ &\leq v(S^i) + v(N \setminus S^i) \\ &< v(N) \qquad \text{Strict Superadditivity of } (N,v) \end{aligned}$$

Thus i's offer is inefficient. This contradicts the premise that σ is efficient with probability 1 proving the claim made in this step.

Step 2. Suppose $\eta(\delta) = v(S^i) - \sum_{j \in S^i} b_j(N,\sigma) > 0$. We construct a profitable deviation for i. Construct a feasible deviation (N,d) where

$$\begin{aligned} \forall j \in S^i, &\quad d_j = u_j(\delta, N, i) + \frac{\epsilon}{|S^i|} \qquad 0 < \epsilon < \eta(\delta) \\ &\quad d_k = u_k(\delta, N, i) - \epsilon \\ \forall j \in N \setminus \{k \cup S\}, &\quad d_j = u_j(N,i) \end{aligned}$$

For this deviation, the credibility of i's threat to implement S^i is preserved. The requirement for this is

$$\begin{aligned} & v(S) - \sum_{j \in S^i \setminus i} d_j > b_i(N,\sigma) \\ & \leftrightarrow v(S) > \sum_{j \in S^i} b_j(N,\sigma) + \frac{|S^i| - 1}{|S^i|}\epsilon \\ & \leftrightarrow \eta(\delta) > \frac{|S^i| - 1}{|S^i|}\epsilon \end{aligned}$$

which is true. So the deviation (N,d) in which i by bribing $S^i \setminus i$ enlists their support in further taking advantage of k is not deterred as the credibility of i's threat to implement S^i is unaffected by this deviation. This contradicts the premise that σ is an SSPE. We have proved that $\forall S \subsetneq N, \sum_{j \in S} b_j(N,\sigma) \geq v(S)$. Q.E.D.

References

Alfaro, L. (2014): "Sovereign Debt Restructuring: Evaluating the Impact of the Argentina Ruling," *Harvard Business Law Review (Forthcoming)*.

Binmore, K., A. Rubinstein, and A. Wolinsky (1986): "The Nash bargaining solution in economic modelling," *The RAND Journal of Economics*, 176–188.

Britz, V., P. J.-J. Herings, and A. Predtetchinski (2010): "Non-cooperative support for the asymmetric Nash bargaining solution," *Journal of Economic Theory*, 145, 1951–1967.

——— (2014): "On the convergence to the Nash bargaining solution for action-dependent bargaining protocols," *Games and Economic Behavior*, 86, 178–183.

Chatterjee, K., B. Dutta, D. Ray, and K. Sengupta (1993): "A noncooperative theory of coalitional bargaining," *The Review of Economic Studies*, 60, 463–477.

Chaturvedi, R. (2013b): "Efficient Coalitional Bargaining with Noncontingent Offers," .

Compte, O. and P. Jehiel (2010): "The coalitional Nash bargaining solution," *Econometrica*, 78, 1593–1623.

Dutta, B. (1990): "The egalitarian solution and reduced game properties in convex games," *International Journal of Game Theory*, 19, 153–169.

Dutta, B. and D. Ray (1989): "A concept of egalitarianism under participation constraints," *Econometrica: Journal of the Econometric Society*, 615–635.

Harsanyi, J. (1974): "An equilibrium-point interpretation of stable sets and a proposed alternative definition," *Management science*, 20, 1472–1495.

Hornbeck, J. (2010): *Argentina's Defaulted Sovereign Debt: Dealing with The" Holdouts"*, DIANE Publishing.

Krishna, V. and R. Serrano (1996): "Multilateral bargaining," *The Review of Economic Studies*, 63, 61–80.

Mas-Colell, A., M. D. Whinston, J. R. Green, et al. (1995): *Microeconomic theory*, vol. 1, Oxford university press New York.

Moldovanu, B. and E. Winter (1994): "Core implementation and increasing returns to scale for cooperation," *Journal of Mathematical Economics*, 23, 533–548.

——— (1995): "Order independent equilibria," *Games and Economic Behavior*, 9, 21–34.

Nash Jr, J. F. (1950): "The bargaining problem," *Econometrica: Journal of the Econometric Society*, 155–162.

OKADA, A. (1996): "A noncooperative coalitional bargaining game with random proposers," *Games and Economic Behavior*, 16, 97–108.

PERRY, M. AND P. J. RENY (1994): "A noncooperative view of coalition formation and the core," *Econometrica: Journal of the Econometric Society*, 795–817.

ROSENMÜLLER, J. (1981): *The theory of games and markets*, North-Holland Amsterdam.

RUBINSTEIN, A. (1982): "Perfect equilibrium in a bargaining model," *Econometrica: Journal of the Econometric Society*, 97–109.

SELTEN, R. (1980): *A noncooperative model of characteristic-function bargaining*, Institut für Mathematische Wirtschaftsforschung an der Universität Bielefeld.