

Real Exchange Rate Overshooting in Real Business Cycle and Overlapping Wage Contract Model

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Abstract

The objective of this paper is to establish the ability of a Real Business Cycle (RBC) model to account for the behaviour of the real exchange rate, using Indian data. We calibrate the dynamic general equilibrium open economy model (Minford, Sofat 2004) based on optimising decisions of rational agents, using annual data for India. The first order conditions from the households' and firms' optimisation problem are used to derive the behavioural equations of the model. The interaction with the rest of the world comes in the form of uncovered real interest rate parity and current account both of which are explicitly micro-founded. The paper discusses simulation results of 1 percent productivity growth shock, which shows the real exchange rate appreciates and then goes back to a new equilibrium (lower than the previous one), producing a business cycle. Thus the behaviour of the real exchange rate may be explicable within the RBC context. Finally we test our model and evaluate statistically whether our calibrated model is seriously consistent with the real exchange rate data, using bootstrapping procedure. We bootstrap our model to generate pseudo real exchange rate series and find that the ARIMA parameters estimated for the data lie within the 95% confidence limits constructed by bootstrapping. Then we follow the same

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procedure for the overlapping wage contract model (We add an overlapping wage contract equation to the RBC model of Minford, Sofat (2004) to introduce nominal rigidity). We find the same result for the nominal rigidity version of the RBC model that the ARIMA parameters estimated for the actual data lie within the 95% confidence limits constructed by bootstrapping. So we conclude that the behaviour of the Indian real exchange rate (US \$ / Indian Rupees) can be explained within an RBC as well as overlapping wage (OLW) contract model.

1.Introduction

In this paper we explore the ability of a Real Business Cycle (RBC) model to account for the behaviour of the real exchange rate using Indian data. Our argument is that RBC can reproduce the univariate properties of the real exchange rate. A large number of studies have examined movements in the real exchange rate and found that they exhibit swings away from various definitions of purchasing power parity. Many studies have found evidence of reversion to PPP but very slow reversion. One commonly used model to explain the behaviour of the real exchange rate is the Balassa Samuelson model based on differing productivity trends. Suppose a country experiences productivity growth primarily in its traded goods sector and the law of one price (LOP) holds among traded goods and the nominal exchange rate remains constant. Productivity growth in traded goods sector will lead to wage rises in that sector without necessity for price rises. Hence traded goods prices can remain constant and LOP can continue to hold with the unchanged nominal exchange rate. But workers in the non-traded goods sector will also demand comparable pay rises and this will lead to a rise in the price of non-tradeables and hence an overall rise in the consumer price index (CPI). Since the LOP holds among traded goods, and by assumption, the nominal exchange rate has remained constant, this means the upward movement in domestic CPI will not be matched by a movement in the nominal exchange rates so that, if PPP initially held, the domestic currency must now appear overvalued on the basis of comparison made using CPI expressed in a common currency at the prevailing exchange rate. The important assumption is that productivity growth is much higher in the traded goods sector. Relative price of non-tradeables may rise even in the case of balanced growth of the two sectors of the economy, as long as the non-traded goods sector is more labour intensive relative to the

traded goods sector. The Harrod Balassa Samuelson condition is that relatively higher productivity growth in the tradeables sector will tend to generate a rise in relative price of non-tradeables. The percentage change in the relative price of non-tradeables is determined only by production side of the economy, while the demand factors do not affect the real exchange rate in the long run. If the degree of capital intensity is the same across the traded and non-traded sectors, then the percentage change in relative prices is exactly equal to the productivity differential between the two sectors. If the non-traded sector is less capital intensive than the traded sector, then even in the situation of balanced productivity growth in the two sectors, the relative price of non-tradeables will rise.

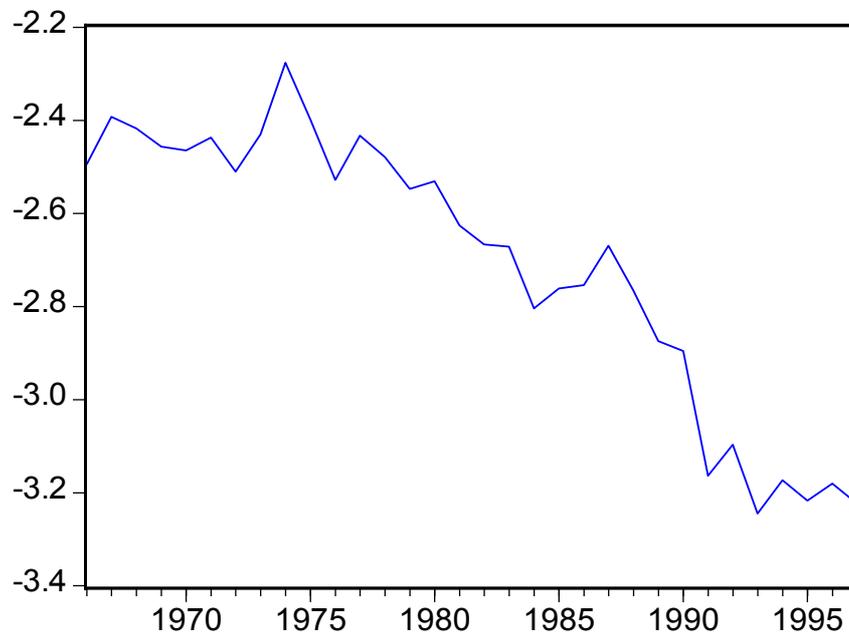
In other words, the Harrod Balassa Samuelson model suggests that the long run equilibrium real exchange rate should depend on the productivity of tradeables and non-tradeable sectors in home and foreign economies. Given perfect labour mobility, changes in relative productivity across sectors lead to changes in relative prices. Since technological innovation is most likely to be concentrated in the tradeable goods sector, countries with higher long-run growth rates should have higher relative prices of non-tradeable goods as well as higher valued currencies.

More recently studies that have allowed for non-linear adjustment have found that the speed of reversion is much greater and becomes of similar order to that for other macro variables such as output and inflation.

We begin by looking at the empirical evidence on the Indian real exchange rate (US \$ / Rs) for the period 1966 –1997. The univariate final form equation is best described by an ARIMA(1,1,1) process. The series is therefore not actually mean-reverting but

integrated of order 1. Our main aim is to see whether our calibrated RBC model can generate the same univariate behaviour.

FIGURE 1. Bilateral Real Exchange Rate between India and US (US Dollar/ Indian Rupees)



The paper develops as followed. Section 2 describes the model, section 3 describes the model solution, algorithm, steady state equations and descriptions of the data. Section 4 describes simulation of the RBC model. Section 5 explains the significance of an overlapping wage (OLW) contract in a rational expectation framework and also the simulation of the OLW model. Section 6 describes the data pattern and bootstrapping and the final section concludes.

2. Model

In this section we describe the characteristic features of an open economy RBC model as developed by Minford and Sofat (2004). This is important for our purpose as this

helps us to generate the structural equations of the model to be calibrated for the Indian economy.

Consider an economy populated by identical infinitely lived agents who produce a single good as output and use it both for consumption and investment. We assume that money is irrelevant in this model. To simplify the notation we abstract from population growth and represent all variables in per capita terms. We assume that there are no market imperfections i.e. no frictions or transaction costs. At the beginning of each period t , the representative agent chooses a) the commodity bundle necessary for consumption during the period, b) the total amount of leisure that she would likely to enjoy during the period and c) the total amount of factor inputs necessary to carry out production during the period. All of these choices are constrained by fixed amount of time available and the aggregate resource constraint that agents face. During the period t , the model economy is influenced by various random shocks. In an open economy goods can be traded but for simplicity it is assumed that these do not enter in the production process but are only exchanged as final goods. The consumption, C_t in the utility function below is composite per capita consumption, made up of agents' consumption of domestic goods, C_t^d and their consumption of imported goods C_t^f . The composite consumption function can be represented as an Armington aggregator of the form

$$C_t = \left[\omega (C_t^d)^{-\rho} + (1 - \omega) (C_t^f)^{-\rho} \right]^{\frac{-1}{\rho}} \quad (1)$$

where ω is the weight of home goods in the consumption function and σ , the elasticity of substitution is equal to $\frac{1}{1 + \rho}$

The consumption-based price index that corresponds to the above specification of preference, denoted P_t is derived as

$$P_t = \left[\omega^{\frac{1}{1+\rho}} (P_t^d)^{\frac{\rho}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}} (P_t^F)^{\frac{\rho}{1+\rho}} \right] \quad (2)$$

where P_t^d is domestic price level and P_t^F is the foreign price level in domestic currency

Given the specification of the consumption basket, the agent's demand for home and foreign goods are a function of their respective relative price and composite consumption

$$C_t^d = \left(\frac{P_t^d}{\omega P_t} \right)^{-\left(\frac{1}{1+\rho}\right)} C_t \quad (3)$$

$$C_t^f = \left(\frac{P_t^F}{(1-\omega)P_t} \right)^{-\left(\frac{1}{1+\rho}\right)} C_t \quad (4)$$

In a stochastic environment a consumer is expected to maximise her expected utility subject to her budget constraint. Each agent's preferences are given by

$$U = \text{Max} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right] \quad 0 < \beta < 1 \quad (5)$$

where β is the discount factor, C_t is consumption in period t , L_t is the amount of leisure time consumed in period t and E_0 is the mathematical expectational operator.

The essential feature of this structure is that agent's tastes are assumed to be constant over time and is not influenced by exogenous stochastic shocks. The preference ordering of consumption sub-sequences $[(C_t, L_t), (C_{t+1}, L_{t+1}), \dots]$ does not depend on t or on consumption prior to time t . We assume that $u(C, L)$ is increasing in (C, L) and concave $u'(C, L) > 0$, $u''(C, L) < 0$. We also assume that $u(C, L)$ satisfies Inada type conditions: $u'(C, L) \rightarrow \infty$ as $c \rightarrow 0$ and $u'(C, L) \rightarrow 0$ as $c \rightarrow \infty$, $u'(C, L) \rightarrow \infty$ as $l \rightarrow 0$ and $u'(C, L) \rightarrow 0$ as $l \rightarrow \infty$

2.1 The Representative Household

The model economy is populated by a large number of identical households who make consumption, investment and labour supply decisions over time. Each household's objective is to choose sequence of consumption and hours of leisure that maximises its expected discounted stream of utility. We assume a time separable utility function of the form

$$U(C_t, 1 - N_t) = \theta_0 (1 - \rho_0)^{-1} C_t^{(1-\rho_0)} + (1 - \theta_0) (1 - \rho_2)^{-1} (1 - N_t)^{(1-\rho_2)} \quad (6)$$

where $0 < \theta_0 < 1$ and $\rho_0, \rho_2 > 0$ are substitution parameters.

This sort of functional form is used for example by McCallum and Nelson (1999a).

The advantage of using this specification is that it does not restrict elasticity of substitution between consumption and leisure to unity. Barro and King (1984) note that time-separable preference ordering of this form would not restrict the sizes of intertemporal substitution effects. However time separability constrains the relative size of various responses such as those of leisure and consumption to relative price and income effects.

Individual economic agents view themselves as playing a dynamic stochastic game.

Changes in expectation about future events would generally affect current decisions.

Individual choices at any given point of time are likely to be influenced by what agents believe would be their available opportunity set in the future. Each agent in our model is endowed with a fixed amount of time, which she spends on leisure L_t and / or work N_t . If total endowment of time is normalised to unity, then it follows that $N_t + L_t = 1$ or $L_t = 1 - N_t$ (7)

Let us assume \bar{l} is the normal amount of leisure which is necessary for an agent to sustain her productivity over a period of time. If an agent prefers more than normal

amount of leisure say U_t , she is assumed to be unemployed ($U_t = (1 - N_t) - \bar{l}$) in this framework. An agent who chooses U_t is entitled to get an unemployment benefit μ_t . It is assumed that $\mu_t < v_t$ (i.e. the consumer real wage as defined below) so that there is an incentive for the agent to search for a job. With the introduction of unemployment benefit substitution between work and leisure is higher.

The representative agent's budget constraint is

$$(1 + \phi_t)C_t + \frac{b_{t+1}}{1 + r_t} + \frac{Q_t b_{t+1}^f}{1 + r_t^f} + \frac{p_t S_t^p}{P_t} = (1 - \tau_{t-1})v_{t-1}N_{t-1} + \mu_{t-1} \left[(1 - N_{t-1}) - \bar{l} \right] + b_t + Q_t b_t^f + \frac{(p_t + d_t)S_t^p}{P_t} \quad (8)$$

where p_t denotes present value of share, $v_t = \frac{W_t}{P_t}$ is real consumer wage, $w_t = \frac{W_t}{P_t^d}$ is

producer real wage. Consumption and labour income are taxed at rates ϕ_t and τ_t respectively, both of which are assumed to be stochastic process. Also

$(1 + \phi_t)C_t^d = \frac{M_t^{dp}}{P_t^d}$ that is representative agent's real demand for domestic money is

equal to consumption of domestic goods inclusive of sales tax. In a similar way, the agent's real demand for foreign money is equal to consumption of foreign goods

inclusive of sales tax $(1 + \phi_t)C_t^f = \frac{M_t^{Fp}}{P_t^F}$ This follows from the fact that consumption

in this framework is treated as a cash good i.e. cash-in-advance constraint is binding only in the case of consumption. Investment is treated as a credit good. b_t^f denotes foreign bonds, b_t domestic bonds, S_t^p demand for domestic shares and Q_t is the real exchange rate.

In a stochastic environment the representative agent maximises her expected discounted stream of utility subject to her budget constraint. The Lagrangian associated with this problem is

$$U = \text{Max} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\theta_0 (1 - \rho_0)^{-1} C_t^{(1-\rho_0)} + (1 - \theta_0) (1 - \rho_2)^{-1} (1 - N_t)^{(1-\rho_2)} \right] \right\} +$$

$$\lambda_t \left\{ (1 - \tau_{t-1}) w_{t-1} N_{t-1} + \mu_{t-1} \left[(1 - N_{t-1}) - \bar{l} \right] + b_t + Q_t b_t^f + \frac{(p_t + d_t) S_t^p}{P_t} - (1 + \phi_t) C_t \right.$$

$$\left. - \frac{b_{t+1}}{1 + r_t} - \frac{Q_t b_{t+1}^f}{(1 + r_t^f)(1 - rp)} - \frac{p_t S_t^p}{P_t} \right\}$$

.....(9)

Where λ is Lagrangian multiplier, $0 < \beta < 1$ is the discount factor and $E(\cdot)$ is the mathematical expectations operator.

First order conditions of household's optimisation problem are given in the appendix

2.2 The Representative Firm

Firms rent labour and buy capital inputs from households and transform them into output according to a production technology and sell consumption and investment goods to households and government. The interaction between firms and household is crucial, as it provides valuable insights for understanding the fluctuations of macroeconomic aggregates such as output, consumption and employment.

The technology available to the economy is described by a constant returns to scale production function:

$$Y_t = Z_t f(N_t, K_t)$$

$$Y_t = Z_t N_t^\alpha K_t^{1-\alpha}$$

where $0 \leq \alpha \leq 1$, Y_t is aggregate output per capita, K_t is capital carried over from previous period (t-1), and Z_t reflects the state of technology.

It is assumed that $f(N,K)$ is smooth and concave and it satisfies Inada-type conditions i.e., the marginal product of capital (or labour) approaches infinity as capital (or labour) goes to 0 and approaches 0 as capital (or labour) goes to infinity.

The capital stock evolves according to $K_{t+1} = (1 - \delta)K_t + I_t$

Where δ is the depreciation rate and I_t is the gross investment. In a stochastic environment the firm maximises present discounted stream, V , of cash flows, subject to the constant returns to scale production technology, i.e.

$$\text{Max} V = E_t \sum_{i=0}^T d_{it}^i (Y_t - K_t(r_t + \delta) - w_t N_t^d) \quad \text{subject to } Y_t = Z_t f(N_t, K_t). \text{ Here } r_t \text{ and}$$

w_t are rental rates of capital and labour inputs used by the firm, both of which are taken as given by the firm. Output of the firm depends not only on capital and labour inputs but also on Z_t . First order conditions of the firm's optimisation problem are given in the appendix. The relevant equations about the government and the foreign sector are also given in the appendix.

2.3 Behavioural equations of the model

First order conditions from the household's and firm's optimisation problem are used to derive the following behavioural equations of the model.

i) Consumption C_t ; solves for r_t

$$(1+r_t) = \frac{1}{\beta} \left(\frac{C_t}{E_t[C_{t+1}]} \right)^{-\rho_0} \left(\frac{1+\phi_{t+1}}{1+\phi_t} \right)$$

$$r_t = \frac{1}{\beta} \left(\frac{C_t}{E_t[C_{t+1}]} \right)^{-\rho_0} \left(\frac{1+\phi_{t+1}}{1+\phi_t} \right) - 1$$

where $C_t = \left[\omega (C_t^d)^{-\rho} + (1-\omega) (C_t^f)^{-\rho} \right]^{\frac{-1}{\rho}}$

ii) Money supply \bar{M}_t^d ; solves for P_t^d

$$\bar{M}_t^d = (1+\phi_t) C_t^d P_t^d + \bar{G}_t P_t^d$$

$$P_t^d = \frac{\bar{M}_t^d}{(1+\phi) C_t^d + \bar{G}_t}$$

where $P_t = \left[\omega^{\frac{1}{1+\rho}} (P_t^d)^{\frac{\rho}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}} (P_t^f)^{\frac{\rho}{1+\rho}} \right]$

iii) Demand for shares S_{t+1}^p

$$S_{t+1}^p = \bar{S}_t ; \quad b_{t+1} = b_{t+1}^p \text{ implied}$$

iv) Present value of share

$$p_t = E_t \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1+r_t)^i} \left(\frac{P_t}{P_{t+i}} \right)$$

v) Production function $Y_t = Z_t N_t^\alpha K_t^{(1-\alpha)}$

vi) Demand for labour

$$N_t^d = \left(\frac{\alpha Z_t}{w_t} \right)^{\left(\frac{1}{1-\alpha} \right)} K_t$$

vii) Capital

$$K_{t+1} = (1-\alpha) \frac{Y_t}{r_t + \delta}$$

viii) GDP identity, Y_t ; solves for C_t

$$Y_t = C_t + I_t + G_t + NX_t$$

Where NX_t is net exports

ix) Investment

$$K_{t+1} = (1-\delta)K_t + I_{t+1}$$

x) Wage w_t :

$$w_t = w_t^*$$

xi) Evolution of b_t ; government budget constraint:

$$b_{t+1} = (1+r_t)b_t + PD_t - \frac{\Delta \bar{M}_t}{P_t}$$

xii) Equilibrium wage, w_t^* ; w_t^* is derived by equating demand for labour, N_t^d , to the supply of labour N_t^s , where

$$(1-N_t^s) = \left\{ \frac{\theta_0 C_t^{-\rho_0} \left[(1-\tau_t) \exp \left(\log w_t^* - \frac{(1-\omega)^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}}} \log Q_t \right) - \mu_t \right]}{(1-\theta_0)(1+\phi_t)(1+r_t)} \right\}^{\frac{-1}{\rho_2}}$$

where Q_t is the real exchange rate, $(1-\omega)^{\frac{1}{1+\rho}}$ is the weight of domestic prices in the CPI index.

xiii) Dividends are surplus corporate cash flow:

$$d_t \bar{S}_t = Y_t - N_t^s w_t - K_t (r_t + \delta)$$

$$d_t = \frac{Y_t - N_t^s w_t - K_t (r_t + \delta)}{\bar{S}_t}$$

xiv) Primary deficit PD_t

$$PD_t = G_t + \mu_t \left(1 - N_t^s - \bar{l} \right) - \tau_{t-1} v_{t-1} N_{t-1}^s - \phi_{t-1} C_{t-1} - T_{t-1}$$

xv) Tax T_t

$$T_t = T_{t-1} + \gamma^G (PD_{t-1} + b_t r_t) + \varepsilon_t$$

xvi) Exports

$$\log EX_t = \sigma_1 \log(1 - \omega^f) + \log C_t^F + \sigma_1 A^f \log Q_t$$

$$\text{where } A^f = \frac{(\omega^f)^{\frac{1}{1+\rho}}}{(\omega^f)^{\frac{1}{1+\rho}} + (1 - \omega^f)^{\frac{1}{1+\rho}}}$$

xvii) Imports IM_t

$$\log IM_t = \sigma \log(1 - \omega) + \log C_t - \sigma A \log Q_t$$

$$\text{where } A = \frac{\omega^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}}}$$

xviii) UIP condition

$$r_t = r_t^f + E_t \Delta \log Q_{t+1} + \varepsilon_{UIP}$$

where r^f is the foreign real interest rate

xix) Net exports

$$NX_t = EX_t - IM_t$$

xx) Evolution of foreign bonds b_t^f

$$b_{t+1}^f = (1 + r_t^f) b_t^f + NX_t$$

xxi) Nominal exchange rate, S_t

$$\log S_t = \log Q_t - \log P_t^f + \log P_t$$

xxii) Evolution of household debt D_{t+1}

$$D_{t+1} = (1 + r_t) D_t - Y_{t-1} + (1 + \phi_t) C_t + \tau_t v_t N_t^s + T_t$$

xxiii) Household transversality condition

$$Y_{T-1} - r_T D_T - \phi_T C_T - \tau_T v_T N_T^s - T_T = C_T$$

Exogenous processes in the RBC model

1) $\Delta \ln Z_t = \varepsilon_{1,t}$

2) $\Delta \tau_t = \varepsilon_{2,t}$

3) $\Delta \phi_t = \varepsilon_{3,t}$

4) $\Delta \mu_t = \varepsilon_{4,t}$

5) $\Delta \ln \bar{M}_t = \varepsilon_{5,t}$

6) $\Delta \ln P_t^f = \varepsilon_{6,t}$

7) $\Delta \ln C_t^F = \varepsilon_{7,t}$

8) $\Delta \ln r_t^f = \varepsilon_{8,t}$

Initial value of ε_i

ε_i are parameters for defining exogenous random processes ;

| | |
|-----------------|---|
| ε_1 | 1 |
| ε_2 | 0 |
| ε_3 | 0 |
| ε_4 | 0 |

| | |
|-----------------|---|
| ε_5 | 1 |
| ε_6 | 0 |

Values of Coefficients

| | | |
|--|------------|--------|
| Output elasticity of production | α | 0.65 |
| Discount factor | β | 0.97 |
| Depreciation rate | δ | 0.0125 |
| Fraction of elasticity of goods substitution | ρ_0 | 1.20 |
| Money multiplier | θ_0 | 0.50 |
| Degree of seignorage | γ^G | 0.05 |
| Fraction of elasticity of goods substitution | ρ_2 | 1.00 |
| Weight of home goods in consumption function | ω | 0.70 |
| Fraction of elasticity of | ρ | -0.50 |

| | | |
|---|------------|-------|
| goods substitution | | |
| Weight of foreign goods in consumption function | ω^f | 0.70 |
| RER sensitivity to demand for labour | h | 0.80 |
| Fraction of elasticity of goods substitution | ρ_3 | -0.50 |
| Elasticity of import substitution | σ | 2.0 |
| Elasticity of export substitution | σ_1 | 2.0 |

Note: Values of the coefficients used in the model have been calibrated from the recent literature.

3.1 CALIBRATION

Having explained the structural equations, we can now proceed to calibration for the Indian economy. In doing so, numerical values need to be assigned to the structural parameters of the models. These values such as the output elasticity of the production factors, the degree of risk aversion or the elasticity of intertemporal substitution are taken from some casual empirical characteristics pertaining to the Indian economy. The exogenous stochastic processes should also be calibrated. However it is hard to find information from a real economy concerning the stochastic structure of technology shocks, shocks to preferences, error of controlling money growth or tax revenues or the correlations among them. For this purpose, persistence properties in actual time series data can be used to calibrate some aspects of the model. For instance, in the simplest business cycle model, an AR(1) model is assumed for productivity shocks, with the coefficients generally chosen so that the simulated output series exhibits persistence similar to GNP series in actual economies. A quite different strategy seeks to use the simulated time series to estimate some or all structural parameters through a formal method like the Maximum Likelihood Method (MLM). These more standard econometric procedures choose values for all

parameters by optimising a given criterion- the likelihood of the data, in the case of MLM. This procedure has two main advantages. It avoids possibly arbitrary selection of parameter values and it provides a measure of dispersion that can be used to evaluate the goodness of fit of model to data.

As initial starting values, the parameters in models have been calibrated to match historical data. This method has proven to produce low bias.

3.2 Model Solution and Algorithm

In solving our model, we are forced by its complexity and non-linearity to use a computer algorithm. A well-behaved rational expectations model has a unique solution. To obtain this solution, the solving procedure sets the terminal condition that beyond some terminal date N , all the expectational variables are set to their equilibrium values. It is necessary for the terminal date to be large, in order to reduce the sensitivity of the model to the variations in the terminal date. The justification for the terminal condition is that non-convergent behaviour of the system would provoke behaviour by government or other economic agents (transversality condition) different from that assumed as normal in the model and that would eliminate the divergence.(Minford, 1979). As pointed out by Matthews and Marwaha (1979), the actual value of the terminal condition can be derived from the long run equilibrium condition of the model. In some cases, the steady-state properties of the model can be used to choose the terminal conditions of the model, although several other methods can be easily used.

There are several iterative methods, but the most common is the Gauss-Seidel method. This iterative method is built in the program developed by Matthews (1979) and Minford et al (1984) called RATEXP which has been used to get the model solution. The computer program typically uses a backward –solving (dynamic

programming) technique. However, unlike the classical dynamic programming, the solution vector is approached simultaneously for all $t = 1, 2, \dots, T$, but convergence follows a backward process. The problem lies in that the model must firstly obtain a dynamic solution for a given time span using initial guess values of the expectational variables. These initial values are then adjusted in an iterative manner until convergence is obtained. After checking for equality between expectations and solved forecasts, the initial expectations set is gradually altered until convergence is obtained. In effect this endogenises the expectational variables in that period. Our model is highly non-linear, consequently a larger number of iterations are required as compared to linear models. It should be noted that in general a non-linear model does not have a unique reduced form. When a non-linear model is solved in a deterministic manner the solution values of the endogenous variables are not in general equal to their expected values. A correct solution requires stochastic simulation.

In order to understand how the algorithm works, consider a set of simultaneous non-linear structural equations written in implicit form

$$F\{y(t), y(t-1), x(t), u(t)\} = 0 \quad \dots(i)$$

Where, $y(t)$ is a vector of endogenous variables, $y(t-1)$ is a vector of lagged endogenous variables, $x(t)$ is a vector of exogenous variables and $u(t)$ is a vector of stochastic shocks with mean zero and constant variances. $F(.)$ represents a set of functional form. Setting the disturbance terms equal to their expected values and solving for the reduced form, we have

$$y_t = H\{x(t), y(t-1)\} \dots(ii)$$

where $H(.)$ is the reduced form functional form. Partitioning equation (ii) so as to distinguish between endogenous variables on which expectations are formed $y(2)$ and the others $y(1)$, we have

$$y_1(t) = h_1\{x(t), y_1(t-1)\} \quad \dots(iia)$$

$$y_2(t) = h_2\{x(t), y_2(t-1), E[y_2(t+j)/t]\} \dots \text{(iiib)}$$

where $E[y_2(t+j)/t]$ denotes the rational expectation of y_2 formed in period $t+j$ based on information available at t . Our program uses starting values for the vector $E[y_2(t+j)/t]$ which, together with values for the fully exogenous variables, are assumed to extend over the whole solution period. The algorithm ensures that the expectational values stored in the vector $E[y_2(t+j)/t]$ converge to the values predicted by the model for y_2 in period $t+j$.

For simplicity, let us assume that the solution period extends from $t = 1, \dots, T$ and that expectations are formed for one period ahead only.

The convergence of the expectational values towards the model's predicted values follows a Jacobi algorithm, which can be described as

$$E[y_2(t, k+1)/t-1] = E[y_2(t, k)/t-1] + q\{y_2(t, k)/t-1\} \dots \text{(iv)}$$

$$0 < q < 1, t = 1, 2, \dots, T$$

for the k^{th} iteration, with the objective of minimising the residual vector $R(t)$, defined as

$$R(t) = \text{abs}\{y_2(t) - E[y_2(t)/t-1]\} < L, \quad t = 1, 2, \dots, T \dots \text{(v)}$$

where q is the step length and L is some pre-assigned tolerance level.

Since $E[y_2(t)/t-1]$ is stored in period $t-1$, the end period expectational variable remains undetermined. We require a value for $y_2(T+1)$ which lies outside the domain of the solution period. The technique used in our program consists of imposing a set of terminal conditions on the rationally expected variables. In a rational expectations model, the forward expectations terms tend to induce unstable roots. The use of terminal conditions has the effect of setting the starting values of the unstable roots to zero asymptotically, thereby ruling out unstable paths.

In sum, the complete algorithm can be described in the following series of steps

Step 1. Solve the model given initial values for the expectational variables.

Step 2. Check for convergence.

Step 3. Adjust expectational variables

Step 4. Re-solve the model given new iterated values of the expectational variables.

3.3 Steady State Equations of the Model

The steady state of an economy is its rest point when the variances of all shocks are zero and the levels of consumption, labour, stock of capital and inventories are constant. The study of steady state is important as it characterises the long run features of the economy. The steady state equations of the model are given below.

$$\beta = \frac{1}{1+r_t}$$

$$\bar{M} = (1+\phi)CP + GP$$

$$p = \frac{d}{1+r}$$

$$Y = ZN^\alpha K^{(1-\alpha)}$$

$$N = \frac{\alpha Y}{w^*}$$

$$K = \frac{(1-\alpha)Y}{r+\delta}$$

$$Y = C + I + G + NX$$

$$I = \delta K$$

$$w = w^*$$

$$rb = -PD + \frac{\Delta M^d}{P^d}$$

$$1 - N = \left(\frac{\theta_0 C^{-\rho_0} \left[(1 - \tau) \exp \left(\log w^* - \frac{(1 - \omega)^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}}} \log Q \right) - \mu \right]}{(1 - \theta_0)(1 + \phi)(1 + r)} \right)^{\frac{1}{\rho_2}}$$

$$d = \frac{Y - w^* N - (r + \delta)K}{\bar{S}}$$

$$PD = G + \mu \left(1 - N - \bar{l} \right) + \pi^d M^d - \tau v N - \phi C - T$$

$$\log EX = \sigma_1 \log(1 - \omega^f) + \log C^F + \sigma_1 \left(\frac{(\omega^f)^{\frac{1}{1+\rho}}}{(\omega^f)^{\frac{1}{1+\rho}} + (1 - \omega^f)^{\frac{1}{1+\rho}}} \right) \log Q$$

$$\log IM = \sigma \log(1 - \omega) + \log C - \sigma \left(\frac{\omega^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}}} \right) \log Q$$

$$r = r^f + \Delta \log Q$$

$$r^f b^f = NX$$

$$\log S = \log Q - \log P^f + \log P$$

$$Y - (1 + \phi)C - \tau N - T = rD$$

3.4 DESCRIPTION OF DATA

Data on Indian interest rate (money market rate), domestic price level (consumer price index), output (gross domestic product), capital, consumption, investment, government expenditure, primary deficit, exports, imports, and nominal exchange rates have been taken from International financial Statistics (IFS). Data on real exchange rate have been generated by using the following equation.

$$\log S = \log Q - \log P^f + \log P$$

Data on labour supply have been collected from Economic Survey of India. The source for the wage data (manufacturing) is the United Nations Yearbook. We divide the wage data by consumers' price index (obtained from IFS) to get consumers' real wage and by producers' price index (obtained from IFS) to get producers' real wage. We subtract government revenue (obtained from IFS) from government expenditure (obtained from IFS) to get data on domestic bond. Data on foreign bonds have been generated by using the foreign bond evolution equation. Data on lump sum tax, labour income tax and consumption tax are not easily available. So we take the ratio of tax revenues to non-agricultural GDP to get a proxy for the lump sum tax as agricultural income is tax free in India. We use the ratio of revenues from personal income tax (obtained from Economic Survey of India) to non-agricultural GDP (obtained from

Economic Survey of India) as a proxy for the labour income tax. We take the ratio of indirect tax revenue (obtained from Economic Survey of India) to consumption (obtained from Economic Survey of India) as the proxy for consumption tax. In India unemployed people do not get any unemployment benefit from the government. They stay with their families. We assume that a family spends 20% of the consumer real wage for an unemployed person (as we assume that 20% of the consumer real wage is sufficient for an unemployed person to live at subsistence level in India.) So we take 20% of the consumer real wage as the proxy for unemployment benefit. The data on household debt have been generated by using the household debt evolution equation. Productivity is calculated as solow residual. Data of foreign (US) consumption, foreign (US) interest rate (Federal Fund Rate) and foreign (US) price (Consumer price index) have been taken from IFS.

4.1 SIMULATIONS

Once the model has been solved numerically, one can analyse the characteristics of the transition of the model to its steady state. This may arise either because initially the economy is outside steady state or because some structural change is introduced (it could be a policy intervention) altering the steady state. This type of analysis is crucial to evaluate the possible effects of change in policy rules, i.e. of policy interventions and to assess the overall properties of the model.

Standard simulation methods consist of comparing the solution of the model with one where one or more of the exogenous variables are perturbed. Comparing the base and the perturbed solutions gives an estimate of the policy multipliers if the exogenous variables perturbed is a policy instrument. In other words, comparing the results of the simulation experiments with those obtained in the base run provides valuable information regarding the effects of policy changes on the economy.

There is also question of selection of the length of the simulation period. The period should be long enough for the effect of changes to work through the model. This is especially important in models which contain long lags or slow rate of adjustment.

Darby et al. (1999) lists two advantages of having a long simulation period. First, when solving non-linear rational expectation models it is important to ensure that the terminal date for the simulation is sufficiently far in the future so that the simulation is unaffected by the choice of the terminal date. Second, simulating the model over a long period makes it easier to observe the long run solution of the model.

Our simulation starts in 1968 and end in 1997 using annual Indian data. Results of our simulation exercise are reported in graphical form. The graphs show the percentage deviation of a particular variable-real output, price level and so on- from the base line path.

4.2 Results

The effects of both demand and supply shocks on the behaviour of output, consumption, capital stock, investment, employment, price level, real wage, real interest rate, imports, exports and real exchange rate is examined by deterministically simulating the calibrated model using the extended path method discussed earlier.

In addition to providing quantitative input to policy analysis these deterministic simulations provide useful insights into the dynamic properties of the model. For the baseline simulation- that is, the simulation with no change in policy instruments-the endogenous variables are set so as to track the actual historical values perfectly. This is done by adding residual to each equation. The residuals are computed using the future expectations of the endogenous variables generated by the model using an overlapping forecast.

4.3 Calibration and simulation of the RBC model

1% per annum productivity growth shock

Our simulations start in 1968 and end in 1997 using annual Indian data. We begin by running a deterministic productivity growth shock through our model calibrated to annual data for India. This deterministic simulation is done in order to establish the basic order of magnitude and shape of the response function of the real exchange rate to the workhorse RBC technology shock.

The response profile is attractive in exhibiting a pronounced cycle. We consider a deterministic productivity growth shock - a 5-year rise of productivity growth rate by 1% per annum. Productivity grows at 1% in the first year, 2% in the second year, 3% in the third year and so on till the fifth year when it grows at 5%. After that it is permanently 5% above the base. Results of our simulation exercise are reported in graphical form in figure 2.

The productivity growth raises income and stimulates a stream of investments to raise the capital stock in line. Output cannot be increased without an increase in labour supply and capital which takes time. Thus the real interest rate must rise to reduce demand to the available supply. The rising real interest rate violates uncovered real interest parity (URIP) which must be restored by a rise in the real exchange rate

relative to its expected future value. This rise is made possible by the expectation that real exchange rate will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival of stream of sufficient capital and so output, the real exchange rate also moves back to equilibrium. This gives us a business cycle in the real exchange rate. This new equilibrium represents a real depreciation on the previous steady state since output is now higher and must be sold on world markets by lowering its price.

In the next section we explain the significance of an overlapping wage contract in a rational expectation framework and add an overlapping wage contract equation to the RBC model (Minford, Sofat 2004) and run the simulations for a deterministic productivity growth shock for the overlapping wage (OLW) contract model.

5.1 Significance of an overlapping wage contract in a rational expectation framework

According to Friedman, suppliers of labour at the beginning of an inflationary period underestimate the price level that will prevail over the period of the work contract, accordingly overestimate the real wage and offer a greater supply of labour at the prevailing nominal wage than they would if expectations were correct. The result is employment in excess of equilibrium level and a trade-off between output and unanticipated inflation. However the expectational errors cannot persist so that employment returns to its equilibrium level and unemployment returns to its natural rate as expectations adjust to reality.

The widespread use of adaptive expectations suggested that an ever-accelerating rate of inflation could maintain an unemployment rate below the natural rate. The accelerationist version of natural rate hypothesis had two important consequences.

First, by making the short run trade-off depend on expectational errors it brought to the fore the question of the optimality of the natural rate. Second, the reliance of the accelerationist hypothesis on expectational errors made it possible that some expectations mechanism other than adaptive expectations would imply that there is no trade-off usable by policy makers. Rational expectation is that hypothesis.

Under rational expectations, the expected price level will change in a manner dependent on the monetary supply rule. If monetary policy accommodates inflationary shocks, the expected price level will rise. If monetary policy counteracts inflationary shocks, the expected price level may be lower than the level expected for this period. The only way in which monetary policy can affect output is by creating a difference between the actual price level and the expected price level. If monetary authority has superior information to private economic agents, say because it receives data more rapidly than they do, it can affect the behaviour of output.

One of the assumptions required for anticipated monetary policy to have no effect on output in a rational expectation framework is that agents are able to act on their information set. If private agents cannot respond to new information by changing their consumption, wage-price decisions, etc., as quickly as the public sector can change any (at least one) of its controls, then scope emerges for systematic stabilisation policy to have real effects. This insight was developed by Fischer (1977a, b) and Phelps and Taylor (1977) in the context of multi-period non-contingent wage or price contracts.

Suppose all wage contracts run for two periods and the contract drawn in period t specifies nominal wages for period $t + 1$ and $t + 2$ (in the manner of Fischer). At each period of time, half the labour force is covered by a pre-existing contract. As long as the contracts are not contingent on new information that accrues during the contract period, this creates the possibility of stabilisation policy. Firms respond to changes in

their environment like unpredictable changes in demand which were unanticipated at the time of pre-existing contract, by altering output and employment at the pre-contracted wage. Only contracts which are up for renewal can reflect prevailing information. If the monetary authorities can respond to new information that has accrued between the time two-period contract is drawn up and the last period of operation of the contract, then systematic stabilisation policy is possible. The overlapping wage contract equation which we add to the RBC model (Minford, Sofat 2004) is the following.

$$\log w_t = \frac{1}{2}(E_{t-1} \log w_t^* + E_{t-2} \log w_t^*) - \frac{1}{2}(\log P_t - E_{t-1} \log P_t) - \frac{1}{2}(\log P_t - E_{t-2} \log P_t)$$

5.2 Calibration and Simulation of the OLW model

1% per annum productivity growth shock

We consider a deterministic productivity growth shock to the OLW model – a 5-year rise of productivity growth rate by 1% per annum. Productivity grows at 1% in the first year, 2% in the second year, 3% in the third year and so on till the fifth year when it grows at 5%. After that it is permanently 5% above the base. Results of our simulation exercise are reported in graphical form in figure 2.

Growth in productivity increases output, stimulates investment to raise capital stock. Output cannot be increased without an increase in labour supply and capital which takes time. Thus the real interest rate must rise to reduce demand to the available supply. The rising real interest rate violates uncovered real interest parity (URIP) which must be restored by a rise in the real exchange rate relative to its expected future value. This rise is made possible by the expectation that real exchange rate will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. Real exchange rate moves back to equilibrium as real interest rates fall with the rise in capital and output. Rise in output is more in the overlapping wage

contract model as compared to the productivity shock in the real business cycle model. The reason is that in the overlapping wage contract model employment rises more as forced by the wage contract. Investment also rises more in OLG model as compared to the RBC model. When the wage contract gets over, the model behaves like the RBC model where real wage is equal to equilibrium real wage ($w = w^*$).

6.1 Data patterns

Let us begin by looking at the bilateral real exchange rate between India and US (US Dollar / Indian Rupees). The path of the real exchange rate is presented in Figure 1. The univariate final form equation is in fact best described by an ARIMA (1,1,1) process; the series therefore is not actually mean reverting but integrated of order 1. Our main aim is to see whether our calibrated RBC model can generate the same univariate behaviour.

In this section we perform Augmented Dickey Fuller test and Phillips Perron test to check stationarity of the real exchange rate series. Using both the Augmented Dickey Fuller test and Phillips Perron test, we find that India's real exchange rate vis-à-vis US (US \$ / Indian Rupees) is an I(1) series. Table 1 reports the results. The real exchange rate series in levels fails to reject the null hypothesis of non-stationarity at 5% significance level, using both ADF and PP test statistics. When we perform the test with the first difference of the series we can easily reject the null at 5 percent.

Table 1: Test for Non-stationarity of the Real Exchange Rate

Unit root test (with trend and intercept)

| | Level | First difference |
|--------------------|---------|------------------|
| ADF Test Statistic | -3.4223 | -6.7618 |

| | | |
|-------------------|---------|---------|
| PP Test Statistic | -3.4168 | -7.2648 |
|-------------------|---------|---------|

Having established the non-stationarity of the series we now estimate the best fitting ARIMA process to the real exchange rate, using annual data from 1966 to 1997. The results in Table 2 indicate that an ARIMA (1,1,1) best describes the data. The best fitting ARIMA regression results are shown in Table 3.

Table 2. Best Fitting Real Exchange Rate ARIMA

| | Coefficient | P-value |
|-------|-------------|---------|
| AR(1) | -0.6671 | 0.0024 |
| AR(2) | 0.1544 | 0.7600 |
| MA(1) | 0.5809 | 0.0329 |
| MA(2) | -0.9411 | 0.9466 |

Table 3. Best Fitting ARIMA Results for Rupees Real Exchange Rate (US Dollar/Indian Rupees)

Dependent Variable: D(Q)
Method: Least Squares

Sample: 1966 1997
Included observations: 32
Convergence achieved after 17 iterations
Backcast: 1965

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|--------------------|-------------|-----------------------|-------------|-----------|
| C | -0.033574 | 0.016389 | -2.048597 | 0.0496 |
| AR(1) | -0.667182 | 0.200509 | -3.327443 | 0.0024 |
| MA(1) | 0.580969 | 0.259303 | 2.240505 | 0.0329 |
| R-squared | 0.258434 | Mean dependent var | | -0.034958 |
| Adjusted R-squared | 0.207292 | S.D. dependent var | | 0.110286 |
| S.E. of regression | 0.098192 | Akaike info criterion | | -1.714728 |
| Sum squared resid | 0.279607 | Schwarz criterion | | -1.577315 |
| Log likelihood | 30.43565 | F-statistic | | 5.053223 |
| Durbin-Watson stat | 2.224469 | Prob(F-statistic) | | 0.013097 |
| Inverted AR Roots | -.67 | | | |
| Inverted MA Roots | -.58 | | | |

6.2 Bootstrapping

Our objective is to check whether we can generate the facts of the real exchange rate (US \$ / Indian Rupees) such as we find them, assuming that our model and its error processes are true. We want to find the sampling variability implied by the model - to find the 95% confidence limits around the real exchange rate ARIMA parameters.

One approach is to linearise the model, which would allow us to map it to a VARMA and in principle compute reduced form standard errors for each parameter. However the reliability of the standard errors would be open to question given our small sample size. Analytical computation of the confidence limits is also not possible given the non-linear nature of the model.

Comparison of our model with the ARIMA we have estimated on the actual data cannot be done via deterministic simulation. We want to replicate the stochastic environment to see whether our estimated ARIMA equations could have been generated within it. We do this via bootstrapping the RBC model with its error processes. In the RBC model, the error in the UIP equation is basically the risk premium. In the equations that are identities we have residuals either due to

measurement errors or due to approximations made in the model. They are treated as fixed elements and are not bootstrapped. Having obtained the residuals, we determine the best fitting data generating process for them, to obtain the i.i.d. shocks in our error processes. In our model we also have the exogenous processes- productivity, labour income tax, consumption tax, money supply, government expenditure, foreign prices, foreign consumption and foreign interest rates. To replicate the real exchange rate with its unit root we need unit root drivers in the system which are coming from the exogenous processes all of which have been modelled as random walks.

We generate the sampling variability within the model by the method of bootstrapping the model's estimated residuals; this permits us to find the 95% confidence limits around the real exchange rate ARIMA regression parameters. The idea is to create pseudo data samples (here 500) for the real exchange rate. We draw the vectors of i.i.d shocks in our error processes with replacement, by drawing vectors for the same time period we preserve their contemporaneous cross-correlations; we then input them into their error processes and these in turn into the model to solve for the implied path of real exchange rate over the sample period.

We run ARIMA regressions on all the samples to derive the implied 95% confidence limits for all the coefficients. Finally we compare the ARIMA coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals.

The comparison informs us whether the data rejects the model. We find that the ARIMA coefficients estimated from the actual data lie within the 95% confidence limits constructed by bootstrapping. The results in Table 4 validate the hypothesis that the real exchange rate behaviour is explicable within the RBC framework and the OLG framework.

Table 4. Confidence Limits from our Model for Real Exchange Rate ARIMA

| | Estimated | 95% Confidence Limits Lower (derived from RBC) | 95% Confidence Limits Upper (derived from RBC) | 95% Confidence Limits Lower(derived from OLW) | 95% Confidence Limits Upper (derived from OLW) |
|-------|-----------|---|---|---|---|
| AR(1) | -0.6671 | -0.75486 | 0.656537 | -0.79034 | 0.63692 |
| MA(1) | 0.5809 | -1.46681 | 0.664147 | -1.46672 | 0.683383 |

7. Conclusion

We find that the real exchange rate appreciates as a result of a deterministic productivity growth shock and then falls back to a lower equilibrium, producing a business cycle and the expected simulation properties. The productivity growth raises income and stimulates a stream of investments to raise the capital stock in line. Output cannot be increased without an increase in labour supply and capital which takes time. Thus the real interest rate must rise to reduce demand to the available supply. The rising real interest rate violates uncovered real interest parity (URIP) which must be restored by a rise in the real exchange rate relative to its expected future value. This rise is made possible by the expectation that real exchange rate will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival of stream of sufficient capital and so output, the real exchange rate also moves back to equilibrium. This gives us a business cycle in the real exchange rate. This new equilibrium represents a real depreciation on the previous steady state since output is now higher and must be sold on world markets by lowering its price.

We can conclude whether our model could be consistent with the facts by asking whether it could have generated the patterns we find in the actual real exchange rate data. To do this we generate the sampling variability within the RBC and OLW model by the method of bootstrapping the model's estimated residuals. This allows us to find the 95% confidence limits around the real exchange rate ARIMA regression parameters. The AR and MA coefficients estimated with the actual data lie within the 95% confidence limits generated by the method of bootstrapping. This validates our hypothesis that the real exchange rate behaviour is explicable within RBC and OLW framework.

Appendix

Some important equations to understand the RBC model (Minford, Sofat 2004)

The first order conditions of household's optimisation problem:

The first order conditions of household's optimisation problem with respect to

$C_t, N_t, b_t, b_t^f, S_t^p$ are :

$$(1 - \rho_0)\theta_0(1 - \rho_0)^{-1}C_t^{-\rho_0} = \lambda_t(1 + \phi_t) \dots \dots \dots (10)$$

$$(1 - \rho_2)(1 - \theta_0)(1 - \rho_2)^{-1}(1 - N_t)^{-\rho_2} = \beta E_t \lambda_{t+1} [(1 - \tau_t)v_t - \mu_t] \dots \dots \dots (11)$$

$$\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} \dots \dots \dots (12)$$

$$\frac{\lambda_t Q_t}{1 + r_t^f} = \beta E_t \lambda_{t+1} Q_{t+1} \dots \dots \dots (13)$$

$$\frac{\lambda_t p_t}{P_t} = \beta E_t \lambda_{t+1} \left(\frac{p_{t+1} + d_{t+1}}{P_{t+1}} \right) \dots \dots \dots (14)$$

The first of the above equations (equation 10) equates the marginal utility of domestic consumption to shadow price of output. Sales tax impinges on this equation. The second equates the marginal disutility of labour to labour's marginal product-the real wage. The marginal product of labour is affected both by tax on labour and the

unemployment benefit. From the representative household's first order condition we know that supply of labour is positively related to the net-of-tax real wage and negatively related to the unemployment benefit. If the after-tax real wage is temporarily high, substitution effect overpowers the income effect. The increase in work effort raises employment and output. On the other hand unemployment benefit negatively impinges upon supply of work effort. These equations which are stochastic analogue of the well-known Euler equations, which characterises the expected behaviour of the economy, determine the time path of the economy's values of labour, consumption and investments (in financial assets).

Substituting equation (8) in (6) yields

$$(1 + r_t) = \left(\frac{1}{\beta} \right) \left(\frac{C_t}{C_{t+1}} \right)^{-\rho_0} \left(\frac{1 + \phi_{t+1}}{1 + \phi_t} \right) \dots (15)$$

Substituting (6) and (8) in (7)

$$(1 - N_t) = \left\{ \frac{\theta_0 C_t^{-\rho_0} [(1 - \tau_t) v_t - \mu_t]}{(1 - \theta_0)(1 + \phi_t)(1 + r_t)} \right\}^{-\frac{1}{\rho_2}} \quad (16)$$

where v_t (consumer real wage) enters labour supply equation so that

$$\log v_t^* = \log W^* - \left[\frac{\omega^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}}} \log P_t^d + \frac{(1 - \omega)^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}}} \log P_t^F \right]$$

Also given that $\log W_t = \log w_t + \log P_t^d$, (producer real wage) and using

$\log Q_t = \log P_t^F - \log P_t^d$, then

$$\log v_t^* = \log w_t^* - \frac{(1 - \omega)^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}}} \log Q_t$$

Therefore (12) becomes

$$(1 - N_t) = \left\{ \frac{\theta_0 C_t^{-\rho_0} \left[(1 - \tau_t) \exp \left(\log w_t^* - \frac{(1 - \omega)^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}}} \log Q_t \right) - \mu_t \right]}{(1 - \theta_0)(1 + \phi_t)(1 + r_t)} \right\}^{-\frac{1}{\rho_2}}$$

.....(17)

If each household can borrow an unlimited amount at the going interest rate , then it has an incentive to pursue a Ponzi game. The household can borrow to finance current consumption and then use future borrowing to roll over the principal and pay all of the interest. To prevent the household from playing a Ponzi game it is further assumed that the household's decision rule is subject to a transversality condition

$$Y_{T-1} - r_T D_T - \phi_T C_T - \tau_T v_T N_T^s - T_T = C_T \quad (18)$$

Substituting (8) in (10) gives

$$p_t = \left(\frac{p_{t+1} + d_{t+1}}{1 + r_t} \right) \frac{P_t}{P_{t+1}} \quad (19)$$

Using $p_{t+1} = \frac{p_{t+2} + d_{t+2}}{(1+r_{t+1})} \frac{P_{t+1}}{P_{t+2}}$ in above yields

$$p_t = \left(\frac{p_{t+2} + d_{t+2}}{(1+r_t)(1+r_{t+1})} \right) \left(\frac{P_t}{P_{t+2}} \right) + \left(\frac{d_{t+1}}{1+r_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \quad (20)$$

using the arbitrage condition and by forward substitution the above gives

$$p_t = \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1+r_t)^i} \left(\frac{P_t}{P_{t+i}} \right) \quad (21)$$

Equation (21) states that the present value of share is simply discounted future dividends.

In small open economy models the domestic real interest rate is equal to the world real interest rate, which is taken as given. Further it is assumed that the economy has basically no effect on the world rate because, being a small part of the world, its affect on the world savings and investment is negligible. These assumptions imply that the real exchange rate for the small open economy is constant. However we are modelling a medium sized economy. In our set up the economy is small enough to continue with the assumption that world interest rates are exogenous but large enough for the domestic rate to deviate from the world rate. In our model real exchange rates are constantly varying. To derive the uncovered interest parity condition equation (12) is substituted into (13)

$$\frac{1+r_t}{1+r_t^f} = \frac{Q_{t+1}}{Q_t} \quad (22)$$

In logs this yields to

$$r_t = r_t^f + E_t \Delta \log Q_{t+1} \quad (23)$$

The Government

In this framework it is assumed that the government spends current output according to a non-negative stochastic process that satisfies $G_t \leq Y_t$ for all t . The variable G_t denotes per capita government expenditure at t . It is also assumed that government expenditure does not enter the agents objective function. In case of equilibrium business cycle models embodying rational expectations, output is always at the desired level. Given the information set, agents are maximising their welfare subject to their constraints. Since there are no distortions in this set-up government expenditure may not improve welfare through its stabilisation program. This is why government expenditure has been excluded from the representative agent's utility function. The state also pays out unemployment benefits μ_t which leads to higher substitution between work and leisure.

The government finances its expenditure by collecting taxes on labour income τ_t and taxes on consumption ϕ_t which are assumed to be stochastic processes. Also it issues debt, bonds b_t each period which pays a return next period. Then it collects

seigniorage, i.e. $\frac{M_{t+1}^d - M_t^d}{P_t^d}$ which is assumed to act as a lump-sum tax, leaving real

asset prices and allocation unaltered and is assumed to be a stochastic process.

Since tax on labour income reduces the after-tax return accruing to an agent from supplying labour in market, it is likely to affect her choice as to how much of labour to supply at a given point of time. By reducing the take-home wage, the labour

income tax reduces the opportunity cost of leisure, and there is a tendency to substitute leisure for work. This is the substitution effect and it tends to decrease labour supply. At the same time tax reduces the individual's income. Given that leisure is a normal good, this loss in income leads to a reduction in consumption of leisure, ceteris paribus. The income effect tends to induce an individual to work more. It is the relative strengths of the income and substitution effects which would ultimately determine whether an agent would work more or less.

Tax on consumption are similar to income tax in the sense that they are imposed on flows generated in the production of current output.

The government budget constraint is

$$G_t + b_t + \mu_t \left[(1 - N_t) - \bar{l} \right] = \tau_{t-1} v_{t-1} N_{t-1} + \phi_{t-1} C_{t-1} + \frac{b_{t+1}}{1 + r_t} + \frac{M_{t+1}^d - M_t^d}{P_t^d}$$

(24)

where b_t is real bonds and P_t^d is the domestic price level. Note that

$\tau_{t-1} v_{t-1} N_{t-1} + \phi_{t-1} C_{t-1}$ is the total tax revenue collected by the state. Also the

government faces a cash-in-advance constraint i.e.,

$$P_t^d G_t \leq M_t^{dg} \quad (25)$$

where M_t^{dg} is government's demand for domestic money. Here we assume that the government has some bias, i.e. it consumes only domestic goods.

Firm's Optimisation Problem

The technology available to the economy is described by a constant-returns to scale production function:

$$Y_t = Z_t f(N_t, K_t) \quad (26)$$

The capital stock evolves according to $K_{t+1} = (1 - \delta)K_t + I_t$ (27)

$$Max V = E_t \sum_{i=0}^T d_{it}^i (Y_t - K_t(r_t + \delta) - w_t N_t^d) \quad (28)$$

Subject to (26)

The firm optimally chooses capital and labour so that the marginal products are equal to price per unit of input. The first order conditions with respect to K_t and N_t^d are as follows

$$K_t = \frac{(1 - \alpha)Y_t}{r_t + \delta} \quad (29)$$

$$N_t^d = \left(\frac{w_t}{\alpha Z_t} \right)^{\left(\frac{1}{\alpha-1} \right)} K_t \quad (30)$$

The non-negativity constraint applies i.e. $K_t \geq 0$. Firms own the capital stock and choose investment and domestic labour.

The Foreign Sector

The response of trade balance to shocks on the terms of trade has preoccupied trade theorists for decades. In open economies a country's investment and consumption plans are no longer constrained by its own production frontier. As in Armington (1969), demands for products in this framework are distinguished not only by their kind but also by their place of production. The Armington assumption that home and foreign goods are differentiated purely because of their origin of production has been workhorse of empirical trade theory.

In a stochastic environment the representative agent maximises her expected discounted stream of utility subject to her budget constraint. In order to derive the real exchange rate and hence the balance of payments explicitly from micro-foundations we take into account the consumption constraint on agent

$$P_t C_t = P_t^d C_t^d + P_t^f C_t^f \quad (31)$$

Consumption function is an Armington aggregator of the form

$$C_t = \left[\omega (C_t^d)^{-\rho} + (1 - \omega) (C_t^f)^{-\rho} \right]^{-\frac{1}{\rho}} \quad (32)$$

where C_t is composite per capita consumption, made up of C_t^d , agents consumption of domestic goods and C_t^f , their consumption of imported goods and ω is the weight of home goods in the consumption function. The utility based price index corresponding to the above consumption function is of the form

$$P_t = \left[\omega^{\frac{1}{1+\rho}} (P_t^d)^{\frac{\rho}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}} (P_t^f)^{\frac{\rho}{1+\rho}} \right] \quad (33)$$

Now the Lagrangian associated with the agent's maximisation subject to the budget as well as consumption constraint is

$$U = \text{Max} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\theta_0 (1 - \rho_0)^{-1} C_t^{(1-\rho_0)} + (1 - \theta_0) (1 - \rho_2)^{-1} (1 - N_t)^{(1-\rho_2)} \right] \right\} +$$

$$\lambda_t \left\{ \begin{aligned} & (1 - \tau_{t-1}) v_{t-1} N_{t-1} + \mu_{t-1} \left[(1 - N_{t-1}) - \bar{l} \right] + b_t + Q_t b_t^f + \frac{(p_t + d_t) S_t^p}{P_t} - (1 + \phi_t) C_t - \frac{b_{t+1}}{1 + r_t} \\ & - \frac{Q_t b_{t+1}^f}{(1 + r_t^f)(1 - rp)} - \frac{p_t S_t^p}{P_t} \end{aligned} \right\}$$

$$+ \lambda_t^c \{P_t^d C_t^d + P_t^F C_t^f - P_t C_t\}$$

$$(34)$$

The first order conditions with respect to C_t^d and C_t^f are

$$\theta_0 C_t^{-\rho_0} \frac{\partial C_t}{\partial C_t^d} - \lambda_t (1 + \phi_t) \frac{\partial C_t}{\partial C_t^d} - \lambda_t^c P_t^d + \lambda_t^c P_t \frac{\partial C_t}{\partial C_t^d} \quad (35)$$

$$\theta_0 C_t^{-\rho_0} \frac{\partial C_t}{\partial C_t^f} - \lambda_t (1 + \phi_t) \frac{\partial C_t}{\partial C_t^f} - \lambda_t^c P_t^F + \lambda_t^c P_t \frac{\partial C_t}{\partial C_t^f} \quad (36)$$

Dividing equation (36) by equation (35) we have

$$\frac{P_t^F}{P_t^d} = \frac{\frac{\partial C_t}{\partial C_t^f}}{\frac{\partial C_t}{\partial C_t^d}} \quad (37)$$

$$\text{or } \frac{P_t^F}{P_t^d} = \frac{1 - \omega}{\omega} \left(\frac{C_t^d}{C_t^f} \right)^{1+\rho} \quad (38)$$

Now we can write equation (38) as

$$Q_t = \frac{1 - \omega}{\omega} (F)^{1+\rho} \quad (39)$$

$$\text{where } Q_t = \frac{P_t^F}{P_t^d} \text{ and } F = \frac{C_t^d}{C_t^f}$$

Elasticity of substitution between home goods and imported foreign goods is given by

$$\sigma = \left(\frac{\partial F}{\partial Q} \right) \left(\frac{Q}{F} \right) = \frac{1}{1+\rho} \quad (40)$$

Substituting (40) in (39) we have real exchange rate

$$Q_t = \frac{1-\omega}{\omega} \left(\frac{C_t^d}{C_t^f} \right)^{\frac{1}{\sigma}} \quad (41)$$

To the extent that home and imported goods are not perfect substitutes, σ will take some finite value. The lower the estimated σ means the less the substitution between the two goods. In other words the greater the degree of product differentiation, the smaller is the elasticity of substitution between the products.

From the real exchange rate equation, we can derive import equation for our economy. Taking logs of equation (41) we have

$$\log IM_t = \sigma \log \left(\frac{1-\omega}{\omega} \right) + \log C_t^d - \sigma \log Q_t \quad (42)$$

Note that $IM_t = C_t^f$

To derive the import function we need to substitute out for $\log C_t^d$ from household's expenditure minimisation we know

$$C_t^d = \left(\frac{P_t^d}{\omega P_t} \right)^{-\left(\frac{1}{1+\rho} \right)} C_t \quad (43)$$

Taking logs

$$\log C_t^d = \sigma \log \omega + \sigma \log P_t - \sigma \log P_t^d + \log C_t \quad (44)$$

Now substituting equation (44) in equation (42), we have

$$\log IM_t = \sigma \log(1 - \omega) + \log C_t - \sigma A \log Q_t \quad (45)$$

$$A = \frac{\omega^{\frac{1}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}}} \quad (46)$$

The equation states that imports into the country are positively related to the total consumption in the home country and negatively related to the real exchange rate, i.e. as Q_t increases (i.e., the currency depreciates), import demand falls.

Now an Armington aggregator consumption function and a corresponding real exchange rate equation exists for the foreign country as well.

$$C_t^F = \left[\omega^f (C_t^{df})^{-\rho_3} + (1 - \omega^f) (C_t^{ff})^{-\rho_3} \right]^{\frac{1}{\rho_3}} \quad (47)$$

$$Q_t^f = \left(\frac{1 - \omega^f}{\omega^f} \right) \left(\frac{C_t^{df}}{C_t^{ff}} \right)^{\frac{1}{\sigma_1}} \quad (48)$$

where C_t^F is the composite consumption of the foreign country, C_t^{df} is the foreign country's consumption of own goods, C_t^{ff} is foreign country's consumption of home goods, ω^f is the weight of foreign country's own goods in its composite consumption function, Q_t^f is the real exchange rate for the foreign country and $Q_t^f = \frac{1}{Q_t}$.

$\sigma_1 = \frac{1}{1 + \rho_3}$ is the elasticity of substitution between home goods, i.e. home exports and foreign country's own goods.

Taking logs of equation (48)

$$\log EX_t = \sigma_1 \log \left(\frac{1 - \omega^f}{\omega^f} \right) + \log C_t^{df} + \sigma_1 \log Q_t \quad (49)$$

Note that $EX_t = C_t^{df}$ and $Q_t^f = \frac{1}{Q_t}$

To derive the export function we need to substitute out for $\log C_t^{df}$. As before, from the foreign household's expenditure minimisation we know

$$C_t^{df} = \left(\frac{P_t^{df}}{\omega^f P_t^*} \right)^{-\left(\frac{1}{1+\rho_3} \right)} C_t^F \quad (50)$$

where P_t^* is the foreign CPI of the form

$$P_t^* = \left[(\omega^f)^{\frac{1}{1+\rho_3}} (P_t^{df})^{\frac{\rho_3}{1+\rho_3}} + (1 - \omega^f)^{\frac{1}{1+\rho_3}} (P_t^D)^{\frac{\rho_3}{1+\rho_3}} \right] \quad (51)$$

where P_t^{df} is the foreign country's own price level and P_t^D is the domestic price level in foreign currency.

Taking logs of equation (50)

$$\log C_t^{df} = \sigma_1 \log \omega^f + \sigma_1 \log P_t^* - \sigma_1 \log P_t^{df} + \log C_t^F \quad (52)$$

Substituting equation (52) in equation (49)

$$\log EX_t = \sigma_1 \log(1 - \omega^f) + \log C_t^F + \sigma_1 A^f \log Q_t \quad (53)$$

$$A^f = \frac{(\omega^f)^{\frac{1}{1+\rho}}}{(\omega^f)^{\frac{1}{1+\rho}} + (1 - \omega^f)^{\frac{1}{1+\rho}}} \quad (54)$$

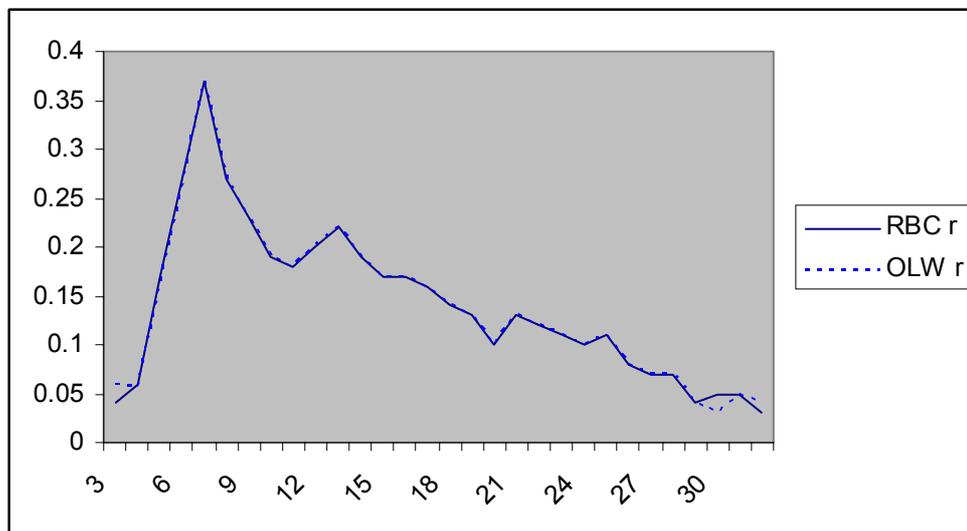
The equation states that export of the home country is a positive function of the total consumption in the foreign country and also a positive function of the real exchange rate. If Q_t increases, i.e. home currency depreciates then exports will increase. In the model home and foreign agents need foreign and home money respectively, in order to transact with each other. The foreign agents need home money to buy our exports, but get home money for imports as well as our purchase of foreign bonds. So their net supply of foreign money is equal to net exports plus sales of foreign bonds i.e. balance of payments surplus. This surplus is equal to home agents net demand for foreign money, who get foreign money from firms exporting to foreign agents and need foreign money for imports and purchases of foreign bonds. So if home agents adjust their sales of foreign bonds, then all balances. In equilibrium it is assumed that exports and imports are equal and hence the agents would have no tendency to change their asset position. In disequilibrium the changes between domestic and foreign bonds will depend upon net exports.

$$NX_t = EX_t - IM_t \quad (55)$$

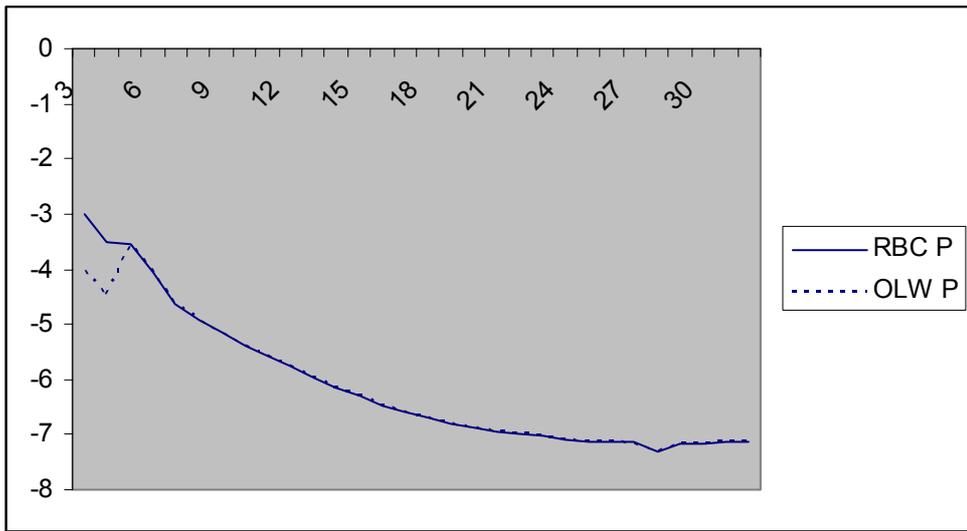
Foreign bonds thus evolve over time according to the following equation.

$$b_{t+1}^f = (1 + r_t^f)b_t^f + NX_t \quad (56)$$

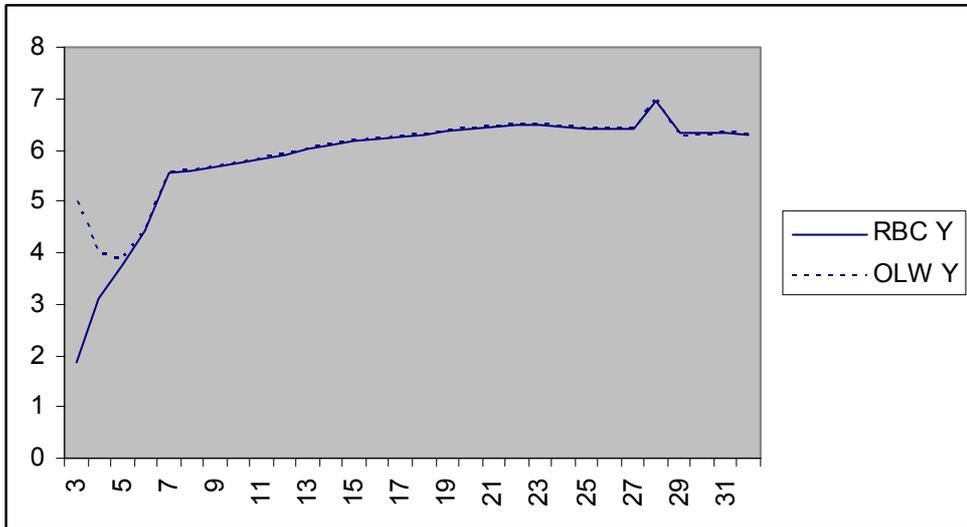
FIGURE 2. 1% Per Annum Productivity Growth Shock in RBC and OLW Model



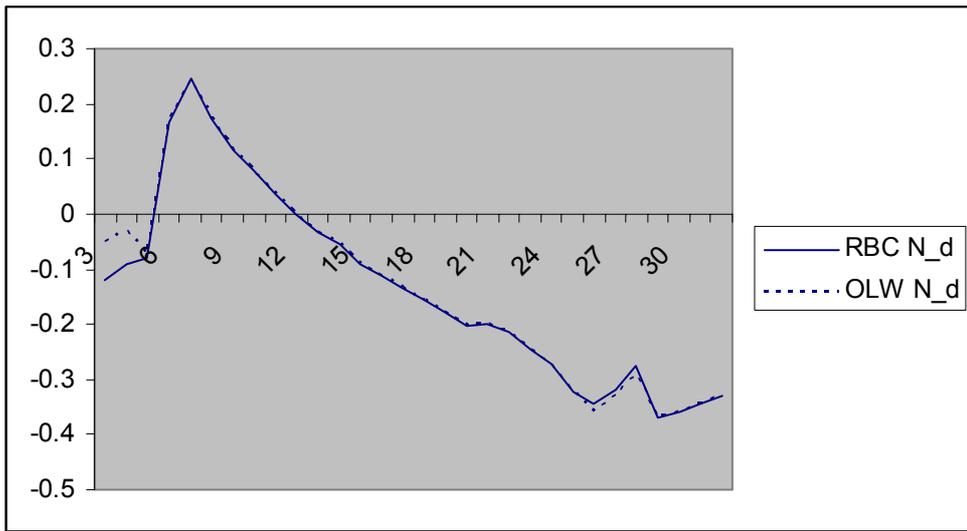
Real interest rate



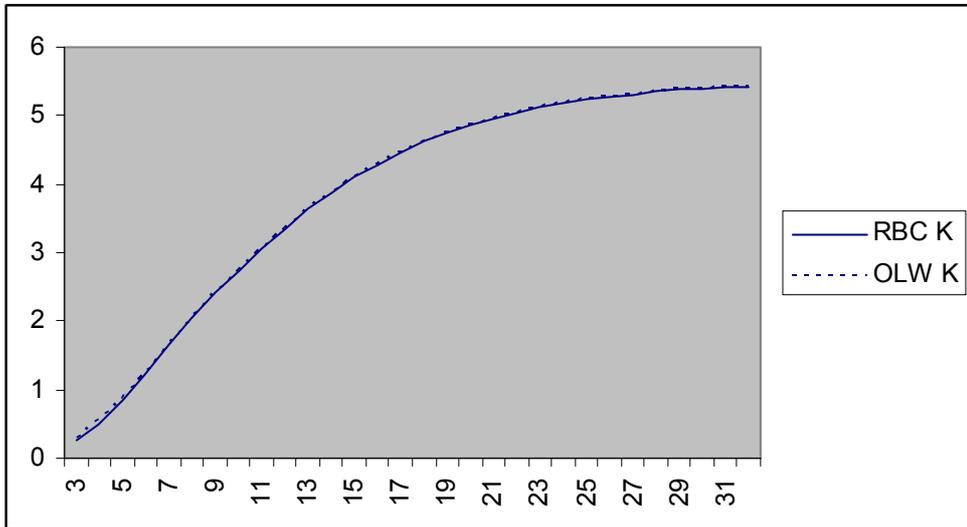
Price level



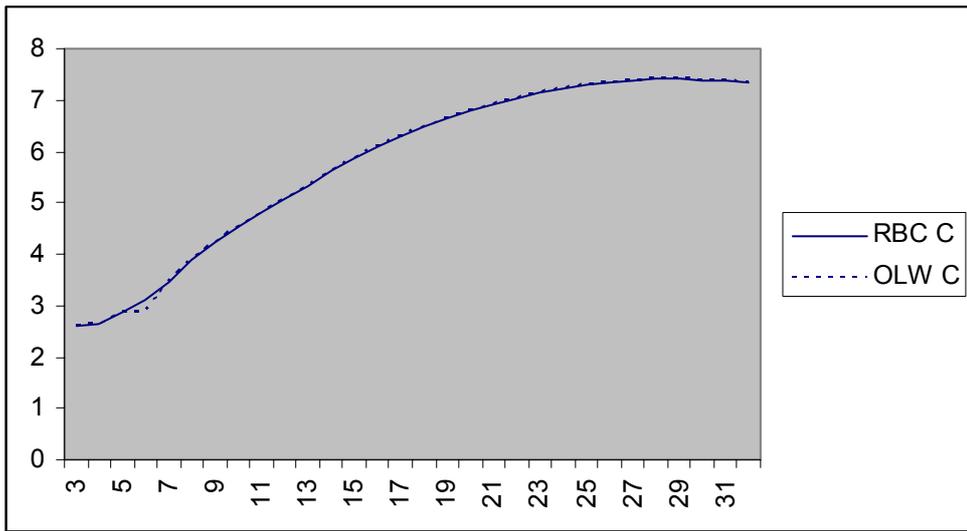
Output



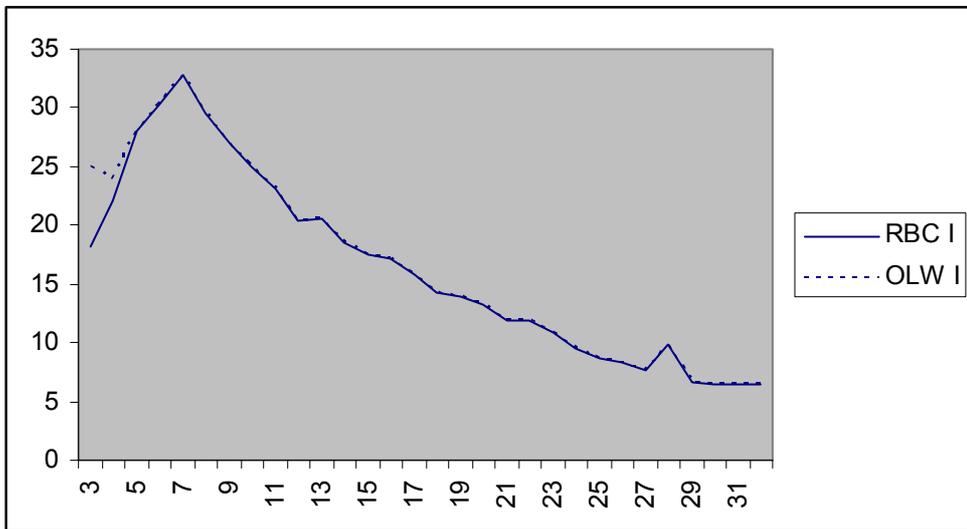
Employment



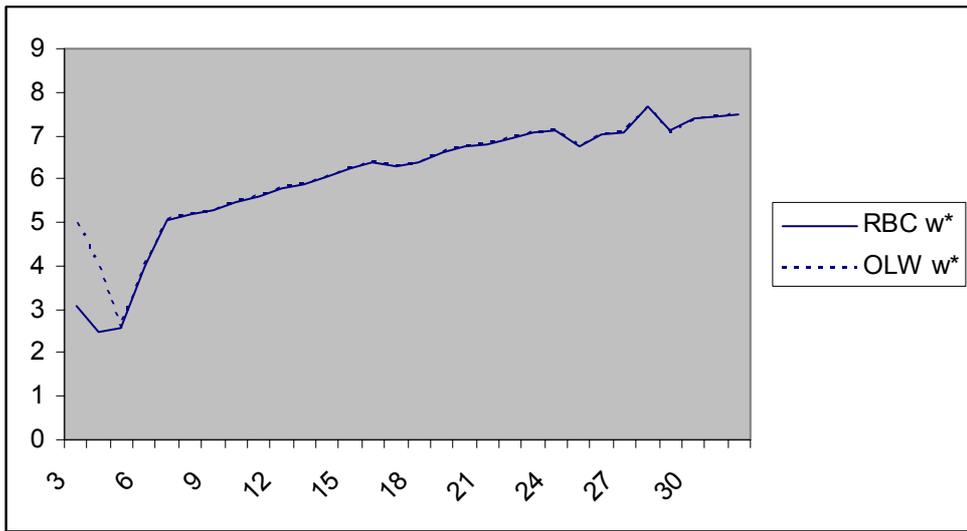
Capital stock



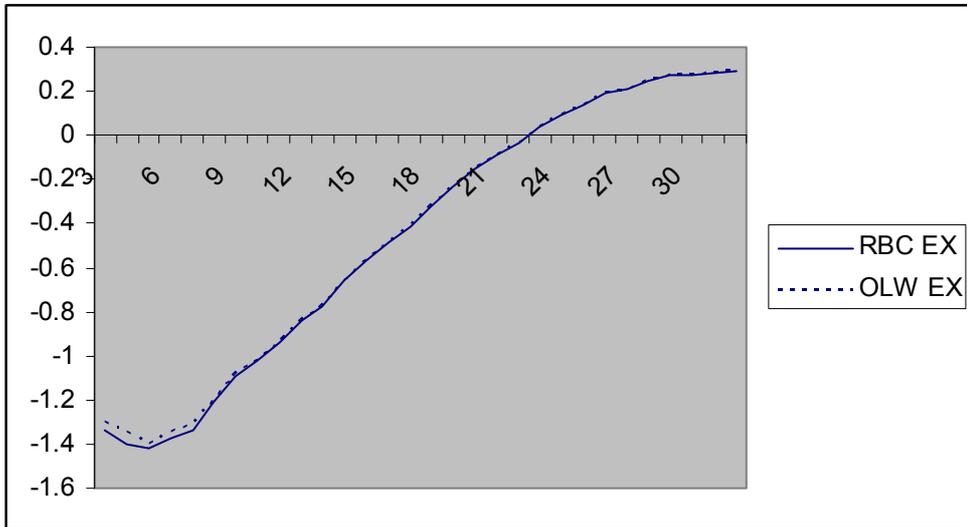
Consumption



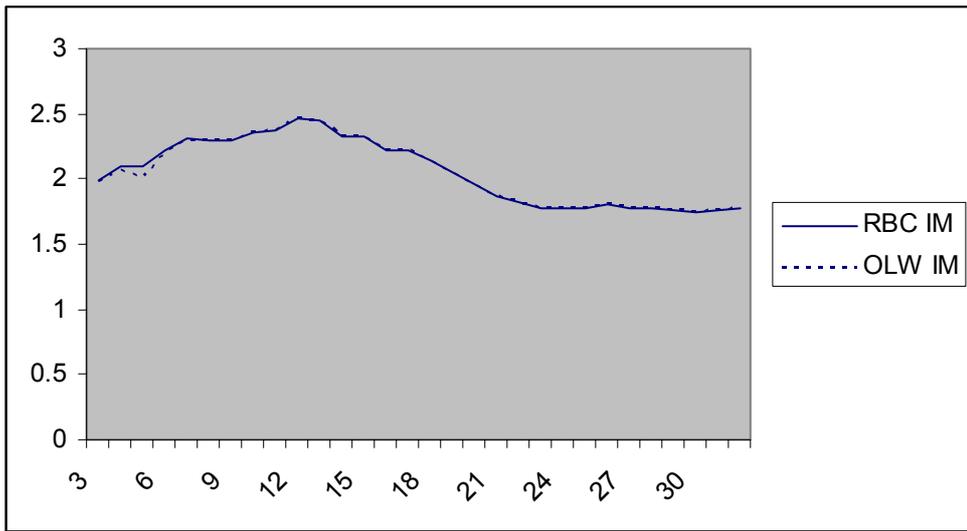
Investment



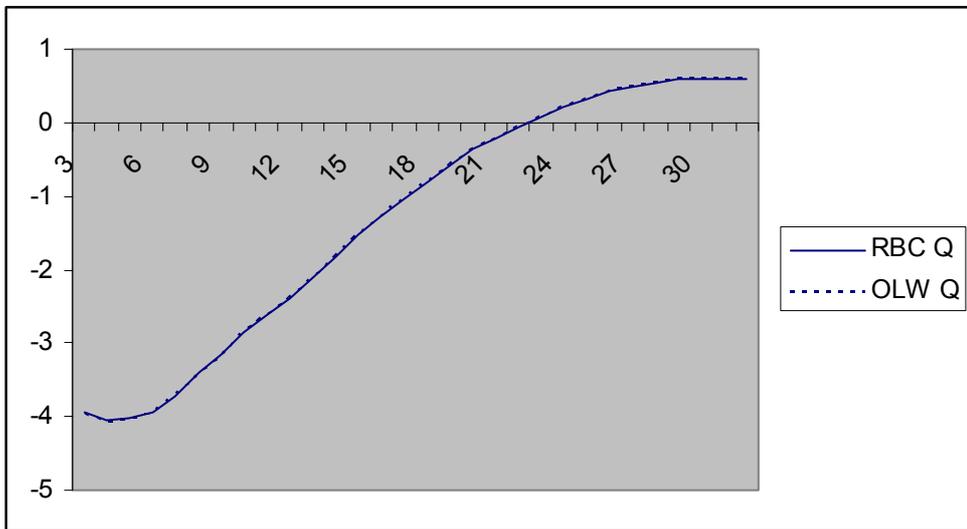
Real wage



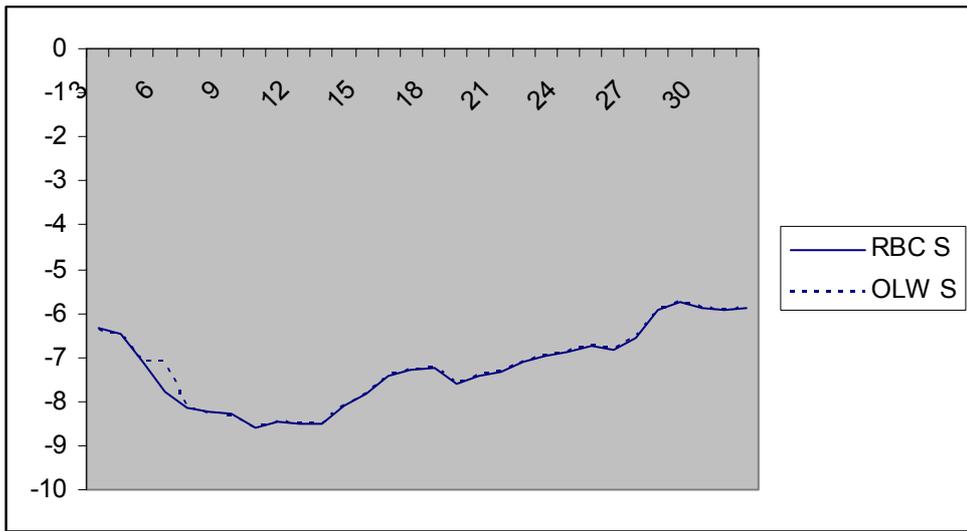
Export



Import



Real exchange rate



Nominal exchange rate

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