Segmented Financial Markets and Exchange Rate Dynamics in a Small Open Economy*

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Abstract

This paper studies optimal exchange rate policies when there are shocks to the financial sector in a small open economy with segmented asset markets. In such an environment, we contrast the welfare implications of fixed exchange rate and flexible exchange rate regime. We show that the ranking of the regime can be mapped to the risk sharing that the rule allows relative to what optimal policy calls for. Flexible exchange rates, by facilitating greater risk sharing, welfare dominates the fixed regime. Further, we also examine the volatility of the equity prices under these regimes. We show that volatility of equity prices is higher under fixed exchange regime as compared to the flexible regime.

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Keywords: Exchange rate regime, Equity prices, Segmented Asset Markets

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1 Introduction

There is a growing interest in how monetary policy should respond to shocks originating in the financial sector. In this paper we examine this issue in a small open economy where only a section of the economy has access to financial markets (i.e., asset markets are segmented).

Whether the nature of shocks has any bearing on the choice of the exchange rate regime has remained an important issue in the open economy literature. In the context of open economies, following the pioneering works of Mundell (1968), and Fleming (1962), the debate has traditionally focused on the choice between fixed and flexible exchange rates. The text book prescription on this subject has been that in an environment of sticky prices, flexible exchange rates work better when the shocks arise in the goods market while fixed exchange rates work better if the shocks originate in the money market. The intuition behind this "Mundell-Fleming" model is simple. Fixed exchange rates insulate the real side of the economy from disturbances in the money market while flexible exchange rates better insulate against real shocks by allowing the terms of trade to adjust relatively quickly.

More recently, Lahiri-Singh-Vegh (LSV) (2006) abstract from price rigidities and instead focus on exchange rate policy when financial markets are segmented. In a milieu where only a fraction of agents have access to financial markets they show that the Mundell-Fleming dictum is turned on its head. Specifically, they show that fixed exchange rates are optimal when shocks originate in the goods market whereas flexible exchange rates dominate when shocks are monetary. Interestingly, while most of the debate has centered on aggregate supply and monetary shocks, there is little work on the choice of the monetary policy regime when the small open economy is faced with financial sector shocks. This is the gap that we attempt to address in this paper.

We first study optimal devaluation rate in a small open economy with segmented financial markets subject to financial sector shocks. Next, we consider two classic simple rules: fixed exchange rates, monetary targeting. Our objective is to identify the simple rule that in terms of welfare is closest to the optimal monetary policy. We show analyically that flexible exchange rates welfare dominate fixed exchange rates.

Next, we examine the volatility of share prices under the alternate monetary policy regimes. Following the global financial crises of 2008, there has been a growing debate on the link between monetary policy and asset prices. Specifically, the literature has focused on whether monetary policy should target asset prices. Proponents of an asset price targeting approach argue that a central bank should actively counteract excessive asset price increases (Blanchard 2000, Bordo and Jeanne 2002, Borio and Lowe 2002, Borio and White 2003, Cecchetti et al. 2000 and Goodhart 2000). Others such as (Bean 2003, Bernanke 2002, Bernanke and Gertler 1999, 2001) have argued against it. We instead compare the volatility of share prices in fixed and floating exchange rate regimes. Our analysis demonstrates that share prices are more volatile under a pegged regime. Table 1 show that the volatility of share prices for a select group of countries under both fixed and flexible exchange rate regimes. Consistent with our results, the share prices under fixed exchange rate regimes are on average almost 20% higher than that found under fixed exchange rate regimes.

Table 1:Small Open Economy: Source Bloomberg

IMF Classification	Country	Stock Volatility
Fixed Exchange Rate	Hongkong	29%
	Denmark	18%
	Singapore	19%
	China	29%
Floating Exchange Rate	Australia	13%
	Japan	13%

This paper as in LSV abstracts from any nominal rigidity and focuses on a standard monetary model in which the only friction is that an exogenously-given fraction of the population called traders can access asset markets. Importantly, any money injections are absorbed exclusively by the traders. An increase in the money growth therefore acts as an inflation tax on the non-participants (non-traders) by redistributing resources in favor of the traders. In addition to money injections traders have access to domestic and foreign bonds as well as stocks. The only source of uncertainty in the model is the dividend shock in the stock market. As in Zervou (2013), trading households receive a share of the stochastic dividend tree in proportion to the amount of stocks they hold.

The policymaker attaches equal weight to the utilities of both the trader and non-trader. Optimal policies that maximize the overall utility of the economy can be summarized as follows. A positive dividend shock puts upward pressure on the prices of shares and the consumption of the trader. The policymaker responds by appreciating of the currency thereby lowering the inflation tax. This in turn increases the consumption of the non-trader by raising their purchasing power. Thus optimal policy facilitates increased risk sharing between the trader and the non-trader. The redistribution of resources in favor of the non-trader reduces the demand for stocks by the trader and mitigates some of the price increase.

We next turn to the simple rules, fixed exchange and flexible exchange rate. Under fixed exchange rates prices are constant and there is no redistribution in favor of the non-trader. Hence the entire shock is borne by the trader and there is no risk sharing. This also results in share prices rising significantly under this regime. By contrast the currency appreciates under the flexible exchange rate regime fecilitating risk sharing between the trader and non-trader households. To summarize asset prices and consumption of traders is most volatile under a fixed exchange rate regime. Consumption of the non-trader is higher and more volatile under flexible exchange rates. On balance woing to the reduced risk sharing and higher volatility of the consumption of the trader, the fixed exchange rate regime is welfare dominated by the flexible exchange rate regime.

This paper is linked to a growing volume of literature that has sought to examine monetary policy in the presence of segmented asset markets. (See, among others, Alvarez and Atkeson (1997), Alvarez, Lucas, and Weber (2001) and Chatterjee and Corbae (1992)). However almost all of these papers abstract away from shocks to the financial sector.

Our work in this paper builds on Zervou (2013). In the context of a closed economy with segmented asset markets they demonstrates that optimal policy is contractionary when there are positive shocks to the financial sector and expansionary when there are negative shocks. They also compute welfare numerically under certain popular simple rules. One drawback with this approach is that since welfare is a function of the extent of segmentation, it is not completely evident if the welfare ranking will unabigously hold for all values of segmentation. Our focus on the other hand is on the choice of the exchange rate regime in the context of an open economy. Unlike the Zervou paper we compare welfare anayltically across regimes.

Our paper is also closely linked to the work by LSV. As discussed they show that the Mundell-Fleming results are reversed when the source of the friction is in the asset markets as opposed to the product markets. Specifically, they demonstrate that when there are real shocks to the economy then fixed exchange rates outperform flexible exchange rates. By contrast, we show that if the real shocks to the economy originate in the financial sector then flexible exchange rates are preferable. In otherwords we demonstrate that the Mundell-Fleming hypothesis would continue to hold as long as the source of the real shock is in the financial sector.

The rest of the paper is organized as follows. In Section 2.1 we present our basic model composed of the different economic agents, and the constraints they face and in Section 2.3 we discuss the equilibrium properties and asset pricing features of our model. In Section 3 we arrive at an optimal policy, and derive model dynamics under fixed and flexible exchange rate regime. In Section 4 we conduct a numerical analysis to illustrate the dynamics under a financial shock. In section 5 we analytically compute the welfare under two regimes and Section 6 concludes. Technical details of the model are provided in the Appendix.

2 The Model Economy

2.1 Environment

The basic model closely follows LSV, extended to include financial sector shocks in the spirit of Zervou (2013). Consider a small open economy perfectly integrated with world goods markets. There is a unit measure of households who consume an internationally-traded good. The world currency price of the consumption good is fixed at one. The households' intertemporal utility function is

$$W_t = E_0 \sum_{s=t}^{\infty} \beta^{t-s} u(c_s) \tag{1}$$

where β is the households' time discount factor, c_s is consumption in period s, while E_t denotes the expectation conditional on information available at time t. In addition to goods markets, there are four asset markets: stock, nominal domestic bond, foreign real bond, and money market. Markets are segmented in the sense only a fraction λ of the population, called traders, have access to the stock and bond markets, where $0 \leq \lambda \leq 1$. The rest,

$(1-\lambda)$, called non-traders, can only hold domestic money as an asset.

The households face a cash-in-advance constraint. As is standard in these models, the households are prohibited from consuming their own endowment. We assume that a household consists of a seller-shopper pair. While the seller sells the household's own endowment, the shopper goes out with money to purchase consumption goods from other households.

The only source of uncertainty in the model is the dividend shock in the stock market. The stock market is introduced similarly to Zervou (2013), wherein participating agents receive a share of the stochastic dividend tree in proportion to the amount of stocks they hold. Essentially, every period while the non-traders receive a fixed endowment $y^N$of the consumption good, the traders receive an endowment y^T and a share of the stochastic real dividend ϵ_t. Specifically, the dividend process ϵ_t is given by:

$$\epsilon_t = \bar{\epsilon} + \eta_t \tag{2}$$

where $\bar{\epsilon}$ is the mean and η_t is an *iid* shock with zero mean and variance σ_η^2. The total output in the economy is therefore given by

$$y_t \equiv \epsilon_t + \lambda y^T + (1-\lambda) y^N \tag{3}$$

The mean output in the economy can therefore be written as $\bar{y} \equiv \bar{\epsilon} + \lambda y^T + (1-\lambda) y^N$. The timing is as follows. First, dividend shocks are realized at the beginning of every period. Consequently, only traders have a risky component in their income, while non-traders get only the fixed endowment. Second, the household splits. Sellers of both households stay at home and sell their endowments for local currency. Shoppers of the non-trading households are excluded from the asset market and, hence, go directly to the goods market with their overnight cash to buy consumption goods. Shoppers of trading households first carry the cash held overnight to the asset market where they trade in domestic and foreign bonds, stocks and receive any money injections for the period. After rebalancing her portfolio the trader household heads to the goods market. After all trades for the day are completed and markets close, the shopper and the seller are reunited at home.

2.2 Households

2.2.1 Trader Households

The traders begin any period with assets in the form of money balances bond and stock holdings carried over from the previous period. Asset markets open first where the shopper of the trader household rebalances the household's asset position and also receives the lump sum transfers from the government. Thus, for any period t, the accounting identity for the asset market transactions of a trader household

$$\begin{aligned}\hat{M}_t^T = M_t^T + \frac{T_t}{\lambda} + (1+i_{t-1})\frac{B_t}{\lambda} - \frac{B_{t+1}}{\lambda} \\ + S_t(1+r)f_t - S_t f_{t+1} + q_t z_t - q_t z_{t+1}\end{aligned} \tag{4}$$

where $\hat{M}_t^T$ denotes the money balances with which the trader leaves the asset market and M_t^T denotes the money balances with which the trader entered the asset market. B denotes aggregate one-period nominal government bonds, i is the interest rate on these nominal bonds, f are foreign bonds (denominated in terms of the consumption good), r is the exogenous and constant world real interest rate, T are aggregate (nominal) lump-sum transfers (i.e., negative taxes) from the government, q is the price of a share and z are the number of shares. Armed with this nominal cash $\hat{M}_t^T$, the trader household then proceeds to the goods market to purchase consumption for the period t. The cash in advance constrain for this transaction is given as

$$S_t c_t^T \leq \hat{M}_t^T \tag{5}$$

The trader household also sells its endowment y^T and encashes the dividend ϵ_t, both of which become the cash which it carries over in the next period $t+1$

$$M_{t+1}^T = S_t y^T + S_t z_t \epsilon_t \tag{6}$$

Combining equation (4) and assuming that the cash in advance constraint equation (5) binds[1], gives us the following

$$
\begin{aligned}
S_t c_t^T = M_t^T + \frac{T_t}{\lambda} \qquad & +(1+i_{t-1})\frac{B_t}{\lambda} - \frac{B_{t+1}}{\lambda} \\
& + S_t(1+r)f_t - S_t f_{t+1} + q_t z_t - q_t z_{t+1}
\end{aligned}
\tag{7}
$$

A trader chooses c_t, B_{t+1}, f_{t+1} and z_t to maximize (1) subject to the constraints of equation (6) and equation (7). Combining the first order condition yields

$$
U'\left(c_t^T\right) = \beta(1+r)E_t\left[U'\left(c_{t+1}^T\right)\right] \tag{8}
$$

$$
\frac{U'\left(c_t^T\right)}{S_t} = \beta(1+i_t)E_t\left[\frac{U'\left(c_{t+1}^T\right)}{S_{t+1}}\right] \tag{9}
$$

$$
\frac{U'(c_t^T)}{S_t} q_t = \beta E_t\left[\frac{U'\left(c_{t+1}^T\right)}{S_{t+1}}\left(q_{t+1} + S_t \epsilon_t\right)\right] \tag{10}
$$

Equations (8), (9), determine the optimal holdings of the foreign and domestic bond. Combining the two equations one obtains the uncovered interest parity condition. Equation (10) equates the marginal cost of purchasing a stock with the marginal benefit of holding this asset.

2.2.2 Non-Trader Households

The non-trader household doesn't have access to asset markets. She receives an endowment of y^N every period t and uses cash, M^N, carried over from the previous period to procure current period consumption. The cash in advance constraint for the non-trader is is given by

$$
S_t c_t^N \leq M_t^N \tag{11}
$$

The non-trader also sells her endowment for cash which she carries over the next period

$$
M_{t+1}^N = S_t y^N \tag{12}
$$

[1] This is a standard assumption in the literatures, for instance see Alvarez et al. (2001), Alvarez et al. (2002) Appendix A, Lahiri et al. (2007) Appendix A.1. Relaxing this assumption only introduces more complexity to the analysis without altering any results. Having assumed this, once the equilibrium values of the state variables are solved, one can set parameter values conditional on which the assumption will hold.

2.2.3 Government

The government in this economy holds foreign bonds (reserves) which earn the world rate of interest r. Thus, the government's budget constraint is given by

$$M_{t+1} - M_t = S_t h_{t+1} - S_t(1+r)h_t + (1+i_{t-1})B_t - B_{t+1} + T_t \tag{13}$$

where h_t is the foreign bonds that the government enters with, in period t. As in LSV, the money supply can be altered through open market operations, through interventions in the foreign exchange market, or through transfers. Importantly, monetary policy impacts only traders directly as they are the only ones in the economy with access to asset markets.

2.3 Equilibrium & Asset Pricing

In the money market, the equilibrium condition is given by

$$M_t = \lambda M_t^T + (1-\lambda)M_t^N \tag{14}$$

If we combine equations (6), (12) and (14) we get the relationship that $M_{t+1} = S_t y_t$. This is akin to the quantity theory equation where the velocity of money is unity. Equivalently, we can write this as

$$S_t = \frac{M_t}{y_t} = \frac{M_t}{\bar{y} + \epsilon_t - \bar{\epsilon}} \tag{15}$$

It follows from (15) that

$$\frac{S_t}{S_{t-1}} = \left(\frac{M_t}{M_{t-1}}\right)\left(\frac{y_{t-1}}{y_t}\right) \tag{16}$$

where $\frac{S_t}{S_{t-1}} = 1+\theta_t$ is the devaluation rate at time t, $\left(\frac{M_t}{M_{t-1}}\right) = 1+\mu_t$ is the rate of growth of money supply and $\left(\frac{y_{t-1}}{y_t}\right) = \frac{1}{1+g_t}$ is the growth rate in output. Note, since we assume purchasing power parity holds θ_t is also the rate of inflation in the economy. In the stock market, the total shares of the firm which are distributed among the traders should sum upto unity, i.e.

$$\lambda z_{t+1} = 1 \tag{17}$$

Combining equations (5), (7), (11), (12), (13), and (14) we get the following flow constraint for the aggregate economy (current account). This is also the goods market equilibrium obtained by combining (3), (14), (17)

$$\lambda c_t^T + (1-\lambda)c_t^N = y_t + (1+r)k_t - k_{t+1} \tag{18}$$

where $y_t = \epsilon_t + \lambda y^T + (1-\lambda)y^N$, is the total output in the economy in real terms and $k \equiv h + \lambda f$ denote the total foreign bonds in the economy.

Next we proceed to solve consumption of traders and non-traders. Using equation (11) and equation(12) we obtain consumption of non-traders as

$$c_t^N = \frac{1}{1+\theta_t} y^N \tag{19}$$

It follows from (19), that an increase in the inflation tax θ, redistributes resources away from the non-trader and thereby lowers her consumption. To solve for the consumption of traders, we use the trader's flow constraint equation (7) and substitute in it the equations (15) and (13), to get

$$c_t^T = y^T + \frac{\epsilon_t}{\lambda} + (1+r)\frac{k_t}{\lambda} - \frac{k_{t+1}}{\lambda} + \frac{1-\lambda}{\lambda}(\bar{y} - \bar{\epsilon} - \frac{\bar{y}-\bar{\epsilon}}{1+\theta_t}) \tag{20}$$

The first two terms represent the total endowment that each trader receives. The last component $\frac{1-\lambda}{\lambda}(\bar{y} - \bar{\epsilon} - \frac{\bar{y}-\bar{\epsilon}}{1+\theta_t})$ captures the redistribution caused due to changes in monetary policy in the economy. An increase in the depreciation rate θ_t, would raise the inflation tax leading to a redistribution of $\frac{1-\lambda}{\lambda}(\bar{y} - \bar{\epsilon} - \frac{\bar{y}-\bar{\epsilon}}{1+\theta_t})$ from non-traders to traders. Note that $\bar{y} - \bar{\epsilon}$, is the endowment of the non-trader household, which stays constant throughout.

3 Optimal monetary policy and simple monetary rules

Having characterized the decentralized equilibrium, we are now set to evaluate and compare alternative monetary policies when the economy is subject to shocks in the financial sector. We begin by evaluating optimal monetary policy wherein the policymaker sets the devaluation rate in order to maximize total welfare. The three simple rules,fixed exchange rates, monetary targeting and asset price targeting are addressed thereafter.

3.1 Optimal Monetary Policy

In the analysis of optimal policy, the central bank assumes the role of a social planer who is interested in maximizing the aggregated welfare in the economy. The objective function of the central bank therefore becomes

$$\max_{\varepsilon_t} E_0 \sum_{0}^{\infty} \beta^t \{\lambda u(c_t^T) + (1-\lambda) u(c_t^N)\} \tag{21}$$

subject to the economy wide constraint as given in equation (18) for each time period t. The first order conditions imply that

$$\lambda \frac{\partial u\left(c_t^T\right)}{\partial c_t^T} \frac{\partial c_t^T}{\partial \theta_t} + (1-\lambda) \frac{\partial u\left(c_t^N\right)}{\partial c_t^N} \frac{\partial c_t^N}{\partial \theta_t} = 0 \tag{22}$$

Substituting for $\frac{\partial c_t^T}{\partial \theta_t}$ and $\frac{\partial c_t^N}{\partial \theta_t}$ from equations (19) and (20) and substituting in (22) we get

$$\frac{\frac{\partial u\left(c_t^N\right)}{\partial c_t^N}}{\frac{\partial c_t^T}{\partial \theta_t}} = 1$$

In other words a policymaker who attaches equal weight to the trader and non-trader equates their consumption at the margin. Using (19) and (20) we consumption under optimal policy for the two groups is given by (see Appendix (A.2))

$$c_t^{T*} = c_t^{N*} = rk_t + \bar{y} + (1-\beta)(\epsilon_t - \bar{\epsilon}) \tag{23}$$

Next we solve for the optimal devaluation rate-the rate of devaluation which equalizes the consumption across the two groups

$$\theta_t^* = -\frac{(1-\beta)\epsilon_t + \beta\bar{\epsilon} + rk_t}{rk_t + \bar{y} + (1-\beta)(\epsilon_t - \bar{\epsilon})} \tag{24}$$

Further, evaluating the derivative of the depreciation rate with respect to the dividend shock (evaluated around $\bar{\epsilon}$)

$$\frac{\partial \theta^*}{\partial \epsilon_t} = -\frac{(1-\beta)(\bar{y} - \bar{\epsilon})}{\bar{y}^2} < 0 \tag{25}$$

It follows from (25) that optimal policy calls for an appreciation in the currency in response to a positive real dividend shock in the stock market. Essentially, in the absence of policy intervention, consumption of traders would rise in response to the shock while consumption of the non-traders would remain unaffected. The policymaker, who attaches equal weight to the consumption of both groups responds by appreciating the currency causing a fall in the inflation tax. This increases the purchasing power of the non-trader allowing them to increase their current consumption. Optimal policy therefore felicitates increased risk sharing between the two groups. Further, it follows from equation (10) that the appreciation of the currency in response to a positive dividend shock stabilizes stock prices. Intuitively, the positive dividend shock causes an increase in demand for assets as traders seek to smooth their consumption. However, the increased risk sharing brought about by the lowering of the inflation tax mitigates some of this increased demand thereby stabilizing stock prices.

3.2 Fixed Exchange Rates

Under fixed exchange rates, the monetary authority sets a constant path of the exchange rate equal to $\bar{S}$. In particular, we assume that the nominal exchange rate is fixed at

$$\bar{S} = \frac{\bar{M}_0}{y_0} = \frac{M_t}{y_t} \tag{26}$$

where $\bar{M}_0$ is the total money supply and y_0 is the total output at $t = 0$. It follows from (26) that the money supply varies directly with movements in the exchange rate. To understand this consider for example a positive shock to output. It follows from equation (26) , that for a given money supply, this will cause the exchange rate to appreciate. To stabilize the exchange rate the monetary authority responds by buying foreign bonds and increasing the money supply. Given that the exchange rate and hence prices are fixed, it follows from equation (19) that the consumption of non-traders is fixed and given by

$$c_t^{N,fix} = \bar{y} - \bar{\epsilon} \tag{27}$$

It follows that under fixed exchange rates, the non-trader is completely insulated from the shocks in stock market. This implies that the trader bears the full impact of the dividend shocks arising in the stock market. Combining equations (3), (20) and (26) the consumption of the trader households can be written as (see Appendix (A.3))

$$c_t^{T,fix} = \frac{r}{\lambda}k_t + \frac{\beta}{\lambda}\bar{y} + \frac{(1-\beta)}{\lambda}y_t - (\frac{1-\lambda}{\lambda})(\bar{y} - \bar{\epsilon}) \tag{28}$$

The consumption of the trader depends upon the current period and the previous period dividend shock. An increase in the current period dividend causes real output to rise hence prices to fall and the exchange rate to appreciate. In order to stabilize the currency the central bank intervenes by buying up foreign bonds and increasing the money supply in the economy. Since only traders have access to these money injections it raises their current consumption. On the other hand an increase in the dividend in the previous period increases the amount of money balances carried over by traders into the current period. This raises demand for consumption in the current period causing the currency to depreciate. The policymaker responds by reducing the money supply thereby reducing the consumption of the traders. Notice, from equation (10) that unlike optimal policy where the appreciation of the exchange rate helped stabilize asset prices, here the stock price respond by the full amount of the shock. It follows that stock prices under fixed exchange rates are more volatile than under optimal monetary policy.

3.3 Flexible Exchange Rates

Under flexible exchange rates, the monetary authority fixes the total money supply

$$M_t = \bar{M}_0 \tag{29}$$

We assume that the initial nominal balances across the two types of agents is the same and is given by $M_0^N = M_0^T = \bar{M}_0$. From the quantity equation (15), the exchange rate is given by

$$S_t = \frac{\bar{M}_0}{(\bar{y} - \bar{\epsilon}) + \epsilon_t} \tag{30}$$

Combining (19) and (30), the consumption of non-traders under flexible exchange rates can be written as

$$c_t^N = (\bar{y} - \bar{\epsilon}) \frac{(\bar{y} - \bar{\epsilon}) + \epsilon_t}{(\bar{y} - \bar{\epsilon}) + \epsilon_{t-1}} \tag{31}$$

Unlike fixed exchange rates, the consumption of the non-traders is no longer insulated from the shocks to the dividend under this regime. Specifically, the consumption of the non-trader varies positively with shocks to dividend in the current period and negatively with shock to the dividend of the previous period. Intuitively, a positive shock to dividend in period $(t-1)$, decreases prices in that period and from (12) reduces the amount of cash balances non-traders carry into period t. Consequently, consumption falls in period t. On the other hand, an increase in dividend in the current period, increases consumption of the non-trader by reducing the burden of the inflation tax they face.

To arrive at the consumption of traders we iterate equation (20) yielding (see Appendix (A.4))

$$c_t^{T,flex} = \frac{r}{\lambda}k_t + \frac{\beta}{\lambda}\bar{y} + \frac{(1-\beta)}{\lambda}y_t - \frac{(1-\lambda)(1-\beta)}{\lambda}(\bar{y}-\bar{\epsilon})\frac{y_{t-1}}{y_t}$$
$$- \frac{\beta(1-\lambda)(1-\beta)}{\lambda}(\bar{y}-\bar{\epsilon})(y_t)(\frac{1}{\bar{y}} + \frac{\sigma_y^2}{\bar{y}^3}) - \frac{\beta^2(1-\lambda)}{\lambda}(\bar{y}-\bar{\epsilon}) \tag{32}$$

Equation.(23) indicates that as participation in the financial markets increase, $\lambda \rightarrow 1$, the consumption under the flexible exchange rate regime converges to the one obtained under optimal policy. The depreciation rate under this regime can be obtained by combining (15) and (16)

$$\theta_t = \frac{\epsilon_{t-1} - \epsilon_t}{(\bar{y}-\bar{\epsilon}) + \epsilon_t} \tag{33}$$

Under a regime of flexible exchange rates, an increase in the current dividend ϵ_t, causes current output to rise resulting in a fall in inflation. On the other hand an increase in the dividend of the previous period, raises current income of the traders causing demand and hence prices to rise.

4 Numerical analysis

Having characterized the decentralized equilibrium, we are now set to evaluate and compare alternative monetary policies under dividend shocks. While many of our results are derived analytically, we rely on numerical computations for obtaining impulse responses and obtaining volatility of asset prices. Following LSV and Zervou (2013), we use the following parameter values for our analysis: $\beta = 0.99$, $\lambda = .0.15$, $\bar{\epsilon} = 2.35$, $\bar{y} = 1$, $\eta = 0.1$, $r = 1.01\%$, $\sigma_\epsilon = 0.8$.

Below, we present equilibrium dynamics under alternative policies under dividend shocks. Following the analysis in (3), we focus on the dynamics of consumption of traders, consumption of non-traders, depreciation rate (inflation) and stock prices.

4.1 Dividend Shocks

Figure 1 plots the responses under optimal policy, fixed exchange rate and flexible exchange rate regime, when there is a one standard deviation positive shock to dividend. Consumption

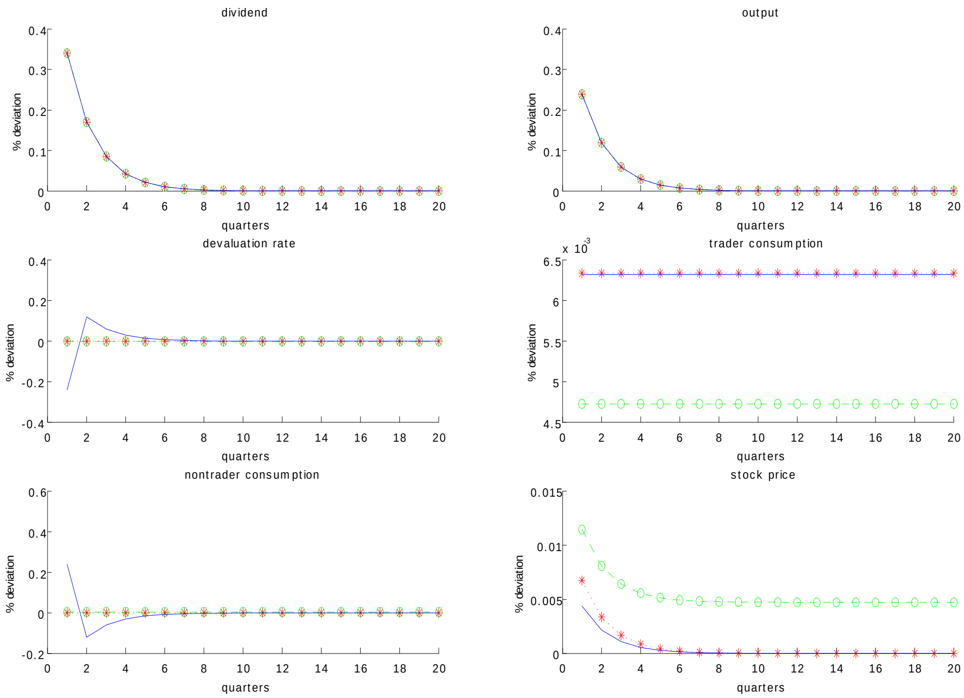

Figure 1: Impulse response functions on a positive dividend shock

of trader, non-trader and stock prices rise and the currency appreciates. Essentially, the shock to dividend raises the consumption of the trader as well as stock price as they demand more of this asset. The policymaker attempts to smooth the consumption of the trader by appreciating the currency. The lower inflation tax increases the consumption of the non-trader.

Under an exchange rate peg, due to fixed prices the consumption of the non-trader is completely insulated from the dividend shock. Prices of shares rise as traders demand more of this asset. Contrary to the fixed exchange rate regime, a positive shock to dividend results in appreciation of the exchange rate under both the asset price targeting regime and the flexible exchange rate regime. As discussed, in the previous section the extent of the appreciation under asset price targeting depends upon the extent of market participation. For our benchmark parameters we find that the appreciation is greater under the asset price targeting regime. The higher appreciation under the asset price targeting regime also implies that inflation tax is lower and consumption of the non-traders is higher under this regime.

Table 2: Relative Volatility with respect to Output

Relative Standard Deviation	Optimal	Fixed	Flexible
Traders Consumption: $\frac{\sigma_{CT}}{\sigma_y}$	0.0247	0.1649	0.1645
Non Traders Consumption: $\frac{\sigma_{CN}}{\sigma_y}$	0.0248	0.0000	0.3213
Devaluation Rate: $\frac{\sigma_\theta}{\sigma_y}$	0.0022	0.0000	0.3213
Stock Price: $\frac{\sigma_{\hat{q}}}{\sigma_y}$	11.0256	1.9604	1.2728

Table 2 compares the volatilities of select variables across regimes for our benchmark parameters. Consistent with our discussion above the consumption of traders and share prices are most volatile under fixed exchange rate rates. On the other hand the lower inflation tax under the flexible exchange rate regime results in consumption of non-traders being most volatile under this regime.

5 Welfare under alternative policies

Our goal here is to evaluate analytically the welfare implications under the fixed and flexible exchange rate. We compute welfare by comparing the unconditional expectation of life-time welfare at time $t = 0$.In order to keep the welfare analytically tractable, we assume a quadratic utility specification as before. Our disscussion in this section follows LSV closely with the welfare function given by

$$W^{i,j} = E\{\sum_{t=0}^{\infty} \beta^t \left[c^{i,j} - \eta \left(c^{i,j}\right)^2\right]\} \; i = \text{T,N} \qquad j = \text{flex, fix} \tag{34}$$

where

$$W^j = \lambda W^{T,j} + (1-\lambda) W^{N,j} \tag{35}$$

As in LSV, Equation (34) gives the welfare for each agent under a specific exchange rate regime where the relevant consumption for each type of agent is given by the consumption functions derived above for each regime. Equation. (35) is the aggregate welfare for the economy under each regime which is the sum of the regime specific individual welfares weighted by their population shares. The assumption of quadratic utility implies that the expected welfare can be written as

$$E\{c - \eta c^2\} = E(c) - \eta[E(c)]^2 - \eta Var[c] \tag{36}$$

where where $Var(c)$ denotes the variance of consumption and $\dot{\eta} > 0$ is a parameter.

Proposition 1 *In a small open economy with segmented financial markets, if the only source of uncertainty is shocks to financial sector (dividend) then the flexible exchange rate regime dominates the fixed exchange rate regime.*

We provide a detailed proof in the Appendix (A.5). A sketch of the proof is provided below. The difference in welfare across the two regimes can be written as

$$\begin{aligned}
\Delta W^{flex-fix} &= \sum_{t=0}^{\infty} \beta^t [\lambda\{(E_0(c_t^{T,flex}) - E_0(c_t^{T,fix})) \\
&\quad -\eta((E_0(c_t^{T,flex}))^2 - (E_0(c_t^{T,fix}))^2) \\
&\quad -\lambda\eta(Var(c_t^{T,flex}) - Var(c_t^{T,fix}))\} \\
&\quad +(1-\lambda)\{(E_0(c_t^{N,flex}) - E_0(c_t^{N,fix})) \\
&\quad -\eta((E_0(c_t^{N,flex}))^2 - (E_0(c_t^{N,fix}))^2) \\
&\quad -\lambda\eta(Var(c_t^{N,flex}) - Var(c_t^{N,fix}))\}]
\end{aligned}$$

where under the fixed exchange rate regime the following hold

$$\begin{aligned}
E_0(c_t^{N,fix}) &= \bar{y} - \bar{\epsilon} \\
Var(c_t^{N,fix}) &= 0 \\
E_0(c_t^{T,fix}) &= \frac{1}{\lambda}\bar{y} - (\frac{1-\lambda}{\lambda})(\bar{y} - \bar{\epsilon}) \\
Var(c_t^{T,fix}) &= (t+1)(\frac{(1-\beta)}{\lambda})^2 \sigma_y^2
\end{aligned} \tag{37}$$

Similarly under the flexible exchange rate regime

$$\begin{aligned}
E_0(c_t^{N,flex}) &= (\bar{y} - \bar{\epsilon})(1+\delta) \\
Var(c_t^{N,flex}) &= (\bar{y} - \bar{\epsilon})^2(1+\delta)(2\delta) \\
E_0(c_t^{T,flex}) &= \frac{1}{\lambda}\bar{y} - \frac{1}{(1-\beta)}\Psi(1+\delta) \\
Var(c_t^{T,flex}) &= (t+1)\{\frac{(1-\beta)^2}{\lambda^2}\delta\bar{y}^2 - \Omega^2\delta\bar{y}^2\} + O(\delta^2)
\end{aligned} \tag{38}$$

where δ, Ψ are parameters defined in the appendix. For reasonable parameter values we show in the appendix that $W^{flex} > W^{fix}$. It is also easy to show that $\Delta W^{flex-fix} = 0$ for $\lambda = 1$, meaning that the two regimes are welfare equivalent under complete market participation. Flexible exchange rates by mimicking optimal policy clearly dominates the fixed regime. Under fixed exchange rate there is no risk sharing between the trader and the non-trader. The lower expected income of the non-trader coupled with higher volatility in consumption of the trader leads to a lower degree of overall welfare when compared to the flexible exchange rate regime. Our results are contrary to those obtained by LSV who show that when an economy is subject to real shocks fixed exchange rates outperform flexible exchange rates. Essentially, in their framework, while expected consumption is identical across the two regimes volatility of consumption for non-traders is lower under the peg. This leads to the pegged regime outperforming the flexible regime. The cruical difference between the two papers arises in the nature of the real shocks that hit the economy. In the LSV paper the real shock hits both the trader and the non-trader household whereas in our case it hits only the trader household. Thus while the flexible exchange rate regime felicitates risk sharing thereby improving welfare in our framework it increases volatility of the non-trader household and reduces welfare in their paper.

6 Conclusion

This paper extends the segmented asset markets framework to study how financial sector shocks affect exchange rate policy in a small open economy. Our results show that the differences in welfare across fixed and flexible exchange rate regimes can be mapped with the financial risk sharing the regimes allow relative to what optimal policy calls for. Flexible exchange rates welfare dominates the pegged regime by felicitating greater risk sharing between traders and non-traders. In addition we also examine the volatility of share prices under the alternate regimes. We show that asset prices and consumption of financial market participants is most volatile under a pegged regime. On the other hand consumption of the non- participants is most volatile under flexible exchange rates. To conclude our analysis suggest that in the case where real shocks originate in the financial sector, the standard

Mundell-Fleming prescription continues to hold under asset market segmentation. A useful extension of the model will be to carry out this exercise when the market segmentation is endogenous. Intuitively the nature of monetary policy regime itself should influence the extent of market participation. This in turn would impact the risk sharing across regimes. We leave this for future research.

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A Appendix

A.1 Random Walk

Through the Euler equation for foreign bonds we have

$$U'\left(c_t^T\right) = \beta(1+r)E_t\left[U'\left(c_{t+1}^T\right)\right] \tag{A.1}$$

From the assumption of quadratic utility and substituting $\beta(1+r) = 1$, we find that the consumption of trader is a random walk such that

$$c_t^T = E_t(c_{t+1}^T) \tag{A.2}$$

Irrespective of the monetary policy regime, this random walk nature will always hold true for the trader. Later, we show that when the policy regime is optimal, the consumption of the non-trader also becomes a random walk and is equal to the consumption of the trader.

A.2 Optimal Policy (State-Contingent)

Under optimal policy, the first best allocation is that $c_t^T = c_t^N$. Consumption of trader follows a random walk, such that $c_t^T = E_t\{c_{t+1}^T\}$. Therefore, the consumption of non-trader, under optimal policy is also a random walk, such that $.c_t^N = E_t\{c_{t+1}^N\}$. From the accounting identity we have

$$\lambda c_t^T + (1-\lambda)c_t^N = y_t + (1+r)k_t - k_{t+1} \tag{A.3}$$

Since this holds for every t, it must also hold under expectations.

$$\sum_{t=t}^{\infty} E_t[\lambda c_t^T + (1-\lambda)c_t^N] = \sum_{t=t}^{\infty} y_t + (1+r)k_t - k_{t+1} \tag{A.4}$$

Solving it recursively we find that

$$c_t^{T,opt} = c_t^{N,opt} = rk_t + (1-\beta)y_t + \beta\bar{y} \tag{A.5}$$

It follows that

$$E_0(c_t^{T,opt}) = E_0(c_t^{N,opt}) = rE_0(k_t) + \bar{y} \tag{A.6}$$

Further, we know that at optimality

$$c_t^T = y_t + (1+r)k_t - k_{t+1} \tag{A.7}$$

Equating this with the earlier expression, we get the motion for k_t as

$$k_{t+1} = k_t + \beta y_t - \beta\bar{y} \tag{A.8}$$

Immediately, it follows that $E_0(k_{t+1}) = E_0(k_t) = k_0 = 0$. And $Var(k_t) = t\beta^2\sigma_y^2$. Substituting these, we get

$$E_0(c_t^{T,opt}) = E_0(c_t^{N,opt}) = \bar{y} \tag{A.9}$$

and

$$Var(c_t^{T,opt}) = Var(c_t^{N,opt}) = (t+1)(1-\beta)^2\sigma_y^2 \tag{A.10}$$

A.3 Fixed Exchange Rate

Under fixed exchange rates, the consumption of the non-trader is given by

$$c_t^{N,fix} = \bar{y} - \bar{\epsilon} \tag{A.11}$$

Trivially then, $E(c_t^{N,fix}) = \bar{y} - \bar{\epsilon}$ and $Var(c_t^{N,fix}) = 0$. The consumption of traders is given by:

$$c_t^{T,fix} = \frac{y_t}{\lambda} + (1+r)\frac{k_t}{\lambda} - \frac{k_{t+1}}{\lambda} - \frac{(1-\lambda)(\bar{y} - \bar{\epsilon})}{\lambda} \tag{A.12}$$

Upon recursive algebra,

$$c_t^{T,fix} = r\frac{k_t}{\lambda} + \frac{\beta}{\lambda}\bar{y} + \frac{(1-\beta)}{\lambda}y_t - (\frac{1-\lambda}{\lambda})(\bar{y} - \bar{\epsilon}) \tag{A.13}$$

It follows that

$$E(c_t^{T,fix}) = rE(\frac{k_t}{\lambda}) + \frac{1}{\lambda}\bar{y} - (\frac{1-\lambda}{\lambda})(\bar{y} - \bar{\epsilon}) \tag{A.14}$$

and

$$Var(c_t^{T,fix}) = r^2 Var(\frac{k_t}{\lambda}) + (\frac{(1-\beta)}{\lambda})^2\sigma_y^2 \tag{A.15}$$

Further

$$\frac{k_{t+1}}{\lambda} = \frac{k_t}{\lambda} + \frac{\beta}{\lambda}y_t - \frac{\beta}{\lambda}\bar{y} \tag{A.16}$$

Its easy to see that under fixed exchange rates, $E(\frac{k_{t+1}}{\lambda}) = E(\frac{k_t}{\lambda}) = \frac{k_0}{\lambda} = 0$. Also, this makes it consistent with the accounting identity under expectations. Further, recursively, it yields

$$\frac{k_t}{\lambda} = \frac{k_0}{\lambda} + \frac{\beta}{\lambda}\sum_{s=0}^{t-1} y_s - \frac{\beta}{\lambda}\bar{y}t \tag{A.17}$$

$$Var(\frac{k_t}{\lambda}) = (t)(\frac{\beta}{\lambda})^2\sigma_y^2 \tag{A.18}$$

Substituting in the consumption equation

$$E_0(c_t^{T,fix}) = \frac{1}{\lambda}\bar{y} - (\frac{1-\lambda}{\lambda})(\bar{y} - \bar{\epsilon}) \tag{A.19}$$

$$Var(c_t^{T,fix}) = (t+1)(\frac{(1-\beta)}{\lambda})^2\sigma_y^2 \tag{A.20}$$

A.4 Flexible Exchange Rates

Under flexible exchange rates, the consumption of the non-traders is given by

$$c_t^{N,flex} = (\bar{y} - \bar{\epsilon})\frac{y_t}{y_{t-1}} \tag{A.21}$$

To a second order approximation, and assuming that the distribution of y_t is IID over time and also symmetric, we have that $E_0(c_t^{N,flex}) = (\bar{y} - \bar{\epsilon})(1 + \delta)$ where $\delta = \sigma_y^2/\bar{y}^2$. Also, $Var(c_t^{N,flex}) = (\bar{y}-\bar{\epsilon})^2(1+\delta)(2\delta)$.Correspondingly the consumption of traders can be written as

$$c_t^{T,flex} = \frac{y_t}{\lambda} + (1+r)\frac{k_t}{\lambda} - \frac{k_{t+1}}{\lambda} - \frac{(1-\lambda)(\bar{y}-\bar{\epsilon})}{\lambda}\frac{y_t}{y_{t-1}} \tag{A.22}$$

and can be recursively solved to

$$\begin{aligned} c_t^{T,flex} &= r\frac{k_t}{\lambda} + \frac{(1-\beta)}{\lambda}y_t + \frac{\beta}{\lambda}\bar{y} - \frac{(1-\lambda)(\bar{y}-\bar{\epsilon})(1-\beta)}{\lambda}\frac{y_t}{y_{t-1}} \\ &- \frac{(1-\lambda)\beta(1-\beta)(\bar{y}-\bar{\epsilon})}{\lambda}\frac{\bar{y}}{y_t} - \frac{(1-\lambda)\beta^2(\bar{y}-\bar{\epsilon})}{\lambda}(1+\delta) \end{aligned} \tag{A.23}$$

$$E_0(c_t^{T,flex}) = rE_0(\frac{k_t}{\lambda}) + \frac{1}{\lambda}\bar{y} - \frac{(1-\lambda)(\bar{y}-\bar{\epsilon})}{\lambda}(1+\delta)$$

where $E_0(\frac{k_t}{\lambda})$ is computed below. Let $\frac{(1-\lambda)(\bar{y}-\bar{\epsilon})(1-\beta)}{\lambda} = \Psi > 0$, then $\frac{(1-\lambda)\beta(1-\beta)(\bar{y}-\bar{\epsilon})}{\lambda} = \beta\Psi$ and $\frac{(1-\lambda)\beta^2(\bar{y}-\bar{\epsilon})}{\lambda} = \frac{\beta^2}{(1-\beta)}\Psi$ and we can write the above equation as

$$c_t^{T,flex} = r\frac{k_t}{\lambda} + \frac{(1-\beta)}{\lambda}y_t + \frac{\beta}{\lambda}\bar{y} - \Psi\frac{y_t}{y_{t-1}} - \beta\Psi\frac{\bar{y}}{y_t} - \frac{\beta^2}{(1-\beta)}\Psi(1+\delta) \tag{A.24}$$

and the expected consumption of the trader can be written as

$$E_0(c_t^{T,flex}) = rE_0(\frac{k_t}{\lambda}) + \frac{1}{\lambda}\bar{y} - \frac{1}{(1-\beta)}\Psi(1+\delta)$$

Further, the variance of the trader's consumption under flexible prices will be given by

$$\begin{aligned} Var(c_t^{T,flex}) &= r^2Var(\frac{k_t}{\lambda}) + \frac{(1-\beta)^2}{\lambda^2}\sigma_y^2 + \Psi^2Var(\frac{y_t}{y_{t-1}}) + (\beta\Psi)^2Var(\frac{\bar{y}}{y_t}) \\ &- 2r\Psi Cov(\frac{k_t}{\lambda}, \frac{y_t}{y_{t-1}}) - \frac{2(1-\beta)\Psi}{\lambda}Cov(y_t, \frac{y_t}{y_{t-1}}) \\ &- \frac{2(1-\beta)\beta\Psi}{\lambda}Cov(y_t, \frac{\bar{y}}{y_t}) + 2\beta\Psi^2Cov(\frac{y_t}{y_{t-1}}, \frac{\bar{y}}{y_t}) \end{aligned} \tag{A.25}$$

for $t \geq 1.Var(c_0^{T,flex}) = \frac{(1-\beta)^2}{\lambda^2}\sigma_y^2 + (\beta\Psi)^2\bar{y}\delta(1-\delta) + \frac{\Psi^2}{\bar{y}^2}\sigma_y^2 + \frac{2(1-\beta)\beta\Psi}{\lambda}\bar{y}\delta$
$-2\frac{(1-\beta)}{\lambda}\frac{\Psi}{\bar{y}} + 2\beta\Psi^2(-\delta)$, upon assuming that $y_t = \bar{y}$ for $t < 0$.

Equating the two expressions for consumption of traders, we find the motion for k_t as

$$\frac{k_{t+1}}{\lambda} = \frac{k_t}{\lambda} + \frac{\beta y_t}{\lambda} - \frac{\beta}{(1-\beta)}\Psi\frac{y_t}{y_{t-1}} + \beta\Psi\frac{\bar{y}}{y_t} - \frac{\beta}{\lambda}\bar{y} + \frac{\beta^2}{(1-\beta)}\Psi(1+\delta) \tag{A.26}$$

Immediately it follows that $E_0(\frac{k_{t+1}}{\lambda}) = E_0(\frac{k_t}{\lambda})$ and this is also consistent with the accounting identity in the intertemporal form. Upon recursive algebra, it follows that

$$\frac{k_t}{\lambda} = \frac{k_0}{\lambda} + \frac{\beta}{\lambda}\sum_{s=0}^{t-1} y_s - \frac{\beta}{(1-\beta)}\Psi\sum_{s=0}^{t-1}\frac{y_s}{y_{s-1}} + \beta\Psi\sum_{s=0}^{t-1}\frac{\bar{y}}{y_s} - \frac{\beta}{\lambda}\bar{y}t + \frac{\beta^2}{(1-\beta)}\Psi(1+\delta)t \tag{A.27}$$

Without the loss of generality we can assign $\frac{k_0}{\lambda} = 0$. It follows that $E_0(\frac{k_t}{\lambda}) = 0$ and

$$E_0(c_t^{T,flex}) = \frac{1}{\lambda}\bar{y} - \frac{1}{(1-\beta)}\Psi(1+\delta)$$

$$\begin{aligned} Var(\frac{k_t}{\lambda}) &= (\frac{\beta}{\lambda})^2\sigma_y^2 t + (\frac{\beta}{(1-\beta)}\Psi)^2(1+\delta)(2\delta)(t-1) + (\frac{\beta}{(1-\beta)}\Psi)^2\delta + (\beta\Psi)^2\delta(1-\delta)t \\ &- 2\frac{\beta}{\lambda}\frac{\beta}{(1-\beta)}\Psi\sum_{s=1}^{t-1}Cov(y_s, \frac{y_s}{y_{s-1}}) - 2\frac{\beta}{\lambda}\frac{\beta}{(1-\beta)}\Psi\frac{\sigma_y^2}{\bar{y}} + 2\frac{\beta}{\lambda}\beta\Psi\sum_{s=0}^{t-1}Cov(y_s, \frac{\bar{y}}{y_s}) \\ &- 2\frac{\beta}{(1-\beta)}\beta\Psi^2\sum_{s=1}^{t-1}Cov(\frac{y_s}{y_{s-1}}, \frac{\bar{y}}{y_s}) - 2\frac{\beta}{(1-\beta)}\beta\Psi^2(-\delta) \\ &- 2\frac{\beta}{(1-\beta)}\beta\Psi^2\sum_{s=0}^{t-2}Cov(\frac{y_{s+1}}{y_s}, \frac{\bar{y}}{y_s}) - 2\frac{\beta}{\lambda}\frac{\beta}{(1-\beta)}\Psi\sum_{s=0}^{t-2}Cov(y_s, \frac{y_{s+1}}{y_s}) \end{aligned} \tag{A.28}$$

$$\begin{aligned} Var(\frac{k_t}{\lambda}) &= (\frac{\beta}{\lambda})^2\sigma_y^2 t + (\frac{\beta}{(1-\beta)}\Psi)^2(1+\delta)(2\delta)(t-1) + (\frac{\beta}{(1-\beta)}\Psi)^2\delta + (\beta\Psi)^2\delta(1-\delta)t \\ &- 2\frac{\beta}{\lambda}\frac{\beta}{(1-\beta)}\Psi(1+\delta)(\delta)\bar{y}(t-1) - 2\frac{\beta}{\lambda}\frac{\beta}{(1-\beta)}\Psi\delta\bar{y} + 2\frac{\beta}{\lambda}\beta\Psi(-\bar{y}\delta)(t) \\ &- 2\frac{\beta}{(1-\beta)}\beta\Psi^2(-\delta)(1+\delta)(t-1) - 2\frac{\beta}{(1-\beta)}\beta\Psi^2(-\delta) \\ &- 2\frac{\beta}{(1-\beta)}\beta\Psi^2\delta(1-\delta)(t-1) - 2\frac{\beta}{\lambda}\frac{\beta}{(1-\beta)}\Psi(-\delta)\bar{y}(t-1) \end{aligned} \tag{A.29}$$

The above can be simplified to

$$\begin{aligned} Var(\frac{k_t}{\lambda}) &= (\frac{\beta}{\lambda})^2\delta\bar{y}^2 t + (\frac{\beta}{(1-\beta)}\Psi)^2(2\delta)(t-1) + (\frac{\beta}{(1-\beta)}\Psi)^2\delta + (\beta\Psi)^2\delta t - 2\frac{\beta}{\lambda}\frac{\beta}{(1-\beta)}\Psi(\delta)\bar{y}(t-1) \\ &- 2\frac{\beta}{\lambda}\frac{\beta}{(1-\beta)}\Psi\bar{y}\delta + 2\frac{\beta}{\lambda}\beta\Psi(-\bar{y}\delta)(t) - 2\frac{\beta}{(1-\beta)}\beta\Psi^2(-\delta)(t-1) - 2\frac{\beta}{(1-\beta)}\beta\Psi^2(-\delta) \\ &- 2\frac{\beta}{(1-\beta)}\beta\Psi^2\delta(t-1) - 2\frac{\beta}{\lambda}\frac{\beta}{(1-\beta)}\Psi(-\delta)\bar{y}(t-1) + O(\delta^2) \end{aligned} \tag{A.30}$$

$$Var(\frac{k_t}{\lambda}) = (\frac{\beta}{\lambda})^2\delta\bar{y}^2 t + (\frac{\beta}{(1-\beta)}\Psi)^2(2\delta)(t-1) + (\frac{\beta}{(1-\beta)}\Psi)^2\delta + (\beta\Psi)^2\delta t$$
$$-2\frac{\beta}{\lambda}\frac{\beta}{(1-\beta)}\Psi\bar{y}\delta - 2\frac{\beta}{\lambda}\beta\Psi(\bar{y}\delta)(t) + 2\frac{\beta}{(1-\beta)}\beta\Psi^2(\delta) + O(\delta^2)$$

Further, $Var(\frac{y_t}{y_{t-1}}) = (1+\delta)(2\delta), Var(\frac{\bar{y}}{y_t}) = \delta(1-\delta),\ Cov(\frac{k_t}{\lambda}, \frac{y_t}{y_{t-1}}) = \beta\Psi,\ Cov(y_t, \frac{y_t}{y_{t-1}}) = \bar{y}\delta(1+\delta)$,

$Cov(y_t, \frac{\bar{y}}{y_t}) = -\bar{y}\delta,\ Cov(\frac{y_t}{y_{t-1}}, \frac{\bar{y}}{y_t}) = -\delta(1+\delta)$. Substituting these in the expression for $Var(c^{T,flex})$ we get

$$Var(c_t^{T,flex}) = r^2 Var(\frac{k_t}{\lambda}) + \frac{(1-\beta)^2}{\lambda^2}\sigma_y^2 + \Psi^2(1+\delta)(2\delta) + (\beta\Psi)^2\delta(1-\delta) - 2r\Psi\beta\Psi$$
$$-\frac{2(1-\beta)\Psi}{\lambda}\bar{y}\delta(1+\delta) + \frac{2(1-\beta)\beta\Psi}{\lambda}\bar{y}\delta - 2\beta\Psi^2\delta(1+\delta) \tag{A.31}$$

This can be simplified to

$$Var(c_t^{T,flex}) = r^2 Var(\frac{k_t}{\lambda}) + \frac{(1-\beta)^2}{\lambda^2}\delta\bar{y}^2 + (1-\beta)\Psi^2(2\delta) + (\beta\Psi)^2\delta - 2(1-\beta)\Psi^2 - \frac{2(1-\beta)^2\Psi}{\lambda}\bar{y}\delta + O(\delta^2)$$

Upon rearranging, one can write this as

$$Var(c_t^{T,flex}) = (t+1)\{\frac{(1-\beta)^2}{\lambda^2}\delta\bar{y}^2 - \Omega^2\delta\bar{y}^2\} + O(\delta^2)$$

Where $\Omega = 0$ for $\lambda = 1.and\ \Omega = f(\frac{1-\lambda}{\lambda})$. Also,

$$E_0(c_t^{T,flex}) = \frac{1}{\lambda}\bar{y} - \frac{1}{(1-\beta)}\Psi(1+\delta)$$

We will use these results in the next section, when we compare the welfare differences across the fixed and flexible regime.

A.5 Welfare comparison Fixed vs Flexible

As before, the welfare is defined as:

$$W = E\{\sum_{t=0}^{\infty} \beta^t W_t\}$$

The quadratic utility implies that the expected welfare can be written as $E\{c-\eta c^2\} = E(c)-\eta[E(c)]^2 - \eta Var[c]$, where $Var[c]$ denotes the variance of consumption. Upon substituting for the respective consumption moments, we find that

$$\begin{aligned}\Delta W^{flex-fix} &= \frac{\lambda}{1-\beta}[-\frac{\Psi\delta}{1-\beta}] - \frac{\eta\lambda}{1-\beta}[-\frac{2\Psi\delta}{1-\beta}(\frac{1}{\lambda}\bar{y} - \frac{\Psi}{1-\beta})] + \frac{(1-\lambda)}{1-\beta}\delta(\bar{y}-\epsilon) \\ &- \frac{\eta(1-\lambda)}{1-\beta}[\delta(\bar{y}-\bar{\epsilon}) + 2(\bar{y}-\bar{\epsilon})^2\delta - \frac{\eta\lambda[-\Omega^2\delta\bar{y}^2]}{(1-\beta)^2} - \frac{\eta(1-\lambda)}{1-\beta}[(\bar{y}-\bar{\epsilon})^2(1+\delta)(2\delta)]\end{aligned}$$

We can rewrite the above as

$$\Delta W^{flex-fix} = \{2(\bar{y}-\bar{\epsilon})^2 + \phi(1-\beta)\}\lambda^2 - \{2\phi(1-\beta) + 2(\bar{y}-\bar{\epsilon})\bar{y}\}\lambda + \{2(\bar{y}-\bar{\epsilon})\bar{\epsilon} + \phi(1-\beta)\} = 0$$

where $\phi = \{\frac{(1-\lambda)}{\lambda}\}^2\Omega > 0$. This equation has two roots, $\lambda = 1$ and $\lambda = \frac{2(\bar{y}-\bar{\epsilon})+\phi(1-\beta)}{2(\bar{y}-\bar{\epsilon})^2+\phi(1-\beta)} > 1$ for $\bar{\epsilon} > (2/3)\bar{y}$. For reasonable parameter values this will be true and therefore, $W^{flex} > W^{fix}$ for most parameter values. It is also easy to see that $\Delta W^{flex-fix} = 0$ for $\lambda = 1$, meaning that the two regimes are welfare equivalent under complete market participation and also, $\Delta W^{flex-fix} > 0$ for the extreme case of $\lambda = 0$.

A.6 Stock Price Volatility

The stock price from equation (10) can be written as

$$\begin{aligned}\hat{q}_t = E_t \sum_{a=1}^{\infty} \beta^a * 1 - \eta\{\bar{y} + (\epsilon_{t+a} - \bar{\epsilon}) + (1+r)\frac{k_{t+a}}{\lambda} - \frac{k_{t+a+1}}{\lambda} \\ + \frac{1-\lambda}{\lambda}(\bar{y} + (\epsilon_{t+a} - \bar{\epsilon}) - \frac{(\bar{y}-\bar{\epsilon})}{1+\theta_{t+a}})\} \\ * /1 - \eta\{\bar{y} + (\epsilon_t - \bar{\epsilon}) + (1+r)\frac{k_t}{\lambda} - \frac{k_{t+1}}{\lambda} \\ + \frac{1-\lambda}{\lambda}(\bar{y} + (\epsilon_t - \bar{\epsilon}) - \frac{(\bar{y}-\bar{\epsilon})}{1+\theta_t})\} \\ * \frac{1}{1+\theta_{t+1}}\epsilon_{t+a-1}\}\end{aligned} \tag{A.32}$$

For the optimal policy, the stock price is given as

$$
\begin{aligned}
\hat{q}^*(x_t) = E_t \sum_{a=1}^{\infty} \beta^a & \frac{1 - 2\xi(rk(x_{t+a}) + \bar{y} + (1-\beta)(\epsilon_{t+a} - \bar{\epsilon})}{1 - 2\xi(rk(x_t) + \bar{y} + (1-\beta)(\epsilon_t - \bar{\epsilon}))} \\
& * \frac{(rk(x_{t+a}) + \bar{y} + (1-\beta)(\epsilon_{t+a} - \bar{\epsilon})}{\bar{y} - \bar{\epsilon}} \epsilon_{t+a-1}
\end{aligned} \tag{A.33}
$$

For the fixed exchange rate regime, using equation () and equation () we can write the expression for the real stock price as

$$
\begin{aligned}
\hat{q}_t^{fixed} = E_t \sum_{a=1}^{\infty} & \beta^a * 1 - \eta\{\bar{y} + (\epsilon_{t+a} - \bar{\epsilon}) + (1+r)\frac{k_{t+a}}{\lambda} - \frac{k_{t+a+1}}{\lambda} \\
& + \frac{1-\lambda}{\lambda}(\bar{y} + (\epsilon_{t+a} - \bar{\epsilon}) - (\bar{y} - \bar{\epsilon}))\} \\
& * /1 - \eta\{\bar{y} + (\epsilon_t - \bar{\epsilon}) + (1+r)\frac{k_t}{\lambda} - \frac{k_{t+1}}{\lambda} \\
& + \frac{1-\lambda}{\lambda}(\bar{y} + (\epsilon_t - \bar{\epsilon}) - (\bar{y} - \bar{\epsilon}))\} \\
& * \frac{1}{1+\theta_{t+1}} \epsilon_{t+a-1}\}
\end{aligned} \tag{A.34}
$$

For the flexible exchange rate, using equation () we can write the expression for the real stock price as

$$
\begin{aligned}
\hat{q}^{flex}(x_t) = E_t \sum_{a=1}^{\infty} & \beta^a * 1 - \eta\{\bar{y} + (\epsilon_{t+a} - \bar{\epsilon}) + (1+r)\frac{k_{t+a}}{\lambda} - \frac{k_{t+a+1}}{\lambda} \\
& + \frac{1-\lambda}{\lambda}(\bar{y} + (\epsilon_{t+a} - \bar{\epsilon}) - \frac{(\bar{y} - \bar{\epsilon})}{y_{t+a-1}} y_{t+a})\} \\
& * /1 - \eta\{\bar{y} + (\epsilon_t - \bar{\epsilon}) + (1+r)\frac{k_t}{\lambda} - \frac{k_{t+1}}{\lambda} \\
& + \frac{1-\lambda}{\lambda}(\bar{y} + (\epsilon_t - \bar{\epsilon}) - \frac{(\bar{y} - \bar{\epsilon})}{y_{t-1}} y_t)\} \\
& * \frac{y_{t+a}}{y_{t+a-1}} \epsilon_{t+a-1}\}
\end{aligned} \tag{A.35}
$$

We then compute the stock price volatility through numerical analysis.