Predictability and Power in Legislative Bargaining*

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Abstract

This paper examines the relationship between the concentration of political power in legislative bargaining and the predictability of the process governing the recognition of legislators. Our main result establishes that, for a broad class of legislative bargaining games, if the recognition procedure permits the legislators to rule out some minimum number of proposers one period in advance, then irrespective of how patient the individual legislators are, Markovian equilibria necessarily deliver all economic surplus to the first proposer. This result illustrates how the predictability of future power transitions may lead to great inequity, and offers a cautionary message for the benefits of transparency in the allocation of bargaining power. We extend our result to general coalitional settings with non-transferable utility, and show that risk-averse players are made strictly worse off by the early resolution of uncertainty. We show that the result is robust to a number of variations in the game (including perturbations over time, efficient default options, endogenous bargaining protocols, private learning, heterogeneous priors, and inequity-averse preferences) and derive comparative statics to show that the rents captured by the first proposer are increasing in the predictability of bargaining power. We find that amendment procedures can mitigate such inequities, and thereby offer a strong theoretical rationale for such open-rule procedures.

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Contents

1 Introduction

Motivation: Multilateral bargaining is a ubiquitous aspect of decision making within social groups. For example, in the political sphere, it governs the distribution of pork, the design of international treaties, and the formation of governments in parliamentary systems. Negotiators routinely attempt to steer these processes towards their own objectives, not only by organizing coalitions to support or block particular proposals, but also by maneuvering for control of the process through which they and others bring forth proposals. The relative power of the negotiators determines how agreements are reached and who benefits most from them.

In this paper, we investigate the ways in which the predictability of *future* bargaining power affects multilateral negotiations. An individual's willingness to join a coalition in support of a proposal today, rather than one aiming to block the proposal and extend negotiations, depends on how much bargaining power she expects to have if negotiations continue. If she is optimistic about her future bargaining power, she will only support proposals that are favorable to her; in contrast, if she expects to have little bargaining power in the future, she should be willing to support current proposals that provide her with modest benefits. The proposer, when designing his proposal today, accounts for the beliefs that players have about the future, and on that basis, shapes his winning coalition. We study both positively the bearing that the predictability of future bargaining power has for negotiations, and normatively, whether the group is made better or worse off by the early resolution of uncertainty.

Proposer Power and Predictability: We examine these issues in the framework of noncooperative multilateral bargaining, building on Rubinstein (1982) and Baron and Ferejohn (1989): rejection of a proposal leads to costly delay, and hence bargaining power flows from the ability to make proposals (or "set the agenda"). Most models of legislative bargaining treat proposer power as a primitive in which uncertainty as to who makes a proposal in period t is resolved *immediately* before the proposal is made, and not any earlier.[1] While this assumption can be defended as a stylized attempt to capture the somewhat inscrutable nature of the various protocols by which proposers are actually selected, it limits analysis to a setting in which the selection of each proposer is *entirely* random, and no information bearing on the identity of the period-t proposer is revealed over time. But in practice, important elements of the processes governing the recognition of proposers may be non-random, and information concerning random elements may be revealed in advance of the period-t proposer's selection. Examples include:

1. Proposers may be pre-announced. For instance, an otherwise inscrutable (and hence apparently random) chair may always identify the next proposer when recognizing the current one.

[1] In Baron and Ferejohn (1989) and its many applications, recognition is i.i.d. In the related literature subsection, we review recent research on history-dependent recognition protocols, in which players may learn from the history of proposers so far, but do not obtain any further information. Our work involves both a canonical history-dependent recognition process and more importantly, a canonical information structure through which players learn about bargaining power above and beyond the history of recognition to that date.

2. Based on the rules governing multilateral bargaining, it may be possible to rule out certain candidates for the proposer in advance. For example, the rules may require that proposers belong to pertinent committees or have seniority. Alternatively, the rules may prevent members of the same party from being recognized twice in a row or before representatives from all other parties have had a turn, or they may specify that the proposer for period t must be selected from a slate of nominees who are announced at an earlier point in time.

3. The selection of the period-t proposer may depend upon strategic choices that are themselves predictable in equilibrium, e.g., if choice of the proposer is up to a chair who is elected in advance, and who has a known "pecking order" of favorites, or if members can compete for the right to propose through costly effort or the expenditure of political capital.

4. Members of the legislature may observe who is falling in and out of favor with the group, or perhaps even the electorate, and on that basis, draw inferences of who may have the opportunity to make a legitimate proposal tomorrow.

In light of these possibilities, we have three distinct motivations for studying the impact of predictability on negotiations.

First, it appears to us to be a realistic possibility that players obtain some information about who shall have bargaining power in the future. To the extent that the institutional arrangements described above and those like it are seen in groups and legislatures, it would be useful from the standpoint of *positive theory* to identify the qualitative implications of predictability.

Second, we view this question as having *normative importance*: when designing rules and constitutions that govern multilateral bargaining, institutional architects must decide whether to make transitions in power more or less predictable. The predictability of a process may be naturally connected to its transparency, and these institutional architects may have three separate motives for having this form of transparency. First, they may worry that future political volatility impedes negotiations today and makes risk-averse players worse off, whereas resolving uncertainty about future power at an earlier date may lead the group to make better choices. Second, they may view a predictable and transparent bargaining process as being more immune to lobbying, bribing, and corruption, and better than an opaque process in which bargaining power is allocated through "back-door deals." Finally, institutional architects may worry that in the absence of strict procedures or information about bargaining power, players become overconfident and optimistic about their future bargaining power, and that such optimism delays agreement. By contrast, public information about who shall have power in the future constrains the beliefs that players may hold and thereby fosters agreement. For these three reasons (and perhaps more), institutional architects may be comparing information structures to ask the same question as our study here: what are the benefits and costs of being transparent about future transitions in bargaining power?

Third, we view this to be a fundamental theoretical question: how are multilateral negotiations influenced by the early resolution of uncertainty? Typically, questions of timing are posed in the

sphere of decision theory in which an individual values more information to less because that gives him a better ability to fine-tune his choices (Blackwell 1953) or has intrinsic preferences for when uncertainty resolves (Kreps and Porteus 1978). It is well-known that in strategic settings, players need not be better off with the revelation of more information,[2] but such analyses are often posed in a static context. We re-visit this classical issue in an important dynamic setting, investigating how negotiations are influenced by the timing of resolution of uncertainty.

Our Approach and Results: In each period, one of n legislators proposes a division of a fixed surplus, and needs the support of $q < n$ legislators for the proposal to pass. We consider a canonical class of recognition processes in which the identity of the proposer in period may depend on a history of random shocks and past proposers. Our main departure from the literature is that we explicitly model information about future bargaining power and compare different information structures. For each period $t > 0$, public information about period-t bargaining power may be revealed in earlier time periods. We vary the degree to which information makes period-t bargaining power easy to predict. The information structures that we compare are rich and not confined to any particular parametric structure.

We find that even a limited degree of predictability has stark implications: under a condition described in the next paragraph, the first proposer receives the *entire* prize, irrespective of negotiators' discount factors, in every subgame perfect equilibrium of the finite horizon game, and every Markov Perfect equilibrium of the infinite horizon game. Predictability is detrimental not only to equity but also to efficiency if players are risk-averse: equilibrium agreements are utilitarian inefficient, and these risk-averse players may *ex ante* Pareto prefer an institutional arrangement without any early resolution of uncertainty.

The predictability condition that gives rise to this result is simple and intuitive. A bargaining process exhibits *one-period predictability of degree d* if, for each history, at least d players can be ruled out as the next proposer. Our main result, Theorem 1, establishes that if the bargaining process is one-period predictable of degree q, then the first proposer captures all of the surplus. (Recall that passage of a proposal requires $q < n$ favorable votes.)

More important than the exact form of our main result is its elementary logic, which applies robustly across a range of settings. At its essence are challenges of commitment and competition. When a voting rule is non-unanimous, a proposer can exclude any other negotiator from a winning coalition, and this possibility constrains the amount of surplus any particular participant anticipates obtaining from that proposer. More information about bargaining power at period $t + 1$ helps the period-t proposer identify the most profitable negotiators to exclude: those who expect to be the proposer tomorrow have incentives to block the current proposal and extend negotiations, while others have no reason to reject offers unless they expect the next proposer to treat them more favorably.

[2]For example, Hirshleifer (1971) identifies how participants in an insurance market are made worse off by the release of public information. More recently, Morris and Shin (2002) and Angeletos and Pavan (2007) describe the subtle implications of public information when players have coordination motives.

But the proposer at $t+1$ cannot commit to such favors: since she will be able to predict bargaining power at $t+2$ prior to making her offer, she will, in turn, act on her own incentives to form the cheapest winning coalition by proposing to share the surplus with participants who do not expect to be recognized at $t+2$. Iterating this logic, the period-t proposer can secure the support of those who are ruled out from being the next proposer by offering them arbitrarily small shares of the surplus, even if they expect to be recognized in subsequent periods. Because the number of such individuals is sufficient to form a winning coalition, the first proposer extracts all of the surplus.

This strategic force is robust and general. The proposer captures nearly the entire surplus even if all players are recognized with strictly positive probability but q legislators are recognized with sufficiently small probability or if the bargaining process today is predictable, but may not be from tomorrow onwards with sufficiently small probability. The result seamlessly extends to environments with general coalition structures (allowing for weighted voting rules), non-transferable utility, heterogeneous priors, and those in which players can influence current and future bargaining power through costly maneuvers. Across these settings, when future bargaining power is sufficiently predictable, the shares offered by the first proposer are unaffected by heterogeneity in discount factors, recognition probabilities, voting weights, risk-aversion, and the costs of maneuvers. Perhaps surprisingly, a weaker but qualitatively similar result emerges even if the default option is not $(0,\ldots,0)$ but some efficient division of the surplus, players privately learn about their bargaining power, or players have preferences that are averse to inequality.

Comparative Statics and Broader Implications: We describe the implications of predictability in a stark case because it transparently illustrates the underlying strategic force. But such an analysis raises the concern that a small dose of predictability has no impact. We show that this is not the case using a tractable class of models (falling within our framework) to derive results for the spectrum of possibilities between no predictability (as in Baron and Ferejohn 1989) and perfect one-period predictability (in which next period's proposer is revealed one period in advance). Fixing the recognition rule to be one of i.i.d. uniform probability, we vary the fineness of the information obtained in each period t about the identity of the next proposer. Increasing the number of players who can be excluded, on the basis of information, from being the proposer tomorrow (the degree of one-period predictability) monotonically increase the first proposer's payoff until q players can be excluded, at which point she captures the entire surplus. This setting generates a closed-form solution in large legislatures (as $n \to \infty$), and we find that then, only the limiting proportional degree of one-period predictability (d/n) and the limiting proportional voting rule (q/n) matter. This result has the consequence that if the degree of predictability trails the size of a winning coalition by a constant number of legislators, the proposer still captures nearly all of the surplus, even though the condition for our main result no longer applies.

We view these results as bearing a nuanced message for the value of transparency in group negotiations. One sometimes troubling aspect of legislative and multilateral bargaining is that bargaining power is decided in "back door deals," vulnerable to lobbying and corruption, and is generally opaque

to all of the participants in negotiations. One anticipates that transparency in these settings may lead to fairer negotiations. But transparency in the sense of having more information about who (stochastically) shall have more bargaining power in the future leads to less equitable outcomes because it helps the proposer discriminate among people when building her coalition. In this context, opacity about the distribution of future bargaining power fosters equity.[3]

Our results also speak to the motive for transparency that emerges from players having biased beliefs. One concern about opaque institutions in which players have little information about procedures is that there is no guarantee that they shall share the same beliefs about the institution. This concern is directly relevant to this context: a player may become optimistic about her future bargaining power, either because she likes to think wishfully (Akerlof and Dickens 1982; Brunnermeier and Parker 2005) or because she realizes that her bargaining power reflects upon her status, and so she has *ego-motivation* (Köszegi 2006) to distort her belief. Optimism can generate bargaining delays (Babcock and Loewenstein 1997; Yildiz 2003, 2004), especially in large groups (Ali 2006). A predictable bargaining process, by contrast, disciplines players' beliefs so as to foster immediate agreement. Our results raise the flag that such measures may come at the cost of equity, and we describe these tradeoffs in Section 7.

Insofar as players in real-world negotiations may learn about their bargaining power before the actual proposer is selected, we find it troubling that the first agenda-setter can so effectively capitalize on the predictability of the bargaining process. What institutional measures mitigate such inequity? Clearly, one mechanism is to offer players veto power (which we describe in Section 5.2), but more interestingly, we show in Section 8 that a simple and relatively ubiquitous institutional measure substantially improves equity: give players the right to amend a proposal before it is put to a vote. Such "open-rule" negotiations feature in many legislatures, and our theory offers a strong rationale for this bargaining structure: when bargaining and amendment power are predictable, an amendment process has a no delay Markov Perfect equilibrium in which the first proposer is forced to share $\frac{\delta}{1+\delta k}$ with each of her k amenders. If the amendment process is universal so that all players may amend the proposal, then each player obtains approximately an even split as $\delta \to 1$. The sharp contrast of our results for closed and open rule bargaining offers two insights. First, to the extent that bargaining power in practice is predictable, this result strengthens the normative case for an amendments process. Second, it provides one reason as to why even if bargaining power in practice is predictable, we see more egalitarian sharing than implied by our main result.[4]

[3]We should offer a caveat to this interpretation. *Prima facie*, it may appear that the ubiquity of back door deals corresponds closely to the Baron-Ferejohn model in which all uncertainty about bargaining power in period t is resolved in period t. We are not convinced that this is correct, not least because the setting in which recognition is determined by a process of influencing leaders and power brokers would be exactly one in which a legislator forms expectations about her future power based on whether she could access power brokers. This setting is best modeled with private information, and we show in Section 5.5 that a qualitatively similar result applies when q players privately learn that they won't be the proposer tomorrow.

[4]We thank Attila Ambrus for this suggestion.

Outline: Section 2 conveys the intuition for our results through some simple examples. We present our framework in Section 3 and our main results in Section 4. In Section 5, we consider various extensions to our results. We develop comparative statics on imperfect predictability in Section 6. We consider the benefits and costs of predictability when players are optimistic about bargaining power in Section 7. Section 8 describes the implications of open rule negotiations. Section 9 concludes. Omitted proofs are in Appendix A.

1.1 Related Literature

Our analysis illustrates how predictable bargaining power combined with a non-unanimous voting rule ("excludability") confers extreme power to those with short-term control over agenda setting. Perhaps surprisingly, neither predictability nor excludability in isolation engenders such inequality. Rubinstein (1982) models an alternating-offer protocol in which future bargaining power is perfectly predictable but requires a unanimity voting rule; in both bilateral and multilateral settings, stationary equilibria yield approximately equal division as $\delta \to 1$ (Osborne and Rubinstein 1990). Baron and Ferejohn (1989) model excludability but assume that future bargaining power is completely unpredictable; as a result, the proposer shares roughly half the surplus with the winning coalition. Combining these two institutional features generates a starkly different prediction.

An immediate way for a bargaining process to have predictability of degree q is for q or more players to never be recognized, independent of history. Such concentration of proposal power necessarily gives rise to significant inequality, even if players are heterogeneous in their patience or voting power (Kalandrakis 2006). But such circumstances transparently establish these q players as being entirely powerless throughout the game, and those with proposal power as being dictators who can form winning coalitions with powerless players. Future bargaining power is predictable in these models only because it is exactly the same as current bargaining power and so no information is needed. Our main result is different because we demonstrate that the first proposer receives the entire prize even if (i) his probability of being selected in any future period is low, (ii) it is predictable that *someone else* (or some other group of individuals) will make the next proposal, and (iii) subsequent proposers are not at all predictable today. A player appropriates the entire surplus in our framework even when his political power is fleeting and transient, rather than being persistent.

A bargaining process may be predictable, even without signals being revealed over time, for particular history-dependent recognition rules; we are certainly not the first to study non i.i.d. rules. One strand of work directly extends Rubinstein (1982) to multiplayer settings by assuming that bargaining power deterministically rotates through the group and the voting rule is unanimity.[5] Another strand studies Markov recognition processes,[6] i.e., that in which the recognition probability in period $t+1$ depends only on the identity of the period-t proposer. Since we permit recognition in period t to depend on the full history of past proposers and not merely the past proposer, both of the

[5] See Herrero (1985), Binmore (1986), Asheim (1992), and Kultti and Vartiainen (2010).
[6] See Kalandrakis (2004a) and Britz, Herings and Predtetchinski (2010).

aforementioned recognition processes are special cases of those we study. Merlo and Wilson (1995), Yildiz (2003, 2004), and Cho and Duggan (2009) consider as rich a class of recognition processes as we do but focus on different issues and settings. Ortner (2013) and Simsek and Yildiz (2014) study durable bargaining power where the proposer today is very likely to be the proposer tomorrow, and focus on when deadlines and elections can generate delay.

Our work departs from this prior body of work in an important way: none of these papers study information about bargaining power whereas our focus is precisely on comparing *information structures* while holding a recognition process fixed. Our approach permits us to say that for a fixed recognition process, the first proposer exploits information about bargaining power tomorrow to capture a larger share of the surplus and that more predictability leads to more unequal and perhaps worse outcomes. Since the previous literature does not model information, players in these models learn about future bargaining power only by observing who is recognized beforehand. Therefore, bargaining power becomes "more predictable" in period $t+1$ in these models only by varying the actual primitive recognition process in period $t+1$ so as to concentrate that power (conditional on the history of recognition). Our analysis shows how that even political processes that are not concentrated can be exploited when players obtain information about future bargaining power.

We extend our results to environments where players can maneuver for recognition, or a committee chair (with predictable preferences) can select who to recognize. These results connect with the recent research on endogenous bargaining power (Yildirim 2007; Diermeier, Prato and Vlaicu 2013), and we show that if players can predict the outcomes of a game of strategic maneuvering, the first proposer may capture the entire surplus. A related result has been proven in the research on "rejector-friendliness," initiated by Chatterjee, Dutta, Ray and Sengupta (1993). Suppose that the first rejector of a proposal (according to the sequential voting order) in period t is recognized with probability μ in the next period, whereas every other player is recognized with uniform probability. Ray (2007) offers an interesting example in which the first proposer can capture the entire surplus when $\mu = 0$ and unanimous consent is required.[7] Our results are complementary insofar as our setting is fundamentally different: bargaining power is potentially non-stationary and independent of prior voting decisions, rather than the other way around, and we focus on the role of information and early resolution of uncertainty, which has no counterpart in this literature.

Our results speak to the importance of commitment: if the second proposer could commit to an equitable distribution of surplus, others would reject any exploitative offer made by the first proposer, and so instead he too would propose a more equitable outcome.[8] Relatedly, Bernheim,

[7] See Kawamori (2013) and Britz, Herings and Predtetchinski (2014) for extensions to this result.

[8] As an illustration, suppose that the proposer at $t=1$, p^1, can commit to an offer to be made at $t=1$ before the proposer at $t=0$, p^0, makes an offer. The legislator p^1 has a incentive to ensure that she is the most expensive member of a minimal winning coalition at $t=0$. She can do at least as well as committing to an offer providing 0 to $q-1$ players (including the first proposer), $\frac{1}{n-q} - \epsilon$ to herself, and equal shares for the remaining $n-q-1$ players. The legislator p^0 then chooses a policy in which he obtains $q-2$ players for free and proposer p^1 at a cost of $\frac{\delta}{n-q} - \delta\epsilon$. It is however, unclear as to what would emerge as an equilibrium outcome, or whether commitment leads to an indeterminacy problem when the entire sequence of potential proposers can commit to their proposals ahead of time (Roessler, Shelegia and Strulovici 2014).

Rangel and Rayo (2006) prove that in negotiations with an evolving status quo, a mildly predictable recognition process can provide the *last* proposer with dictatorial power. Note the contrast between these implications and that of Diermeier and Fong (2011), in which a single agenda-setter's inability to commit to future proposals limits the surplus she can extract; in our case, it is the inability of future agenda-setters to commit that permits the current agenda-setter to extract surplus.

Our interest is motivated by thinking about a particular form of transparency in negotiations: as to whether transitions in future bargaining power should be revealed today. Recent papers in bargaining (e.g. Hörner and Vieille 2009; Kaya and Liu 2014) have studied an important but very different form of transparency that relates to *past actions* rather than the future: should buyers today be able to observe past offers that were rejected by a seller, or should these offers be hidden? The strategic issues that emerge involve signaling and adverse selection, whereas most of our analysis (barring that of Section 5.5) focuses on issues of public information.

2 Examples

We illustrate the implications of predictable recognition processes using a series of examples.

Example 1: One-Period-Ahead Revelation. Our starting point is the closed rule divide-a-dollar model of Baron and Ferejohn (1989), where in each period, each of Alice, Bob, and Carol, has a 1/3 probability of being the proposer independently of the past, legislators are equally patient, and legislative approval requires a simple majority. In Baron and Ferejohn (1989), the identity of the period-$(t+1)$ proposer is revealed at the beginning of period $t+1$. Suppose instead that this uncertainty is resolved a period before: the identity of the period-$(t+1)$ proposer is known at the beginning of period t, prior to the period-t proposer making a proposal.

First consider a two-period model in which proposals can be made at $t \in \{0,1\}$, and if no agreement is reached by $t=1$, the dollar is destroyed. Suppose, at $t=0$, Alice and Bob are known to be the proposers at $t=0$ and $t=1$ respectively. The unique sub-game perfect equilibrium (henceforth SPE) outcome emerges from backward induction: if agreement is not reached at $t=0$, Bob proposes at $t=1$ to keep the entire dollar for himself, and at least one other player votes in favor. Thus, at $t=0$, every player other than Bob has an expected continuation value of 0 if no agreement is reached immediately. So the unique SPE outcome involves Alice proposing to keep the entire dollar for herself at $t=0$, and Carol votes in favor, since her payoff from rejecting the offer is 0.[9]

Naturally, the same conclusion applies in longer finite-horizon settings: each proposer is able to capture the entire surplus since everyone other than the next proposer expects to receive 0 if there

[9]This behavior does not rely on the fortuitous resolution of indifference among voters. Suppose that players at the voting stage vote sequentially or eliminate weakly dominated strategies at the voting stage. There cannot exist any SPE in which these offers are rejected with positive probability because the current proposer can always break indifference by offering members of the winning coalition arbitrarily small shares.

is delay. In the infinite horizon setting, there exists an SPE in which the first proposer receives the entire prize: in every period, the selected legislator proposes to keep the prize for himself, and all legislators (except for the next proposer, who is known) vote in favor. Given the continuation equilibrium, rejecting the proposal would simply shift the prize from the current proposer to the next proposer, which is of no benefit to other legislators. Thus, such behavior is sequentially rational.

Because multilateral bargaining games with infinite horizons give rise to folk theorems (Baron and Ferejohn 1989; Osborne and Rubinstein 1990), the literature focuses on Markov Perfect Equilibria (henceforth MPE). The concept implies that players can condition proposals and voting strategies only on variables that are directly payoff-relevant (rather than indirectly relevant through others' strategies)—i.e., the identities of the current and next proposers, and (when voting) the proposal currently on the table—and not on past proposals or voting decisions.

Our main result is that the first proposer captures the entire surplus in *all* MPE of the infinite horizon model. The following sketches the proof for this special case. Suppose each MPE involves agreement in every period (a claim we prove in Lemma 1). As before, suppose the proposers at $t \in \{0, 1\}$ are known to be Alice and Bob, and towards a contradiction that Alice has to share at least $\epsilon > 0$ to secure the support of Carol, who knows she will not be recognized at $t = 1$. Carol is willing to reject lower offers from Alice only if there is some realization of the $t = 2$ proposer for which Bob offers Carol at least ϵ/δ with strictly positive probability. But Bob makes such an offer only if gaining Alice's vote is at least as costly. So, each of Alice and Carol demands at least ϵ/δ to vote for Bob's proposal at $t = 1$, even though at least one of them is known to not be the proposer at $t = 2$. Therefore, there must be some realization of the $t = 3$ proposer at $t = 2$ such that the $t = 2$ proposer offers at least ϵ/δ^2 to another player with strictly positive probability. Since the same argument applies at $t = 3, 4, \ldots$, and $\delta < 1$, there must be a contingency under which a proposer eventually offers more than the entire surplus to another player with strictly positive probability, which is plainly a contradiction.

This result shows how the early resolution of uncertainty is detrimental to equity. When utility is non-transferable, the early resolution of uncertainty may be harmful to efficiency. Specifically suppose that for each player i, her payoff from accepting a share x_i is $\sqrt{x_i}$. Consider the equilibrium as $\delta \to 1$. In the setting of Baron and Ferejohn (1989)—with symmetric i.i.d. recognition and without predictability—each player expects to obtain $\frac{4}{5}$ if recognized and $\frac{1}{5}$ if she is included as a coalition partner (which happens with probability $\frac{1}{2}$ when another player is recognized). Her *ex ante* expected payoff, evaluated prior to the resolution of uncertainty at $t = 0$, is $\frac{1}{\sqrt{5}}$. By contrast, with one-period-ahead revelation, each player expects to obtain 1 if the proposer and 0 otherwise, leading to a strictly lower ex ante expected payoff of $\frac{1}{3}$. Reducing the volatility of future political power therefore makes all risk-averse players *ex ante* worse off.

Example 2: Rotating Recognition. Consider a recognition rule that perpetually cycles from Alice to Bob to Carol; this is the conventional generalization of Rubinstein's alternating-offer protocol to 3 or more players (Osborne and Rubinstein 1990). Unlike the previous example, players have

perfect foresight of future bargaining power for each future period and not merely one period in advance. Suppose that approval requires a simple majority.

As before, constructing an MPE that delivers this outcome is straightforward: each proposer proposes to keep the entire surplus, and every player known not to be the proposer tomorrow votes in favor of every proposal. No player has an incentive to deviate because the current proposer and one other player commonly know that neither will be the proposer tomorrow. Although Carol anticipates that she has all bargaining power at $t = 2$, she capitulates at $t = 0$ because she expects Alice to do the same at $t = 1$ when Bob proposes to extract the entire surplus.

No other outcome emerges in any MPE. Suppose towards a contradiction that Alice offers $\epsilon > 0$ to either Bob or Carol. Carol's vote must cost at least $\epsilon > 0$, which can happen only if Bob offers her at least ϵ/δ at $t = 1$ with strictly positive probability. But Bob would be willing to do that only if Alice's vote at $t = 1$ costs him at least the same. By the same logic, Alice's vote at $t = 1$ can be so expensive only if Carol offers her ϵ/δ^2 with strictly positive probability at $t = 2$. We see that as $t \to \infty$, some proposer must offer another player more than the entire surplus with strictly positive probability, yielding a contradiction.

Compare this outcome to that which emerges with a unanimity rule: an MPE involves a split of $\frac{1}{1+\delta+\delta^2}$ for Alice, $\frac{\delta}{1+\delta+\delta^2}$ for Bob, and $\frac{\delta^2}{1+\delta+\delta^2}$ for Carol. When Carol can veto agreements until $t = 2$, she retains power at $t = 0$ even if she is known to not be the proposer at $t = 1$. By contrast, when Carol knows that Bob cannot commit to including her in his coalition, she is willing to accept any offer at $t = 0$. This contrast highlights the role of *excludability*.[10]

Example 3: Nominations. Suppose that the period-t proposer is selected randomly (with equal probabilities) in period t from a set of n^* nominees, N_t^*. The nominees are in turn selected in period $t-1$ from the full set of legislators, and immediately announced. For concreteness, we assume that nominees are also selected randomly (with equal probabilities), but the particular selection process is in fact irrelevant. Finally suppose that the list of nominees is not too long: $n^* \leq \frac{n-1}{2}$.

We see that the following is an equilibrium: in period t, the selected legislator proposes to keep the entire prize for himself, and the proposal passes with the support of legislators belonging to $N \backslash N_{t+1}^*$. Members of that group are willing to vote for the proposal because they understand that rejecting it would simply shift the entire prize from the current proposer to some member of N_{t+1}^*. Because N_{t+1}^* has fewer than $\frac{n-1}{2}$ members, $N \backslash N_{t+1}^*$ has at least $\frac{n+1}{2}$ members, and therefore the proposal passes. The intuition for uniqueness is also essentially identical to that before.

This special case has immediate implications also for procedural rules that favor legislators based on their seniority or membership in special committees: if proposals must come from a small subset

[10]This example also has implications for systems in which bargaining power rotates through parties. Suppose every legislator belongs to one of P parties, where $P \geq 3$, and that, despite these party affiliations, each legislator is concerned only about his own constituents. Let n_j (with $\sum_{j=1}^{P} n_j = n$) denote the number of legislators belonging to party j. By convention, list the parties so that $n_1 > n_2 > ... > n_P$. Also assume $n_1 < \frac{n}{2} - 1$, so that no party has a majority. Consider a recognition rule that cycles through the parties, starting with the largest. The first proposer from the largest party receives the entire prize in every MPE, for the same logic as above.

of the entire legislature, the first proposer necessarily captures the entire surplus.

3 The Model

3.1 Environment

Consider a group of players, $\mathcal{N} = \{1, ..., n\}$, who are bargaining over the division of a fixed payoff (normalized to unity); i.e., the policy space is $\mathcal{X} \equiv \{x \in [0,1]^n : \sum_{i \in \mathcal{N}} x_i = 1\}$. Proposals can be made at discrete points of time in $\mathcal{T} \equiv \{t \in \mathbb{N} : t \leq \bar{t}\}$, where $\bar{t} \leq \infty$ is the deadline for bargaining. In each period, a player is recognized to propose a policy in $\mathcal{X}$. If the group approves the proposal according to the voting rules described below, the game ends and the policy is implemented. If the group rejects the proposal, play proceeds to the next period unless $t = \bar{t}$, in which case the game ends and each player obtains a payoff of 0.

Within period t, events unfold as follows:

1. Information concerning the identity of current and future proposers is revealed, and the proposer for time t, p^t, is determined. The proposer p^t makes a proposal.
2. Legislators vote on the proposal.

Details concerning each of these stages follow.

Stage 1: Information and Recognition. The selection of proposer at time t may depend both on random events, as in Baron and Ferejohn (1989), and on institutional rules that constrain the possible sequences of proposers. Formally, consider a canonical probability space $(\Omega, \mathcal{F}, \mu)$ (where Ω is the state space, $\mathcal{F}$ is a σ-algebra, and μ is a probability measure) encompassing all uncertainty pertaining to the bargaining process, and let $\omega \in \Omega$ denote the generic state of nature. For every $t \in \mathcal{T}$, define $h_P^t \equiv (p^\tau)_{\tau \in \mathcal{T}, \tau \leq t}$ as the history of proposers, and let H_P^t denote the set of possible proposer histories. The recognition rule is a sequence of functions $\widetilde{P}^t : H_P^{t-1} \times \Omega \to \mathcal{N}$, where $\widetilde{P}^t$ governs the selection of p^t, the proposer in period t. Of course, for a process of that type, the state of nature recursively determines the entire sequence of proposers. Hence we can rewrite the recognition rule more compactly as a stochastic process $(P^t)_{t \in \mathcal{T}}$, where each P^t is $\mathcal{F}-$measurable and maps Ω to $\mathcal{N}$. This formulation is canonical insofar as it allows for every deterministic and stochastic recognition protocol that is independent of players' actions.[11]

In stage 1 of period t, the players commonly observe a signal σ^t, where $\sigma^t(\omega)$ is $\mathcal{F}$-measurable. For each t, we can represent the information structure induced by the stochastic processes $(\sigma^\tau, P^\tau)_{\tau \in \mathcal{T}, \tau \leq t}$ as a partition, $\mathcal{S}^t$, of the state space Ω. The partition identifies states of nature that generate exactly the same signals and history of proposers through period t. Formally, $\mathcal{S}^t$ satisfies two requirements:

[11] In Section 5.4, we enrich it further by allowing for the possibility that players can influence recognition through political maneuvers.

(i) it partitions Ω and therefore $\bigcup_{s^t \in \mathcal{S}^t} s^t = \Omega$; and (ii) for each $s^t \in \mathcal{S}^t$, $\{\omega, \omega'\} \subset s^t$ if and only if $\sigma^\tau(\omega) = \sigma^\tau(\omega')$ and $P^\tau(\omega) = P^\tau(\omega')$ for every $\tau \leq t$.

The framework we use to describe uncertainty concerning future proposers and the revelation of pertinent information embeds all possibilities, with the restriction that revealed information is never forgotten; in other words, the signal structure generates a sequence of partitions $\{\mathcal{S}^t\}_{t \in \mathcal{T}}$ that are weakly finer over time. For example, the framework encompasses the extreme possibilities that the recognition order is known in advance, and that no information concerning the period-t proposer is revealed prior to period t. Between these extremes, we place no restrictions on the correlation structure governing the selection of proposers and the generation of signals.

Stage 2: Voting. The players vote on the proposal in a fixed sequential order.[12] A proposal is implemented if and only if at least q players (including the proposer) vote in favor. A voting rule is *non-unanimous* if $q < n$.

Payoffs: Players evaluate payoffs according to conventional exponential discounting: player i's discount factor is δ_i. No player is perfectly patient and $\hat{\delta} < 1$ denotes an upper-bound on their discount factors. If proposals at every period t in $\mathcal{T}$ are rejected, each player obtains a payoff of 0. If proposal x is implemented at time t, player i's payoff is $u_i(x, t) \equiv \delta_i^t x_i$.

3.2 Solution Concept

It is well-known that when five or more players bargain, every division can be supported as the outcome of a subgame perfect equilibrium in the infinite horizon (Baron and Ferejohn 1989; Osborne and Rubinstein 1990). The literature generally avoids the implications of this "folk theorem" by restricting attention to equilibria that are stationary. Because our model is inherently non-stationary, featuring history-dependent recognition processes and information revelation, we adopt a solution-concept in which players can condition on these payoff-relevant features. We restrict attention to Markov Perfect Equilibria, which prescribe the same continuation strategies at all structurally indistinguishable nodes (i.e., those at which the same information pertinent to the selection of subsequent proposers has been revealed).

Formally, at the proposal stage of period t, the *structural state* consists of the proposer's identity and all information bearing on the selection of future proposers. In our framework, s^t encapsulates that state in period t; recall in particular that it encodes the identities of all proposers through and including period t, as well as all signals pertaining to the identities of future proposers. For the voting stage of period t, the state consists of (s^t, x^t), where x^t is the period-t proposal.[13] Let $\mathcal{S}_i^t$ be the set

[12] Sequential rationality combined with voting in a sequential order implies that for every equilibrium, there exists an outcome-equivalent equilibrium in which each player eliminates weakly dominated strategies at the voting stage and "votes as if pivotal."

[13] As is conventional, we ignore the voting history at stage 2 of period t up to the time that player i votes. Although an equilibrium must specify behavior for every voting history, it is well-known that the requirement of subgame

of all $s^t \in \mathcal{S}^t$ consistent with player i being the proposer in period t, i.e., in which for every $\omega \in s^t$, $P^t(\omega) = i$. A Markov Perfect Equilibrium is an SPE in which we can write each player's equilibrium strategy as a sequence of functions $\left(\xi_P^{i,t}, \xi_V^{i,t}\right)_{t\in\mathcal{T}}$ such that $\xi_P^{i,t} : \mathcal{S}_i^t \to \Delta\mathcal{X}$ is player i's randomization over proposals when recognized in period t in structural state s^t, and $\xi_V^{i,t} : \mathcal{S}^t \times \mathcal{X} \to \Delta\{\text{yes}, \text{no}\}$ is player i's randomization over whether to vote for or against a policy $x \in \mathcal{X}$ proposed in period t and structural state s^t.

Our focus on MPE rules out equilibrium strategies for which choices depend on past proposals and votes, inasmuch as those actions have no direct structural implications for the continuation game,[14] but we permit (in principle) equilibrium behavior to condition on time, the identity of the proposer today and those in the past, and all information concerning the identity of future proposers. The state space for the MPE changes from one period to the next, and in light of the information that players receive about ω, the state space is very rich. These are departures from the literature's conventional focus on *stationary equilibria*, but MPE is the appropriate generalization when the past offers information about bargaining power in the future.

We have two motivations for studying this class of equilibria. First, adopting an appropriate generalization of the solution concept that is widely used in the literature facilitates transparent comparisons with existing results and highlights the implications of predictability. Second, Markovian strategies are the simplest possible form of behavior consistent with equilibrium rationality.[15] Every equilibrium must condition choices on variables that alter structural features of the continuation game, and non-Markovian equilibria are more complex because choices *also* depend on variables that have no structural implications for the continuation game. Because non-Markovian equilibria require legislators to follow different continuation strategies in structurally identical circumstances, sustaining any such equilibrium presumably requires more coordination. Yet, because every MPE ends in immediate agreement (as we show in Lemma 1), there is no efficiency motive for selecting a more complex equilibrium. Thus, the complex coordination required for a non-Markovian equilibrium is never in the legislators' mutual interests.

3.3 Some Examples

This framework subsumes numerous examples of bargaining processes:

(i) Baron and Ferejohn (1989): Let $\Omega = \mathcal{N}^{|\mathcal{T}|}$, and $P^t(\omega) = \omega_{t+1}$. The signal at time t is $\sigma_t(\omega) = \omega_{t+1}$. The measure μ is the "uniform" measure on Ω.

(ii) *One-period-ahead revelation*: The process is as above, except the signal at time t is $\sigma_t(\omega) = \omega_{t+2}$, so that the period-$(t+1)$ proposer is identified prior to the period-t proposal.

perfection selects the same outcome in the sequential voting game that would emerge if players voted simultaneously and sincerely for their preferred alternatives.

[14] As is conventional, our restriction is on equilibria, and not strategies. Players have the option to consider non-Markovian deviations, and for a strategy profile to be an MPE, such deviations have to be unprofitable.

[15] In the canonical legislative bargaining environment, Baron and Kalai (1993) prove that the Markovian equilibrium is the simplest equilibrium based on an automaton notion of complexity.

(iii) *Fixed Order*: The role of proposer rotates through the players in a fixed order. In this case, Ω is a singleton. For the case in which bargaining power rotates in an ascending order of player labels, let $P^t(\omega) = Rem(t/n) + 1$, in which *Rem* is the remainder function. The signal at time t is $\sigma_t(\omega) = \omega$.

(iv) *Nomination*: At most $n - q$ nominees for the period-t proposer are determined randomly in period $t' < t$, and the proposer is then chosen randomly from the nominees in period t. In this case, players partially learn the identity of the period-t proposer in period t', and then fully learn it in period t.

(v) *Markov Transition*: For a state space Ω, let the stochastic process $\{P^t\}_{t\in\mathcal{T},t>1}$ be such that for every $t \geq 1$, P^t is a Markov Chain on $\mathcal{N}$. A framework without any early resolution of uncertainty is $\sigma_t(\omega) = (P^0(\omega), \ldots, P^t(\omega))$, whereas that with the one-period-ahead revelation property is $\sigma_t(\omega) = (P^0(\omega), \ldots, P^{t+1}(\omega))$.

4 Bargaining with a Closed Rule

We formally define predictability and examine its implications in closed rule negotiations.

4.1 Predictability

In each period, players forecast the distribution of future bargaining power based on all information accumulated up to the present. Suppose that in stage 1 of period t, the sequence of signals and proposers indicate that the underlying state of the world ω is in the event s^t. Player i is recognized at time $t+1$ if and only if ω is in

$$\widehat{\Omega}_i^{t+1}(s^t) \equiv \left\{\omega \in s^t : P^{t+1}(\omega) = i\right\},$$

which has probability $r_i^{t+1}(s^t) \equiv \mu\left(\widehat{\Omega}_i^{t+1}(s^t) \mid s^t\right)$. The *losers* are those players who have 0 probability of being the proposer at $t+1$ conditional on all that is known at the proposal stage in period t:

$$L^{t+1}(s^t) \equiv \left\{i \in \mathcal{N} : r_i^{t+1}(s^t) = 0\right\}.$$

We use the cardinality of this set as a metric for predictability.

Definition 1. *The bargaining process exhibits* ***one-period predictability of degree*** **d** *if* $|L^{t+1}(s^t)| \geq d$ *for all* s^t *in* $\mathcal{S}^t$ *and* t *in* $\mathcal{T}$.

One-period predictability of degree d means that by the time the proposer is selected in period t, at least d players have been ruled out as the proposer for period $t+1$. Plainly, one-period predictability of degree d implies the same for any degree $d' < d$. We can classify the examples

from Section 3.3 as follows: process (i) (Baron-Ferejohn) does not exhibit one-period predictability of degree d for any strictly positive d, processes (ii) and (iii) exhibit one-period predictability of degree $n-1$, and process (iv) exhibits one-period predictability of degree q. Note that one-period predictability of degree d has no implications for two-period predictability (defined in the analogous way). For example, though process (ii) exhibits one-period predictability of degree $n-1$, it does not exhibit two-period predictability of degree d for any $d > 0$. The degree of predictability for a given bargaining process depends on both the underlying recognition process and the information structure. Different information structures for the same recognition process can lead to different levels of predictability, as we see in comparing examples (i) and (ii). Our results for this class of models do not require an ability to predict bargaining power in any period but the next.

4.2 Main Result

In this section, we state and prove our main result.

Theorem 1. *Suppose the voting rule is non-unanimous, requiring $q < n$ votes for a proposal to pass. If the bargaining process exhibits one-period predictability of degree q, the proposer selected at $t = 0$ captures the entire surplus in every MPE.*

This result illuminates the implications of combining predictability of the bargaining process with a non-unanimous voting rule: the current proposer can extract all of the surplus by forming a winning coalition consisting only of those who definitely will not make a proposal in the next period. Perhaps surprisingly, the members of this minimal winning coalition may expect to have bargaining power two periods hence, but in equilibrium, the votes of such individuals are bought cheaply since they expect to obtain no surplus in the next period. Thus, one-period predictability of degree q confers complete power.

Constructing an MPE that delivers this division of surplus is straightforward. Suppose that in each period t, and in each structural state s^t, the proposer p^t offers to share nothing, each player in $L^{t+1}(s^t)$ accepts all offers, and any other players accepts an offer if and only if his share exceeds his discounted continuation value. No proposer or voter will have a strict incentive to deviate from this strategy profile.

Of course, the theorem makes the much stronger claim that all MPE generate this outcome. The proof makes use of two properties of MPE, stated formally (and proven) below as Lemmas 1 and 2: all MPE end in immediate agreement, and the proposer never offers strictly positive surplus to more than $q-1$ other players (the smallest group needed to achieve a winning coalition). Once these preliminary steps are established, the proof for our main result has a clean argument, sketched below.

Specifically, suppose towards a contradiction that the first (period 0) proposer, player i, does not capture the entire surplus. Let player j be a member of player i's minimal winning coalition to whom i offers (weakly) more than she does to anyone else, and let x_j^0 denote this share. Because she chooses to exclude the other $(n-q)$ players and include player j in her minimal winning coalition, each of

the excluded players must be more expensive to buy out; i.e., each has an expected discounted continuation value that weakly exceeds x_j^0. Thus, at least $(n-(q-1))$ players have expected discounted continuation values no less than x_j^0. If the bargaining process exhibits one-period predictability of degree q, then at least one person within that set (call her player k) has no chance of being recognized in the next period. Player k necessarily derives all of her continuation value from the payoff she expects to receive when someone else serves as the proposer in period 1. Thus, in some structural state in period 1, the period-1 proposer must offer player k a payoff of at least $x_j^0/\hat{\delta}$.

The same logic, of course, holds for the aforementioned state in period 1. So by induction, there is some period-2 state in which the proposer offers some player a payoff of at least $x_j^0/\hat{\delta}^2$. Iterating this argument, we see that, for every t, there exists a structural state in which the proposer offers another player a share of at least $x_j^0/\hat{\delta}^t$. Because $\hat{\delta} < 1$, at least one player eventually obtains a share that exceeds the maximal feasible payoff, which is an obvious contradiction.

Necessity: The preceding logic shows that one-period predictability of degree q suffices for the first proposer to capture the entire surplus. Is this condition also necessary? We describe a setting in Section 6 where the first proposer cannot capture the entire surplus if the degree of one-period predictability is strictly less than q. Nevertheless, for that setting, greater predictability confers greater power: the first proposer's share is strictly increasing in the degree of predictability until $d = q$, at which point he captures the entire surplus.

However, as a general matter, one-period predictability of degree q is not the tightest possible condition for ensuring that the first proposer receives the entire surplus. One way to weaken this condition is to employ a "proposer-specific" notion of predictability. Suppose in particular that, in each structural state s^t, one can rule out $q-1$ players *other* than the current proposer, p^t, as the proposer for $t+1$. Then the current proposer captures the entire surplus even if the degree of one-period predictability is less than q. We return to this possibility in Section 5.2.

A Formal Proof of the Main Result: To prove Theorem 1, we first establish that every pure or mixed MPE must yield immediate agreement, and that every equilibrium proposal is directed towards the cheapest minimal winning coalition. For these purposes, it is useful to introduce some additional notation for players' continuation values in an MPE. For every t in $\mathcal{T}$ and every structural state s^t in $\mathcal{S}^t$, let $p^t(s^t)$ denote the proposer in period t. Moreover, for $t < \bar{t}$, let $V_i^{t+1}(s^t)$ denote the expected continuation value of player i at the beginning of period $t+1$ (before Stage 1) after the rejection of an offer in period t and structural state s^t; for the finite-horizon setting ($\bar{t} < \infty$), let $V_i^{\bar{t}+1}(s^{\bar{t}}) \equiv 0$. For a coalition $C \subseteq \mathcal{N}$, let $W_C^t(s^t) \equiv \sum_{i \in C} \delta_i V_i^{t+1}(s^t)$ represent the sum of discounted continuation values for the coalition. Denote the lowest cost of a coalition of size $q-1$ as

$$\underline{W}^t(s^t) \equiv \min_{\substack{C \subseteq \mathcal{N} \setminus \{p^t(s^t)\}, \\ |C| = q-1}} W_C^t(s^t),$$

the associated set of coalitions that achieve the minimum cost by

$$C^t(s^t) \equiv \left\{C \subseteq \mathcal{N} \setminus \left\{p^t(s^t)\right\} : |C| = q-1 \text{ and } W_C^t(s^t) = \underline{W}^t(s^t)\right\},$$

and the cheapest policies required to secure the support of such coalitions as

$$\mathcal{X}^t(s^t) \equiv \left\{x \in \mathcal{X} : \exists C \in C^t(s^t) \text{ such that } x_i = \delta_i V_i^{t+1}(s^t) \forall i \in C \text{ and } x_{p^t(s^t)} = 1 - \underline{W}^t(s^t)\right\}.$$

Every coalition C in $C^t(s^t)$ includes $q-1$ players (other than the proposer) with the weakly lowest discounted continuation value. The set of policies $\mathcal{X}^t(s^t)$ is that which offers discounted continuation values to these cheapest minimal winning coalition partners, 0 to others, and the rest to the proposer $p^t(s^t)$. Observe that the maximum offered to any player other than $p^t(s^t)$ is the same for all proposals in $\mathcal{X}^t(s^t)$: i.e., there exists $\overline{x}^t(s^t)$ such that for every offer $x \in \mathcal{X}^t(s^t)$, $\overline{x}^t(s^t) = \max_{i \neq p^t(s^t)} x_i$.

Lemma 1 (Immediate Agreement)**.** *For every t in $\mathcal{T}$ and structural state s^t in $\mathcal{S}^t$, every MPE proposal offered with strictly positive probability is accepted with probability* 1*.*

Proof. Suppose there is a structural state s^t in $\mathcal{S}^t$ such that an equilibrium proposal offered with strictly positive probability in period t, x', is rejected with strictly positive probability. Select some $x \in \mathcal{X}^t(s^t)$ and let C in $C^t(s^t)$ be an associated minimal winning coalition (excluding the proposer). Define a proposal x^ϵ for small $\epsilon \geq 0$ in which $x_i^\epsilon = x_i + \epsilon$ for every $i \in C$, $x_i^\epsilon = 0$ for every $i \notin C \cup \{p^t(s^t)\}$, and the proposer keeps the remainder. In equilibrium, the proposal x^ϵ is accepted by all members of C with probability 1 if $\epsilon > 0$. Observe that because $\sum_{j \in \mathcal{N}} V_j^{t+1}(s^t) \leq 1$, and $\hat{\delta} < 1$,

$$\sum_{i \in C} \delta_i V_i^{t+1}(s^t) + \delta_{p^t(s^t)} V_{p^t(s^t)}^{t+1}(s^t) \leq \hat{\delta} \sum_{j \in \mathcal{N}} V_j^{t+1}(s^t) < 1. \tag{1}$$

Therefore, for sufficiently small ϵ, the proposer's share of $1 - \sum_{i \in C} \delta_i V_i^{t+1}(s^t) - (q-1)\epsilon$ exceeds her discounted continuation value of $\delta_{p^t(s^t)} V_{p^t(s^t)}^{t+1}(s^t)$. Conditional on the equilibrium proposal x' being rejected, the proposer is strictly better off deviating to x^ϵ for sufficiently small $\epsilon > 0$. Conditional on the equilibrium proposal x' being accepted, the proposer's share can be no greater than that she obtains when offering x (otherwise a winning coalition would not support it). Since proposal x' is rejected with strictly positive probability, she is strictly better off offering x^ϵ for sufficiently small $\epsilon > 0$. Therefore, no equilibrium offer is rejected with strictly positive probability. □

Lemma 2 (Minimal Winning Coalition)**.** *For every t in $\mathcal{T}$ and structural state s^t in $\mathcal{S}^t$, every MPE proposal offered with positive probability provides positive payoffs only to members of the cheapest minimal winning coalition: $x \in \mathcal{X}$ is an MPE proposal in s^t only if $x \in \mathcal{X}^t(s^t)$.*

Proof. Any proposal in which the proposer shares less than $\underline{W}^t(s^t)$ with others is rejected with probability 1, and so Lemma 1 rules out such MPE proposals. If the proposer shares strictly more than $\underline{W}^t(s^t)$ with others, deviating to the proposal x^ϵ defined in the proof of Lemma 1 is strictly profitable for sufficiently small $\epsilon > 0$. □

Proof of Theorem 1. Let the structural state in Stage 1 of period 0 be s^0, and consider $\overline{x}^0(s^0)$, the highest equilibrium share that the proposer $p^0(s^0)$ offers to any player other than herself with strictly positive probability. Suppose towards a contradiction that $\overline{x}^0(s^0) > 0$. By Lemmas 1-2, we know that every MPE offer is made to a minimal winning coalition and accepted. Consider the set of players whose support cannot be secured for shares less than $\overline{x}^0(s^0)$:

$$H^0(s^0) \equiv \left\{ i \in \mathcal{N} \backslash \{p^0(s^0)\} : \delta_i V_i^1(s^0) \geq \overline{x}^0(s^0) \right\}.$$

$H^0(s^0)$ must have cardinality of at least $n-(q-1)$, because otherwise proposer $p^0(s^0)$ could form a cheaper coalition without having to offer $\overline{x}^0(s^0)$ to any player. Since the bargaining process exhibits one-period predictability of degree q, $H^0(s^0) \bigcap L^1(s^0)$ is non-empty. Consider a generic player i in $H^0(s^0) \bigcap L^1(s^0)$: player i definitely will not be the proposer in the next period, and his continuation value must therefore reflect an offer he receives. So there exists some structural state $s^1 \in \mathcal{S}^1$ such that the associated proposer offers player i at least $\frac{\overline{x}^0(s^0)}{\delta_i} \geq \frac{\overline{x}^0(s^0)}{\hat{\delta}}$ with strictly positive probability. Therefore, the highest share offered by that proposer to another player, $\overline{x}^1(s^1)$, is no less than $\frac{\overline{x}^0(s^0)}{\hat{\delta}}$.

The same logic applies in the structural state s^1 at $t=1$. So by induction, there exists a sequence of states $\{s^t\}_{t \in \mathcal{T}}$ such that for each t, $s^t \in \mathcal{S}^t$, and $\overline{x}^t(s^t) \geq \frac{\overline{x}^0(s^0)}{\hat{\delta}^t}$. If $\overline{t} = \infty$, $\hat{\delta} < 1$ implies that $\overline{x}^t(s^t)$ eventually exceeds 1; if $\overline{t} < \infty$, the same argument implies that the proposer in the final period offers a strictly positive share to another player. In both cases, we have reached a contradiction. □

5 Extensions

In this section, we consider several extensions of our framework, pointing out that (i) the proposer captures nearly all of the surplus if he can "almost" rule out q other players; (ii) our analysis can be generalized to encompass more general coalitional structures and non-transferable utility; (iii) a qualitatively similar result applies when players have access to an efficient default option; (iv) our findings generalize to settings in which the recognition of proposers depends not only on procedural rules and random events, but also on political maneuvering; (v) there exists an MPE in which the first proposer captures the entire surplus if players learn about bargaining power privately; and (vi) a qualitatively similar result applies when players are inequity-averse.

5.1 Robustness: Almost-Persistent Virtual Predictability

A feature of our main result is that it requires a degree of certainty, both concerning q individuals who will not make the next proposal for sure, and that such predictability perfectly persists throughout the game. Neither requirement is essential, which we illustrate by perturbing both features of our setting, and show that the first proposer still captures nearly the entire surplus. Throughout this exercise, we fix a non-unanimous voting rule that requires $q < n$ votes for a proposal to pass.

Formally, for every $\epsilon \in [0,1)$, we define the set of *almost losers*: as $L_\epsilon^{t+1}(s^t) \equiv \left\{ i \in \mathcal{N} : r_i^{t+1}(s^t) \leq \epsilon \right\}$.

In other words, based on all available information available at the period t proposal stage, the probability that any player in $L_\epsilon^{t+1}(s^t)$ will be recognized as the period $t+1$ proposer is at most ϵ. Let $\mathcal{P}_{d,\epsilon}^t = \{s^t \in \mathcal{S}^t : |L_\epsilon^{t+1}(s^t)| \geq d\}$ be the set of structural states in period t such that d players can be ϵ-virtually excluded from being the proposer tomorrow.

Definition 2. *The bargaining process exhibits* $(\mathbf{1-\rho})$***-persistent one-period*** $\boldsymbol{\epsilon}$***-predictability of degree*** $\mathbf{d}$ *if* $\mathcal{P}_{d,\epsilon}^0 = \mathcal{S}^0$ *and for every* $t \geq 1$, $Pr(s^t \in \mathcal{P}_{d,\epsilon}^t \mid s^{t-1}) \geq 1-\rho$ *for all* $s^{t-1} \in P_{d,\epsilon}^{t-1}$.

The above definition perturbs one-period predictability of degree d both at each time and over time. At $\rho = 0$ and $\epsilon = 0$, the above definition collapses to one-period predictability of degree d (modulo probability-0 events). Now consider $\epsilon > 0$: at every history, no player can be ruled out for sure from being tomorrow's proposer but at least d players may be ϵ-virtually excluded. Now consider $\rho > 0$: at $t = 0$, players can ϵ-virtually exclude d players from being the proposer at $t = 1$, and in every period $t \geq 1$, players anticipate that if in period t, they could ϵ-virtually exclude d players from being the proposer tomorrow, then they can do the same at period $t+1$ with probability of at least $(1-\rho)$. This appears to us to be the natural way to evaluate the sensitivity of our results to predictability stochastically failing at future dates, although we know of no precedents for this form of robustness in a dynamic game.[16] We note that this is a strong form of robustness since it does not impose any restrictions on tail events: the process may be unpredictable for almost all histories.[17]

Since we study ϵ and ρ close to 0, we limit attention to $\epsilon + \rho < 1$.

Theorem 2. *If the bargaining process exhibits p-persistent one-period* ϵ*-predictability of degree q, then in every MPE, the proposer selected at* $t = 0$ *does not offer more than* $\frac{\hat{\delta}(\epsilon+\rho)}{1-\hat{\delta}(1-\epsilon-\rho)}$ *to any other player, which converges to* 0 *as* $\epsilon, \rho \to 0$.

Thus, if there are sufficiently many players who are unlikely to be recognized, and it is sufficiently likely that this continues over time, the first proposer captures almost the entire surplus in every MPE. The logic of the argument closely resembles that given for Theorem 1: we show that if the proposer at $t = 0$ offers any player a share that exceeds the bound in the result, then there must exist some future state in which the proposer offers at least one player more than the entire surplus. In contrast to the proof of Theorem 1, the maximum amount offered to an almost loser does not increase geometrically; however, when initialized at a level exceeding $\frac{\hat{\delta}(\epsilon+\rho)}{1-\hat{\delta}(1-\epsilon-\rho)}$, it grows according to an "expansive" mapping that upon repeated iteration escapes the feasible set.[18]

[16] The closest analogue is the notion of p-cohesion defined by Morris (2000). Consider a directed graph in which for each period t, each state s^t is a node, and a directed link from s^t to s^{t+1} reflects that s^{t+1} is refined from s^t after the observation of σ^{t+1}, P^{t+1}. The bargaining process exhibits $(1-\rho)$-persistent one-period ϵ-predictability of degree d if and only if the group of nodes $\cup_{t\in\mathcal{T}}\mathcal{P}_{d,\epsilon}^t$ is $(1-\rho)$-cohesive.

[17] For example, consider a bargaining process that begins with predictability of degree q but at each date, it transitions to that with predictability of degree 0 with probability ρ conditional on not having done so already. Therefore, the probability that the bargaining process retains any predictability decays exponentially with time so it follows that almost all structural states are those in which the bargaining process has no predictability.

[18] It is infeasible to construct an MPE in this setting. If the set of signals is discrete, then so is the structural state space. In that case, the framework reduces to a special case of the setting in Duggan (2014), in which an MPE is proven to exist.

5.2 Coalitional Bargaining, Non-Transferable Utility, and Veto Power

Our main result generalizes to settings with more complex coalitional structures and non-transferable utility. Suppose the policy space is $\mathcal{X}$, and player i's stage payoff from policy x is $u_i(x_i)$ where, for each i, $u_i(\cdot)$ is strictly increasing, continuous, and concave, with $u_i(0) = 0$. Player i's discount factor is δ_i, and perpetual disagreement yields a payoff of 0. Say that a coalition of players is *decisive* if approval of an offer by all members of the coalition results in its implementation. Let $\mathcal{D} \subset 2^{\mathcal{N}}$ be the set of all *decisive coalitions*. As is conventional, we assume that $\mathcal{D}$ satisfies *monotonicity*: if D is decisive and $D \subseteq D'$, then D' is decisive. This general structure provides tremendous flexibility for modeling coalitional power; e.g., it encompasses settings in which players have unequal voting weights, as well as those with individual or coalitional veto power.

We generalize our notion of one-period predictability as follows:

Definition 3. *The bargaining process exhibits* ***one-period decisive predictability*** *if for all s^t in $\mathcal{S}^t$ and t in $\mathcal{T}$, there exists a decisive coalition D in $\mathcal{D}$ such that:*

(*a*) *D includes the proposer at time t, $p^t(s^t)$, and*
(*b*) *every other player in D definitely will not be recognized at $t+1$, i.e., $D \setminus \{p^t(s^t)\} \subseteq L^{t+1}(s^t)$.*

For the special class of anonymous aggregation rules—i.e. D is in $\mathcal{D}$ if and only if $|D| \geq q$—one-period decisive predictability actually *weakens* one-period predictability of degree q, insofar as it requires only that $q-1$ players *other than the current proposer* definitely will not be the next proposer. The vital implication of predictability is that the current proposer can form a decisive coalition with players who definitely will not be recognized in the next period; for Theorem 1, we invoked predictability of degree q (rather than $q-1$) so as to avoid stating a condition that depends on the identity of the proposer. The following theorem generalizes our earlier result:

Theorem 3. *If the bargaining process exhibits one-period decisive predictability, the proposer selected at $t = 0$, player i, obtains a payoff of $u_i(1)$ in every MPE.*

Observe that Theorem 3 implies Theorem 1; its greater generality requires a less direct proof and an additional intermediate result. We have to address the constraints that a proposer faces in choosing a decisive coalition, and that utility is non-transferable. In particular, it is no longer the case that all those excluded from an equilibrium coalition necessarily have lower discounted continuation value than those who are included. Instead, the argument proceeds by analyzing the cost of each coalition, normalized by each player's utility function. We establish that coalitions formed with players who definitely will not be the next proposer must have zero cost; otherwise, there is some state in which a proposer offers more than is feasible.

Two interesting implications follow from the risk-aversion permitted in Theorem 3. First, heterogeneity in risk-aversion, as captured by the concavity of $u_i(\cdot)$, may be less important in negotiations when bargaining power is predictable and unanimity is not required, offering a contrast to notions

of power described by Binmore, Rubinstein and Wolinsky (1986). Second, players may be worse off with predictability. Suppose that the setting is such that all utility functions are identical and strictly concave: in these cases, sharing promotes aggregate welfare but with predictability, a high degree of inequality still prevails. As illustrated by Example 1, players may be worse off in the sense of *ex ante* welfare, and may Pareto prefer for there to be no predictability. Thus, the early resolution of uncertainty may have negative consequences for both equity and efficiency in negotiations.

This setting can also be used to highlight the importance of veto power with weaker notions of predictability. Veto rights can moderate a proposer's power because a player with veto power can delay agreement even if she definitely will not be the next proposer. We show that with weaker notions of predictability, the first proposer may not capture the entire surplus, but she shares the surplus only with veto players. Suppose that passage of a proposal requires the support of players $1, \ldots, k$ and at least $q - k$ of the remaining $n - k$ players, where necessarily $k \leq q < n$.

Theorem 4. *If the bargaining process exhibits one-period predictability of degree q, then in every MPE, the proposer selected at $t = 0$ shares the surplus only with veto players.*

A player without veto power obtains a strictly positive payoff *only if* she is the first proposer. The contrast between the shares obtained by veto and non-veto players is more stark than in the framework without any predictability; in that latter setting, some non-veto players may receive positive shares even when another player is the proposer. As to whether a veto player receives a strictly positive share depends upon details of the recognition process, but it is easy to show that with a symmetric recognition probability and the "One-Period-Ahead Revelation" environment in Example 1, each veto player is offered at least $\frac{\delta}{n(1-\delta)+\delta k}$ regardless of the identity of the first proposer.

5.3 Efficient Default Options

So far, we have assumed that in the event of perpetual disagreement, each player obtains a payoff of 0. While this assumption features in much of the literature on multilateral bargaining, it may appear to play a powerful role in our results (at least in the finite-horizon). We show that a qualitatively similar result applies even if the default option selected in the event of disagreement is efficient.

We view this as being important for two distinct reasons. First, in many kinds of negotiations, recent agreements serve as the default option, and so there is reason to believe that disagreements may not destroy surplus.[19] Second, since we can always fold continuation values into the default option, this extension permits us to evaluate behavior when players commonly know that at some point in the future, the game may change so as to no longer be predictable. In proving that our results extend, we restrict attention to a simple-majority rule and an odd number of players.

Let us begin with the finite-horizon: suppose that if the players do not accept a proposal by

[19] Motivated by this possibility, a burgeoning literature (e.g. Baron 1996; Kalandrakis 2004b; Bernheim, Rangel and Rayo 2006; Diermeier and Fong 2011; Anesi and Seidmann 2014) has studied bargaining dynamics with an endogenous status quo.

period $\bar{t} \in \{2, 3, \ldots\}$, then a default option x^D is implemented. Order players by their share of the default option so that $x_1^D \leq x_2^D \leq \ldots \leq x_n^D$. We make two assumptions about the default options:

- **Genericity**: For each pair of distinct players i and j, $x_i^D \neq x_j^D$.
- **Majority Improvements**: For every minimal winning coalition C, $\sum_{i \in C} x_i^D < 1$.

Genericity rules out the knife-edge condition in which two distinct players receive an identical default option. Majority improvements ensures that for every minimal winning coalition, there is a distribution of surplus that improves upon the default option. Note that we permit equilibrium agreements to be utilitarian inefficient since we do not impose $\sum_{i \in \mathcal{N}} x_i^D \leq 1$.

We assume that $\delta_i = 1$ for each player i—so that there is no delay cost from waiting for the disagreement outcome—and perfect one-period predictability. We denote the identity of the period-$(t+1)$ proposer anticipated in state s^t in period t by $p^{t+1}(s^t)$.

Theorem 5. *If there are at least* 7 *players and* 3 *periods, the first proposer captures the entire surplus in every SPE, and every other player obtains* 0*.*

Players can wait for a default option that offers strictly positive shares to at least $n-1$ of the players, and yet, strategic forces push a majority to capitulate to a proposal that offers them nothing. The logic is that at the final period, $\bar{t}$, the proposer includes only those $(n-1)/2$ players with the lowest disagreement shares. Should there be disagreement at the penultimate period, there are $(n-1)/2$ players who anticipate a payoff of 0. The penultimate proposer, $p^{\bar{t}-1}$, then offers positive shares to at most one other player (if she is among the $(n-1)/2$ excluded in the final period), which must be either player 1 or player 2. Thus, at period $\bar{t}-2$, at least $n-3$ players expect zero payoffs if they disagree today. Because $n \geq 7$, this is a large enough coalition for the proposer $p^{\bar{t}-2}$ to capture the entire surplus. By induction, the first proposer captures the entire surplus.

As we alluded to above, this result is important for two reasons. First, it indicates that a variant of our result survives with an efficient default option in the focal case of simple-majority rule.[20] Second, it permits us to capture environments in which there is perfect one-period predictability only initially: suppose that players anticipate that after $\bar{t}$, the environment changes to one without predictability (e.g. Baron and Ferejohn 1989). Folding equilibrium continuation values into the default option x^D, our result implies that if the subsequent equilibrium values are asymmetric, the first proposer captures the entire surplus.

Our results do not hinge on backward induction from a known deadline at $t = 0$ and apply even if the deadline is uncertain. Consider an infinite-horizon game with a random deadline. The state ω encodes the deadline: the final period is $\bar{t}(\omega) < \infty$, which is an $\mathcal{F}$-measurable function. We assume

[20] Indeed, the result further suggests that even in a setting with multiple sessions of negotiations (each with its own pie), and an endogenous status quo, there may exist an equilibrium analogous to that of Kalandrakis (2004b) in which the first proposer in every session captures the entire pie. Perhaps such equilibria could be used to enforce the sharing of surplus, analogous to the constructions of Bowen and Zahran (2012) and Anesi and Seidmann (2014). We defer a fuller understanding of these implications for future work.

that for all ω in Ω, $\bar{t}(\omega) \geq 2$, so it is common knowledge that negotiations proceed for at least three periods. Players receive information about the deadline through the signal received in period t, $\sigma^t(\omega)$, and based on this information, they form a partition over the states of nature.

Definition 4. *The deadline is* one-period predictable *if in each period t, and for all ω such that $\bar{t}(\omega) = t+1$ and ω' such that $\bar{t}(\omega') > t+1$, there does not exist a member of the partition $\mathcal{S}^t$ that contains both ω and ω'.*

One-period predictability of the deadline guarantees that players know, one-period in advance, as to whether the next period of negotiations is the final period. We view this as a modest requirement, ruling out settings in which players cannot anticipate a deadline even in the preceding period.

Theorem 6. *If the deadline is one-period predictable, there exists a pure strategy MPE in which the first proposer captures the entire surplus.*

These results demonstrate that predictability can lead to extreme outcomes even if players choose efficient policies when they disagree, and are not being driven by (nor fragile to the removal of) the assumption that the default option is inefficient.

5.4 Political Maneuvers

Our results extend seamlessly to environments in which players can maneuver for bargaining power or otherwise influence the selection of future proposers. Suppose that in each period t, prior to the arrival of information and the selection of a proposer, each player i (potentially including a Chair, denoted $i = 0$, in addition to the negotiators) chooses a (potentially) costly maneuver m_i^t from some set M_i, and that the entire history of maneuvers up to that point (in addition to past random shocks and proposers) influences recognition in period t.

In this setting, it is useful to distinguish between two forms of predictability. The first is *unconditional predictability*, defined as follows: at the end of period t, it is possible to rule out a fixed set of q players as the next proposer irrespective of period $t+1$ maneuvers. Theorem 1 applies to this setting with only slight modification. A weaker notion is that of *conditional predictability*, defined as follows: at the end of period t, it is possible to rule out some set of q players as the next proposer for each profile of period $t+1$ maneuvers (where the set may depend on the maneuver profile). Under that condition, our main result follows for all pure strategy MPE: because players can accurately predict future maneuvers in any such equilibrium, the period-t proposer can still form a winning coalition with players who will definitely not be the next proposer. Since the logic of these arguments mirror that of Theorem 1, we relegate formalizations to the Appendix.

As an application, suppose a Chair is endowed with the power to choose the proposer at the outset of each period from a set of eligible candidates (which may be history-dependent). Because the bargaining process satisfies conditional predictability of degree $n-1 \geq q$, the first proposer captures the entire surplus in every pure strategy MPE. While randomization on the part of the

Chair could overturn this conclusion, a deterministic choice is more intuitively compelling when the Chair has favorites among the negotiators; i.e., for every policy x and pair i and j such that $x_i \neq x_j$, the Chair has strict preferences between x, and the policy $x_{i \leftrightarrow j}$ that exchanges the allocations for i and j. Thus, a Chair who fails to project inscrutability may have to choose between highly unequal distributions of surplus even if she favors equality.[21]

5.5 Private Learning

Our work is motivated by our desire to understand the costs and benefits of a predictable bargaining process, in which participants are publicly informed about future transitions of bargaining power. But in opaque legislative institutions—in which bargaining power is allocated through back door deals—a legislator may not know who else has or hasn't been able to access power brokers, but in many cases, she may know if she has been unable to do so. We show that a qualitatively similar (but weaker) result applies when players privately learn about bargaining power.

Formally, suppose that in addition to the public signal σ^t, each player i observes a private signal σ_i^t. For each t, the information structure generates a partition $\tilde{\mathcal{S}}_i^t$ for player i in which $\tilde{s}_i^t \in \tilde{\mathcal{S}}_i^t$ is a generic member of player i's partition. The information structure is common knowledge. Let a structural state be $\tilde{s}^t = (\tilde{s}_1^t, \ldots, \tilde{s}_n^t)$, which encapsulates the information possessed by each player, and let $\tilde{\mathcal{S}}^t$ be the set of possible structural states. We consider a similar solution-concept to that before: proposal equilibrium strategies by player i condition on her information $\tilde{s}_i^t$, and voting strategies by player i condition on information $\tilde{s}_i^t$ and the proposal on the table.

Parallel to our definitions in Section 4, let $\tilde{r}_i^{t+1}(\tilde{s}^t) \equiv \mu\left(\widehat{\Omega}_i^{t+1}(\tilde{s}_i^t) \mid \tilde{s}_i^t\right)$. The *privately informed losers* are those players who have 0 probability of being the proposer at $t+1$ conditional on all that is known at the proposal stage in period t: $L^{t+1}(s^t) \equiv \left\{i \in \mathcal{N} : \tilde{r}_i^{t+1}(\tilde{s}^t) = 0\right\}$.

Definition 5. *The bargaining process exhibits* ***one-period private predictability of degree*** **d** *if* $|L^{t+1}(\tilde{s}^t)| \geq d$ *for all* $\tilde{s}^t$ *in* $\tilde{\mathcal{S}}^t$ *and* t *in* $\mathcal{T}$.

One-period private predictability of degree d requires that each of at least d players privately learns in period t as to whether she has a strictly positive probability of being the proposer in period $t+1$. Unlike one-period predictability of degree d, the identity of these d players is not commonly known. For a non-unanimous q voting rule, it is straightforward to construct an MPE in which the first proposer captures the entire surplus: suppose that each proposer offers to take all of the surplus, and each privately informed loser votes in favor of each proposal. No player has any incentive to deviate from this profile. Thus, the following result applies.

Corollary 1. *If the bargaining process exhibits one-period private predictability of degree q, then there exists an MPE where the proposer selected at* $t = 0$ *captures the entire surplus.*

[21] Even if the negotiators are not entirely sure of the Chair's preferences, similar conclusions would follow provided they can confidently rule out a sufficient number of possibilities.

Of course, other MPE may also exist, and it would be difficult to construct the entire set at this level of generality. But our point is that the potential for inequality remains even if learning is private and bargaining power is allocated through an opaque mechanism.

5.6 Inequity Aversion

The starkness of our theoretical result and its extreme inequality may lead one to question whether players who have social preferences or are fairness-minded would behave differently. We anticipate that our results would fail for numerous models of social preferences, in particular those that motivate giving in dictator games. But perhaps surprisingly, a qualitatively similar result applies even if players are averse to the inequality described in our preceding results: following Fehr and Schmidt (1999), suppose that player i's payoff from accepting proposal x is

$$x_i - \frac{\alpha}{n-1}\sum_{j\neq i}\max\{x_j - x_i, 0\} - \frac{\beta}{n-1}\sum_{j\neq i}\max\{x_i - x_j, 0\},$$

where $\alpha > \beta > 0$, and $\beta < \frac{1}{n-1}$. For simplicity, we assume that players share a common discount factor δ, bargain over an infinite-horizon, and eachobtains a payoff of 0 in the event of perpetual disagreement. We consider the "One-Period-Ahead-Revelation" setting described in Example 1 of Section 3.3: in each period, each player has a $1/n$ probability of being the proposer, independently of the past, and the period-$(t+1)$ proposer is revealed in period t. Consider a MPE of this environment: it is straightforward to establish that agreement is immediate and offers are made to a minimal winning coalition. Suppose that in equilibrium the proposer p^t offers y to each member of his coalition, and includes player $i \neq p^{t+1}$ in his coalition. Player i's payoff from accepting the equilibrium proposal is

$$U = y - \frac{\alpha}{n-1}(1-qy) - \frac{\beta}{n-1}(n-q)y, \tag{2}$$

in which the first term is player i's selfish payoff, the second term is her loss from disadvantageous inequality with respect to the proposer, and the third is her loss from advantageous inequality. By contrast, her payoff from rejecting the equilibrium proposal is

$$\delta\left[\frac{1}{n}(-\frac{\alpha}{n-1}) + \frac{n-1}{n}\left\{\frac{q-1}{n-1}U + \left(\frac{n-q}{n-1}\right)\left(-\frac{\alpha}{n-1}\right)\right\}\right]. \tag{3}$$

The first term represents the payoff from being *excluded* from the coalition in the event that she is identified in period $t+1$ as the period-$(t+2)$ proposer (which occurs with probability $1/n$).[22] The second term represents the complementary event: with probability $(q-1)/(n-1)$, proposer p^{t+1} includes player i in her coalition and offers her U, and with probability $(n-q)/(n-1)$, she is

[22] Observe that with inequity averse preferences, the payoff from receiving a share of 0, independently of how others divide the surplus, is $-\frac{\alpha}{n-1}$.

excluded, in which case she obtains the exclusion payoff. Because player i must be just indifferent between accepting and rejecting a proposal, equating (2)-(3) implies that

$$U = -\frac{n-q+1}{n-\delta(q-1)}\left[\frac{\delta\alpha}{n-1}\right]$$
$$y = \frac{\alpha n(1-\delta)}{((n-1)+\alpha q-\beta(n-1))\,(n-\delta(q-1))}$$

The above computations imply the following corollary.

Corollary 2. *As $\delta \to 1$, the first proposer captures the entire surplus in every MPE.*

We highlight interesting strategic features of this equilibrium. At $\delta = 0$, the proposer makes the same offer that she would in a one-shot bargaining game. Since her coalition partners value the equity of the $(0,\ldots,0)$ default option, she has to offer a strictly positive share to gain their acceptance. For $\delta > 0$, competitive forces permits the proposer to offer less utility to each coalition partner: each coalition partner fears being excluded from future coalitions, even more than she would were her preferences purely selfish because she dislikes disadvantageous inequality. Thus, for every $\delta > 0$, each coalition partner's payoff, U, is strictly negative, and so all but the proposer would strictly prefer perpetual disagreement to the equilibrium agreement. Even more starkly, each player's *ex ante* payoff (prior to recognition at $t = 0$) converges to $\frac{1}{n}(1-\alpha-\beta)$ as $\delta \to 1$ so that if $\alpha+\beta > 1$, the players unanimously prefer perpetual disagreement to the equilibrium outcome. The inability to commit to perpetual disagreement leads all players to be strictly worse off.[23]

6 Comparative Statics: Imperfect Predictability

This section examines a tractable subclass of the environments subsumed by our framework, with the object of illuminating the spectrum of possibilities between one-period predictability of degree zero, as in Baron and Ferejohn (1989), and of degree q, as in Theorem 1. We show that increases in the degree of predictability increase the expected share captured by the first proposer.

Suppose that proposer recognition is governed by an i.i.d. process, and that in every period, each player has a $1/n$ chance of being recognized. Suppose that players receive a potentially informative signal about the period-t proposer in period $t-1$. We represent the signal realization by its induced posterior belief: for a vector of probabilities λ (such that $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$) governing the selection of the period-t proposer, the signal reveals which player i is assigned to which probability λ_j.[24] For simplicity, we suppose that all players share a common discount factor of δ. The Baron and Ferejohn

[23] This paradoxical result is not driven by the game being infinite-horizon, and a similar result applies in finite-horizon environments in which players could reach the $(0,\ldots,0)$ outcome in a finite sequence of moves. Specifically, for every $\epsilon > 0$, there exists a SPE in sufficiently long games for sufficiently high δ in which first proposer captures $1-\epsilon$.

[24] It is straightforward to formulate a signal space with $n!$ possible realizations that induces this family of posterior beliefs.

(1989) framework corresponds to the special case where $\lambda_1 = \ldots = \lambda_n = \frac{1}{n}$, which exhibits one-period predictability of degree 0. Example 1 in Section 2 ("One-Period-Ahead Revelation") corresponds to $0 = \lambda_1 = \ldots = \lambda_{n-1} < \lambda_n = 1$, which exhibits one-period predictability of degree $n-1$.

We characterize the MPE for this model as follows. In period t, prior to recognition, let w_i denote the continuation value for a player who will be selected as the period-(t) proposer with probability λ_i. In equilibrium, proposers assemble the cheapest possible minimal winning coalitions by including the players who are least likely to be recognized in period $t+1$. The continuation value is recursively computed as the solution to:

$$w_i = \lambda_i \underbrace{\left(\frac{n-q+1}{n} \left(1 - \delta \sum_{j=1}^{q-1} w_j \right) + \frac{1}{n} \sum_{k=1}^{q-1} (1 - \delta \sum_{j=1, j\neq k}^{q} w_j) \right)}_{\text{Expected Proposer Surplus}} + (1-\lambda_i) \underbrace{\delta \left(\sum_{j=1}^{q-1} \frac{w_j}{n} + \frac{(q-1)w_q}{n(n-1)} \right)}_{\text{Included in Coalition}}.$$

The first term in this expression represents the player's continuation value conditional upon being recognized as the period-(t) proposer. It encompasses two distinct possibilities that depend upon information about period-$(t+1)$ bargaining power: either she is among the $n-q+1$ players most likely to be recognized, in which case she purchases the cheapest $q-1$ votes (i.e., those of the players least likely to be recognized), or she is among the $q-1$ players least likely to be recognized, in which case she purchases the $q-1$ cheapest votes other than her own. The second term represents the player's continuation value conditional upon another player being recognized as the proposer. It reflects the same considerations: the player assigned the recognition probability λ_q in period $t+1$ is included in the minimal winning coalition in period t if and only if the current proposer is assigned a weakly lower probability in period $t+1$. This recursive formulation generates n linear equations with n unknowns, and consequently has a unique solution.

We use this approach to derive the closed-form solution of the three player game.

Example 1. *Suppose there are three players who make decisions based on simple majority rule. As $\delta \to 1$, the expected share to the first proposer converges to*

$$\frac{2}{3}(1-w_1) + \frac{1}{3}(1-w_2) = \frac{2\lambda_1 + \lambda_2 + 3}{6\lambda_1 + 3\lambda_2 + 3}.$$

When $\lambda_1 = \lambda_2 = \frac{1}{3}$, the preceding term is $\frac{2}{3}$, which coincides with the solution in Baron and Ferejohn (1989). Notice that the above term is decreasing in both λ_1 and λ_2 so greater predictability monotonically increases the first proposer's share.

The proposer's ability to capture rents also depends on the relative power of the other players. Holding fixed λ_3, the first proposer's share increases with the disparity of power between the other two players. Indeed, the first proposer's share can be re-written as $\frac{9-3\lambda_3-(\lambda_2-\lambda_1)}{15-9\lambda_3-3(\lambda_2-\lambda_1)}$, which is increasing in both λ_3 and $\lambda_2 - \lambda_1$. Intuitively, greater inequality in predicted bargaining power decreases the cost of buying the vote of the weakest coalition partner.

We can use this approach to investigate the implications of one-period predictability of degree d

for $d < q$, in the special case where d players learn one period in advance that they definitely will not be the next proposer, while the remaining $n - d$ players learn that they are equally likely to be the next proposer; formally, $\lambda_1 = \ldots = \lambda_d = 0$ and $\lambda_{d+1} = \ldots = \lambda_n = \frac{1}{n-d}$. We can write continuation values as $\underline{w}$ for players $1, \ldots, d$ and $\overline{w}$ for players $d+1, \ldots, n$. To solve for these values, we compute $\underline{w}$ recursively and then make use of the fact that all continuation values must sum to 1 (since there is no delay). Relegating the algebra to the Appendix, we find that

$$\overline{w} = \frac{(n-1)(n-\delta d)}{n((n-1)(n-d) - \delta d(n-q))}, \text{ and}$$
$$\underline{w} = \frac{\delta(n(q-(d+1))+d)}{n((n-1)(n-d) - \delta d(n-q))}.$$

The above terms are strictly positive for non-unanimous rules ($q < n$) when the degree of predictability d is strictly less than q. The first proposer's expected share is

$$1 - \delta + \delta(n-q)\overline{w} + \delta/n.$$

Using this solution, we can determine the effect of d, the number of players who definitely will not make the next offer, on the proposer's payoff for the case of $d \leq q$:

Theorem 7. *Suppose the voting rule is non-unanimous, and the bargaining process exhibits one-period predictability of degree $d \leq q$. For every $\delta > 0$, the share obtained by the first proposer is strictly increasing in d.*

Thus, the conceptual message of Theorem 1 generalizes beyond the case of $d \geq q$: when $d \leq q$, greater one-period predictability (measured according to the degree d) implies greater power to the first proposer. We emphasize that this offers a message about *information structures* and *transparency about the future* since we are fixing the recognition process throughout this exercise: information helps a proposer discriminate among members of the group on the basis of their bargaining power tomorrow and form a coalition with those least likely to have power.

Our second result characterizes the proposer's (approximate) share in large legislatures. Consider a sequence of games $(G_n)_{n=3,4,\ldots}$ such that game G_n has n players, requires q_n votes for approval of a proposal, and exhibits one-period predictability of degree d_n. We say that the sequence is *convergent* if there exists α_v and α_p such that $q_n/n \to \alpha_v$ and $d_n/n \to \alpha_p$. Our next result identifies the proposer's limiting share in a convergent sequence of games.

Theorem 8. *Consider a convergent sequence of games $(G_n)_{n=3}^{\infty}$ in which α_v is the limiting proportional voting rule and α_p is the limiting proportional degree of one-period predictability. For every $\epsilon > 0$, there exists $\overline{n}$ such that if $n > \overline{n}$, the share of the surplus captured by the first proposer is within ϵ of $1 - \frac{\delta(\alpha_v - \alpha_p)}{1-\alpha_p(1+\delta(1-\alpha_v))}$ if $\alpha_p \leq \alpha_v$, and 1 otherwise.*

This expression shows how one-period predictability of a less-than-decisive degree influences the first proposer's share in the limit. For $\alpha_p = 0$, the proposer's share corresponds to that found by Baron

and Ferejohn (1989). Increases in the limiting degree of one-period predictability (as measured by α_p) improve the outcome for the first proposer, consistent with the conceptual message of Theorem 7. Moreover, for $\alpha_p < \alpha_v$, the proposer's share is a *convex* function of α_p.[25]

Two additional implications of Theorem 8 merit emphasis. First, even if the votes required for passage (q_n) exceed the degree of one-period predictability (d_n), the proposer's share will converge to unity as the legislature becomes arbitrarily large provided that the difference $q_n - d_n$ grows slower than linear, i.e., is $o(n)$. Second, as the voting rule converges to unanimity, the first proposer's limiting share converges to $1-\delta$ irrespective of the degree of one-period predictability. Thus, we see once again that the source of the proposer's power is the combination of a predictable bargaining process and the ability to exclude players from a minimal winning coalition.

7 Optimism and Overconfidence

A broad literature on psychology and economics emphasizes how, in the face of uncertainty, a player may optimistically expect uncertainty to resolve in her favor. One rationale for such beliefs is that people enjoy thinking wishfully (Akerlof and Dickens 1982; Brunnermeier and Parker 2005); yet an even stronger force in this context is that one's bargaining power reflects one's status and strength and so a player may distort her beliefs to cater to her ego (Köszegi 2006). A number of authors have argued that optimism can generate delays in bargaining (Babcock and Loewenstein 1997; Yildiz 2003, 2004), particularly in multilateral settings (Ali 2006). One benefit of a predictable bargaining process is that information about future bargaining power may "discipline" players' beliefs. We evaluate the costs and benefits of transparency, extending our results to settings with heterogeneous beliefs.

We study finite-horizon environments ($\bar{t} < \infty$) in which the equilibrium outcome is selected by iterated conditional dominance, so as to sidestep challenges of equilibrium selection that emerge with heterogeneous priors.[26] The state space Ω describes all uncertainty and μ denotes nature's distribution. Each player i has her own beliefs about how the bargaining process shall unfold represented by a *conditional probability system* $\mu_i(\cdot \mid \cdot) : \mathcal{F} \times \cup_{t\in\mathcal{T}}\mathcal{S}^t \to [0,1]$: for each structural state $s^t \in \cup_{t\in\mathcal{T}}\mathcal{S}^t$ reached during the game, the belief $\mu_i(\cdot \mid s^t)$ describes player i's conditional beliefs.[27]

Definition 6. *Player i's conditional belief is* ***absolutely continuous with respect to the truth*** *if for every $s^t \in \cup_{t\in\mathcal{T}}\mathcal{S}^t$, $\mu(F \mid s^t) = 0$ implies that $\mu_i(F \mid s^t) = 0$ for every $F \in \mathcal{F}$.*

We view absolute continuity to be a modest condition: when the past history of recognition or public information encoded in s^t unambiguously rules out certain events from occurring, the players

[25] The second derivative of the proposer's share with respect to α_p is $\frac{2\delta(1-\alpha_v)(1-\delta\alpha_v)(1+\delta(1-\alpha_v))}{(1-\alpha_p(1+\delta(1-\alpha_v)))^3} > 0$.

[26] Dekel, Fudenberg and Levine (2004) and Yildiz (2007) discuss tensions of assuming that players' beliefs about the play of nature are distorted but players' beliefs about the play of others is not. Particular to this setting, Ali (2006) discusses how when players are extremely optimistic, there is a continuum of equilibria of the infinite-horizon in which behavior is independent of past proposals and votes, but is nevertheless non-stationary.

[27] A CPS satisfies Bayes' Rule whenever possible, but also summarizes a player's conditional belief about the future when she observes a history of recognition and signals to which she had ascribed zero probability. See Myerson (1991) for a formal definition.

commonly recognize that this is the case. This condition leads to a result similar to Theorem 1: if a bargaining process exhibits one-period predictability of degree q, then for each state s^t, there exist at least q players such that for each player i in this set of players, the true probability that player i is recognized in period $t+1$, $\mu\left(\widehat{\Omega}_i^{t+1}(s^t) \mid s^t\right) = 0$. Being absolutely continuous with respect to the truth forces each player j to share the same conditional belief about player i being the next proposer. Therefore, in every state s^t, there are at least q players for whom all players ascribe probability 0 to being the next proposer. The logic of Theorem 1 implies the following corollary.

Corollary 3. *If the bargaining process exhibits one-period predictability of degree q, and each player's conditional belief is absolutely continuous with respect to the truth, then the first proposer captures the surplus in every SPE.*

The above result implies that even if players lack common priors, unambiguous information that categorically rules out a player from being the proposer tomorrow is sufficient to generate immediate agreement and for the first proposer to capture the entire surplus.[28]

We compare this outcome to that which arises without predictability. For the sake of tractability, we restrict attention to games in which each player ascribes probability p to herself being the next proposer, independent of the past, and players share a common discount factor δ. This setting captures delays motivated by nearby deadlines (Yildiz 2003; Simsek and Yildiz 2014) and extreme optimism (Ali 2006). In a SPE, the following behavior emerges: in the final period, $\bar{t}$, players always agree to the final proposer capturing the entire surplus. Since each player believes herself to be that proposer with probability p, she would have to be offered at least δp in the penultimate period $\bar{t}-1$. Therefore, agreement is possible for a q-voting rule if and only if $\delta q p \leq 1$, and otherwise, bargaining is delayed. Yildiz (2003) and Ali (2006) show that the delay near the end of the game is of length

$$L(\delta, q, p) = \left\lceil \frac{\log q + \log p}{\log(1/\delta)} \right\rceil - 1,$$

where $\lceil \cdot \rceil$ is the ceiling function, and the equilibrium cost of delay is in the interval $\left[\frac{\delta qp-1}{\delta qp}, \frac{qp-1}{qp}\right)$. Therefore, in games with a short deadline ($\bar{t} < L(\delta, q, p) + 2$), the bargaining outcome is delayed, and after that inefficient delay, a single player captures the entire surplus. The group is then better off by having a bargaining process with one-period predictability of degree q: the outcome is equally inequitable, but there is no delay in reaching that agreement.

By contrast, in long games ($\bar{t} \geq L(\delta, q, p) + 2$), the benefits of adopting a predictable bargaining process are nuanced. Proposition 4 of Ali (2006) establishes that if $\frac{\delta(np-1)(q-1)}{n-1} \leq 1$, agreement is necessarily immediate (in a symmetric SPE). Predictability has no benefits then and leads to a less equitable distribution. If $\frac{\delta(np-1)(q-1)}{n-1} > 1$, agreement may be delayed, but since the delayed agreement

[28] In settings in which all players have strictly positive probability of being recognized, but the bargaining process exhibits one-period ϵ-predictability of degree q, a stronger condition would be needed to ensure that the first proposer captures the entire surplus.

would not feature a single player capturing the entire surplus, the costs of delay would have to be weighted against the cost of inequality of the entire surplus going to a single player.

8 The Benefits of Open-Rule Negotiations

Many legislatures employ "open rule" procedures that allow for amendments and require a motion to bring any (possibly amended) proposal to a vote. If bargaining processes in practice are predictable (either because of public or private learning), our results offer a strong theoretical rationale for a system of amendments, and offer a cautionary message of how curtailing the amendment system may concentrate power and generate very unequal agreements. An amendment system weakens the first proposer's ability to capitalize on predictability, especially if the amendment process is open to a large number of legislators. This result also shed light of why we may see less stark divisions of surplus than our results for closed-rule negotiations would imply.

We model open-rule bargaining by generalizing the framework of Baron and Ferejohn (1989). For simplicity, we take the number of legislators to be odd and assume that the voting rule is simple majority rule. At the beginning of period 0, the first proposer p^0 names a policy x in $\mathcal{X}$. A slate of k distinct amenders $A^0(p^0) = (a_1^0, \ldots, a_k^0)$ is then drawn at random (with equal probabilities) from $\mathcal{N}\backslash\{p^0\}$. First a_1^0 chooses whether to offer an amendment or move the proposal. To offer an amendment, a_1^0 names an alternative policy x' in $\mathcal{X}\backslash\{x\}$. The legislature then votes between x and x'. Period 0 ends and period 1 begins, with the winning policy (either x or x') serving as the proposal on the table. A new slate of amenders ($A^1(p^0)$ or $A^1(a_i^0)$) is chosen, and the process starts over. If instead a_1^0 moves the proposal, a_2^0 is recognized, and must likewise either offer an amendment or join the pending motion. As long as all prior amenders join the motion, the process moves sequentially through $A^0(p^0)$. If every amender joins the motion, then the policy x is put to a vote. Should a strict majority vote in favor, the policy is implemented; otherwise, period 0 ends and period 1 begins with the random selection of a new proposer p^1, as well as amenders $A^1(p^1)$. Players discount payoffs across (but not within) periods at a common rate ($\delta < 1$), and consequently incur the costs of delay whenever a proposal is amended or rejected.

Baron and Ferejohn (1989) study a special case of this open-rule procedure in which the slate of amenders comprises a single individual ($k = 1$), and the amender in period t also serves as the proposer in period t if there is no proposal on the table. We consider, as they do, a symmetric recognition process in which each player has the same probability of becoming a proposer, and conditional on the choice of proposer, each list of amenders is drawn from the remaining players with uniform probabilities. Because our objective is to determine whether an amendment process counters the effects of a predictable bargaining process, we put amendments and proposal-making on equal footing: both are perfectly predictable one period in advance.

Formally, we assume that the bargaining process exhibits *perfect one-period predictability*, defined as follows: in each period t, players know the identities of the proposer in period $t+1$ (who becomes

active only if proposal in period t is moved and then rejected) and the set of amenders in period $t+1$ for each possible contingency. The following result characterizes the extent to which an open rule moderates the tendencies identified in Theorem 1.

Theorem 9. *There exists a pure strategy MPE that reaches the following agreement without delay: the first proposer offers* $\frac{\delta}{1+\delta k}$ *to each amender and* 0 *to every other player, keeping* $\frac{1}{1+\delta k}$ *for herself.*

The open-rule process ensures that a proposer shares surplus with those who can amend her proposal, and agreement is reached immediately.[29] The special case of a single amender ($k = 1$), studied by Baron and Ferejohn (1989), deserves emphasis: here a proposer offers $\frac{\delta}{1+\delta}$ to the amender and keeps $\frac{1}{1+\delta}$ for himself, coinciding with the result of two-player bargaining in Rubinstein (1982). For the other extreme case in which $k = n - 1$, the proposer shares $\frac{\delta}{1-\delta+\delta n}$ with each other player, and obtains a slight advantage of $\frac{1}{1-\delta+\delta n}$; as $\delta \to 1$, the equilibrium splits converge to equal shares. Generally, a more universal amendment procedure—defined as the number of distinct players who can amend the proposal on the table—leads to more egalitarian sharing of surplus.

These results juxtaposed with those for the closed rule offer support for open-rule negotiations. Studying a setting in which bargaining power is completely unpredictable, Baron and Ferejohn (1989) finds that the amendment procedure may improve equity but comes at the cost of delay. Our results indicate that to the extent that real-world negotiations feature elements of learning and predictability, a system of amendments may substantially improve equity without generating inefficient delays.

9 Concluding Remarks

In practice, bargaining power flows from a variety of sources. In many settings, it is reasonable to expect that future bargaining power is predictable, at least to some extent. The central observation motivating our analysis is that such predictability can dramatically influence the outcomes of multilateral negotiations when passage of a proposal does not require unanimous consent. Predictability of future power becomes a critical source of current power, one that can dominate the effects of heterogeneity in patience, risk-aversion, or voting weights. Predictability need not be perfect to influence negotiations. On the contrary, a modest degree of predictability ensures that the first proposer receives the entire surplus, and below that threshold greater predictability implies a larger share for the proposer. Thus, our theory yields implications that are both testable and useful for understanding why certain groups divide resources less equally than others. Our results also offer normative insights into institutional design; for example, they explain how the early resolution of uncertainty of bargaining power can generate inequality and inefficiency in certain contexts. Amendment processes can limit the power of the first proposer and thereby promote more equitable outcomes.

[29] Through an argument that involves comparing the best and worst possible equilibrium payoffs for a proposer, one can show that the MPE outcome is unique among all those in which the proposer shares surplus only with amenders. We conjecture that this is the unique MPE outcome more generally, but have not yet proven that every MPE outcome offers 0 to those not in $p^0 \cup A^0(p^0)$.

In this first step towards understanding the impact of predictability on negotiations, we have restricted attention to pure distributive problems. In this setting, each individual is indifferent between all outcomes for which she receives the same share, irrespective of how the residual share is distributed among other negotiators. If externalities are present, one can reformulate policies as points in utility space and proceed as in Section 5.2 and Section 5.6. However, in some instances, natural restrictions on the policy space will defeat the logic of Theorem 3. To illustrate, suppose the parties are negotiating over the level of a public good, and no side-payments are possible. In that case, the policy space is one-dimensional, and it is natural to assume that each player has single-peaked preferences, so that a Condorcet winner exists. With majority rule and standard (unpredictable) bargaining processes, the negotiated outcome cannot stray far from the Condorcet winner as players become patient (Jackson and Moselle 2002; Cho and Duggan 2009), and we conjecture that a similar result holds even with predictable bargaining processes. As a general matter, when the negotiators have preferences that are more congruent than is the case for the settings studied herein, they may naturally coordinate so as to block the first proposer from exploiting power that would otherwise flow from an ability to predict future bargaining strengths.

A Appendix

A.1 Omitted Proofs

Proof of Theorem 2 on p. 19. We first describe the function that we use as a lower bound on the amount a proposer must share with at least one other party. Consider the function $f : \Re \to \Re$ defined by $f(y) \equiv \frac{y-\hat{\delta}(\epsilon+\rho)}{\hat{\delta}(1-\epsilon-\rho)}$. Observe that f has a unique fixed point, namely $\hat{y} = \frac{\hat{\delta}(\epsilon+\rho)}{1-\hat{\delta}(1-\epsilon-\rho)}$. The function f is both strictly increasing and *expansive*: for each $y > \hat{y}$, an induction argument establishes that

$$f^k(y) - \hat{y} = \left(\frac{1}{\hat{\delta}(1-\epsilon-\rho)}\right)^k (y - \hat{y}).$$

Since $\hat{\delta}(1-\epsilon-\rho)$¡1, it follows that for each $y > \hat{y}$, there exists a finite $\overline{k}$ such that for every $k > \overline{k}$, $f^k(y) > 1$. We use this observation to prove this result.

Let the structural state in Stage 1 of period 0 be s^0 and consider $\overline{x}^0(s^0)$, the highest equilibrium share that the proposer $p^0(s^0)$ offers to any player other than herself. Suppose towards a contradiction that $\overline{x}^0(s^0) > \hat{y}$. Because $s^0 \in \mathcal{P}^0_{q,\epsilon}$, an argument identical to that of Theorem 1 implies that there exist a player i in $H^0(s^0) \bigcap L^1_\epsilon(s^0)$. Player i's continuation value $V^1_i(s^0)$ emerges from three events:

(i) he is recognized: the rents that he captures are bounded above by 1, and the probability of this event is bounded above by ϵ.

(ii) the realized structural state in period 1 is not in $\mathcal{P}^1_{q,\epsilon}$: his payoffs are bounded above by 1 and the probability of this event is bounded above by ρ.

(iii) the realized structural state in period 1 is in $\mathcal{P}^1_{q,\epsilon}$, and player i is not recognized: this event occurs with probability at least $1-\epsilon-\rho$, and his payoff is bounded above by the most that he receives in any structural state in $\mathcal{P}^1_{q,\epsilon}$ in which someone else is the proposer, denoted by $\tilde{x}^1_i(s^1)$.

Combining the above implies that $V^1_i(s^0) \leq \epsilon + \rho + (1-\epsilon-\rho)\tilde{x}^1_i(s^1)$. Because the greatest share offered to any non-proposer, $\overline{x}^1(s^1)$ must exceed $\tilde{x}^1_i(s^1)$, and player i's discounted continuation value in state s^0 weakly exceeds $\overline{x}^0(s^0)$, it follows that

$$\frac{\overline{x}^0(s^0)}{\hat{\delta}} \leq V^1_i(s^0) \leq \epsilon + \rho + (1-\epsilon-\rho)\overline{x}^1(s^1)$$

or re-arranging that $\overline{x}^1(s^1) \geq f\left(\overline{x}^0(s^0)\right)$. Since f is strictly increasing and expansive, we are guaranteed that $f\left(\overline{x}^0(s^0)\right) > \overline{x}^0(s^0)$, which is greater than $\hat{y}$. Therefore, the same argument applies in state $s^1 \in \mathcal{P}^1_{q,\epsilon}$. Accordingly, there exists a sequence of states $\{s^t\}_{t\in\mathcal{T}}$ such that for each t, we have $\overline{x}^t(s^t) \geq f^t\left(\overline{x}^0(s^0)\right)$, and $\overline{x}^0(s^0) > \hat{y}$. Our earlier observation implies that if $\overline{t} = \infty$, a proposer eventually offers a share exceeding 1 to another player in some state, or if $\overline{t} < \infty$, a proposer in the final period offers a strictly positive share to another player. In both cases, we reach a contradiction. □

Proof of Theorem 3 on p. 20. We re-define the cost of a coalition, $W^t_C(s^t)$: for a state s^t and coalition $C \subseteq \mathcal{N}$, let $W^t_C(s^t) \equiv \sum_{i\in C} u_i^{-1}\left(\delta_i V_i^{t+1}(s^t)\right)$. Given that $V_i^{t+1}(s^t) \in [0, u_i(1)]$, $\delta_i \in (0,1)$, $u_i(0) = 0$, and u_i is strictly increasing and continuous, we know that $W^t_C(s^t)$ is well-defined. Let

$$\underline{W}^t(s^t) \equiv \min_{\substack{C\subseteq\mathcal{N}\setminus\left\{p^t(s^t)\right\},\\ C\cup\left\{p^t(s^t)\right\}\in\mathcal{D}}} W^t_C(s^t),$$

be the cost of the cheapest decisive coalitions for proposer $p^t(s^t)$, which are in the set

$$C^t(s^t) \equiv \left\{C \subseteq \mathcal{N}\setminus\left\{p^t(s^t)\right\} : C\cup\left\{p^t(s^t)\right\} \in \mathcal{D} \text{ and } W^t_C(s^t) = \underline{W}^t(s^t)\right\}.$$

The proposals that involve creating such coalitions are

$$\mathcal{X}^t(s^t) \equiv \left\{x \in \mathcal{X} : \exists C \in C^t(s^t) \text{ such that } u_i(x_i) = \delta_i V_i^{t+1}(s^t) \forall i \in C \text{ and } x_{p^t(s^t)} = 1 - \underline{W}^t(s^t)\right\}.$$

In an equilibrium, let $a(s^t)$ denote the (undiscounted) average of policies that are selected in the continuation after rejection of the proposal in state s^t. Because u_i is concave for each i and $\delta_i < 1$, we necessarily have $u_i(a_i(s^t)) > \delta_i V_i^{t+1}(s^t)$ for all i. Consequently, for any coalition C, we have $W^t_C(s^t) < \sum_{i\in C} a_i(s^t) \leq 1$. It follows that $1 - \underline{W}^t(s^t) > 0$, and hence that $\mathcal{X}^t(s^t)$ is non-empty.

No Delay: We first extend Lemma 1. Suppose there is a structural state s^t in $\mathcal{S}^t$ such that an equilibrium proposal offered with strictly positive probability, x', is rejected with strictly positive

probability. Select some $x \in \mathcal{X}^t(s^t)$ and let $C \in C^t(s^t)$ be the associated minimal winning coalition (excluding the proposer). Define a proposal x^ϵ for small $\epsilon \geq 0$ in which $u_i(x_i^\epsilon) = u_i(x_i + \epsilon)$ for every $i \in C$, $x_i^\epsilon = 0$ for every $i \notin C \cup \{p^t(s^t)\}$, and keeps $1 - \underline{W}^t(s^t) - (q-1)\epsilon$ for himself (which is feasible in light of the the fact that $1 - \underline{W}^t(s^t) > 0$). In the equilibrium, the proposal x^ϵ must be accepted by all members of C with probability 1 if $\epsilon > 0$. Because $\sum_{i \in \mathcal{N}} u_i^{-1}\left(\delta_i V_i\left(s^t\right)\right) < \sum_{i \in \mathcal{N}} a_i(s^t) \leq 1$ and $\hat{\delta} < 1$,

$$\begin{aligned}
\underline{W}^t(s^t) + u^{-1}\left(\delta_{p^t(s^t)} V_{p^t(s^t)}^{t+1}\left(s^t\right)\right) &= \sum_{i \in C} u_i^{-1}\left(\delta_i V_i^{t+1}\left(s^t\right)\right) + u^{-1}\left(\delta_{p^t(s^t)} V_{p^t(s^t)}^{t+1}\left(s^t\right)\right) \\
&\leq \sum_{i \in \mathcal{N}} u_i^{-1}\left(\delta_i V_i^{t+1}\left(s^t\right)\right) \\
&< 1.
\end{aligned}$$

Therefore, for sufficiently small $\epsilon > 0$, we have

$$x_{p^t(s^t)}^\epsilon = 1 - \underline{W}^t(s^t) - (q-1)\epsilon > u^{-1}\left(\delta_{p^t(s^t)} V_{p^t(s^t)}^{t+1}\left(s^t\right)\right).$$

Thus, conditional on x' being rejected, the proposer is discretely better off deviating to x^ϵ for sufficiently small $\epsilon > 0$. Conditional on x' being accepted, the proposer's share can be no greater than she obtains when offering x. Since proposal x' is rejected with strictly positive probability, she is strictly better off offering x^ϵ for sufficiently small $\epsilon > 0$. Therefore, no equilibrium offer x' can be rejected with strictly positive probability. □

Minimal Winning Coalition: Lemma 2 extends to this setting: if the proposer $p^t(s^t)$ chooses a policy outside $\mathcal{X}^t(s^t)$, then she can profitably deviate to such a policy (plus tiny additional payments to members of the minimal winning coalition) to obtain immediate agreement at a strictly lower cost.

An Additional Lemma: Recall that $\xi_P^{i,t}(s^t)$ is the equilibrium mixed action selected by proposer $p^t(s^t)$ at state s^t: for a proposal x in $\mathcal{X}$, let $\xi_P^{i,t}(s^t)(x)$ denote the equilibrium probability with which proposer $p^t(s^t)$ makes that proposal at state s^t. We prove an additional lemma for this setting bounding the continuation value at time t for the coalition of losers.

Lemma 3. *Consider a time period $t < \bar{t}$ and a structural state s^t. The following relates costs of coalitions across periods:*

$$W_{L^{t+1}(s^t)}^t(s^t) \leq \hat{\delta} \int_{\mathcal{S}^{t+1}} \underline{W}^{t+1}(s^{t+1}) d\mu\left(s^{t+1} \mid s^t\right).$$

Proof. Observe that by definition of $W_C^t(s^t)$, and using $\delta_i \leq \hat{\delta}$,

$$W_{L^{t+1}(s^t)}^t(s^t) = \sum_{i \in L^{t+1}(s^t)} \delta_i V_i^{t+1}(s^t) \leq \hat{\delta} \sum_{i \in L^{t+1}(s^t)} V_i^{t+1}(s^t). \tag{4}$$

Consider any player i in $L^{t+1}(s^t)$: such a player is recognized with probability 0 in period $t+1$. In other words, given s^t, for each feasible continuation structural state in period $t+1$, $s^{t+1} \subset s^t$, player i is distinct from the proposer $p^{t+1}(s^{t+1})$. Therefore, player i can only expect to obtain strictly positive payoffs in period $t+1$ in structural states s^{t+1} in which the proposer $p^{t+1}(s^{t+1})$ makes an offer that offers a strictly positive share to player i. In such a scenario, he is offered the utility of his discounted continuation value, namely $\delta_i V_i^{t+2}(s^{t+1})$. Therefore, for every player i in $L^{t+1}(s^t)$,

$$V_i^{t+1}(s^t) = \int_{\mathcal{S}^{t+1}} \delta_i V_i^{t+2}(s^{t+1}) \sum_{x \in \mathcal{X}^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \xi_P^{i,t+1}(s^{t+1})(x) d\mu\left(s^{t+1} \mid s^t\right). \tag{5}$$

We substitute (5) into (4):

$$\begin{aligned}
W^t_{L^{t+1}(s^t)}(s^t) &\leq \hat{\delta} \sum_{i \in L^{t+1}(s^t)} \int_{\mathcal{S}^{t+1}} \delta_i V_i^{t+2}(s^{t+1}) \sum_{x \in \mathcal{X}^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \xi_P^{i,t+1}(s^{t+1})(x) d\mu\left(s^{t+1} \mid s^t\right) \\
&= \hat{\delta} \int_{\mathcal{S}^{t+1}} \sum_{i \in L^{t+1}(s^t)} \delta_i V_i^{t+2}(s^{t+1}) \sum_{x \in \mathcal{X}^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \xi_P^{i,t+1}(s^{t+1})(x) d\mu\left(s^{t+1} \mid s^t\right) \\
&\leq \hat{\delta} \int_{\mathcal{S}^{t+1}} \sum_{i \in \mathcal{N} \setminus p^{t+1}(s^{t+1})} \delta_i V_i^{t+2}(s^{t+1}) \sum_{x \in \mathcal{X}^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \xi_P^{i,t+1}(s^{t+1})(x) d\mu\left(s^{t+1} \mid s^t\right), \\
&= \hat{\delta} \int_{\mathcal{S}^{t+1}} \sum_{x \in \mathcal{X}^{t+1}(s^{t+1})} \xi_P^{i,t+1}(s^{t+1})(x) \sum_{i \in \mathcal{N} \setminus p^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \delta_i V_i^{t+2}(s^{t+1}) d\mu\left(s^{t+1} \mid s^t\right), \\
&= \hat{\delta} \int_{\mathcal{S}^{t+1}} \sum_{x \in \mathcal{X}^{t+1}(s^{t+1})} \xi_P^{i,t+1}(s^{t+1})(x) \underline{W}^{t+1}(s^{t+1}) d\mu\left(s^{t+1} \mid s^t\right), \\
&= \hat{\delta} \int_{\mathcal{S}^{t+1}} \underline{W}^{t+1}(s^{t+1}) d\mu\left(s^{t+1} \mid s^t\right).
\end{aligned}$$

in which the first line is the substitution, the second line interchanges the sum and integral, the third line uses the fact that for each $s^{t+1} \subset s^t$, $L^{t+1}(s^t)$ is a subset of $\mathcal{N} \setminus p^{t+1}(s^{t+1})$, the fourth line re-arranges terms by interchanging summation, the fifth line uses that by definition, for each x in $\mathcal{X}^{t+1}(s^{t+1})$, $\sum_{i \in \mathcal{N} \setminus p^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \delta_i V_i^{t+2}(s^{t+1}) = \underline{W}^{t+1}(s^{t+1})$, and the sixth line uses the generalized Lemma 2 to note that $\sum_{x \in \mathcal{X}^{t+1}(s^{t+1})} \xi_P^{i,t+1}(s^{t+1})(x) = 1$. □

We now prove the theorem by contradiction. Suppose the state in Stage 1 of period 0 is s^0, and that a policy proposed with positive probability in which the proposer $p^0(s^0)$ offers a strictly positive amount, x, to another player, in which case $\underline{W}^0(s^0) \geq x$. Since the bargaining process exhibits one-period decisive predictability, there exists a set of coalition partners C that excludes $p^0(s^0)$ such that $C \bigcup \{p^0(s^0)\}$ is in $\mathcal{D}$, and C is a subset of $L^1(s^0)$. By definition, $\underline{W}^0(s^0) \leq W^0_C(s^0)$ and by monotonicity, $W^0_C(s^0) \leq W^0_{L^1(s^0)}(s^0)$. Therefore, $W^0_{L^1(s^0)}(s^0)$ must be no less than x. Lemma 3 implies that there must exist a structural state s^1 such that $\underline{W}^1(s^1) \geq x/\hat{\delta}$. Since $\underline{W}^1(s^1)$ is defined to be the cost of the cheapest decisive coalition partners for proposer $p^1(s^1)$, the same argument as above implies that $W^1_{L^2(s^1)}(s^1)$ must also be no less than $x/\hat{\delta}$. Therefore, by induction, there exists

a sequence of states $\{s^t\}_{t\in\mathcal{T}}$ such that for each t, $s^t \in \mathcal{S}^t$, and $\underline{W}^t(s^t) \geq \frac{x}{\hat{\delta}^t}$. If $\bar{t} = \infty$, $\hat{\delta} < 1$ implies that $\underline{W}^t(s^t)$ eventually exceeds $\sum_{i\in\mathcal{N}} u_i(1)$, which is beyond the range of feasible payoffs; if $\bar{t} < \infty$, the same argument implies that the proposer at $\bar{t}$ does not appropriate the entire surplus in some state $s^{\bar{t}}$. In both cases, we have reached a contradiction. □

Proof of Theorem 4 on p. 21. Observe that if $k = q$, Theorem 4 follows from Lemma 2: any proposal in which a proposer offers a strictly positive amount to a non-veto player is not in $\tilde{\mathcal{X}}^t(s^t)$.

Now suppose that $k < q < n$: it must be that there are at least two non-veto players. Observe that for every state s^t, there exists $\tilde{x}^t(s^t)$ such that for every offer $x \in \mathcal{X}^t(s^t)$, $\tilde{x}^t(s^t) = \max_{i\notin(\{p^t(s^t)\}\cup\{1,\ldots,k\})} x_i$. Our claim is that for every $s^0 \in \mathcal{S}^0$, $\tilde{x}^0(s^0) = 0$. Suppose towards a contradiction that $\tilde{x}^0(s^0) > 0$. Consider the set of non-veto players whose support cannot be secured for shares less than $\tilde{x}^0(s^0)$:

$$\tilde{H}^0(s^0) \equiv \left\{i \in \{k+1,\ldots,n\}\backslash\{p^0(s^0)\} : \delta_i V_i^1(s^0) \geq \tilde{x}^0(s^0)\right\}.$$

$\tilde{H}^0(s^0)$ must have cardinality at least $n-(q-1)$ because otherwise proposer $p^0(s^0)$ would be able to form a coalition of veto and non-veto players without having to offer $\tilde{x}^0(s^0)$ to any player. Therefore, $\tilde{H}^0(s^0)\bigcap L^1(s^0)$ is no-empty. Therefore, there must exist some state s^1 such that player i offered at least $\tilde{x}^0(s^0)/\delta_i$, which implies that $\tilde{x}^1(s^1) \geq \tilde{x}^0(s^0)/\hat{\delta}$. By induction (as before), there must then exist a state in which a proposer shares more than the entire surplus (if $\bar{t} = \infty$) or offers a strictly positive share in $\bar{t}$ (if $\bar{t} < \infty$), both of which are contradictions. □

Proof of Theorem 5 on p. 22. At $\bar{t}$, the proposer $p^{\bar{t}}(s^{\bar{t}})$ forms a minimal winning coalition with the $(n-1)/2$ other players who obtain the lowest amount from the default option: because majority improvements are possible,

$$x^D_{p^{\bar{t}}(s^{\bar{t}})} < 1 - \min_{\substack{C\subseteq\mathcal{N}\backslash\left\{p^t(s^t)\right\},\\ |C|=(n-1)/2}} \sum_{j\in C} x_j^D.$$

Therefore, proposer $p^{\bar{t}}(s^{\bar{t}})$ is strictly better off from the acceptance of this proposal than her disagreement payoff, and all players in C are indifferent between accepting and rejecting this proposal (and in equilibrium, they vote to accept it). Observe that regardless of the identity of $p^{\bar{t}}(s^{\bar{t}})$, that proposer never includes any player from $(n+3)/2,\ldots,n$ in her minimal winning coalition. So if agreement has not been reached previously, $i \neq p^{\bar{t}}(s^{\bar{t}})$ and $i \geq (n+3)/2$, then player i's continuation payoff at the beginning of period $\bar{t}$ is 0.

Consider negotiations in the penultimate period, $\bar{t}-1$. There are two cases to consider:

1. $p^{\bar{t}}(s^{\bar{t}-1}) > (n-1)/2$: If there is disagreement today, the next period proposer forms a minimal winning coalition with players $\{1,\ldots,(n-1)/2\}$. Therefore, all players in $\{(n+1)/2,\ldots,n\}\cap$

$L^{\bar{t}}(s^{\bar{t}-1})$ expects 0 payoffs in the event of disagreement today. There are $(n-1)/2$ players in this set. If $p^{\bar{t}-1}(s^{\bar{t}-1}) \leq (n-1)/2$ or $p^{\bar{t}-1}(s^{\bar{t}-1}) = p^{\bar{t}}(s^{\bar{t}-1})$, then she can guarantee passage of a proposal in which she offers ϵ to each player in this set, and therefore, in equilibrium, she captures the entire surplus. Otherwise, proposer $p^{\bar{t}-1}(s^{\bar{t}-1}) > (n-1)/2$ can obtain the agreement of $(n-3)/2$ other players at no cost. She then includes player 1 and obtains $1 - x_1^D$.

2. $p^{\bar{t}}(s^{\bar{t}-1}) \leq (n-1)/2$: If there is disagreement today, the next period proposer forms a minimal winning coalition with other players in $\{1, \ldots, (n+1)/2\}$. Therefore, all players in $\{(n+3)/2, \ldots, n\}$ expect 0 payoffs in the event of disagreement today. There are $(n-1)/2$ players in this set. If $p^{\bar{t}-1}(s^{\bar{t}-1}) \leq (n+1)/2$, then she obtains the entire surplus in equilibrium. Otherwise, proposer $p^{\bar{t}-1}$ can obtain the agreement of $(n-3)/2$ other players at no cost. If $1 \neq p^{\bar{t}}(s^{\bar{t}-1})$, the proposer includes player 1 offering her a share of x_1^D and otherwise, she offers x_2^D to player 2.

Consider negotiations in period $\bar{t}-2$. All players anticipate, in equilibrium, that if there is disagreement today, the only players who may expect a strictly positive surplus are in $\{p^{\bar{t}-1}(s^{\bar{t}-2}), 1, 2\}$, and the remaining $n-3$ players expect zero surplus. Proposer $p^{\bar{t}-2}(s^{\bar{t}-2})$ captures the entire surplus if $n - 3 \geq \frac{n+1}{2}$, which is equivalent to $n \geq 7$. Therefore, the proposer in period $\bar{t}-2$ captures the entire surplus. By induction, the first proposer captures the entire surplus in every SPE. □

Proof of Theorem 6 on p. 23. We construct a pure strategy MPE. Suppose that the state is s^t.

1. For all $\omega \in s^t$, $\bar{t}(\omega) \in \{t, t+1\}$: the proposer $p^t(s^t)$ and other players follow the strategy profile outlined in Theorem 5 for the final and penultimate periods.

2. For all $\omega \in s^t$, $\bar{t}(\omega) > t+1$: the proposer $p^t(s^t)$ offers 0 to each player. Each player votes in favor of any proposal that assures her at least her continuation value, and otherwise rejects. In equilibrium, all players $L^{t+1}(s^t) \cap \{p^{t+1}(s^t), 1, 2\}$ vote to accept the proposal.

Observe that no player has any incentive to deviate in the proposing or voting stages. □

Proof of Theorem 7 on p. 28. We begin by describing the system of equations used to solve for $\underline{w}$ and $\overline{w}$. Consider continuation values in the beginning of period t, prior to recognition and information revelation, and player i such that player i expects to not be recognized in period t. It follows by recursive calculation that

$$\underline{w} = \left(\frac{d}{n}\right)\delta\underline{w} + \left(\frac{n-d}{n}\right)\left(\frac{d(q-d)}{(n-1)(n-d)} + \frac{(n-d-1)(q-1-d)}{(n-1)(n-d-1)}\right)\delta\overline{w},$$

where $\frac{d}{n}$ is the probability that $i \in L^{t+1}(s^t)$; $\left(\frac{n-d}{n}\right)\left(\frac{d}{n-1}\right)\left(\frac{q-d}{n-d}\right)$ is the probability $i \notin L^{t+1}(s^t)$, $p^t(s^t) \in L^{t+1}(s^t)$ and that period-t proposer includes i in the winning coalition; and finally $\left(\frac{n-d}{n}\right)\left(\frac{n-d-1}{n-1}\right)\left(\frac{q-1-d}{n-d-1}\right)$

is the probability $\{i, p^t(s^t)\} \subset \mathcal{N} \backslash L^{t+1}(s^t)$ and the period-t proposer includes i in the willing coalition. Combining this equation with

$$d\underline{w} + (n-d)\overline{w} = 1 \tag{6}$$

yields the solutions in the text. Finally, the first proposer's expected share can be represented as

$$\begin{aligned}
&\left(\frac{n-d}{n}\right)(1 - d\delta\underline{w} - (q-d-1)\delta\overline{w}) + \left(\frac{d}{n}\right)(1 - (d-1)\delta\underline{w} - (q-d)\delta\overline{w}) \\
&= \left(\frac{n-d}{n}\right)(1 - \delta(1-(n-d)\overline{w}) - (q-d-1)\delta\overline{w}) + \frac{d}{n}(1 - \delta(1-(n-d)\overline{w}) + \delta\underline{w} - (q-d)\delta\overline{w}) \\
&= \left(\frac{n-d}{n}\right)(1 - \delta + \delta(n-q)\overline{w} + \delta\overline{w}) + \frac{d}{n}(1 - \delta + \delta(n-q)\overline{w} + \delta\underline{w}) \\
&= 1 - \delta + \delta(n-q)\overline{w} + \frac{\delta}{n},
\end{aligned}$$

where the first line follows from the fact that the first proposer is not a member of $L^1(s^0)$ with probability $(n-d)/n$, and is a member with probability $\frac{d}{n}$; the second line uses (6); the third line simplifies the expression; and the fourth line uses (6) again. The derivative of the proposer's share with respect to d is

$$\frac{\delta(n-1)(n-q)(n-\delta q-(1-\delta))}{((n-1)n - d(\delta(n-q)+(n-1)))^2} > 0$$

which implies that the first proposer's share is strictly increasing in d for $d < q$. □

Proof of Theorem 8 on p. 28. If $\alpha_p > \alpha_v$, then it follows that for sufficiently large n, $d_n > q_n$ in which case Theorem 1 implies that the first proposer captures the entire surplus. Suppose that $\alpha_p \leq \alpha_v$. By our earlier result, the first proposer's share is

$$\begin{aligned}
&1 - \delta + \frac{\delta}{n} + \delta(n-q_n)\overline{w}_n \\
=&1 - \delta + \frac{\delta}{n} + \frac{\delta(n-q_n)(n-1)(n-\delta d_n)}{n(n(n-1) - d_n(\delta(n-q_n)+n-1))} \\
=&1 - \delta + \frac{\delta}{n} + \frac{\frac{\delta(n-q_n)}{n}\frac{(n-1)}{n}\frac{(n-\delta d_n)}{n}}{\frac{(n-1)}{n} - \frac{\delta d_n(n-q_n)}{n^2} - \frac{d_n}{n} + \frac{d_n}{n^2}}
\end{aligned}$$

Taking limits as $n \to \infty$, $q_n/n \to \alpha_v$, and $d_n/n \to \alpha_p$, we obtain

$$1 - \delta + \frac{\delta(1-\alpha_v)(1-\delta\alpha_p)}{1-\delta\alpha_p(1-\alpha_v)-\alpha_p} = 1 - \frac{\delta(\alpha_v - \alpha_p)}{1-\delta\alpha_p(1-\alpha_v)-\alpha_p}.$$

□

Proof of Theorem 9 on p. 32. Define a policy x proposed by player p to be *movable* in period t if $x_j \geq \frac{\delta}{1+\delta k}$ for each j in $A^t(p)$. We write $M^t(p)$ for the set of movable policies by player p in period t. Consider a strategy profile in which:

1. In every period t for which there is no proposal on the table, the proposer p^t offers $\frac{\delta}{1+\delta k}$ to each amender and 0 to all others.

2. When voting on a proposal in period t that has been moved by each amender in A^t, each player votes to accept the proposal unconditionally unless he is either the proposer p^{t+1} or an amender in $A^{t+1}(p^{t+1})$. The proposer in period $t+1$ votes to accept the proposal if and only if he obtains at least $\frac{\delta}{1+\delta k}$, and the amender votes to accept if and only if he obtains at least $\frac{\delta^2}{1+\delta k}$. Define a proposal to be *passable* if it satisfies these conditions.

3. In period t, if the proposal on the table is movable, then each amender moves the proposal. If it is neither movable nor passable, then assuming previous amenders have moved the proposal, each a_i^t offers an amendment to keep $\frac{1}{1+\delta k}$ for himself and share $\frac{\delta}{1+\delta k}$ with each amender in the set $A^{t+1}(a_i^t)$. In the case where the proposal is passable but not movable, let i' denote the last amender for whom the amount offered is strictly less than $\frac{\delta}{1+\delta k}$. For all $i \leq i'$, a_i^t offers the same amendment just described. For all $i > i'$ (if any), a_i^t moves the proposal.

4. When voting in period t between a proposal x proposed by player p and an amendment x' by player p', each player i votes for x if and only if

   - $x \in M^{t+1}(p)$ and $x' \in M^{t+1}(p')$, and $x_i > x'_i$,
   - or $x \in M^{t+1}(p)$ and $x' \notin M^{t+1}(p')$,
   - or $x \notin M^{t+1}(p)$, $x' \notin M^{t+1}(p')$, and i is in $A^{t+1}(p)$.

First, as a preliminary observation, we note that all movable proposals are passable. If $k \geq \frac{n-1}{2}$, then $p^t \cup A^t(p^t)$ has cardinality of at least $\frac{n+1}{2}$, so the current proposer and amenders can pass a proposal with no other support. According to the strategies, all members of that group will vote in favor of a movable proposal, so it is passable. If $k < \frac{n-1}{2}$, the set of players not in $p^{t+1} \cup A^{t+1}(p^{t+1})$ has cardinality of at least $\frac{n+1}{2}$, and can pass a proposal with no other support. According to the strategies, all members of that group will vote in favor of a movable proposal, so it is passable.

We prove that, for this strategy profile, no player has a profitable deviation for any history by considering each of the three roles separately: proposer, amender, and voter.

- **Proposer:** Suppose there is no offer on the table, so the proposer p^t must make an offer: any proposal that offers less than $\frac{\delta}{1+\delta k}$ to a player j in A^t is amended by that player and defeated. Since no proposal accepted in equilibrium in the continuation game offers a higher discounted expected payoff to the proposer p^t than $\frac{1}{1+\delta}$, he has no incentive to deviate to any proposal that offers less to amender j than $\frac{\delta}{1+\delta k}$. Of the proposals that are accepted in equilibrium, the equilibrium proposal maximizes the proposer's payoff.

- **Amender:** Suppose first that the current proposal on the table in period t is movable. The proposal is also passable, so moving it leads to its implementation (given continuation strategies), yielding a payoff of at least $\frac{\delta}{1+\delta k}$ for the amender. Amending the proposal cannot generate a strictly higher payoff for the amender given prescribed behavior in the continuation game.

  Next suppose the current proposal is neither movable nor passable. Moving the proposal results in implementation of some other policy one period hence, with an expected discounted payoff no greater than $\frac{\delta}{1+\delta k}$ (given continuation strategies). By proposing the amendment prescribed by the equilibrium strategies, the amender can achieve a discounted payoff of $\frac{\delta}{1+\delta k}$, which is (weakly) greater.

  Finally suppose the current proposal is passable but not movable. Amender $a^t_{i'}$ (where i' is defined in part 3 of the description of the equilibrium strategies) plainly has a strict incentive to amend the proposal by offering to keep $\frac{1}{1+\delta k}$ for himself and share $\frac{\delta}{1+\delta k}$ with each amender in the set $A^{t+1}(a^t_{i'})$ (given that this proposal will be implemented one period hence, and that no proposal more favorable to i' would be implemented). Anticipating this successful amendment, each amender i playing prior to i' has a strict incentive (by induction) to offer an analogous amendment. For $i > i'$, a^t_i can obtain an immediate payoff not less than $\frac{\delta}{1+\delta k}$ by moving the proposal (because subsequent amenders will move it and it is passable), and cannot obtain a greater discounted payoff by offering an amendment.

- **Voting Decisions:** By construction, players cast votes in favor of the alternative that yields their highest continuation payoff.

□

A.2 Formal Results for Political Maneuvers

We formalize the conclusions discussed in Section 5.4, permitting legislators $i = 1, \ldots, n$ and the Chair, $i = 0$, to choose (potentially) costly maneuvers m_i in each period from some set M_i that has persistent effects on recognition. We describe in order the timing of maneuvers, the recognition rule, the payoff relevant state, the appropriate predictability conditions and our formal results.

Timing: At the beginning of period t, players engage in political maneuvers. Each player i simultaneously chooses an action variable m^t_i from the feasible set of maneuvers, M_i, a non-empty and compact subset of a Euclidean space. We write $M \equiv M_0 \times \ldots \times M_n$. The selected vector of maneuvers in period t is $m^t = (m^t_0, ..., m^t_n)$, which is observed by all players. We let $h^t_m = (m^0, \ldots, m^t)$ denote the full history of maneuvers up to and including that of period t.

After the maneuvers are selected, players proceed to the **Information and Recognition** stage described in Section 3. We let H^t_m denote the set of possible histories of maneuvers up to and including those of time t, and $H_m = \bigcup_{t \in \mathcal{T}} H^t_m$ denote the set of all possible histories of maneuvers.

The period t recognition rule is represented by a deterministic function $\widetilde{P}^t : H_m^t \times H_P^{t-1} \times \Omega \to \mathcal{N}$ in which H_P^{t-1} is the set of possible proposer histories, and Ω is the state space. Because the state of nature and the history of maneuvers recursively determines the entire sequence of proposers, we can write the recognition rule more compactly as $P^t : H_m^t \times \Omega \to \mathcal{N}$. After the revelation of information and recognition, the proposer p^t proposes a policy in $\mathcal{X}$ and other votes in a fixed sequential order. The proposal is implemented if and only at least q players (including the proposer) vote in favor.

Payoffs: We augment each legislator's payoff in Section 3 with that from maneuvers; substantively we assume that no legislator has any interest in prolonging negotiations because he intrinsically enjoys the process of political maneuvering. For a history $h_m^t \in \mathcal{H}_m$, let $v_i : \mathcal{H}_m \to \Re$ represent player i's costs from that history incurred at time t. If offer x is accepted at time t, legislator i's payoff is

$$u_i\left(x, t, h_m^t\right) = \delta^t x_i - \sum_{\tau=0}^{t} \delta_i^\tau v_i\left(h_m^\tau\right).$$

We assume that for all t, and all $h_m^t \in \mathcal{H}_m$, $v_i\left(h_m^t\right) \geq 0$. Thus, maneuvering is (potentially) costly, and prolonging negotiations cannot be motivated by the desire for further maneuvering. For many applications, it suffices to consider a special case of v_i in which the only dimension of the history of maneuvers, h_m^t, that is costly at time t is the current individual maneuver, m_i^t. However, our results also accommodate settings in which the cost of maneuvering is affected by the maneuvers of others and one's own past maneuvers.

For the Chair's preferences, we write

$$u_0\left(x, t, h_m^{(t)}\right) = \delta_0^t W(x) - \sum_{\tau=0}^{t} \delta_0^\tau v_0\left(h_m^\tau\right),$$

in which $W(x)$ represents her payoffs from a policy x. We make no restrictions on v_0.

Markov Perfect Equilibria: We augment our description of structural states and equilibria to account for the possibilities for maneuvering. In the maneuvering stage of period t, let $\tilde{s}_M^t \equiv (h_m^{t-1}, s^{t-1})$ denote all past maneuvers and all that is known after period $t-1$ about future recognition. We write $\tilde{s}_P^t \equiv (h_m^t, s^t)$ as the state at the proposal stage, in which both the maneuvers and information revealed at period t are included. Let S_M^t denote the set of possible states for the maneuvering stage of period t. We let $S_{P,i}^t$ denote the collection of all states for the proposal stage consistent with player i being the proposer. An MPE is an SPE in which each player's equilibrium strategy can be written as a sequence of function $\left(\xi_M^{i,t}, \xi_P^{i,t}, \xi_V^{i,t}\right)_{t \in \mathcal{T}}$ such that $\xi_M^{i,t} : S_M^t \to \Delta M_i$ is player i's randomization over maneuvers in period t in structural state $\tilde{s}_M^t$, $\xi_P^{i,t} : S_{P,i}^t \to \Delta X$ is player i's randomization over proposals when recognized in period t in structural state $\tilde{s}_P^t$, and $\xi_V^{i,t} : S_P^t \times \mathcal{X} \to \Delta\{\text{yes}, \text{no}\}$ is player i's randomization whether to vote in favor of the proposal in period t in structural state s_P^t.

Predictability: Using the above notation, we can extend our notions of predictability to account for political maneuvers. If the profile of maneuvers at $t+1$ is m^{t+1}, then and the sequence of signals identify that the member of the partition $\mathcal{S}^t$ that ω is in is s^t, then player i is recognized at $t+1$ if and only if ω is in

$$\Omega_i^M(h_m^t, m^{t+1}, s^t) \equiv \left\{\omega \in s^t : P^{t+1}\left((h_m^t, m^{t+1}), \omega\right) = i\right\},$$

which has probability $r_i^M(h_m^t, s^t, m^{t+1}) \equiv \mu(\Omega_i^M(h_m^t, m^{t+1}, s^t)|s^t)$. A player is a *loser conditional* on m^{t+1} in structural state $s_P^t = (h_m^t, s^t)$ if in period $t+1$, he is definitely not the proposer if the period-$t+1$ profile of maneuvers is m^{t+1}:

$$L_C^{t+1}(s_P^t, m^{t+1}) \equiv \left\{i \in \mathcal{N} : r_i^M(s_P^t, m^{t+1}) = 0\right\}.$$

The player is an *unconditional loser* if he is not the proposer regardless of m^{t+1}:

$$L_U^{t+1}(s_P^t) \equiv \bigcap_{m^{t+1} \in M} L_C^{t+1}(s_P^t, m^{t+1}).$$

We offer two distinct notions of predictability.

Definition 7. *The bargaining process exhibits one-period* ***unconditional predictability*** *of degree* **d** *if* $\left|L_U^{t+1}(s_P^t)\right| \geq d$ *for all* s_P^t *in* S_P^t *and* t *in* $\mathcal{T}$.

Definition 8. *The bargaining process exhibits one-period* ***conditional predictability*** *of degree* **d** *if* $\left|L_C^{t+1}(s_P^t, m^{t+1})\right| \geq d$ *for all* s_P^t *in* S_P^t, $m^{t+1} \in M$, *and* t *in* $\mathcal{T}$.

With conditional predictability, the players are able to rule out d legislators in period t when they can predict the maneuvers in period $t+1$. Unconditional predictability is stronger (and implies conditional predictability) as the players need not predict the maneuvers played in period $t+1$ to rule out d legislators from being proposer. The following describes the implications of each condition.

Theorem 10. *If the bargaining process exhibits one-period unconditional (respectively conditional) predictability of degree q, the proposer selected at* $t = 0$ *captures the entire surplus in every (respectively every pure strategy) MPE.*

Proof. For every state s_P^t in S_P^t, let $V_i^{t+1}(s_P^t)$ denote the expected continuation value of player i before Stage 1 of the next period, after the rejection of an offer in state s_P^t, and excluding maneuvering costs that have already been incurred (at period t or before). Lemmas 1 and 2 extend to this setting immediately, so in every MPE proposal is accepted with probability 1.

Case 1: *Unconditional Predictability of Degree q*: Constructing $\overline{x}^0(s_P^0)$ and $H^0(s_P^0)$ as in the proof of Theorem 1, it follows that $H^0(s_P^0) \bigcap L_U^1(s_P^0)$ is non-empty. Consider a generic player i

in $H^0(s^0) \bigcap L^1(s^0)$. For a generic player i in $H^0(s_P^0) \bigcap L_U^1(s_P^0)$, his continuation value is a combination of offers that he receives in states in S_P^1 and maneuvering costs that he incurs in period 1. Since maneuvering can be only costly, it must be that there exists some structural state s_P^1 in S_P^1 such that the associated proposer offers player i at least $\frac{\overline{x}^0(s_P^0)}{\delta_i} \geq \frac{\overline{x}^0(s_P^0)}{\hat{\delta}}$, which implies that $\overline{x}^1(s_P^1) \geq \frac{\overline{x}^0(s_P^0)}{\hat{\delta}}$. Induction (as before) implies that there exists a state in which a proposer shares more than the entire surplus (if $\bar{t} = \infty$) or offers a strictly positive share in the final period (if $\bar{t} < \infty$), both of which are contradictions.

Case 2: *Conditional Predictability of Degree q*: Construct $\overline{x}^0(s_P^0)$ and $H^0(s_P^0)$ as in the proof of Theorem 1. The state for maneuvers in period 1, $s_M^1 = (h_m^0, s_P^0)$, which is identical to s_P^0. Since the MPE is in pure strategies, there is a profile of maneuvers m^1 that is chosen in s_M^1 that is perfectly predictable in state s_P^0. Since the bargaining process exhibits predictability of degree q, it follows that $|L_C^1(s^0, m^1)| \geq q$. Since $H^0(s_P^0)$ must have cardinality of at least $n - (q-1)$, $H^0(s_P^0) \bigcap L^C(s^0, m^1)$ is non-empty. It follows exactly as in the argument above that there exists some structural state s_P^1 in S_P^1 such that $\overline{x}^1(s_P^1) \geq \frac{\overline{x}^0(s_P^0)}{\hat{\delta}}$. Induction, as before, implies a contradiction. □

Finally, we note that the example that we discuss in which the Chair selects proposers is that in which $M_0 = \mathcal{N}$, $\theta^t(\omega) = \theta$ for all t and ω, and $P^t(h_m^t, \theta^t) = m_0^t$. This is a bargaining process that satisfies conditional predictability of degree $n-1$, and so in every pure strategy MPE, the first proposer captures the entire surplus.

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