Using and Producing *Ideas* in Computable Endogenous Growth

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Abstract

It is shown, in this paper, that Paul Romer's suggestion to model *algorithmically* the use and production of *ideas* in an endogenous growth model is formally feasible. Such a modelling exercise imparts a natural *algorithmically evolutionary* flavour to growth models. However, it is also shown that policy implications on efficiency are formally *indeterminate* in a precise and *effective*[1] sense. An attempt is also made to determine and delineate the necessary *tacit element* - in the sense of Polanyi - in the development process (emphasized also by Romer), of which growth is a proper subset.

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[1] *Effective* is meant in the strict formal sense of computability theory.

1 Introduction

Paul Romer, in [10] and [11], going beyond even his seminal papers on endogenous growth theory, broached new visions for an understanding of the growth process in widely differing economic systems. He points out that the conventional modelling of growth processes are deficient in their incorporation of the role and genesis of *ideas*. To rectify this deficiency he proposes an economic definition of ideas based, *inter alia*, on a distinction between their *use* and their *production*. These definitions have an *evolutionary* and *algorithmic* underpinning to them; moreover, the *institutional* setting in which ideas are used and produced are also given an evolutionary basis[2].

In this essay an attempt is made to encapsulate some of these imaginative suggestions in an *algorithmic* – i.e., *recursion theoretic* or *computable* – formalism. Thus, in section §2, Romer's suggestions are summarized with such a formalism as the backdrop for the interpretations. Next, in §3, building on the skeletal model developed by Romer, an algorithmic interpretation of his formalism is suggested which makes the empirical implementation of the modified, computable, model immediate. In particular, the framework becomes natural for a *Genetic* (or any other *evolutionary*) *Programming* implementation. This gives the algorithmic formulation an *evolutionary flavour*. A brief concluding section, §4, suggests some of the directions in which the framework in this essay can be expanded and completed.

The policy implications of Romer´s novel framework appears to be the feasibility (and desirability) of *local* Pareto-improving searches for 'better'-practice – rather than the much maligned and over-used concept of 'best-practice' – technologies. I derive two formal propositions, in §3, that suggest the algorithmic infeasibility of the formal, *effective*, implementation of such searches. The results do not, of course, exclude the possibility of efficient non-algorithmic implementation. However, what it *means* to implement anything *non-algorithmically* remains a moot question.

2 Background and Motivation

The economic underpinning of Romer's definition of an *idea* is that it is *nonrival* and *excludable*[3] . Nonrival means the use of an idea by one agent does not deprive other agents from using it. Excludability signifies the feasibility of an

[2] There is a vast and sophisticated economic literature harnessing metaphors from the theories of evolution, neurophysiology and computability. This is evident at the exciting frontiers of growth theory, evolutionary game theory, computable and computational economics and behavioural economics. Much of this serves as the *tacit* background to this essay. I shall, therefore, feel free to utilize *some* of the theoretical technologies and conceptual underpinnings emanating from the theories of evolution, neurophysiology and computability without, in each instant, defining or explaining technical terms.

[3] To put this definition in perspective it can be contrasted with the characterization of a standard public good, which is nonrival but also nonexcludable.

agent preventing other agents from using it[4].

The functional characterization of an idea is based on the dichotomy between its use and its production. Romer explains this dichotomy in a hypothetical economic setting: ideas are used for producing human capital[5]; in turn, human capital is used to produce ideas . This suggests useful definitions in an economic setting in which ideas may enhance the descriptive and explanatory power of (endogenous) growth models. For the formal definition of an idea Romer resorts to the imaginative metaphor of toy chemistry sets[6]. Such sets typically consist of 'a collection of N jars, each containing a different chemical element'. Thus, in a set with N jars there can be at least 2^{K-1} combinations of K elements ($K = 1, 2, ..., N$). If we move from a child's chemistry set to a typical garment factory in the of a developing country we might find that sewing a shirt entails 52 distinct, sequenced, activities. There are, thus, $52! = 10^{68}$ distinct orderings of the sequences in the preparation of a shirt. Now, as Romer perceptively notes:

> "For any realistic garment assembly operation, almost all the possible sequences for the steps would be wildly impractical, but if even a very small fraction of sequences is useful, there will be many such sequences. It is therefore extremely unlikely that any actual sequence that humans have used for sewing a shirt is the best possible one."
>
> [10], p. 69.

Thus:

> "The potential for continued economic growth comes from the vast search space that we can explore. The *curse of dimensionality* [i.e.,2^{K-1} ; or$52! = 10^{68}$] is, *for economic purposes*, *a remarkable blessing*. To appreciate the potential for discovery, one need only consider the possibility that an extremely small fraction of the large number of possible mixtures may be valuable."
>
> ibid, pp. 68-9[7] ; italics added.

There are some formal problems with these imaginative and interesting observations. Firstly, it is clear that Romer is confining his domain of analysis to the integers, natural numbers or the rational numbers; therefore, formal analysis will have to be combinatorial, constructive or recursion theoretic. Convexity

[4] This is where cryptology, in particular public-key cryptology, enters the scheme of things, in particular via patents and patenting laws.

[5] Human capital is, on the other hand, an excludable, rival good.

[6] Economists of my vintage will recall Trevor Swan's brilliant metaphor of meccano sets 'to put up a scarecrow.....to keep off the index-number birds and Joan Robinson herself' ([9], p. 343). Is it a sign of the times that our *metaphors* have '*evolved*' from the *mechanical* to the *chemical*? Where next, then?

[7] This, surely, is also a basis for the kind of 'learning by doing' that emerged from Lundberg's 'Horndal effect' as famously formalized by [1] (but cf. also [4], ch.3) and very much a foundational ingredient of endogenous growth theory.

will be one of the first casualties of working in such a domain; but also compactness. Secondly, there is the perennial question of the *existence* of a *best possible sequence*. Thirdly, even if existence can be proved it is not clear that it can be discovered and implemented in an operational sense – unless existence is proved combinatorially, constructively or recursion theoretically. Fourthly, there may not be any formal way of discovering, formally, even the 'extremely small fraction' of sequences that may well be valuable. Finally, even in the unlikely event that all of these issues can satisfactorily be resolved, there is the real question of the transition from the currently implemented sequence to a 'more valuable region' of the feasible domain. Unless the currently utilized sequence is in the neighbourhood of the 'extremely small valuable fraction' it is unlikely that a transition makes economic sense in the context of a pure growth model with its given institutional background. The point at which development will have to be distinguished from pure growth may well be located in this *transition manifold*, if such a thing can *effectively*[8] be defined, to be somewhat pseudo-mathematical about it.

These problems need not be faced as squarely within the traditional production theoretic framework with its handmaiden, the book of blueprints[9]. In the traditional framework, which is in the domain of *real analysis*, the well defined concepts of the efficient frontier and concomitant best-practice technologies and so on make most, if not all, of the above issues almost irrelevant. But, by the same token, make it impossible to raise the interesting and important issues that Romer is trying to broach. Romer emphasizes *time-sequenced processes*[10] and, hence, must have something more than *the book of blueprints* metaphor for the repository or encapsulation of ideas. I believe that he is trying to open some manageable vistas without trying to peep into all of the contents of Pandora's proverbial box. I believe also that it can be done, although not without changing the mathematical and conceptual framework of analysis. Such is the backdrop for my formal interpretation of the Romer suggestions and the basis for the two propositions I derive in this paper.

To return to Romer's ideas on ideas, the casual empiricism of the above two quotes, underpinned by the metaphor of the child's toy chemistry set and its functions suggests, to him, the analogy of ideas as mixtures; or, as each of the potentially feasible 2^{K-1} mixtures (i.e., each of the $52! = 10^{68}$ ways of sequencing the sewing of a shirt):

> "Within the metaphor of the chemistry set, it is obvious what one means by an idea. Any mixture can be recorded as *a bit string*, *an ordered sequence of 0s and 1s* [of length N] .... [A]n idea

[8] In the recursion theoretic sense.

[9] Obviously, the book must have an 'appendix' instructing the user on the necessity and mode of using the *axiom of choice*. Every indiscriminate reliance on *indexing over a continuum* of agents, technologies etc., is an implicit appeal to the axiom of choice, or one of its noneffective and noconstructive equivalents (cf., [13]). Even more problematically, there is the prior question of the *effective construction* of the book, in the first place - even in principle.

[10] I don't think Böhm-Bawerk or Hayek (of the Pure Theory of Capital) would find themselves in unfamiliar territory in such a conceptual world for production.

> is the increment in information that comes from *sorting* some of the bit strings *into two broad categories*: useful ones and useless ones.............
>
> When a useful mixture is discovered .... the discovery makes possible the creation of economic value. It lets us combine raw materials of low intrinsic value into mixtures that are far more valuable. Once we have the idea, the process of mixing will require its own [Production Function] (specialized capital and labour). For example, the bit string representing nylon requires a chemical processing plant and skilled workers. Important as these tangible inputs are, it is still the idea itself that permits the resulting increase in value. In this fundamental sense, i*deas make growth and development possible*."
>
> ibid, p. 68; italics added.

An immediate formal question is whether *sorting* a *set of ordered sequence* is *effectively feasible*. If sorting is an effectively feasible process, then so will the *process of discovery* be, at least in the above context. Leaving the answer to such a question to the next part, let me move to Romer's next metaphor, which is to get hints on the way to encapsulate, formally, the role played by ideas, defined as *evolving bit-strings*, when 'used to produce human capital'. Here Romer relies on neurophysiological metaphors: ideas, literally, reconfigure the architecture of the neural network representation of what Simon would term the Thinking (Wo)Man (cf., [12], ch.2 and [15])[11]. 'Ideas.....represented as pure pieces of information, as bit strings' (p. 71) enhance the productivity of physical capital solely by a rearrangement of the possible permutations of the constituent elements that go into its manufacture: be it a process, such as sewing a shirt, or a piece of equipment, say a computer. Similarly, they enhance the value of human capital by reconfiguring the physical architecture underlying, say, *thought processes*:

> "Now consider human capital. In my brain there are different physical connections between my neurons. ....[T]he knowledge that reading a software manual [for a new computer and new word-processing software gives] *rearranges connections in my brain* and makes my human capital more valuable. .... The increased value is created by new ideas. Whether it takes the form of a *hardware design*, *software code*, or an instruction manual, an idea is used to mix or arrange roughly the same physical ingredients in ways that are more valuable. And in each case, these ideas *can be represented* as pure pieces of information, *as bit strings*".
>
> ibid, p. 71; italics added.

[11] I don't think there is the slighted hint or suggestion that Romer subscribes to any version of the serial, centralized, strong (or even weak) AI vision when he makes these analogies and invokes such neurophysiological metaphors.

However, Romer does not himself give such a (formal) representation of *ideas as bit strings* in a production function setting that is compatible with the domain suggested explicitly (natural or rational numbers) in these two path-breaking papers. However, heuristically, ideas, represented as bit strings encapsulating 'pure pieces of information', function as inputs into a physical architecture representing human capital and transform its 'wiring', so to speak, in such a way that 'it' is able to process them more effectively, *in some formal and measurable sense*[12]. From standard results in automata and computability theory, going back to the classic works by McCulloch and Pitts, Kleene and others, it is well known that neural network architectures can be given recursion theoretic formalisms as automata of varying degrees of complexity. To be consistent with the standard postulates of rationality in economic theory it is, however, necessary to postulate an architecture that is formally equivalent to a *Turing Machine*[13]. Such an architecture allows rational decision processes to exhibit a kind of formal untamability of ideas. Let me expand on the heuristics of this last comment a little more (to supplement the previous discursive comments).

The inadequacy of the traditional book of blueprints vision of feasible technologies becomes patently evident if any such interpretation is attempted for ideas held by rational economic agents interpreted as *Turing Machines*. The background to this statement is provided by, for example, the *Busy Beaver Turing Machines*[14]. Even if the neurons in a brain are finite, not even the proponents of strong AI would suggest that the world of ideas in any unit can formally be tamed or accessed - unless by magic or the kind of sleight of hand involved in invoking the axiom of choice. Somehow, somewhere, the open-endedness of ideas must assert itself in some kind of *indeterminacy*[15] in models of growth and development. That is why the past can never hold all the secrets to the future. Trivial though this remark may sound, to formally encapsulate it in an interesting and operational way is not easy. And without such a formalism it will not be possible to delimit the range of validity of Romer's fertile ideas. Hence the recursion theoretic formalism of this essay - although this is only one of the reasons.

This completes the background and the intuitive building-blocks[16].

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[12] The ideal way to proceed, at this point, would be to interpret and define information also recursion theoretically, for which there is a well developed tool-kit provided by *algorithmic information theory* (cf. [12], ch. V).

[13] This is fully formalized and discussed in [12], ch. III.

[14] The Busy Beaver Turing Machines, their architecture, perplexities and relevance for economics are fully discussed in [14], ch.3.

[15] Formally, this means undecidabilities or uncomputabilities, neither of which can be encapsulated in the non-algorithmic mathematics of real analysis routinely utilised by the mathematical formalisms in standard production theory.

[16] In fact, and perhaps more importantly, the world of discovery is surely a subset of the world of inventions in the domain of ideas. Hence, this writer conjectures that constructive mathematics, built on intuitionistic foundations, is a better framework in this particular area of economics. I view the recursion theoretic messages of this paper as a halfway house between such an ideal and the current orthodoxy of Bourbakian formalism in standard mathematical economic theory.

3 A Recursion Theoretic Formalism

We can, now, piece together a recursion theoretic formalism. As a preliminary to doing this it is necessary to summarize Romer's production sub-model which is to be embedded in an (endogenous) growth framework. Romer considers output, Y, to be an additive function of a standard production function and a term representing the production of ideas, one for each of, say, n manufacturing sectors[17] as follows[18]:

$$Y = F(K, L) + \sum_{j=1}^{n} G_j(K_j, L_j, H_j, ; A_j) \tag{1}$$

Where, in addition to the standard notation, we have:

H_j : Human capital used in sector (or activity) j;

A_j : 'idea', characterizing sector (or activity) j;

Next, Romer suggests, with characteristic originality, that *new ideas* be formalized as a general *dynamical system* as follows[19]:

$$A(t+1) = S[H_A(t), \left(A_1(t), A_2(t), ..., A_{n(t)}\right), (H_1(t), H_2(t), ..., H_n(t))] \tag{2}$$

This has the following interpretation. The genesis of new ideas is a function of:

$H_A(t)$: Human capital used *exclusively* in searching for the production of new ideas;

$A_i(t)$: The collection of ideas available in a pre-specified economic region at time t ($\forall i = 1, 2, ..., N; N \leq n$)

Finally, the role of ideas in enhancing human capital in a conventional 'learning-by-doing' specification is to be captured in the following way to complete the production sub-model:

$$H(t+1) = \Omega[H(t), A(t)] \tag{3}$$

Two observations regarding the above terse formalism by Romer may well be worth quoting explicitly, simply to add some clarity and economic motivation. Firstly, referring to the above equation (i.e., (3)), Romer suggests that ([10], p. 86):

> "[L]earning how to use a computer software by using it [can be captured by writing] human capital acquisition as a function of

[17] The notation G came about from Romer's original example for 'Garment' manufacturing; eventually it was retained as the generic notation for 'manufacturing'.

[18] All the functions are endowed with the traditional mathematical assumptions in production theory.

[19] Romer formalizes in continuous time; I have, for ease of exposition chosen to use a discrete-time formalism. None of the results in this paper depends on choosing one or the other formalism, even if the transformations are nonlinear. The notation S denotes the activity of 'searching for new ideas'.

the use of specialized human capital on the job in a conventional learning-by-doing specification."

Secondly, the economic underpinnings for the whole of the production submodel is described succinctly and convincingly along the following lines (ibid, p.86):

> "[The above] description of the accumulation of new ideas and new human capital relies on two different kinds of joint product assumptions. Someone with human capital of type j who is employed in activity j produces manufactured good j, produces more human capital of type j, and (occasionally) makes new discoveries of the 'better ways to sew a shirt' variety."

Now, according to the intuitive definitions suggested by Romer (see above, §2):

1. $A_i(t), (\forall i = 1, 2, ..., N)$ are specified as *bit strings*;
2. $H_j(t), (\forall j = 1, 2, ..., n)$, when considered as arguments of $G_j, (j = 1, 2, ..., n)$, are 'neural networks'

(1) is not a serious formal problem; (2), however, requires a formal specification of a 'neural network' that is capable of *computation universality* - i.e., the computing power of *Turing Machines*. If not, it will not be compatible with the standard, minimal, rationality postulates of economic theory (cf., again, [12], ch.III). Then, by the *Church-Turing Thesis*, we can represent each H_j, $j = 1, 2, ..., n$, and H_A as programs (i.e., as algorithms), computationally equivalent to the corresponding *Turing Machine*. Then, by stacking the *bit strings*, A_i, $\forall i = 1, 2, ...N$, we can consider the *prevailing collection of ideas* (at time t) as a program[20] (or algorithm), too. This means the arguments of the function S in (2) are a collection of programs and, thus, *search can be said to be conducted in the space of programs.*

At this point a direct *genetic programming* interpretation of the (computable) search function S makes the dynamical system (2) naturally *evolutionary.* However, the bitstring representing ideas can be retained as the data structures for the programs, partial recursive functions and Turing Machines in (1)~(3). Then, search will be conducted in the space of programs and data structures.

A similar interpretation for (3) is quite straightforward. However, (1) is an entirely different matter. Standard definitions define the arguments of F and K_j and L_j as arguments in G_j on the domain of *real numbers.* Given the algorithmic definitions of H_j and A_j, it is clear that G_j must be a partial recursive function for the whole system (1) – (3) to make algorithmic sense. This means one of two possible resolutions:

[20] Recall that '*[A]n idea is the increment in information that comes from sorting ....*' . In this connection see the illuminative discussion in [6], ch. 4, §1, on 'Skills as Programs' and Bronowski's '**Silliman Lectures**' ([2], ch. 3).

- Either K_j and L_j, $(\forall j = 1, 2, ..., n)$, must be defined as *computable real numbers*; hence, extensive re-definitions of the domain and range of definitions of H_j and A_j from the *computable numbers* to the *computable reals*.
- Or, K_j and L_j defined over the (countable set of) computable numbers.

Either way standard constrained optimization must be replaced either by classical combinatorial optimization or recursion theoretic decision problems, on the one hand; and, on the other hand, one loses the applicability of separating hyperplane theorems[21] and, hence, welfare and efficiency properties of equilibria cannot, in general, be derived by algorithmic methods. These considerations, particularly taking of the second of the above alternative routes, makes it possible to state, as a summary, the following two theorems[22].

Proposition 1 *Given the recursion theoretic interpretation of (2), there is no* ***effective procedure*** *(i.e., no* ***algorithm****) to 'locate' or identify an arbitrary Pareto improving configuration of ideas from the given configuration of initial conditions.*

Proof. The proof of this proposition is based on a simple application of the Rice (or Rice-Shapiro) theorems in classical recursion theory. The dynamical system that is (2), given the recursion theoretic underpinnings to it, can be represented by an appropriate (**Universal**) **Turing Machine** or, equivalently, by an appropriate **Universal Program**. The given initial conditions for the dynamical system (2), corresponds to the initial configurations for a Turing Machine computation or its program equivalent. These initial conditions and configurations correspond, economically, to the *status quo* set of ideas. But by Rice's theorem no nontrivial subset of programs can be **effectively** located by starting from any arbitrary configuration for a Turing Machine. ∎

In other words, there is no a priori local, effective, search procedure – no algorithmic search procedure –that can be used to discover a Pareto-improving set of ideas. Hence, one must resort to satisficing searches – i.e., rely on heuristics (in the senses made clear and famous by Simon).

Proposition 2 *Given an initial, empirically determined, configuration of ideas, represented algorithmically in (2), there is no effective procedure to determine whether S, implemented as a program will* ***Halt*** *(whether at a Pareto-improved configuration or not).*

Proof. The proof of this proposition is an immediate consequence of the **Unsolvability of the Halting Problem for Turing Machines**. The necessary contradiction is obtained by supposing that there is an effective procedure to

[21] More generally, the *Hahn-Banach theorem*.

[22] I shall use two standard results from classical recursion theory in the proofs of the two propositions, below. An excellent source for an exposition of these results is [3].

determine that an empirically given configuration of ideas, used as initial conditions to implement a program for a Turing Machine, will result in a well-defined set of output values. ■

In other words, this proposition suggests that it is impossible to find, by algorithmic means, **definite** answers to questions about the existence of feasible production processes to implement any given set of ideas; only trial and error methods – again, heuristics – can be resorted to, in this computable world, replete with undecidabilities. The latter point can be made more explicit by proving this proposition, alternatively, by exploiting the diophantine properties of recursively enumerable sets and, then, applying the results used in showing the unsolvability of **Hilbert's 10th Problem**.

Remark 3 *The two propositions, together, cast doubts on the 'blessings of the curse of dimensionality to which Romer referred (see above, §2 and [10], pp. 68-9). There are no effective procedures – i.e., algorithms - discoverable a priori and systematically to determine which 'small fraction of the large number of possible mixtures may be valuable." This is why economic development as a planned process, like the evolutionary paradigm itself, is so difficult, bordering on the impossible, to encapsulate in formal growth models, whether endogenous or not.*

I suppose the moral of the algorithmic formulation and the implication of the two propositions are that evolutionary models of growth *à la* Nelson and Winter have been, together with Molière's *M. Jourdain*, speaking prose all along; and Romer is absolutely right, on the basis of his intuitive definitions, to conclude:

- ".....a *trained person* is still the central input in the *process of trial and error*, experimentation, guessing, hypothesis formation, and articulation that ultimately generates a valuable new idea that can be communicated to and used by others." ([10], p.71; italics added).
- "The same arrogance that made people at the turn of the century think that almost everything had already been invented sometimes leads us to think that there is nothing left to discover about the i*nstitutions that can encourage economic development*. .... Just as in a child's chemistry set, *there is far more scope for discovering new institutional arrangements than we can possibly understand*." ([10], pp. 66, 89; italics added).

The question, however, is how to embed Romer's enhanced growth model within an institutional framework that is conducive to development. I suggest that Romer's trained person adds *The Tacit Dimension* ([8] especially, ch.1, pp.1-27; but cf. also [7], especially Part Two), among other things, to his enhanced growth model. Or, as Polanyi may have felicitously summarized, in an imaginary conversation with Romer:

> "*We can know more than we can tell.*"
> [8], p.4; italics in the original.

To that extent the model has to be formally open ended; i.e., with some indeterminacy. However, the indeterminacy is not arbitrary. The above two propositions are an attempt to encapsulate formal indeterminacy in a structured way. Some kind of formal border between what can be known, learned and 'told' - i.e., formally so described - and that which cannot be so described defines the dividing line between the neat and determined world of formal growth models and the messy and evolutionary development process. The skeletal recursion theoretic formalism and interpretation of Romer's *ideas* given above, and the ensuing two propositions, makes it possible to indicate the formal nature of this dividing line. In general, processes that are *recursively enumerable but not recursive* allow the kind of indeterminacy I am suggesting. The proofs of the above two propositions would locate the indeterminate range without actually determining them - to put it somewhat paradoxically.

4 Concluding Notes

It is interesting to note that Paul David ([5]), in a not-unrelated contribution, tackles the broader issue of the role, nature and scope of knowledge in technological change and, hence, in the growth processes of economies. He, too, proposes an interesting dichotomy for the definition of knowledge: *codified* and *tacit* (the latter along lines suggested by Michael Polanyi, op cit[23]). These definitions are also based on an economic setting with an institutional structure that seems to have an algorithmic and evolutionary perspective. Paradoxically, and contrary to received wisdom, it is possible, I believe, to use the notion of *oracle* (or *relative*) *computation*[24] to recursion theoretically formalize tacit knowledge. On the other hand, codified knowledge is straightforward algorithmic knowledge and almost identical, formally, to Romer's concept of idea.

Let me try to explain, in an elementary and heuristic way, the meaning of the above remarks. There is no better way to summarize Polanyi's pioneering attempts to delineate tacit from non-tacit knowledge than his concise but richly evocative statement that '*we can know more than we can tell*'. I have suggested that this statement encapsulates the role of Romer's 'trained person', to whom one must turn to implement production sequences that have somehow been transplanted from one institutional and historical setting to another. There are several interesting examples of such attempts in Romer's essay and I refer the interested reader to peruse it for further enlightenment and explication. The essential point and role of Romer's 'trained person' and David's 'tacit knowledge' is that their expertise cannot be formalized and transplanted; but they are necessary for the operational part of production sequences to function 'efficiently'.

Assume, now, that the 'codified' part has been transplanted in the form of production processes, formalized, as suggested above, recursion theoretically.

[23] See also [6], ch.4, §2.
[24] See [3], ch.10.

An operative, even as part of the formalized production process, may occasionally have to seek the 'trained person's' advice and help on effecting a particular decision at some point in the sequence. How can this role be 'formalized' in the recursion theoretic formalism I have employed above? I believe there is a simple answer although the simplicity belies its combinatorially complex content. The simple answer is to embed the model in its standard recursion theoretic formalism within a framework capable of appealing to an 'oracle' for advice and help, as and when the need arises when nonrecursive problems are encountered.

In other words, as 'codified knowledge' is implemented in the form of transplanted production processes formalized recursion theoretically, the relevant operative will seek the help of the 'trained person' whenever knowledge and skills that are 'known but cannot be told' will be required. This category of knowledge can - and must - include patented knowledge as well. This is almost exactly analogous to a computation process which, from time to time, halts and requests additional, *non-recursive* information before it can proceed. Thus, the rational economic agent as a *Turing Machine* operating or implementing 'codified knowledge' of ideas formalized as 'bit-strings' will, on encountering the need for knowledge that could not or may not be so represented, will appeal to the '*oracle*' for help before proceeding with the computation, decision process and so on. The only non-formal requirement we will have to append here is that which is classically attributed to an *oracle*[25]. Under this interpretation the standard model of oracle or relative computation is more than adequate for the purposes I have in mind. But a full elaboration of these points, in a formally satisfactory way, would require a disproportionate amount of space and, therefore, I reserve it for a different exercise.

An important question for immediate consideration within the framework of the algorithmic formalisms for the Romer (and even the David) model(s) may be that of ensuring *excludability* . Using recursion theoretic cryptographic results, in particular public-key cryptographic methods, it is not difficult to ensure relatively secure excludability. This will circumvent some of the sensitive issues of reverse engineering ([5]), patent violation and so on that bedevil current trading regimes between the developing and the industrially developed economies. But such questions are beyond the scope of the limited exercise attempted in this paper.

[25] To go back to the origins:

> "For the word which I will speak is not mine. I will refer you to a witness who is worthy of credit; that witness shall be the God of Delphi - ..... he is a god, and cannot lie; that would be against his nature."

Apology: The dialogues of Plato (Jowett's translation., 3rd ed., Vol. II, p.113; italics added)

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