Convergence to a First Best Solution in a Model of Land Acquisition

Soumendu Sarkar*
TERI University, New Delhi

Preliminary draft. Please do not circulate.

Abstract

We explore the effect of competition among a large number of sellers in a model of Land Acquisition. Multiple sellers with one unit of land each are located at the nodes of a graph. Two sellers are *contiguous* if they are connected by an edge in the graph. The buyer realizes a positive value only if he can purchase plots that constitute a path of given length. We characterize conditions for different contiguity structures under which the VCG mechanism almost surely results in a budget surplus as the number of sellers become large. JEL Classification: C78, D47.

Keywords: Contiguity, Convergence, Eminent Domain, Land Acquisition, Mechanism Design.

*E-mail: sarkarsoumendu@gmail.com. This paper contains results from my PhD thesis submitted at Indian Statistical Institute, New Delhi. I am grateful to my supervisor Prof. Arunava Sen for his guidance. I am also grateful to Prof. Kalyan Chatterjee and Prof. Debasis Mishra for their generous comments and suggestions. The usual disclaimer applies.

1 Introduction

According to standard microeconomic theory, the *market power* of individual sellers declines as the number of sellers increases. A classic and extreme example of this is the comparison between standard monopoly and Bertrand duopoly: in the former, the market price is above the marginal cost, but in the latter, competition between two identical sellers drives market price down to marginal cost of production. In models of private information, the efficient outcome is often unattainable. A natural question is whether a *second best* mechanism converges to *first best* as the number of agents become large.

In this paper, we investigate the impact of increasing number of sellers in a multilateral exchange problem, viz., Land Acquisition. It refers to situations where a single buyer purchases a set of land plots from multiple landowners. Often plots are required to be contiguous so that large scale construction can take place. This problem has great relevance in many densely populated countries. Large scale construction often requires industry or the government to acquire vast areas of land that are inhabited and often cultivated by many people.

When prices acceptable to the buyer and the sellers is not public knowledge, the buyer has to negotiate with individual sellers who can respond by delaying strategically. This is commonly known as *holdout*. Often, the land acquisition exercise requires intervention from a third party in the form of arbitration, subsidies or coercion through constitutional means like *eminent domain* laws. Coercion by the State may lead to conflicts of social, political and economic significance [1].

In a previous paper, we have modeled the contiguity structure of land plots as graphs (Sarkar, 2014). In this model the buyer demands a path of a given length on a graph where each node represents a plot owned by a seller and every pair of physically adjacent plots are connected by an edge. There we characterize priors for which *Bayesian incentive compatible* (BIC) mechanisms implement the first best. The analysis also identifies the role of critical sellers who lie on all feasible paths. In particular, it is difficult to satisfy these conditions when the number of critical sellers is large.

However, use of BIC mechanisms requires the mechanism designer to have precise information about the underlying priors. There has been emphasis on the construction of mechanisms that are robust with respect to such assumptions following the critique by Wilson (1987)[2]. A natural way to deal with the Wilson Critique is to require mechanisms to be *dominant strategy incentive compatible*, or DSIC. A mechanism is DSIC if no agent can ever gain in terms of ex-post payoffs by misreporting. The corresponding participation condition is *ex-post individual rationality*, or IR. A mechanism is IR if agents always get a positive payoff.

[1]See Chakravorty (2013) for an elaborate historical analysis of land acquisition in India; see Miceli (2011) for an analysis of eminent domain from a law and economics perspective.

[2]See Bergemann and Välimäki (2006) for a survey.

The VCG mechanism is DSIC and IR. However, it is not *budget-balanced*, or BB: the sum of VCG payments can be positive, negative or zero, depending on the profile and the prior. If the sum of payments is positive, a redistribution of the surplus will improve net welfare of agents. If the sum of payments is negative, the mechanism requires an outside subsidy. The VCG mechanism therefore, becomes approximately first-best in the limit if the sum of VCG payments is almost surely positive. In this paper we investigate this issue.

Note that the underlying graph may change depending on the way new sellers are added. In this paper we examine some special cases where these issues can be dealt with. The first of these is a model where new sellers are added consecutively on a line. The second is a star graph where new sellers form additional edges with a fixed hub seller.

Let k be the number of contiguous plots required by the buyer. Priors satisfy the *Trade in the Limit* or the TL condition if the lowest end of the support of the buyer's valuation is greater than k times that of the sellers' valuation. If this condition is satisfied, then trade will almost surely take place in the VCG mechanism as the number of sellers becomes large. We show that TL is a necessary and sufficient condition for almost surely positive VCG surplus in the limit in the model where new sellers are added consecutively on a line. We also show that a stronger condition is required in the star graph model. We then generalize these conditions to sequences of graphs with special properties. We have provided several numerical examples to illustrate these results.

2 Literature

In a classic paper, Myerson and Satterthwaite (1983) showed that ex-post efficiency cannot be attained in a model of bilateral trade with two-sided asymmetric information. But there are other models with asymmetric information where this is not true, e.g., partnership dissolution (Cramton et al., 1987), transfer of one indivisible item from one seller to two prospective buyers (Makowski and Mezzetti, 1993) or extension of this problem to many buyers with unitary demand and many sellers with unitary endowments (Williams, 1999). The efficient mechanisms derived in these papers, however, require information about the underlying priors — they are, therefore, subject to Wilson's Critique discussed above.

Post Myerson and Satterthwaite (1983), many authors have investigated asymptotic efficiency of market mechanisms extensively (Satterthwaite and Williams, 1989b,a; Gresik and Satterthwaite, 1989; McAfee, 1992; Rustichini et al., 1994; Cripps and Swinkels, 2006; Fudenberg et al., 2007; Williams, 1991; Satterthwaite and Williams, 2002). In such models, an increase in the number of agents implies that the incentive of misreporting when others are reporting truthfully becomes smaller. Consequently, incentive compatible mechanisms begin to approximate the first best. The validity of this reasoning depends on the setting. For instance, this is not true in problems involving public goods. See the review article by Jackson (2000) for elucidation.

3 Preliminaries

Essential concepts and notation for our results are presented below. There are n sellers, indexed by i, each holding one unit of an indivisible good (plot). The n indivisible items are located on a graph $G = (N, E)$ where N denotes the set of nodes (plots) and E denotes the set of edges. A pair of nodes is connected by a direct edge if they are physically adjacent to each other. A sequence of connected nodes is called a path. A path is feasible if it contains at least a fixed number k of nodes where $k \leq n$.

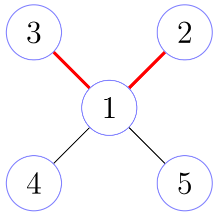

Figure 1: A feasible path in the star graph when $k = 3$

The valuation of each seller i is $v_i \in [\underline{v}, \bar{v}]$. We assume that v_i's are independently and identically distributed random variables with distribution function $F(\cdot)$ and density function $f(\cdot)$. The realization of v_i is observed only by i.

There is one buyer, indexed by 0. Her valuation is $v_0 \in \mathbb{R}_+$ if she acquires a feasible path. We assume that $v_0 \in [\underline{v_0}, \bar{v_0}]$ and $v_0 \sim G(v_0)$. We will assume that F and G have continuous and positive densities $f(\cdot)$ and $g(\cdot)$ in their respective domains.

Valuations of the buyer and the sellers are independently distributed . All valuations are non-negative. Own valuations are private information while the distribution functions F and G are common knowledge. In order to make the problem non-trivial , we make the following assumption:

$$\boxed{\text{ASSUMPTION } \mathbf{NT} : k\underline{v} < \bar{v}_0 \text{ and } k\bar{v} > \underline{v}_0}$$

This assumption ensures that efficiency is a non-trivial issue. If the first part does not hold, then the buyer's valuation for any feasible path will always be less than the sum of valuations of the sellers constituting it. Consequently, trade will never be efficient. If the second part is violated, then the buyer's valuation will always exceed this sum of valuations. Then trade is efficient for any feasible path.

A valuation profile is an $n+1$-vector $v \equiv (v_0, v_1, \ldots, v_n) \in [\underline{v_0}, \bar{v_0}] \times [\underline{v}, \bar{v}]^n$. The j-th component of v is denoted by v_j and the n-vector v_{-j} denotes the profile where the j-th component is dropped from v. Throughout, we will use the subscripts j and $-j$ to indicate "the j-th component" and "all but the j-th component" of a vector respectively. The distribution of the random vector v is called a *prior*, denoted μ. A *land acquisition problem* is a tuple $\langle G, k, \mu \rangle$.

The buyer and the sellers directly report their individual valuations to a mechanism designer. A mechanism consists of an *allocation rule* and a *transfer rule*.

A *deterministic* allocation is an $n+1$-vector x described as follows: for components $i = 1, \ldots, n$, x_i is -1 if seller i sells and 0 otherwise; $x_0 = 1$ if $\sum_{i=1}^{n} |x_i| \geq k$ and 0 otherwise. Let $\mathbb{X}$ be the set of all deterministic allocations. We provide some illustrations below.

Example **1** Suppose $n = 1$ and $k = 1$. Then, $\mathbb{X} = \{(0, 0), (1, -1)\}$.

Example **2** If $n = 2$ and $k = 2$, $\mathbb{X} = \{(0, -1, 0), (0, 0, -1), (1, -1, -1), (0, 0, 0)\}$.

Definition **1 (Allocation Rule)** *An allocation rule* $P : [\underline{v_0}, \bar{v_0}] \times [\underline{v}, \bar{v}]^n \to \mathbb{X}$ *maps a profile of reported values to a deterministic allocation.*

For any agent j, $P_j(v)$ is the j-th component of $P(v)$. In Example 1, suppose that at profile v, P assigns the allocation (1,-1). Then, $P_0(v) = 1$ and $P_1(v) = -1$.

Definition **2 (Transfer Rule)** *A transfer rule* t *is a map* $t : [\underline{v_0}, \bar{v_0}] \times [\underline{v}, \bar{v}]^n \to \mathbb{R}^{n+1}$.

If $t_j(v) > 0$ (resp. $t_j(v) < 0$) then agent j pays (resp. receives) the amount $t_j(v)$.

We make the standard assumption of quasi-linear utilities.

Definition **3 (Payoffs)** *Fix a mechanism* (P, t)*. The (ex post) utility of agent* j *with valuation* v_j *reporting* $\hat{v}_j$ *in mechanism* (P, t) *is*

$$U_j^{(P,t)}(\hat{v}_j, v_{-j} | v_j) = v_j P_j(\hat{v}_j, v_{-j}) - t_j(\hat{v}_j, v_{-j}).$$

Henceforth, we shall fix the mechanism (P, t) and drop the superscript in the notation.

An important requirement for mechanisms is that they induce agents to report their valuations truthfully. Bayesian incentive compatibility ensures that truthful reporting is optimal for each agent and for each valuation in *expectation*. This expectation is computed with respect to the prior distribution of valuations of other agents and on the assumption that other agents are reporting truthfully.

Definition **4 (Dominant Strategy Incentive Compatibility)** *A mechanism is Dominant Strategy Incentive Compatible (DSIC) if for all* j*,*

$$v_j P_j(v_j, v_{-j}) - t_j(v_j, v_{-j}) \geq v_j P_j(\hat{v}_j, v_{-j}) - t_j(\hat{v}_j, v_{-j}) \text{ for all } v_j, \hat{v}_j, \text{ and } v_{-j}.$$

Definition **5 (Ex-post Individual Rationality)** *A mechanism is ex-post individually rational (IR) if for all* j*,*

$$v_j P_j(v_j, v_{-j}) - t_j(v_j, v_{-j}) \geq 0 \text{ for all } v_j.$$

When truthful reporting constitutes an equilibrium, we will simplify notation and write $U_j(v)$ and $U_j(v_j)$ for the ex-post and interim utilities respectively. Henceforth, we will use E rather than $E_j E_{-j}$ to denote expectation taken over profile v. Since the variables are independently distributed, the order of expectation does not matter.

Definition 6 **(Efficiency)** *An allocation rule P is ex post efficient if for all v,*

$$\sum_j v_j P_j(v) \geq \sum_j v_j P'_j(v) \text{ for any allocation rule } P'.$$

Ex-post efficient allocations in our model are defined as follows. Let the feasible paths in G be denoted by $\mathcal{P}_1, \ldots, \mathcal{P}_q$ with $q \geq 1$. Consider a valuation profile v. The sum of valuations in path $\mathcal{P}_i$ will be denoted by $S_i(v)$, $i = 1, \ldots, q$. These sums are ordered as follows: $S_{[1]}(v) \leq \ldots \leq S_{[q]}(v)$. The paths corresponding to these sums are denoted by $\mathcal{P}_{[1]}(v), \ldots, P_{[q]}(v)$ respectively. Efficiency requires trade to take place with sellers in $\mathcal{P}_{[1]}(v)$ if $v_0 > S_{[1]}(v)$; if $v_0 \leq S_{[1]}(v)$ then trade does not occur. This is illustrated in the following example.

Example **3** Consider the graph in Figure 2. Suppose $k = 3$, i.e., there are two feasible paths $\{123\}$ and $\{234\}$. Consider the following valuations : $v_1 = 7$, $v_2 = 5$, $v_3 = 5$ and $v_4 = 8$.

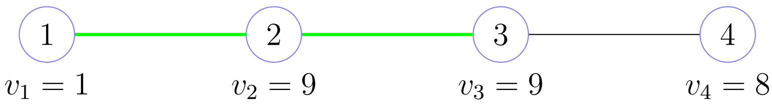

Figure 2: $\mathcal{P}_{[1]}(v)$

Here $\mathcal{P}_{[1]}(v) = \{123\}$ and $S_{[1]}(v) = 19$. Efficiency requires trade with sellers 1, 2 and 3 if $v_0 > 19$.

Note that the efficient rule is not fully specified. These are the cases where there are more than k lowest valuation sellers and the case where the buyer's value is exactly equal to the sum of k lowest seller values. A tie-breaking rule which may involve randomization, is required to fully specify the rule. However, the subsequent analysis will not depend in any way on the choice of the tie-breaking rule. Consequently, we shall abuse notation and refer to the efficient rule as any rule satisfying the condition above and denote it by P^*.

A standard restriction on the transfer payments is that they *balance the budget*, i.e., the mechanism is self-financed and there should be no surplus.

Definition 7 **(Budget Balance)** *A mechanism (P, t) satisfies budget balance if, for all v,*

$$\sum_{j=0}^{n} t_j(v) = 0. \tag{1}$$

In our model, budget balance implies that the buyer pays exactly the sum of all sellers receipts at every valuation profile.

A mechanism achieves the *first-best* if it satisfies efficiency, IIR and BB. A mechanism is *successful* if (a) it is BIC with respect to some prior μ and (b) it achieves the first best.

4 Results

In this Section, we present four sets of results. These are with respect to the LA model, the LAC model with line contiguity, the LAC model with star contiguity and general contiguity structures respectively.

4.1 Convergence in the LA Model

Refer to the sequence of LA models $\langle m, k, \mu \rangle_{m=n}^{\infty}$ in where $n > k$. Let v_0 and $v_1, \ldots, v_m$ be independently distributed in $[\underline{v}_0, \bar{v}_0]$ and $[\underline{v}, \bar{v}]$ respectively. Let the corresponding distribution functions be $G(\cdot)$ and $F(\cdot)$ respectively.

The priors satisfy the *Trade in the Limit* condition, or TL if

$$\underline{v}_0 > k\underline{v}.$$

The following result shows that TL is a necessary and sufficient condition for the VCG surplus to be positive almost everywhere.

Proposition 1 *Consider the sequence of LA models* $\langle m, k, \mu \rangle_{m=n}^{\infty}$ *with* $n > k$. *Then* $\Pr(\sum_{j=0}^{m} t_j^V(v) > 0) \to 1$ *as* $m \to \infty$ *if and only if TL holds.*

Proof: **Only if part:** Suppose $\underline{v}_0 \leq k\underline{v}$. We show that $\sum_{j=0}^{n} t_j^V(v) < 0$ almost whenever trade takes place. The sum of VCG payments at different profiles are listed in the table below which is reproduced from

Table 1: Sum of Payments when $n > k$

Case	**Sum of Payments**	**Sign**
I: $v_0 \geq \underline{v}_0 > \sum_{j=1}^{k} v_{[j]}, A(v) \neq \emptyset$	$\underline{v}_0 - \sum_{h \in A(v)} \left(v_0 - \sum_{j=1, j \neq h}^{k} v_{[j]} \right) - (k - \lvert A(v) \rvert)\, v_{[k+1]}$	≤ 0
II: $v_0 \geq \underline{v}_0 > \sum_{j=1}^{k} v_{[j]}, A(v) = \emptyset$	$\underline{v}_0 - k v_{[k+1]}$	$\lesseqgtr 0$
III: $v_0 > \sum_{j=1}^{k} v_{[j]} \geq \underline{v}_0, A(v) \neq \emptyset$	$\sum_{j=1}^{k} v_{[j]} - \sum_{h \in A(v)} \left(v_0 - \sum_{j=1, j \neq h}^{k} v_{[j]} \right) - (k - \lvert A(v) \rvert)\, v_{[k+1]}$	< 0
IV: $v_0 > \sum_{j=1}^{k} v_{[j]} \geq \underline{v}_0, A(v) = \emptyset$	$\sum_{j=1}^{k} v_{[j]} - k v_{k+1}$	≤ 0
V: $v_0 \leq \sum_{j=1}^{k} v_{[j]}$	0	0

Notice that Cases I and II do not arise when $\underline{v}_0 \leq k\underline{v}$. Further, if trade takes place, $\sum_{j=0}^{m} t_j^V(v) = 0$ only in countably many instances of Case IV. Consequently, $\sum_{j=0}^{m} t_j^V(v) < 0$ at almost every profile where trade takes place.

If part: Suppose $\underline{v}_0 > k\underline{v}$. By assumption NT and the hypothesis, $k\underline{v} < \underline{v}_0 < k\bar{v}$. Therefore, $\underline{v} < \frac{\underline{v}_0}{k} < \bar{v}$. Let $F(\cdot)$ be the c.d.f. of $v_1, \ldots, v_m$. Since it is a monotonic increasing function in $[\underline{v}, \bar{v}]$,

$$F(\underline{v}) < F\left(\frac{\underline{v}_0}{k}\right) < F(\bar{v}),$$
$$\text{i.e., } 0 < F\left(\frac{\underline{v}_0}{k}\right) < 1.$$

Recall from Lemma **??** in that at all profiles v, $\sum_{j=0}^{m} t_j^V(v) \geq \underline{v}_0 - kv_{[k+1]}$. Therefore,

$$\Pr\left(\sum_{j=0}^{m} t_j^V(v) > 0\right) \geq \Pr\left(\underline{v}_0 - kv_{[k+1]} > 0\right). \tag{2}$$

We show that $\Pr\left(\underline{v}_0 - kv_{[k+1]} > 0\right) \to 1$ as $m \to \infty$. Notice that

$$\begin{aligned}\Pr\left(\underline{v}_0 - kv_{[k+1]} > 0\right) &= \Pr\left(v_{[k+1]} < \frac{\underline{v}_0}{k}\right) \\ &= \sum_{r=k+1}^{m} \binom{m}{r} \left\{F\left(\frac{\underline{v}_0}{k}\right)\right\}^r \left\{1 - F\left(\frac{\underline{v}_0}{k}\right)\right\}^{m-r} \\ &= 1 - \sum_{r=0}^{k} \binom{m}{r} \left\{F\left(\frac{\underline{v}_0}{k}\right)\right\}^r \left\{1 - F\left(\frac{\underline{v}_0}{k}\right)\right\}^{m-r}.\end{aligned}$$

For every $r \in \{0, \ldots, k\}$,

$$\binom{m}{r} \left\{F\left(\frac{\underline{v}_0}{k}\right)\right\}^r \left\{1 - F\left(\frac{\underline{v}_0}{k}\right)\right\}^{m-r} = \frac{F\left(\frac{\underline{v}_0}{k}\right)}{r!} \times \frac{m(m-1)\cdots(m-r+1)}{1/\left\{1 - F\left(\frac{\underline{v}_0}{k}\right)\right\}^{m-r}}.$$

The first term of this product is a constant. The second term has a polynomial of degree r in m in the numerator and an exponential term of degree $m - r$ in the denominator. The second term converges to zero as $m \to \infty$, and therefore, each of the $k + 1$ components of the sum converges to zero individually. Therefore, $\Pr\left(\underline{v}_0 - kv_{[k+1]} > 0\right) \to 1$ as $m \to \infty$. ■

Observation **1** In LA, probability of trade taking place is $\Pr(v_0 > \sum_{j=1}^{k} v_{[j]})$. Since,

$$\Pr\left(v_0 > \sum_{j=1}^{k} v_{[j]}\right) \geq \Pr\left(\underline{v}_0 > \sum_{j=1}^{k} v_{[j]}\right),$$

trade takes place almost surely in the limit if $\underline{v}_0 > k\underline{v}$. This is the reason we call it a *trade in the limit* condition.

Observation **2** Since $\Pr\left(v_{[k]} > \underline{v} + \epsilon\right) \to 0$ as $m \to \infty$, if $\underline{v}_0 = k\underline{v}$, the deficit approaches zero. This will not hold if $k\underline{v} > \underline{v}_0$.

4.2 Convergence in the LAC Model with Line Contiguity

A definition of a sequence of LAC problems is required in order to investigate its asymptotic properties. While adding new nodes to a graph G, we have to specify how new edges are constructed. Depending on the initial graph and how new nodes and edges are added, the nature of the graph may change. Initially, we will restrict ourselves to a general asymptotic feature of the VCG mechanism in the LAC model when the underlying graph is a line.

We construct a sequence of line graphs as follows. The graph $L(1)$ consists of a single path of length k. For any natural number $m \geq 1$, the graph $L(m+1)$ is constructed by adding an $m+1$-th node to $L(m)$ via an edge $(m, m+1)$. See Figure 3. This results in a connected acyclic graph, also known as a *tree*. Every path with k nodes is called a feasible path. For a line graph with m nodes, there are $m-k+1$ distinct feasible paths when $m > k$. Let us call these paths $\mathcal{P}_1, \ldots, \mathcal{P}_{m-k+1}$. Valuations of the buyer are drawn independently from prior μ. We assume that v_0 follows c.d.f. $G(\cdot)$ with support $[\underline{v}_0, \bar{v}_0]$ and $v_1, \ldots, v_m$ follow c.d.f. $F(\cdot)$ with support $[\underline{v}_0, \bar{v}_0]$. Let the sums of valuations of sellers on feasible path $\mathcal{P}_i$ for a profile v be denoted as $S_i(v)$. Let us order these sums as $S_{[1]}(v) \leq \ldots \leq S_{[m-k+1]}(v)$ and let the corresponding feasible paths be $\mathcal{P}_{[1]}(v), \ldots, \mathcal{P}_{[m-k+1]}(v)$. The buyer requires a feasible path. Efficiency requires that trade takes place with sellers in $\mathcal{P}_{[1]}(v)$ whenever $v_0 > S_{[1]}(v)$ and trade does not take place otherwise.

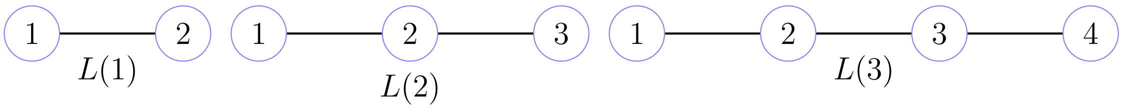

Figure 3: Construction of a sequence of line graphs when $k = 2$

The following result shows that TL is a necessary and sufficient condition for the VCG surplus to be positive almost everywhere.

Proposition 2 *Consider the sequence of LAC models with line contiguity* $\langle L(m), k, \mu \rangle_{m=1}^{\infty}$. *Then* $\Pr(\sum_{j=0}^{m} t_j^V(v) > 0) \to 1$ *as* $m \to \infty$ *if and only if TL holds.*

Proof: **Only if part:** Suppose $\underline{v}_0 \leq k\underline{v}$. We show that $\sum_{j=0}^{m} t_j^V(v) < 0$ almost whenever trade takes place. Table **??** in that lists the different cases for VCG payments is reproduced below. Notice that Cases I and II do not arise when $\underline{v}_0 \leq k\underline{v}$. Further, if trade takes place, $\sum_{j=0}^{m} t_j^V(v) = 0$ only in countably many instances of Case IV. Consequently, $\sum_{j=0}^{m} t_j^V(v) < 0$ almost whenever trade takes place.

If part: Suppose $\underline{v}_0 > k\underline{v}$. By assumption NT and the hypothesis, $k\underline{v} < \underline{v}_0 < k\bar{v}$. Therefore,

$$0 < F\left(\frac{\underline{v}_0}{k}\right) < 1,$$

Table 2: Sum of Payments when $n > k$

Case	**Sum of Payments**	**Sign**
I: $v_0 \geq \underline{v}_0 > S_{[1]}(v), A(v) \neq \emptyset$	$\underline{v}_0 - S_{[1]}(v) - \lvert A(v)\rvert(v_0 - S_{[1]}(v)) - \sum_{\substack{i \in \mathcal{P}_{[1]}(v) \\ i \notin A(v)}}(S_{[1]}(\bar{v}, v_{-i}) - S_{[1]}(v))$	≤ 0
II: $v_0 \geq \underline{v}_0 > S_{[1]}(v), A(v) = \emptyset$	$\underline{v}_0 + (k-1)S_{[1]}(v) - \sum_{i \in \mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i})$	$\lesseqgtr 0$
III: $v_0 > S_{[1]}(v) \geq \underline{v}_0, A(v) \neq \emptyset$	$-\lvert A(v)\rvert(v_0 - S_{[1]}(v)) - \sum_{\substack{i \in \mathcal{P}_{[1]}(v) \\ i \notin A(v)}}(S_{[1]}(\bar{v}, v_{-i}) - S_{[1]}(v))$	< 0
IV: $v_0 > S_{[1]}(v) \geq \underline{v}_0, A(v) = \emptyset$	$-\sum_{i \in \mathcal{P}_{[1]}(v)}(S_{[1]}(\bar{v}, v_{-i}) - S_{[1]}(v))$	≤ 0
V: $v_0 \leq S_{[1]}(v)$	0	0

as before. Recall from Lemma **??** that at all profiles v,

$$\sum_{j=0}^{m} t_j^V(v) \geq \underline{v}_0 + (k-1)S_{[1]}(v) - \sum_{i \in \mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}).$$

We show that

$$\Pr\left(\underline{v}_0 + (k-1)S_{[1]}(v) - \sum_{i \in \mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}) > 0\right) \to 1 \text{ as } n \to \infty.$$

A set of feasible paths $\mathcal{F}(L(m), k, \mu)$ will be called *independent* if no two feasible paths in it share a node, i.e.,

$$\mathcal{P}_i, \mathcal{P}_j \in \mathcal{F}(L(n), k, \mu) \Rightarrow V(\mathcal{P}_i) \cap V(\mathcal{P}_j) = \emptyset,$$

where $V(G)$ is the set of nodes in graph L. Note that we can always construct a non-empty independent set of feasible paths by including $\mathcal{P}_1$ and then including the feasible paths $\mathcal{P}_{k+1}$ if $n \geq 2k$, $\mathcal{P}_{2k+1}$ if $n \geq 3k$ and so on. This set will contain at most $\left[\frac{m-k+1}{k}\right]$ feasible paths where $\left[\frac{m-k+1}{k}\right]$ is the integral part of the improper fraction $\frac{m-k+1}{k}$. Denote this independent set of feasible paths by $\mathcal{F}^*$. See an illustration in Figure 4 below.

Figure 4: Construction of F^* shown with red edges when $k = 2$

At any profile v let the highest valuation of a seller on a feasible path $\mathcal{P} \in \mathcal{F}^*$ be denoted as $\tilde{v}^{\mathcal{P}}$. Each of the $\tilde{v}^{\mathcal{P}}$s are functions of a sample of size k of independent draws from $[\underline{v}, \bar{v}]$. Since all seller values are drawn independently from $F(\cdot)$, $\tilde{v}^{\mathcal{P}_1}, \tilde{v}^{\mathcal{P}_{k+1}}, \ldots, \tilde{v}^{\mathcal{P}_{m-k+1}}$ are independent random variables. Furthermore, $\Pr(\tilde{v}^{\mathcal{P}} \leq x) = F(x)^k$. Order $\tilde{v}^{\mathcal{P}}$, $\mathcal{P} \in \mathcal{F}^*$, as

$\tilde{v}_{[1]} \leq \ldots \leq \tilde{v}_{\left[\frac{m-k+1}{k}\right]}$. Let the corresponding feasible paths be $\tilde{\mathcal{P}}_{[1]}(v), \ldots, \tilde{\mathcal{P}}_{\left[\frac{m-k+1}{k}\right]}(v)$ and the corresponding sums of valuations be $\tilde{\mathcal{S}}_{[1]}(v), \ldots, \tilde{\mathcal{S}}_{\left[\frac{m-k+1}{k}\right]}(v)$. As before,

$$\text{i.e., } 0 < F\left(\frac{\underline{v}_0}{k}\right) < 1.$$

Therefore,

$$0 < F\left(\frac{\underline{v}_0}{k}\right)^k < 1.$$

Since $S_{[1]}(v) \leq \tilde{S}_{[1]}(v)$ and $\tilde{\mathcal{P}}_{[2]}(v)$ does not contain $i \in \mathcal{P}_{[1]}(v)$, $S_{[1]}(\bar{v}, v_{-i}) \leq \tilde{S}_{[2]}(v)$. The following Lemma shows that $\Pr(\underline{v}_0 > S_{[2]}(v)) \to 1$ as $m \to \infty$.

Lemma **1** *Suppose $\underline{v}_0 > k\underline{v}$. Then*

$$Pr(\underline{v}_0 > \tilde{S}_{[2]}(v)) \text{ as } n \to \infty.$$

Proof:

$$\begin{aligned} &\Pr\left(\underline{v}_0 \geq \tilde{S}_{[2]}(v)\right) \geq \Pr\left(\tilde{v}_{[2]} \leq \frac{\underline{v}_0}{k}\right) \\ &= 1 - \left\{1 - F\left(\frac{\underline{v}_0}{k}\right)^k\right\}^m - mF\left(\frac{\underline{v}_0}{k}\right)^k \left\{1 - F\left(\frac{\underline{v}_0}{k}\right)^k\right\}^{m-1}. \end{aligned}$$

Both the second and third term are fractions that converge to zero as $m \to \infty$. Hence the result. ■

Since,

$$\Pr(v_0 > S_{[1]}(v)) \geq \Pr(v_0 > S_{[1]}(\bar{v}, v_{-i})) \geq \Pr(\underline{v}_0 > S_{[1]}(\bar{v}, v_{-i})) \geq Pr(\underline{v}_0 > \tilde{S}_{[2]}(v)), \quad (3)$$

trade almost surely takes place as number of sellers become large. Since $\Pr(\underline{v}_0 > S_1(v)) \to 1$, Case III, IV and V are ruled out almost everywhere in the limit. Further, recall that the set of trade-pivotal sellers at a profile v is

$$A(v) = \{i \in \mathcal{P}_{[1]}(v) : v_0 > S_{[1]}(v), v_0 \leq S_{[1]}(\bar{v}, v_{-i})\}.$$

By Lemma 1, $A(v)$ is empty almost surely for large m. This rules out Case I almost everywhere in the limit. Furthermore, by (3), $\Pr\left(S_{[1]}(\bar{v}, v_{-i}) > S_{[1]}(v)\right) \to 0$. It follows that $\Pr\left(\underline{v}_0 + (k-1)S_{[1]}(v) - \sum_{i \in \mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}) > 0\right) \to \Pr(\underline{v}_0 > S_{[1]}(v))$ which has been shown to approach 1 as $m \to \infty$. ■

The earlier observations on the behavior of $\sum_{j=0}^{n} t_j^V(v)$ when $\underline{v}_0 \leq k\underline{v}$ remain valid.

4.3 The Star Graph

Recall from that a critical seller is a node that is contained in every feasible path. A star graph contains a non-empty set of critical sellers. Consider the following sequence of star graphs with $k = 2$: Let $G^*(1) = \langle\{1, 2\}, \{(1, 2)\}\rangle$ where the first component is the set of nodes V_1 and the second is the set of edges E_1. For any $m \geq 1$, construct $G^*(m + 1)$ as $\langle\{V_m \cup \{m + 2\}, E_m \cup \{(1, m + 2)\}\rangle$. The figure below illustrates this construction.

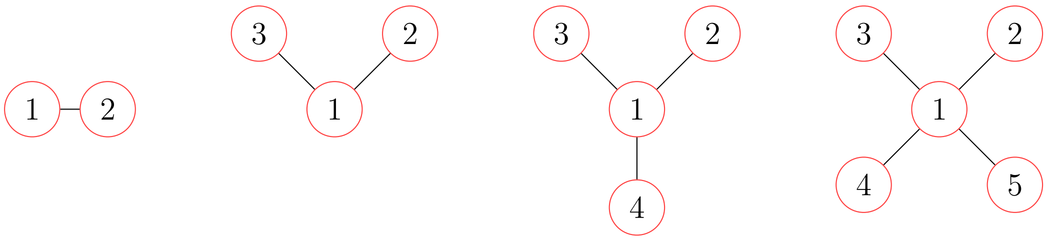

Figure 5: A sequence of star graphs when $k = 2$

A prior satisfies condition TLS1 if

$$\underline{v}_0 > \bar{v} + \underline{v}.$$

It can be interpreted as the counterpart of TL for star graphs with $k = 2$.

Proposition 3 *Consider the sequence:* $\langle G^*(m), 2, \mu\rangle_{m=1}^{\infty}$. *Then* $\Pr(\sum_{j=0}^{m} t_j^V(v) > 0) \to 1$ *as* $m \to \infty$ *if and only if TLS1 holds.*

Proof: **Only if part:** Suppose $\underline{v}_0 \leq \bar{v} + \underline{v}$. The sum of VCG payments in Case II of Table 2 is

$$\begin{aligned}
&\underline{v}_0 + (k-1)S_{[1]}(v) - \sum_{i\in\mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}) \\
&= \underline{v}_0 + \left(v_1 + v_{[1]}^{-\{1\}}\right) - \left(\bar{v} + v_{[1]}^{-\{1\}}\right) - \left(v_1 + v_{[2]}^{-\{1\}}\right) \\
&= \underline{v}_0 - \bar{v} - v_{[2]}^{-\{1\}} \\
&\leq \underline{v} - v_{[2]}^{-\{1\}} \\
&\leq 0,
\end{aligned}$$

where $v_{[i]}^{-\{1\}}$ is the i-th order statistic of the $m-1$ valuations of all sellers other than 1. Since this is the only Case where the VCG sum of payments can be positive, hence the claim.

If part: Let $\underline{v}_0 > \bar{v} + \underline{v}$. Recall that at all profiles v,

$$\begin{aligned}
\sum_{j=0}^{m} t_j^V(v) &\geq \underline{v}_0 + (k-1)S_{[1]}(v) - \sum_{i\in\mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}) \\
&= \underline{v}_0 - \bar{v} - v_{[2]}^{-\{1\}}.
\end{aligned}$$

Therefore,

$$\Pr\left(\sum_{j=0}^{m} t_j^V(v) > 0\right) \geq \Pr\left(\underline{v}_0 - \bar{v} - v_{[2]}^{-\{1\}} > 0\right)$$
$$= \Pr\left(v_{[2]}^{-\{1\}} < \underline{v}_0 - \bar{v}\right).$$

We will show that $\Pr\left(\sum_{j=0}^{m} t_j^V(v) > 0\right) \to 1$. According to assumption NT , $2\bar{v} > \underline{v}_0$. It follows that $\bar{v} > \underline{v}_0 - \bar{v}$. By hypothesis, $\underline{v}_0 - \bar{v} > \underline{v}$. Therefore, $\underline{v} < \underline{v}_0 - \bar{v} < \bar{v}$. This implies,

$$\text{i.e., } 0 < F\left(\underline{v}_0 - \bar{v}\right) < 1.$$

Consequently,

$$\Pr\left(v_{[2]}^{-\{1\}} < \underline{v}_0 - \bar{v}\right)$$
$$= 1 - \left\{1 - F\left(\underline{v}_0 - \bar{v}\right)\right\}^{m-1} - (m-1)F\left(\underline{v}_0 - \bar{v}\right)\left\{1 - F\left(\underline{v}_0 - \bar{v}\right)\right\}^{m-2},$$

which converges to 1 as $m \to \infty$. ■

4.4 A Generalization

The following Propositions extend the results of the earlier subsections to a sequence of graphs under certain conditions.

Let $G(1)$ be a feasible path. For any $G(m)$, $m \geq 1$, let $G(m+1)$ be any arbitrary supergraph of $G(m)$ of order $m+1$. We say that a sequence of graphs $G(m)_{m=1}^{\infty}$ satisfies the *line inclusion* property if for any $m' \geq 1$, we can find a natural number m such that $L(m')$ is a subgraph of $G(m)$.

Proposition 4 *Consider a sequence of graphs $G(m)_{m=1}^{\infty}$ that satisfies the Line Inclusion property. Then for any sequence of LAC problems $\langle G(m), k, \mu\rangle_{m=n}^{\infty}$, the VCG mechanism almost surely results in a surplus if and only if $\underline{v}_0 > k\underline{v}$.*

Proof: This proof is almost identical to that of Proposition 2 and hence omitted. ■

Let $G(1)$ be a connected graph with a nonempty set of critical sellers $c(G(1))$. Let $|c(G)| = C$. These critical sellers form a path of length C, say $\{c_1 c_2 \cdots c_C\}$. For any $G(m)$, $m \geq 1$, let $G(m+1)$ be a supergraph of $G(m)$ such that $G(m+1) = \{V(m) \cup \{m+1\}, E(m) \cup \{x\}\}$ where $x \in \{(m+1, c_1), (c_C, m+1)\}$. In other words, the supergraph adds a new edge at the endpoints of the path $\{c_1 c_2 \cdots c_C\}$. Note that for any $m \geq 1$, $|c(G(m))| = C$. We say that such a sequence $G(m)_{m=1}^{\infty}$ satisfies the *C-preservation* property. Since a graph can have at most $k-1$ critical nodes, $C \leq k-1$. For example, $\langle G^*(m), 2, \mu\rangle_{m=1}^{\infty}$ satisfies 1-preservation property.

PROPOSITION 5 *Let $G(1)$ be a connected graph with a nonempty set of critical sellers $c(G(1))$. Consider a sequence of graphs $G(m)_{m=1}^{\infty}$ that satisfies C-preservation property. For the sequence of LAC problems $\langle G(m), k, \mu\rangle_{m=1}^{\infty}$, the VCG mechanism almost surely results in a surplus if and only if*

$$\underline{v}_0 > C\bar{v} + (k-C)\underline{v}.$$

Proof: First note that for any G of order m in such a sequence,

$$\begin{aligned}
\underline{v}_0 + (k-1)S_{[1]}(v) - \sum_{i\in\mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}) &= \underline{v}_0 + (k-1)\left[\sum_{i\in c(G)} v_i + \sum_{j=1}^{k-C} v_{[j]}^{-c(G)}\right] \\
&\quad - \sum_{i\in c(G)}\left[\bar{v} + \sum_{\substack{j\in c(G)\\ j\neq i}} v_j + \sum_{j=1}^{k-C} v_{[j]}^{-c(G)}\right] \\
&\quad - \sum_{i\notin c(G)}\left[\sum_{i\in c(G)} v_i + \sum_{\substack{j=1\\ j\neq i}}^{k-C+1} v_{[j]}^{-c(G)}\right] \\
&= \underline{v}_0 - C\bar{v} - (k-C)v_{[k-C+1]}^{-c(G)}
\end{aligned}$$

where $v_{[i]}^{-c(G)}$ is the i-th order statistic of the $m-C$ valuations of all non-critical sellers.

Only if part: Suppose $\underline{v}_0 \leq C\bar{v} + (k-C)\underline{v}$. Then

$$\begin{aligned}
\underline{v}_0 + (k-1)S_{[1]}(v) - \sum_{i\in\mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}) &= \underline{v}_0 - C\bar{v} - (k-C)v_{[k-C+1]}^{-c(G)} \\
&\leq (k-C)\left(\underline{v} - v_{[k-C+1]}^{-c(G)}\right) \\
&\leq 0
\end{aligned}$$

Consequently, the sum of payments in Case II of Table 2 cannot be positive. Since this is the only Case where the VCG sum of payments can be positive, hence the claim.

If part: Let $\underline{v}_0 > C\bar{v} + (k-C)\underline{v}$. Recall that at all profiles v,

$$\begin{aligned}
\sum_{j=0}^{m} t_j^V(v) &\geq \underline{v}_0 + (k-1)S_{[1]}(v) - \sum_{i\in\mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}) \\
&= \underline{v}_0 - C\bar{v} - (k-C)v_{[k-C+1]}^{-c(G)}.
\end{aligned}$$

Therefore,

$$\Pr\left(\sum_{j=0}^{m} t_j^V(v) > 0\right) \geq \Pr\left(\underline{v}_0 - C\bar{v} - (k-C)v_{[k-C+1]}^{-c(G)} > 0\right)$$
$$= \Pr\left(v_{[k-C+1]}^{-c(G)} < \frac{\underline{v}_0 - C\bar{v}}{k-C}\right).$$

We will show that $\Pr\left(v_{[k-C+1]}^{-c(G)} < \frac{\underline{v}_0 - C\bar{v}}{k-C}\right) \to 1$. By assumption NT , $k\bar{v} > \underline{v}_0$ it follows that $(k-C)\bar{v} > \underline{v}_0 - C\bar{v}$. By hypothesis, $\underline{v}_0 - C\bar{v} > (k-C)\underline{v}$. Therefore, $\underline{v} < \frac{\underline{v}_0 - C\bar{v}}{k-C} < \bar{v}$. This implies

$$\text{i.e., } 0 < F\left(\frac{\underline{v}_0 - C\bar{v}}{k-C}\right) < 1.$$

Consequently,

$$\Pr\left(v_{[k-C+1]}^{-c(G)} < \frac{\underline{v}_0 - C\bar{v}}{k-C}\right)$$
$$= 1 - \sum_{r=0}^{k-C} \binom{m-C}{r} \left\{F\left(\frac{\underline{v}_0 - C\bar{v}}{k-C}\right)\right\}^r \left\{1 - F\left(\frac{\underline{v}_0 - C\bar{v}}{k-C}\right)\right\}^{m-C-r}.$$

Since each of the components of the sum on the right hand side converges to 0 as $m \to \infty$, $\Pr\left(v_{[k-C+1]}^{-c(G)} < \frac{\underline{v}_0 - \bar{v}}{k-C}\right)$ converges to 1 as $m \to \infty$. ■

5 Discussion

The TL condition can be interpreted as follows. Fix a valuation of the buyer. Then there always exists a tuple of seller valuations for which trade takes place. Note that TL does not mean that trade takes place everywhere. Assumption NT states that $k\bar{v} > \underline{v}_0$, and therefore, there exists a set of valuations for which trade does not take place.

We showed that (a) if TL holds then VCG almost always results in a surplus if the number of sellers is large, both in LA and in the LAC problem with line contiguity; (b) if the corresponding equality holds, then VCG sum of payments converges to zero from the left hand side; (c) if $\underline{v}_0 < k\underline{v}$, then VCG sum of payments never results in a surplus.

The intuition behind these results is clear. Recall that the VCG payment for each agent is interpreted as the externality she imposes on other agents. We have used the statistical fact that the k-th lowest order statistic of a sample of size $n > k$ drawn from any continuous distribution approaches the lower end of its support in probability as n becomes large. It follows that, as the number of sellers become large, the probability that the buyer with a given valuation will find an efficient set of sellers to trade with becomes very high. Further, the probability that the buyer will continue to trade if one seller reports a high valuation becomes very high as well. Therefore, the externality imposed by any successful seller on other agents at a profile becomes small. TL ensures that whenever trade takes place the externality imposed by the successful sellers at a profile is less than that by the buyer.

In Tables 3 and 4, we provide several numerical examples[3] to illustrate this result for the LA problem and the LAC problem with line contiguity when $k = 2$. We generate values for the VCG sum of payments for these problems when valuations are drawn from specific uniform distributions pertaining to cases where (a) TL holds, (b) if the corresponding equality holds and (c) if $\underline{v}_0 < k\underline{v}$. The numerical data confirms our results.

In the LAC problem with star contiguity, the hub of the star graph represents a critical seller, who by definition, is in every feasible path. Recall from that a critical seller who is not trade-pivotal must receive a payment of $\bar{v}$ at a profile where trade takes place. The TLS1 condition can be interpreted as follows. Fix a valuation of the buyer. Then there always exists a tuple of seller valuations such that trade takes place where one of the sellers is critical.

Condition TLS1 requires the buyer to have a very high valuation relative to the sellers. It follows that the presence of critical sellers makes convergence to surplus less likely. We showed that in the problem with star contiguity, (a) if TLS1 holds then VCG almost always results in a surplus if the number of sellers is large; (b) if the corresponding equality holds, then VCG sum of payments converges to zero from the left hand side; (c) if $\underline{v}_0 < \bar{v} + \underline{v}$, then VCG sum of payments never results in a surplus.

Note that TLS1 implies that trade can take place at a profile where the non-critical seller reports a value that is low enough. The statistical facts mentioned above implies that probability of trade taking place becomes high as number of sellers become large. TLS1 ensure that whenever trade takes place the externality imposed by the successful sellers, accounting for the critical one, is less than that by the buyer.

In Table 5, we provide several numerical examples to illustrate this result for the LAC problem with star contiguity. We generate values for the VCG sum of payments when (a) TL holds, (b) TL holds with equality and (c) $\underline{v}_0 < \bar{v} + \underline{v}$.

Table 3: Sum of Payments in the LA model, $k = 2$

m	$v_0 \sim U[200, 300], v_i \sim U[100, 300]$	$v_0 \sim U[200, 300], v_i \sim U[50, 300]$	$v_0 \sim U[50, 300], v_i \sim U[100, 300]$
10	0	-21.266	0
100	-10.898	93.171	-2.858
1000	-0.713	95.429	0
10000	-0.073	99.802	-0.057

We have also shown that the convergence result for line contiguity can be extended to sequences of graphs satisfying the line inclusion property. This property implies that for any

[3]The numerical data presented in these Tables has been generated through programs written in GNU Octave, a high level interactive language for numerical computations.

Table 4: Sum of Payments in LAC with Line Contiguity, $k = 2$

m	$v_0 \sim U[200, 300], v_i \sim U[100, 300]$	$v_0 \sim U[200, 300], v_i \sim U[50, 300]$	$v_0 \sim U[50, 300], v_i \sim U[100, 300]$
10	0	-99.58	0
100	0	28.86	0
1000	-20.087	80.831	0
10000	-1.205	92.203	-2.615

Table 5: Sum of Payments in LAC with Star Contiguity, $k = 2$

m	$v_0 \sim U[400, 1000], v_i \sim U[100, 300]$	$v_0 \sim U[400, 1000], v_i \sim U[50, 300]$	$v_0 \sim U[300, 1000], v_i \sim U[100, 300]$
10	-126.86	13.06	-145.54
100	-5.593	47.422	-20.986
1000	-0.729	49.01	-55.136
10000	-0.05	49.931	-100.11

integer $m > k$, one can always find a graph in the sequence which has the line graph of order m embedded in it.

We also showed that the convergence result for star contiguity is extendable to sequences of graphs satisfying the C-preservation property. Note that an arbitrary sequence of graphs cannot satisfy both line inclusion and preservation together when there is at least one critical seller. In particular, the preservation property implies that one cannot find a path of length more than k.

6 Conclusion

In this paper we focussed on the behavior of VCG surplus as the number of sellers become large but the structure of the underlying graphs are preserved. We showed that under some mild conditions, VCG almost surely results in a surplus for the LA problem and the LAC problem with line contiguity. This condition requires that for any valuation of the buyer, there always exists a profile such that trade takes place. We showed that the corresponding condition changes when we allow for critical sellers like those in a star graph.

References

Bergemann, D. and J. Välimäki (2006): "Information in Mechanism Design," Centre for Economic and Policy Research.

Chakravorty, S. (2013): *The Price of Land: Acquisition, Conflict, Consequence*, Oxford University Press, New Delhi.

Cramton, P., R. Gibbons, and P. Klemperer (1987): "Dissolving a partnership efficiently," *Econometrica*, 615–632.

Cripps, M. W. and J. M. Swinkels (2006): "Efficiency of Large Double Auctions," *Econometrica*, 74, 47–92.

Fudenberg, D., M. Mobius, and A. Szeidl (2007): "Existence of equilibrium in large double auctions," *Journal of Economic Theory*, 133, 550–567.

Gresik, T. A. and M. A. Satterthwaite (1989): "The Rate at Which a Simple Market Converges to Efficiency as the Number of Traders Increase: An Asymptotic Result for Optimal Trading Mechanism," *Journal of Economic Theory*, 48, 304–332.

Jackson, M. O. (2000): "Mechanism theory," *The Encyclopedia of Life Support Systems*.

Makowski, L. and C. Mezzetti (1993): "The possibility of efficient mechanisms for trading an indivisible object," *Journal of Economic Theory*, 59, 451–465.

McAfee, R. P. (1992): "A Dominant Strategy Double Auction," *Journal of Economic Theory*, 56, 434–450.

Miceli, T. J. (2011): *The economic theory of eminent domain: private property, public use*, Cambridge University Press.

Myerson, R. and M. Satterthwaite (1983): "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 29, 265–281.

Rustichini, A., M. A. Satterthwaite, and S. R. Williams (1994): "Convergence to Efficiency in a Simple Market with Incomplete Information," *Econometrica*, 62, 1041–63.

Sarkar, S. (2014): "Mechanism Design for Land Acquisition," Working Paper.

Satterthwaite, M. A. and S. R. Williams (1989a): "Bilateral Trade with the Sealed Bid k-Double Auction: Existence and Efficiency," *Journal of Economic Theory*, 48, 107–133.

——— (1989b): "The Rate of Convergence to Efficiency in the Buyer's Bid Double Auction as the Market Becomes Large," *Review of Economic Studies*, 56, 477–498.

——— (2002): "The Optimality of a Simple Market Mechanism," *Econometrica*, 70, 1841–1863.

Williams, S. R. (1991): "Existence and Convergence of Equilibria in the Buyer's Bid Double Auction," *Review of Economic Studies*, 58, 351–374.

——— (1999): "A Characterization of Efficient, Bayesian Incentive Compatible Mechanisms," *Economic Theory*, 14, 155–180.

Wilson, R. (1987): "Game-Theoretic Approaches to Trading Processes," in *Advances in Economic Theory: Fifth World Congress*, ed. by T. Bewley, Cambridge University Press, chap. 2, 33–77.