# Population Growth and Rising Dowries: The Long-Run Mechanism of A Marriage Squeeze*

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## ABSTRACT

India has experienced a much-documented inflation in dowries since the 1950s, which has been attributed to a spurt in population growth post-World War II. Will recent declines in fertility lead to a reversal of this trend and a regime of bride price? My paper develops a dynamic general equilibrium model of marriage markets, sex-ratio choice and population growth that is used to characterize the long-run relationship between population dynamics and marriage payments in India. I show that in the absence of exogenous sex preferences for offspring, and with no asymmetries between men and women except in desired ages of marriage (of self and spouse), the only sustainable steady state equilibria are characterized by dowry payments and an excess supply of women in the marriage market. This result holds for parameters consistent with marriage market indicators in India.

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# 1. Introduction

In the latter half of the last century, a sharp rise in dowries has been observed in north India and several regions of the south have been observed to switch from bride price to dowry (Epstein (1973), Billig (1991, 1992), Rao (1993)). This rise in dowries has been attributed to a demographic marriage squeeze against women caused by post-World-War-II population growth in India (Caldwell et al (1982, 1983), Billig (1991, 1992), Rao (1993), Bhat and Halli (1999)). A higher rate of population growth results in a larger number of young cohorts relative to old cohorts in the population. If older men marry younger women as has been observed in India, this phenomenon essentially means an excess supply of women relative to men in the marriage market, and leads to a bidding up of the price of grooms, viz. dowry inflation.

Will dowries and the demographic marriage squeeze against women persist in the long run? Population growth before the 1970s was driven by a larger decline in mortality than in birth rates. But fertility and crude birth rates have both been declining since the 1980s. Sex ratios have been falling and there is evidence[1] of active (e.g. female infanticide and foeticide) and passive (e.g. neglect of girls in health care and nutrition) measures adopted by parents to manipulate the sex ratio of their offspring. A sharp drop in the overall sex ratio is observed in the 1980s which is also the period in which sex selective abortion techniques became widely available in India (Hutter et al (1996), Sudha and Rajan (1999)). The lower population growth rates and female-to-male sex ratios associated with declining fertility levels have led to speculation about whether the marriage squeeze against women will be reversed (Bhat & Halli (1999), Das Gupta & Shuzhuo (1999)) which in turn leads to the question of whether bride price may emerge as the dominant form of marriage payments in future.

Whether the marriage squeeze will persist in the long run is of interest for at least two reasons. First, an improvement in the bargaining power of women in the marriage market may be a necessary condition for an improvement in the lot of Indian women who currently face domestic violence and even death if unable to meet the dowry demands of current or potential husbands. Second, there may be positive effects of female decision-making on intra-household allocations (Thomas (1990), Hoddinot and Haddad (1995), Duflo (2003), Pitt et al (2003), Case and Ardington (2005)) that an improvement in women's bargaining power could mobilize. The literature has seen numerous discussions of the marriage

[1] See Sudha and Rajan (1999), Arnold, Kishor and Roy (2002).

squeeze as the primary cause of the Indian dowry inflation (Caldwell et al (1982, 1983), Billig (1992), Rao (1993), Bhat and Halli (1999)) and efforts to test the veracity of the hypothesis (Rao (1993, 2000), Edlund (2000), Dalmia & Lawrence (2005)). But there has been no attempt to model the *long run* impact of the demographic squeeze on marriage market decisions.[2] This paper seeks to fill this gap in the literature by providing a theoretical framework for analyzing the squeeze and its long run implications for marriage payments.

I use a dynamic general equilibrium model[3] to show that under parametric restrictions that are consistent with marriage market indicators in India, the only possible steady state equilibria that may be sustained in the long run are characterized by a marriage squeeze against women and the persistence of dowry.

The result follows from modeling the mechanism of a demographic squeeze as a two-way interaction between marriage market decisions and population dynamics. It is clear that when older men marry younger women, population growth has an impact on marriage payments through its effect on the relative numbers of eligible men and women in the marriage market. However, if parents care about the marriage market returns of their offspring, then expectations of future marriage payments and matching probabilities will determine the optimal number of boys and girls that parents beget. This will, in turn, skew the sex ratio in the population and have an alleviating impact on the nature of the marriage squeeze in future. This two-way link between population dynamics and marriage market decisions forms the core of my analysis.

Using an overlapping generations framework[4], I construct a three-stage model of marriage market bargaining, sex-ratio choice and population evolution. Population dynamics determine the structure of the marriage market in each period. Marriage payments are determined by bargaining among potential partners in the marriage market. Parents choose the optimal sex ratio of offspring based on

[2] Anderson (2005) uses a two-sided matching model to analyze the short run impact of a demographic squeeze, arguing that it should generate dowry *deflation* instead of inflation. Maitra (2006) provides an example which demonstrates, using Anderson's framework, that the dowry payments in the periods of the squeeze could be higher than those prevailing in the initial steady state equilibrium with no population growth. Population growth *can* therefore lead to an increase in dowries, following which the payments will decline if the growth does not persist.

[3] Tertilt (2005) uses a dynamic general equilibrium framework to demonstrate the relationship between marriage payments, fertility levels and savings under different systems of marriage, viz. polygyny versus monogamy. This model does not allow for endogenous sex ratios, however, which are an important feature of marriage markets in India and are responsible for the main result of this paper.

[4] Agents live for only two periods - young and old.

their expectations of future marriage payments. This sex ratio then determines the structure of the population and hence the numbers of brides and grooms in the marriage market in the next period. A steady state general equilibrium is obtained when the age-sex structure of the population is unchanging and the marriage market decisions (marriage payments and sex ratio) are constant over time. I use Pollak's (1987) Birth-Matrix Mating-Rule Model to link the optimal marriage market decisions in each period to the evolution of the population. Since parents have no sex preferences for offspring that are exogenous to the model, the choice of sex ratio depends only on gender-specific expected returns from the marriage market. Further, the only asymmetries between men and women lie in their desired ages of marriage, viz, women prefer to marry early and to marry older men while men prefer to marry late and to marry younger women.

Why are there dowry payments in the only possible steady state equilibria? The answer rests in part on the parametric restrictions imposed by Indian marriage market indicators. A persistent pattern of higher age of first marriage for men in India indicates that it may be optimal for young men to delay marriage at the equilibrium payments they are offered. Meanwhile, the universality of female marriage in India indicates that old women are matched before their younger counterparts. Using parametric restrictions consistent with these observations, my model predicts that there will be more marriage-eligible women than men in a steady state equilibrium and that *young* women are 'squeezed' for partners in each period.[5] Since young women gain sufficiently from marriage to wish to marry young, and reap the marital gains in two periods of marriage, their surplus from marriage is positive. The competition for a spouse that emerges due to the squeeze makes young women bid away this entire surplus from marriage, resulting in dowry payments. Old women pay dowries too because they must match the offers of young women in order to find partners.

How do sex ratios respond to the resultant marriage market incentives? Expectations of future dowry payments make boys more attractive to parents than girls and this skews the sex ratio in favor of males. However, since men choose to delay marriage in equilibrium, their expected return from the marriage market is a single-period return that is discounted. I show that the discount rate must be

[5] If there were more marriage-eligible men than women in the marriage market the equilibrium payment would be a bride price. However, if parents expect sons to pay bride price at marriage, they overproduce daughters and this reverses the marriage squeeze against men! However, in the only possible dowry equilibrium, sons are not over-produced (for reasons explained in the next paragraph), hence the marriage squeeze and dowries can persist.

sufficiently small if the above parametric restrictions hold true and older agents are matched first. This imposes an upper limit on the expected marriage market returns of men and hence an upper limit on the masculinity of the sex ratio. The marriage squeeze against women persists in the steady state equilibrium because the sex ratio, though masculine, is not skewed enough to remove the male advantage in the marriage market.

Does lowering the cost of skewing the sex ratio invalidate this result? With the advent of techniques of sex selective abortion in India, the cost to parents of skewing the sex ratio of offspring is suspected to have fallen. I show using the framework described above, that low costs of sex-ratio choice are a sufficient condition for the above result to hold. This is because a low cost of sex ratio choice does not alter the fact that men marry late and get only a single period of (discounted) marital bliss. This ensures that there is an upper limit on the marriage market returns of men, which allows the marriage squeeze against women to persist in equilibrium.[6]

The results described above make a significant contribution to the literature both for their valuable insights on the workings of Indian marriage markets, as well as for the novelty of the approach from which they stem. They are especially noteworthy for the remarkable accuracy with which they describe the current condition of marriage markets in India.

The rest of this paper is organized as follows. In Section 2, I provide a review of the literature on marriage payments and population growth in India in the last century. Section 3 outlines the theoretical framework of my analysis. Here I present, in order, a model of marriage market bargaining and determination of marriage payments, a model of population growth and a model of sex-ratio choice. I demonstrate, using numerical examples, how the three components described above interact in the determination of a steady state general equilibrium. In Section 4, I impose certain parametric restrictions in my model by considering evidence on Indian marriage market indicators and discuss the properties of steady state equilibria that may be obtained under such constraints. Section 5 repeats the analysis in Section 4 after introducing a parameter reflecting the cost of skewing the sex ratio. Section 6 summarizes the results and concludes the paper.

[6] Low costs of sex ratio choice make parents even more likely to overproduce girls if bride price is expected. This prevents a squeeze against men from being sustained in the long run.

## 2. Related Literature on Marriage Payments and Demographics

Why is the marriage squeeze hypothesis a plausible explanation for the dowry inflation observed in India since the 1950s? How did Indian demographics change during this time and are these patterns consistent with the timing of the dowry inflation? Is there additional evidence that there was indeed a squeeze against women in marriage markets? This section seeks to answer these questions by reviewing the existing literature on dowries and providing an overview of demographic trends in India in the light of the Indian dowry inflation.

Why is the marriage squeeze hypothesis a plausible explanation for the Indian dowry inflation? Anthropologists cite several reasons for the existence of different practices of marriage payments across India, ranging from inheritance laws to kinship structures, modes of production and class structure. The practice of dowry or 'stridhanam' in the 13th and 14th centuries appeared to satisfy the purpose of compensating daughters for their inability to inherit parental wealth (Dalmia and Lawrence (2005)).[7] Another reason commonly cited for the existence of dowry in north India is the norm of female hypergamy (Billig (1991), Caldwell et al (1982, 1983)), whereby women must marry into families of a higher caste and are hence forced to compete for a limited number of men and to pay a price for the desired groom. There are also arguments based on the higher economic contribution of women in the rice producing areas of the south compared with the wheat producing areas of the north (Dalmia and Lawrence (2005)), which leads to the practice of bride price in the former and dowry in the latter. None of these factors changed dramatically enough around the middle of the last century, however, to satisfactorily explain the rapid dowry inflation that has been observed over most of India from that time. Further, during this time, marriage payments have increasingly been made to the families of grooms instead of to the bride, defeating the ancient motive of 'stridhanam' and leading Billig (1992) to argue that the term 'dowry' is but a misnomer for the practice of 'groom price' that emerged at this time. Epstein (1973) favors the explanation that a 'Sanskritization' of lower castes was responsible for the switch to dowry in South India. Sanskritization refers to an emulation of the customs of higher caste Brahmins who have traditionally paid dowry. As Rao (1993) points out, however, "it seems hard to believe that the benefit gained by lower castes in behaving like Brahmins is greater than the

[7] Botticini and Siow (2002) provide a formal model of bequest (to sons) and dowry (to daughters) in virilocal societies.

immense destitution they often face by paying dowries." In fact, in recent times there has been widespread domestic violence and even murder of brides whose families are unable to meet the increasingly high demand for dowry (Bloch and Rao (2002)). Rao (1993) further points out that real dowry payments have increased even among castes that have historically paid dowries, thereby suggesting a general increase in the price of grooms in the marriage market during this time.

A demographic marriage squeeze led by the post-World-War-II spurt in population growth is a possible explanation for the dowry inflation observed in the last century (Caldwell et al (1982, 1983), Billig (1992), Rao (1993), Bhat and Halli (1999)).[8] Higher rates of population growth result in higher numbers of young relative to old cohorts in the population. Since in India younger women marry older men, this results in an excess supply of women relative to men in the marriage market and causes a bidding up of the price of grooms, viz. dowry inflation.

Did Indian demographics change in a way as to generate a squeeze against women? Is the timing of these changes consistent with the timing of the dowry inflation? India embarked on its demographic transition in the 1920s (see Table 1 and Fig 1, after Hutter et al (1996)). At the end of the nineteenth century, both (crude) birth and death rates in India (see Table 2 and Fig. 2) were high at around 50 per 1000 and population growth was low. A negative growth rate was recorded in the period 1911-21 due to high mortality caused by a severe influenza epidemic. A decline in death rates starting in 1921 induced a higher population growth rate and marked the beginning of India's demographic transition. By the 1930s, the population was growing at an average annual exponential growth rate of 1.33% and the demographic transition was well on its way. The timing of the dowry inflation (documented since the 1950s) seems to coincide with the attainment of marriageable age of the babies of the population 'boom', suggesting that a marriage squeeze may indeed be a potential cause of the increasing price of grooms observed ever since.

Is there additional evidence of a squeeze against women in marriage markets? Higher rates of population growth may plausibly cause a marriage squeeze against women, when older men marry younger women and sex ratios are balanced in each period. However, India is notorious for its highly skewed sex ratios and its large numbers of 'missing women'[9]. A glance at Table 3 and overall sex ratios (females

[8] Anderson (2003) shows that modernization can cause dowries to increase in a caste-based society. This is a parallel argument to that of a demographic marriage squeeze - both effects could operate simultaneously to increase dowry payments over time.

[9] After Sen (1992).

per 1000 males) shows that these have been less than 1000 and declining over the last century. The correct indicator of a marriage squeeze, however, is not the overall sex ratio but that of men and women of marriageable age. Caldwell et al (1983) attempt to measure this ratio and conclude that a deficit of 4 million women in the marriage market in 1931 was replaced by a surplus of the same magnitude by 1971. Clearly, population growth in the last century has far outweighed the bias in the sex ratio ensuring that the missing women were not sufficient in number to ease the marriage squeeze against women.

Is this trend likely to continue? The final stage of India's demographic transition began in the 1970s when fertility started to decline. In the late 1980s crude birth rates dropped as well. Will the associated decline in population growth rates and female-to-male sex ratios reverse the marriage squeeze against women? Will bride price emerge as the dominant form of marriage payments in the future? In this paper, I develop an analytical framework that allows me to investigate the answers to these questions.

## 3. The Model

In the subsections that follow, I shall present (in order) a model of marriage market bargaining and determination of marriage payments, a model of population growth and a model of sex-ratio choice. I use an overlapping generations framework to analyze the marriage market and sex-ratio choice of agents and Pollak's (1987) Two-Sex Birth-Matrix-Mating-Rule Model with Persistent Unions to link the marriage market to population growth.

There are two groups of agents in the economy, males and females. Each agent lives for two periods. Agents of the same age and sex are identical. All single, never-married agents are in the marriage market in each period. Remarriage is not permitted. Consistent with the high prevalence (and acceptance) of arranged marriages in India (Dasgupta and Mukherjee (2003), Raman (1981)) I shall assume that parents are responsible for arranging their offspring's marriage[10]. In each period, therefore, parents of single (never-married) agents observe the number of potential spouses of each type (age) in the market and submit their offers of marriage payments and post-payments preferences for partners to a matchmaker[11].

[10] This assumption also enables a separation of marriage decisions and sex ratio choice in the model, as will be clear later.

[11] Since agents live for only two periods, children born to parents of age 1 will be orphans when they reach age 0. Assume that in such cases, parents submit their preferences and payments'

The matchmaker then matches agents based on these post-marriage-payments preferences and a matching rule that will be described below. After marriage, couples 'choose' the number of male and female offspring based on the expected surpluses that their offspring will earn in the marriage market in future.

To demonstrate the effect of population dynamics on marriage market bargaining, I shall at first assume that birth rates and sex ratios are exogenous and fixed. I shall subsequently relax these assumptions by allowing agents to choose the sex ratios of their offspring, incorporating thereby the link from marriage market bargaining to population dynamics.

## 3.1. Marriage Market

### 3.1.1. Preferences

Preferences are common knowledge to all agents. Since parents arrange the marriages of their offspring, these may be interpreted as the utility from marriage ascribed to agents by their parents.

Let $U_i$ denote the period utility of agents of age $i$ when single, where $i = 0$ (*young*), $1$ (*old*). Then,

$$\begin{aligned} U_0(c) &= c \\ U_1(c) &= c - s \end{aligned} \qquad (M1)$$

where $c$ is consumption in the current period and $s$ is the cost of being single in old age. $s$ can be attributed to social pressures to be married and loneliness in old age.

Let $U_i^g$ denote the period utility from marriage of an agent of sex $g$ and age $i$. The specific form of the marital utility function is[12]:

$$\begin{aligned} U_i^f(c,i,a) &= c + K - (i-0)^2 - (a-1)^2 \\ U_i^m(c,i,a) &= c + K - (i-1)^2 - (a-0)^2 \end{aligned} \qquad (M2)$$

where $i = 0$ (*young*), $1$ (*old*), $a$ denotes the age of the spouse at the time of marriage, $c$ denotes consumption in the current period and $K$ ($> 0$) denotes

offer schedule to the matchmaker before they die, so that the latter can arrange the match of the offspring according to parental preferences.

[12] Bergstrom and Lam (1991) use a similar utility function to demonstrate how a marriage squeeze may be absorbed by changing age differentials of spouses. Their utility formulation includes an ideal (own) age of marriage for men and women with the former being higher than the latter. Anderson (2005) also uses marital utility functions similar to $(M2)$.

the utility from marriage, viz. companionship and the social network effects of an extended family. If married young, agents receive a lifetime utility of $[U_0^g(c_0, i, a) + \beta U_0^g(c_1, i, a)]$ (where $\beta$ is the discount factor and $c_t$ denotes the consumption chosen in period $t$) regardless of whether the spouse is living or dead in the second period of marriage. In other words, having been married entitles agents to the social network effects of an extended family even when the spouse is not living.[13]

The marital utility functions $U_i^g$ demonstrate agents' preferences for own and spouse's age at marriage. Other things equal, men prefer to marry at age 1 and women at age 0. Also, men prefer to marry younger women whereas women prefer to marry older men. The motivation for such an utility function for men and women is the observation that in India, the average and minimum ages of marriage of men have been higher than that of women during the last century (Goyal (1988), Mensch et al (2005)). Possible reasons for women preferring to marry young and men preferring young women could be the higher fertility of the latter, and, in a largely patrilocal society such as India, their greater potential to adapt to the ways of the groom's family (Epstein (1973)). Men could prefer to marry later because they seek to maintain a desired age difference between themselves and their spouse as this helps to maintain a favorable balance of power in the relationship (Jensen and Thornton (2003)).[14] Women could prefer to marry older men because of the latter's higher social and economic standing, also a possible reason why men themselves may prefer to postpone marriage in a social setting where they are the primary wage earners. In this model, however, I shall make the simplifying assumption that all agents earn the same wealth in every period, hence I justify women's preference for older men to be a result of the latter's higher standing in society (compared with younger men).

Each agent earns a wealth $w$ in each period. An implication of this assumption is that the wealth earned by an agent in each period is the same whether or not she/he is married. I claim that this is not unrealistic and that any deviation observed in practice is a result of the terms of marriage determined by marriage market bargaining - the focus of this paper. In order to detract from issues of saving and borrowing, I assume that $w$ is perishable and high enough not to be a constraint for marriage payments. The budget constraint is derived in the next

[13] Relaxing this assumption and allowing agents an utility of $c$ when the spouse is not alive, does not change the qualitative results of the paper.

[14] In Bergstrom and Bagnoli (1993), men with good prospects choose to postpone marriage till their success is revealed.

section, after an exposition of the structure of marriage payments.

### 3.1.2. Marriage Payments

Marriage payments $D$ are made in the period of marriage and may not be jointly consumed by both spouses.[15] By convention, let $D > 0$ denote a dowry paid by the bride to the groom and $D < 0$ denote a bride price paid by the groom to the bride. Then the budget constraints in the period of marriage are:

$$\begin{aligned} c &= w - D, \; for\ the\ bride \\ c &= w + D, \; for\ the\ groom \end{aligned} \qquad (M3.1)$$

In all other periods, the budget constraints are:

$$c = w, \; for\ all\ agents$$

Henceforth, I shall denote marriage payments associated with the marriage of a woman of age $i$ and a man of age $j$, by $D_i^j$.

Let $v_i^j$ denote the pre-payments marriage surplus of a woman of age $i$ married to a man of age $j$ and $V_i^j$ denote the pre-payments marriage surplus of a man of age $j$ married to a woman of age $i$. For old agents, this is the utility from marrying an agent of a particular type (age) less the utility of remaining single at the end of the period. Using $(M1)$, $(M2)$ and $(M3)$, I derive,

$$\begin{aligned} v_1^j &= U_1^f(w, 1, j) - U_1 = K + s - 1 - (j-1)^2 \\ V_i^1 &= U_1^m(w, 1, i) - U_1 = K + s - (i-0)^2 \end{aligned} \qquad (M4.1)$$

For young agents, the pre-payments surplus is the lifetime utility from marrying an agent of a particular type less the expected return from postponing marriage to the next period. The latter includes the utility from remaining single now as well as the expectation of marriage returns in the next period (discounted by $\beta$). I shall denote agents' expectations of future marriage returns by $X^g$, where $g$ denotes the gender of the agent. The specific form of $X^g$ depends on demographic

[15] In reality, marriage payments could take the form of explicit wealth transfers or intra-marital arrangements. Here I focus on the monetized value of these payments.

structure.[16] Hence,

$$v_0^j = U_0^f(w,0,j)(1+\beta) - [U_0 + \beta X^f] = (w + K - (j-1)^2)(1+\beta) - [w + \beta X^f]$$
$$V_i^0 = U_0^m(w,0,i)(1+\beta) - [U_0 + \beta X^m] = (w + K - 1 - (i-0)^2)(1+\beta) - [w + \beta X^m] \quad (M4.2)$$

**Definition 1.** *A payment made in a marriage of a woman of age $i$ and a man of age $j$ $(i, j = 0, 1)$ is feasible when $D_i^j \leq v_i^j$ and $-V_i^j \leq D_i^j$. Henceforth, I shall refer to these inequalities as feasibility constraints. Feasibility requires that each agent earns at least as much as her/his reservation utility upon marriage.*

**Definition 2.** *An agent is eligible to marry if he is single and has never been married before.*

**Definition 3.** *A marriage market participant is an eligible agent whose feasibility constraints are satisfied at the offered payments.*

### 3.1.3. Search for Partners and the Matching Rule

Let us focus on a social planner's matching outcome, viz. a matching rule that maximizes the total marital surplus of married agents in each period. In the framework described below, I shall use a specific matching rule that achieves the social planner's matching outcome in equilibrium. The results obained will be true for all matching rules that generate the same equilibrium outcome, viz, that maximize the total marital surplus of married agents.

In each period, parents of eligible agents observe the number of potential partners in the market and submit their schedule of marriage payments along with their post-payments preferences for partners to the matchmaker. The latter then matches agents according to a rule that is specified below and is common knowledge to all agents.

In agents' (post-marriage-payments) ranking of preferences for potential partners, let '1' denote the first preference, '2' denote the second preference and so on. In case of indifference between potential partners who would have taken ranks $\kappa, (\kappa+1), ..., (\kappa+n)$ in the preference ordering, let each of these agents receive a rank of $[\frac{\kappa+(\kappa+1)+...+(\kappa+n)}{n+1}]$. Let $F(x,y)$ denote the rank of male $y$ in female $x$'s preferences and $M(x,y)$ denote the rank of female $x$ in male $y$'s preferences. Also,

[16] See equations (1) and (2) of Appendix D for a derivation of $X^g$ in equilibrium, for a specific demographic case.

let $(x, y)$ denote a match between a woman $x$ and a man $y$.[17] Then, the matching rule is as follows:

i. Matchings $(x, y)$ occur in increasing order of $r(x, y) = F(x, y) + M(x, y)$, i.e. the $(x, y)$ with the minimum $r(x, y)$ is matched first and then the rest in increasing order of magnitude.

ii. In case of equality of $r(x, y)$, matches with the highest total (pre-payments') surplus from marriage occur first. [18]

iii. In case of identical total surplus from marriage, matching is random.

The above rule specifies the order in which the matchmaker pairs agents. Agents who express 'strict' preferences for partners (as represented by $r(x, y)$) are paired first. When agents are indifferent to the type of spouse, groups with higher (pre-payments) marital surpluses are matched first. Among individuals with the same marital surplus, matching occurs at random.

Note that the above matching rule is exhaustive, i.e. it does not leave marriage market participants of both sexes unmatched.

I now define an equilibrium of marriage payments as follows.

**Definition 4.** *An equilibrium (Nash) of marriage payments is a vector of feasible marriage payments $\{D_i^j\}$ from which no agent has an incentive to deviate. Each agent of the same age and sex are identical and hence offers to pay/receive the same marriage payments in equilibrium.*

**Definition 5.** *An equilibrium matching rule is a rule that specifies the order in which agents are matched when the latter offer (Nash) equilibrium marriage payments. This rule typically involves random matches among identical individuals.*

The following propositions are then true:[19]

[17] Note that in this model, the actions of agents $x$ and $y$ are completely identified by their ages. I refrain from labeling $x$ and $y$ as ages, however, to demonstrate the applicability of the matching rule even when agents behave sub-optimally (e.g. if some agents choose to behave differently from others of the same sex and age).

[18] The total marriage surplus in a marriage of a woman $x$ and a man $y$ is the sum of the (pre-payments) marriage surplus that accrues to $x$ from marrying $y$ and that which goes to $y$ from marrying $x$. Since marriage surpluses depend only on sex and age in this model, if $x$ is of age $i$ and $y$ is of age $j$, then the total marriage surplus when they marry is $S_i^j = (v_i^j + V_i^j)$.

[19] A discussion of Propositions 1 and 2 is provided in Appendix A.

**Proposition 1.** *Let $m$ and $f$ denote the number of eligible men and women in the marriage market in any period. Suppose the following demographic conditions are true:*

$(a)$ $m \neq f$, *i.e. there are some agents in the market who are not guaranteed a match that meets the reservation utility*

$(b)$ *The numbers of men and women with the highest (pre-payments) marriage surpluses are not equal to each other or to the total number of prospective partners in the market.*

*Then agents whose types are matched in equilibrium with both types of the opposite sex must offer (receive) payments that make them indifferent to the type of their spouses. Further, if the matching rule leaves some young agents matched and some unmatched, then the former's marriage payments must be such that make them indifferent between marrying now and marrying later.*

**Proposition 2.** *When conditions (a) and (b) above are true, the equilibrium in marriage payments is unique and so is the equilibrium matching rule. When condition (a) or (b) is violated, there may be multiple equilibria in marriage payments but the equilibrium matching rule is unique. In both cases, the unique matching rule matches high (pre-payments) surplus agents before low surplus agents thereby obtaining the social planner's matching outcome in which the total marital surplus of all agents is maximized.*

Note that the equilibrium matching rule satisfies the following axioms:

1. Non-negativity: The number of matches is non-negative.
2. Adding-Up Constraint: The total number of agents in any age-sex category is greater than or equal to the number of matched agents in that demographic category, in each period.
3. Universal scope: The matching rule is defined for all non-zero populations.
4. Continuity: The equilibrium matching rule pairs high-surplus agents first and is continuous when the categories of high-surplus agents do not change over time.

5. Homogeneity: The equilibrium matching rule is homogeneous of degree one, when the categories of high-surplus agents do not change over time.

Pollak (1987) assumes the above properties of the matching rule in establishing the existence of stable population equilibria in a Birth-Matrix-Mating-Rule Model, described in the next section.

### 3.1.4. Example 1

To demonstrate how the marriage market functions, consider a simple static model in which the discount factor $\beta = 0$. Then the pre-payments marriage surpluses of women can be derived from $(M4.1)$ and $(M4.2)$ as:

$$\begin{aligned} v_0^0 &= K - 1 && (E1.1) \\ v_0^1 &= K \\ v_1^0 &= K + s - 2 \\ v_1^1 &= K + s - 1 \end{aligned}$$

From $(M4.1)$ and $(M4.2)$, the pre-payments marriage surpluses of men are:

$$\begin{aligned} V_0^0 &= K - 1 && (E1.2) \\ V_0^1 &= K + s \\ V_1^0 &= K - 2 \\ V_1^1 &= K + s - 1 \end{aligned}$$

Let $f_i$ $(m_j)$ denote the number of eligible women of age $i$ (men of age $j$) in the marriage market $(i, j = 0, 1)$. Let $(i, j)$ denote a match between a woman of age $i$ and a man of age $j$.

Consider the case where $f_0 < m_1 < m_1 + m_0 < f_0 + f_1$

This is an example where the total number of eligible women in the marriage market exceeds the total number of eligible men, so women have to bid for men in equilibrium. Let us analyze the equilibrium marriage payments that will result from the bargaining.

Suppose $0 < s < 1$, $K > 2$. Then,

$$\begin{aligned} v_0^1 &> v_1^1 > v_0^0 > v_1^0 > 0, && (E1.3.1) \\ V_0^1 &> V_1^1 > V_0^0 > V_1^0 > 0, \\ S_0^1 &> S_1^1 > S_0^0 > S_1^0 \end{aligned}$$

where $S_i^j = v_i^j + V_i^j$ is the total marriage surplus of a match $(i, j)$.

The ordering of $S_i^j$ in $(E1.3.1)$ ensures that the matchmaker pairs old men before young men and young women before old women, when agents are indifferent to the age of their spouse.

Then the equilibrium marriage payments will be:

$$
\begin{aligned}
D_0^0 &= K + s - 3 ** \\
D_0^1 &= K + s - 2 > 0 \\
D_1^0 &= K + s - 2 > 0 \\
D_1^1 &= K + s - 1 > 0
\end{aligned}
$$

$**$ *no* $(0,0)$ *matches in equilibrium*

Discussion: In this example, the total number of eligible women exceeds the total number of eligible men. Thus women will have to bid for their partners in equilibrium. The values of parameters $K$ and $s$ ensure that young women are the 'high-surplus' female agents (see $(E1.3.1)$). This means that young women can outbid older women for a match. This, and the knowledge of the matching rule (which matches high-surplus agents first) ensures that the high-surplus younger women need only offer the amounts that make potential spouses indifferent to the age of the women they marry, i.e. $D_0^j = D_1^j - 1$. How much do older women offer? Since older women are low-surplus agents, they are matched after younger women when the latter bid to make men indifferent to the age of the women they marry. This does not leave enough men in the market for all older women. Hence older women bid away their entire marriage surplus, $v_1^j$, as dowry.

Notice that there are two types of competition among agents in excess supply, here women. In this example, *within-group competition* among older women makes them give away their entire marital surplus as dowry. This happens because the parameter values, matching rule and marriage market demographics ensure that there are not enough men for all the older women in the market. The high-surplus younger women do not engage in within-group competition since the matching rule ensures that they are matched first (when they offer enough to make men indifferent to the age of women) and there are enough men for all women of age 0 $(f_0 < m_1)$. However, *between-group competition* implies that they have to match the offers made by older women, which is why younger women also pay a positive dowry in equilibrium.

At the equilibrium payments, all agents are indifferent to the age of their spouses. Thus the post-marriage-payments preference rankings submitted to the matchmaker look like:

| $F(i,j)$ | 0 | 1 | | $M(i,j)$ | 0 | 1 |
|---|---|---|---|---|---|---|
| 0 | 1.5 | 1.5 | | 0 | 1.5 | 1.5 |
| 1 | 1.5 | 1.5 | | 1 | 1.5 | 1.5 |

So, $r(0,0) = r(0,1) = r(1,0) = r(1,1) = 3$

Hence, the equilibrium matching rule specifies that matching will take place in the order of decreasing surpluses in $(E1.3.1)$, viz. that the high-surplus agents (older men and younger women) will be matched before others. Matching among identical agents (of the same age and sex) is random. In equilibrium all men and all younger women are matched. Some older women are left unmatched at the end of the period.

### 3.2. Population Dynamics

This section introduces the link from population dynamics to marriage market decisions. I use Pollak's (1987) Birth-Matrix Mating-Rule (BMMR) Model with Persistent Unions, to model the evolution of the population and connect it to marriage market decisions modeled in the previous section. As Pollak (1986, 1987, 1990) demonstrates, the existence, uniqueness and dynamic stability of population equilibria are often hard to establish analytically. I shall use computational techniques and general equilibrium analysis to show that under certain (realistic) parametric restrictions, it is possible to narrow down the characteristics of dynamic steady state equilibria quite effectively.

The main constructs of Pollak's (1987) model are a matrix of female births to couples of each type $(i,j)$, a (male/female) sex ratio at birth $\sigma$ and a mating rule that specifies the number of matches $\mu_{ij}$ of each type $(i,j)$ in each period. Using these, he shows that the evolution of the population vector and the 'old unions' vector (defined below) over time can be expressed as a mapping[20],

$$(F_0^t, F_1^t, M_0^t, M_1^t, u_{old}^t) = \phi(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1}) \qquad (P1)$$

where $F_i^t$ ($M_j^t$) denotes the number of females of age $i$ (males of age $j$) in the population in time $t$ and $u_{old}^t$ (the 'old unions' vector) denotes the vector of married agents in the population at the *beginning* of period $t$.[21]

[20] See Appendix B.1 for a derivation of $\phi$ in the context of this model.

[21] Agents in the 'old unions' vector are not in the pool of eligible marriage market participants in period $t$. Matches that occur in period $t$ enter the old unions vector in $(t+1)$.

**Definition 6.** *A stable population equilibrium in the above model is a vector* $(\widehat{F}_0, \widehat{F}_1, \widehat{M}_0, \widehat{M}_1, \widehat{u}_{old})$ *and a scalar* $\widehat{r}$ *such that* $[(1+\widehat{r})\widehat{F}_0,\ (1+\widehat{r})\widehat{F}_1,\ (1+\widehat{r})\widehat{M}_0,\ (1+\widehat{r})\widehat{M}_1,\ (1+\widehat{r})\widehat{u}_{old}] = \phi(\widehat{F}_0, \widehat{F}_1, \widehat{M}_0, \widehat{M}_1, \widehat{u}_{old})$. *In keeping with standard demographic nomenclature, the population is 'stable' since its age-sex structure is unchanging. A stable population equilibrium is non-trivial when its size is not zero.*

**Definition 7.** *Eligible marriage market participants* $(f_0^t, f_1^t, m_0^t, m_1^t)$ *are in a stable population equilibrium when in each period, this vector replicates itself upto a constant factor.*

I make the following assumptions in linking Pollak's model to the model of marriage markets derived in the previous section. I assume that maximum total fertilities of couples are exogenously given. In other words, the total number of children is not determined by the components of the model, viz. population dynamics or marriage market returns. I also assume that all children are born in the first period of marriage of the couple. For all other than $(0,0)$ matches, this has to be true since one or the other spouse dies at the end of the first period of marriage. This assumption states that $(0,0)$ couples are impatient to conceive their children and partake of their benefits. Pollak's (1987) original model allows remarriage - I redefine the mapping $\phi$ to incorporate the assumption of no remarriage made in a previous section.

The analysis in this section assumes that birth rates and the sex ratio at birth $\sigma$ are constant over time. I also assume that $\sigma$ is the same for all couples. In the next section, birth rates and $\sigma$ will be endogenous and I shall allow $\sigma$ to vary by couple.

The following proposition is proved in Appendix B.2:

**Proposition 3.** *: When the total population is in a stable population equilibrium growing at the rate* $(1+\widehat{r})$*, the eligible population in the marriage market must also be in a stable (population) equilibrium growing at the same rate.*

Proposition 3 implies that in a stable 'total' population equilibrium, the age-sex composition of eligible marriage market participants is also fixed over time. Hence agents' expectations of future matching probabilities will be constant over time too.

I now define a steady state equilibrium in the marriage market.

**Definition 8.** *The marriage market is in a steady state equilibrium when agents' Nash equilibrium choices of decision variables (e.g. marriage payments) are constant over time. When there are multiple possible equilibria in decision variables, the marriage market is in a steady state equilibrium when the* expected *values of the latter are constant over time.*

In each period, old agents' choice of marriage payments is determined by their marriage market returns in the current period. Young agents determine their optimal marriage payments by looking at their current as well as expected future returns from the marriage market. All decisions are informed by a knowledge of the matching rule. The evolution of the state variables $(f_0^t, f_1^t, m_0^t, m_1^t)$ over time depends on the births in each period and the matching rule (which serves to remove eligible agents from the pool of marriage market participants in the next period). The set of optimization problems and the evolution of the state variables are summarized in Appendix C.

In a steady state equilibrium of the marriage market, the optimal marriage payments determined by the above procedure must be constant over time. Example 2 provides a numerical illustration of such an equilibrium.

### 3.2.1. Example 2

To demonstrate a steady state equilibrium of the marriage market, assume for the moment that sex ratios and birth rates are exogenous. Suppose that the total fertility of a couple depends only on the age of the woman in the couple and that young women are more fertile than older women.

Let $\sigma = 1.6$, $b_0 = 1.6$ and $b_1 = 1$, where $b_i$ is the number of female children born to mothers of age $i$ and $\sigma$ is the (male/female) sex ratio at birth.

Suppose that parameters are such that the matching rule pairs older men before younger men and older women before younger women when agents are indifferent to the age of their spouses.[22] Elementary arithmetic will show that $f_1^t = 0$ and $f_0^t = m_1^t$ will define a stable population equilibrium of marriage market participants. Both the total population and the marriage market population will be growing at the rate $(1 + \widehat{r}) = 1.6$. This is an example of a case where the population is growing but a sex ratio greater than unity ensures that the number of men and women in the marriage market are exactly equal, so that all agents find a partner in their lifetime.

[22] See Appendix D for a derivation of conditions that justify this assumption.

Suppose that parameters are such that young men prefer to postpone marriage at the current marriage payments. There will be multiple equilibria in marriage market payments in this case, which will on average be a bride price (see Appendix D for a detailed derivation). The intuition of the result is as follows: when young men postpone marriage at the offered payments, the number of eligible men and women willing to marry are exactly equal. Women are willing to offer a dowry upto the amount that makes them indifferent between marrying now and marrying later. However, older men have just this one period to make a match and since the numbers of potential spouses are exactly equal, may be coerced to pay a bride price of the amount of their entire marriage surplus! There is therefore a *range* of feasible marriage payments compatible with equilibrium due to the lack of credible threat points of both parties. The upper limit of this range is the dowry that makes young women indifferent between marrying now versus later (young women's marital surplus), and the lower limit is the bride price that reduces old men's post-payments' marital surplus to zero. Suppose that in such a scenario, agents approach the matchmaker and request her to draw a random marriage payment for each match from a uniform distribution over the feasible range. It is then easy to show that the expected value of marriage payments corresponds to a bride price. This is because the (absolute value of the) lower limit of the range of payments is the pre-payments' marriage surplus of older men who are high-surplus agents. This exceeds the upper limit of the range, viz. the marriage surplus of the low-surplus young women. The expected value is therefore a bride price.

If parameters are such that young men are willing to marry young, then the bride price paid in equilibrium will be even higher, since the outside option of women has now improved from no marriage in the current period to marriage to a younger man. Old men now have to compete with and match the offers of young men (who will bid up the bride price to the point that *their* expected surplus from marriage is reduced to zero) so as to make young women indifferent to the age of the men they marry.

### 3.3. Choice of Sex Ratio

This section completes the link between population dynamics and marriage market decisions by presenting a model of choice of (offspring's) sex ratio. This decision of parents is informed by population dynamics and expectations of marriage market returns, and feeds back into the former via the BMMR Model described in the previous section.

I have assumed that parents are responsible for arranging their offspring's marriage. After marriage, agents may choose the sex composition of offspring based on what the latter's expected returns will be in the marriage market. The assumption of arranged marriage separates marriage decisions and sex ratio choice in every period, since these decisions are made by different sets of agents. Assume that rearing children is costless. The utility function of each married agent is then[23]:

$$U^{marr} = c + E_f b_f + E_m b_m - (b_f - b_m)^2 \qquad (F0.1)$$

where $E_g$ is the expected marriage market surplus of an offspring of gender $g$, $b_g$ is the number of offspring of gender $g$ and $c$ is consumption.

Notice that there is a cost of choosing to skew the sex ratio of offspring to anything other than 1.[24] This reflects the cost of accessing technology such as amniocentesis and sex-selective abortion or the psychological cost of infanticide or neglect. Notice also that agents do not have an exogenous sex preference in this model. The choice of sex ratio depends purely on the incentives generated in the marriage market. It is possible to make a case for incorporating a sex preference for boys when they face better labor market opportunities than girls and are hence more likely to care for their parents in old age. In this model, however, men and women earn the same wealth in every period, hence this reason for preferring sons does not hold. All costs and benefits of children are 'public goods' and accrue equally to both parents. Also note that consumption $c$ is determined by the perishable wealth $w$ that each agent earns in every period and the marriage payment that the parents of the agent commit to when they marry away their offspring.[25] Hence $c$ is not a decision variable for married agents but is determined by $w$ and the terms of marriage formalized by their parents. $E_f$ and $E_m$ are determined by agents' expectations of the relative numbers of marriage

[23] Siow and Zhu (2002) use a similar utility function of parents, albeit in a dynamic setup. The assumption that all children are born in the first period of marriage reduces sex ratio choice to a static problem.

[24] An alternative form of $U^{marr}$ may be used: $U^{marr} = c + E_f b_f + E_m b_m - 2(b_f - \theta_f)^2 - 2(b_m - \theta_m)^2$ where $\theta_g$ represents the number of children of gender $g$ that are born to a couple 'naturally' (i.e. without intervention). Since $\theta_f = \theta_m$ in the aggregate, using this form of $U^{marr}$ will not change the results of the paper.

[25] Think of offspring as being the 'property' of parents as long as they are single. Thus the incomes $w$ that children earn are also the property of parents as long as the former are unmarried. When arranging a marriage, parents commit to transfer (or receive) a part of $w$ (earned by the children) as marriage payment on their behalf.

market participants in the future, and the marriage payments that will have to be paid at that time. In a steady state marriage market equilibrium, $E_f$ and $E_m$ will be the same over time ensuring that couple ($c$)-specific birth rates $b_{gc}$ ($g = f, m$) and sex ratios $\frac{b_{mc}}{b_{fc}}$ are also constant over time.[26]

Formally, $E_g$ ($g = f, m$) is defined as follows:

Denote the *total* utility that a woman expects to receive over her lifetime by $\widetilde{E}_f$.Then,

$$\begin{aligned}\widetilde{E}_f &= \overline{p}_0[(w+K)(1+\beta) - ED_0^1] + (1-\overline{p}_0)\overline{p}_1[w + \beta\{w + K - 1 - ED_1^1\} \\ &\quad +\{1 - \overline{p}_0 - (1-\overline{p}_0)\overline{p}_1\}[w + \beta(w - s)]\end{aligned}$$

where $\overline{p}_i$ is the probability that a woman of age $i$ finds a partner.[27]

If a woman cannot find a partner in her lifetime, she gets $w + \beta(w - s)$. Hence the total *surplus* that a daughter is expected to receive over her lifetime is

$$E_f = \widetilde{E}_f - [w+\beta(w-s)] = \overline{p}_0[K(1+\beta)+\beta s - ED_0^1] + \beta(1-\overline{p}_0)\overline{p}_1[K+s-1-ED_1^1] \quad (F0.2)$$

By a similar derivation,

$$E_m = \overline{q}_0[(K-1)(1+\beta) + \beta s + ED_0^0] + \beta(1-\overline{q}_0)\overline{q}_1[K + s + ED_0^1] \quad (F0.3)$$

where $\overline{q}_j$ is the probability that a man of age $j$ finds a partner.

Notice that optimization behavior of agents will ensure that $E_f \geq 0,\ E_m \geq 0$.

Assume that the maximum total fertility of a couple is exogenous and depends only on the age of the woman. Denote $\rho_i$ to be the maximum total fertility of a couple with a woman of age $i$. Assume that $\rho_0 > \rho_1$, i.e. younger women are more fecund. Assume, as before, that couples have all their children in their first period of marriage.[28] This reduces fertility choice to a static problem. For a couple with

[26] Birth rates and sex ratios will depend on the total fertility of the couple. A steady state is characterized by constancy of couple-specific values of the same over time.

[27] Note that I do not break $\overline{p}_i$ into the probabilities of matching with different types in equilibrium. This is not necessary since when agents are matched with more than one type in equilibrium, they must be indifferent between them (see Proposition 1). When matched with only one type in equilibrium, $\widetilde{E}_g$ must contain the probability of matching with this type and the returns from this marriage.

[28] As explained earlier, all but $(0, 0)$ couples have to bear their children in the first period of marriage, because one or the other spouse dies at the end of it. This assumption states that $(0, 0)$ couples also bear their children without delay due to impatience.

a woman of age $i$, the optimal sex-ratio is determined as follows:

$$\underset{b_f, b_m}{Max} \; c + E_f b_f + E_m b_m - (b_f - b_m)^2 \qquad (F1.1)$$

subject to the constraints,

$$\begin{aligned} b_f + b_m &\leq \rho_i \\ b_f &\geq 0 \\ b_m &\geq 0 \end{aligned} \qquad (F1.2)$$

The following proposition is proved in Appendix E:

**Proposition 4.** *In all non-trivial equilibria couples choose to have as many offspring as their total fertility allows, ensuring that young mothers have more offspring than old mothers. Maternal-age ($i$)-specific sex ratios $\sigma_i$ (male/female) of offspring are determined as follows:*

*when* $|E_f - E_m| < 4\rho_1$,

$$\sigma_i = \frac{4\rho_i - (E_f - E_m)}{4\rho_i + (E_f - E_m)} \; \epsilon \; (0, \infty) \; for \; i = 0, 1 \qquad (F6)$$

*when* $4\rho_1 < |E_f - E_m| < 4\rho_0$,

$$\sigma_0 = \frac{4\rho_0 - (E_f - E_m)}{4\rho_0 + (E_f - E_m)} \; \epsilon \; (0, \infty) \qquad (F7.1)$$

$$\begin{aligned} \sigma_1 &= 0 \; if \; E_m < E_f \\ \sigma_1 &= \infty \; if \; E_m > E_f \end{aligned} \qquad (F7.2)$$

*Further, there is no non-trivial steady state equilibrium compatible with the condition* $|E_f - E_m| > 4\rho_0$.

Notice that at the interior solutions $(F6)$ and $(F7.1)$, $\sigma_i$ increases (decreases) with decline in total fertility $\rho_i$ if $E_f - E_m < 0$ $(E_f - E_m > 0)$. In other words, a reduction in fertility skews the sex ratio in favor of offspring with higher expected marriage market returns.

I shall now define a steady state general equilibrium.

**Definition 9.** *A steady state general equilibrium is obtained when the following conditions are true:*

*i. the total population and the eligible marriage market population are in stable population equilibrium, and*

*ii. the marriage market is in a steady state equilibrium, viz. marriage payments and sex-ratio choices are unchanging over time.*

*A steady state general equilibrium is* non-trivial *when the size of the total population is non-zero.*

Example 3 demonstrates the existence of a steady state general equilibrium as defined above.

### 3.3.1. Example 3

Consider the following parameter values: $\varphi_0 = 9,\ \varphi_1 = 3,\ K = 0.5,\ s = 5,\ \beta = 0.5$

Then a non-trivial steady state general equilibrium exists and has the following characteristics[29]:

1. Young men are not willing to marry at the equilibrium marriage payments because $K$ is too small $(K < 1)$.

2. The equilibrium matching rule matches old men and women first when agents are indifferent to the age of their spouse.

3. In the stable population equilibrium, $\overline{q}_0 = 0,\ \overline{q}_1 = 1,\ \overline{p}_0 = 0.403,\ \overline{p}_1 = 1$. That is, in every period, young men refrain from marrying and old men are matched with all old women and some young women. Also, the stable population grows at the rate $(1 + \widehat{r}) = 1.922$, so $\widehat{r} = 0.922$. This is derived at the optimal birth rates and sex ratios derived in (5) below.

4. The equilibrium marriage payments are $D_0^1 = \frac{K+2\beta}{1-\beta} = 3,\ D_1^1 = \frac{K+1+\beta}{1-\beta} = 4$. Hence the equilibrium marriage payments are dowries.

5. The optimal maternal-age-specific birth rates and sex ratios are:

   $b_{f0} = 4,\ b_{m0} = 5,\ \sigma_0 = 1.25$

   $b_{f1} = 1,\ b_{m1} = 2,\ \sigma_1 = 2$

[29] See Appendix F for a detailed derivation.

## 4. The Indian Scenario

Since high-surplus agents are matched first in equilibrium, the matching rule associated with a steady state general equilibrium will be determined by the pre-payments marriage surpluses of agents. These depend on model parameters $(K,\ s,\ \beta)$ and the age-sex composition of the marriage market in equilibrium. What parameter values and matching rule are appropriate for the Indian case? One way to determine the answer is by looking at data on particular marriage market indicators in India and ascertaining the parameter values in the model that generate predictions consistent with these.

One of these indicators is the universality of female marriage in India. Table 4, from Goyal (1988, pp. 17) lists estimates of the percentage of single females in different age groups by birth cohort. The proportion of single females in the age-group 35-39 is 0.5% for cohorts 1931-36 to 1946-51. Since the Indian population started growing from the 1930s these are the cohorts that would be the first to experience the demographic squeeze. Yet we see that most women in these cohorts find partners during their lifetime. This suggests, in my model, that the matching rule is such that *older* women are matched first when men are indifferent to the age of their spouses. It is easy to see why - if young women exceed the number of men in the marriage market (due to population growth), there will clearly be some women who do not find a match when young.[30] If in the next period young women again exceed the number of men in the marriage market and are matched before old women, then it must be the case that none of the old women in the market in this period find a match in their lifetime. Hence it seems reasonable to assume that parameter values are such that the matching rule matches old women before young women (when men are indifferent between the two) and that this fits the Indian case better than if young women were paired first.

Another useful set of indicators are overall and juvenile sex ratios and their behavior over time. Figure 3, from Mayer (1999) and Hutter et al (1996) shows that the sex ratio (female/male) in India has been steadily falling throughout the twentieth century. Bhat and Halli (1999) estimate juvenile sex ratios (male/female) in 1911 and 1981 and find that these have increased over the period too. Sudha and

[30] Since the overall sex ratio in India has been masculine throughout the last century, these young women would have all found a match if young men were willing to marry them. But then, the age at which men marry should have declined over time. This has not been observed in India. The ages at marriage of both men and women have been rising with a narrowing of the age gap at marriage (Mensch et al (2005), Bhat and Halli (1999)).

Rajan (1999) also find sex ratios at birth to have become more 'masculine' in the period 1981-91 and report a worsening female mortality disadvantage during this time. In the context of my model, women would not be in surplus in the marriage market if the juvenile sex ratio (male/female) were greater than one and men were willing to marry young. The latter phenomenon could generate a demographic marriage squeeze against men and result in bride price instead of dowry, violating the evidence on rising dowries in India in the latter half of the twentieth century. Further, the minimum age of first marriage for men in India has been persistently higher than that of women despite an increasing trend of delayed marriage for both sexes over the last century (Mensch et al (2005)). These indicators suggest that parameter values may be such that young men in India choose to postpone marriage at the current marriage payments. In the analysis that follows, I assume this to be the case.

Despite the inherent difficulties of analytically deriving the properties of population equilibria in Pollak's (1987) model, it is possible to characterize steady state equilibria in the general equilibrium model presented here, under the parametric restrictions imposed by the above.

There are five possible demographic configurations that may be obtained in a non-trivial steady state general equilibrium. These are:

$$\begin{aligned} (a)\ f_1^t &> m_1^t > 0 \\ (b)\ f_1^t &= m_1^t < f_1^t + f_0^t \\ (c)\ f_1^t &< f_1^t + f_0^t = m_1^t \\ (d)\ f_1^t &< f_1^t + f_0^t < m_1^t \\ (e)\ f_1^t &< m_1^t < f_1^t + f_0^t \end{aligned} \tag{I1}$$

Proposition 5 is proved in Appendix G.

**Proposition 5.** *Suppose parameter values are such that young men postpone marriage at the offered payments, and old women are matched first when men are indifferent to the age of their spouse. Also suppose that marriages are arranged according to the parental preferences of agents represented by $(M2)$. The only demographic configuration that is consistent with a non-trivial steady state general equilibrium is $(e)$ (see $(I1)$) and the equilibrium marriage payment is a dowry. The aggregate male-to-female sex ratio at birth $(\sigma)$ in this equilibrium is greater than 1.*

Here is the intuition of Proposition 5, proved formally in Appendix G.

Suppose $(a)$ is true in a steady state general equilibrium. This is possible only if the overall sex ratio $\sigma$ is less than 1 (viz. more women than men in any generation).This is because, if old women are paired first by the matchmaker (as assumed), then $f_1^t > m_1^t$ implies $f_1^t = F_0^{t-1}$ (since none of the young women can find a match in any period). Also, if young men postpone marriage (as assumed), then $m_1^t = M_0^{t-1}$. Hence, $(a)$ implies $\sigma^{t-1} = \frac{M_0^{t-1}}{F_0^{t-1}} = \frac{m_1^t}{f_1^t} < 1$ and since $\sigma^{t-1} = \sigma^t = \sigma$ in a steady state equilibrium, $(a)$ can be sustained only when the equilibrium sex ratio $\sigma < 1$. Now consider the marriage payments consistent with an equilibrium of the form $(a)$. Within-group competition among old (high-surplus) females would lead to the payment of a dowry that is equal to the latter's entire surplus from marriage. At these high payments and the expected probabilities of matching consistent with $(a)$, the expected returns from marriage are higher for men ($E_m > E_f$), so it is *not* worthwhile for parents to have more daughters than sons. Hence, a $\sigma$ *greater* than 1 is generated in all non-trivial equilibria, demonstrating that $(a)$ cannot be sustained in a steady state equilibrium.

By an argument similar to the one presented above, $(b)$ would be true in a steady state equilibrium only if $\sigma = 1$. However, when the demographic structure is of the form $(b)$, there are multiple equilibria in marriage payments since the number of high-surplus agents (old men and old women) are equal in number implying that neither group has a credible threat point in marriage market bargaining. The lower limit of payments, $D_1^1$, is the dowry that makes old men indifferent to the age of their spouse (recall that there are young women in the marriage market also) and the upper limit is the payment that reduces the surplus of old women to zero. I assume that in such circumstances, the matchmaker draws the actual payments associated with each match from a uniform distribution over the feasible range. The average payment, $ED_0^1$, is then a dowry. At this payment, however, sons have a higher expected return from the marriage market, so parents prefer to have more sons than daughters and $\sigma > 1$. This shows that $(b)$ cannot be sustained in a steady state equilibrium either.

If $(c)$ were true in a steady state equilibrium, the resulting marriage market demographics would be $f_1^t = 0$ (because all women are matched when young) and $f_0^t + f_1^t = f_0^t = m_1^t$. Such a demographic structure may be replicated in every period, only if the equilibrium growth rate of the population, $(1 + \widehat{r})$, is exactly equal to the sex ratio, $\sigma$.[31] Note also that since women must bear children in the

[31] To see why, suppose that the population is growing in equilibrium so that $(1 + \widehat{r}) > 1$. This will lead to more younger women in the population than older men, if $\sigma \leq 1$. The numbers of

first period of marriage and since there are no old women among eligible marriage market participants, the equilibrium growth rate of the population must be equal to the number of girls born to young women. These observations imply that $\sigma = \frac{b_{m0}}{b_{f0}} = (1+\widehat{r}) = b_{f0}$ which further implies $b_{f0}^2 = b_{m0}$. I show in Appendix G that at the expected marriage payments implied by $(c)$, $(b_{f0}^2 = b_{m0})$ cannot be satisfied for a set of parameters in the relevant range, viz. $K > 0,\ s > 0$ and $\beta \ \epsilon \ (0, 1)$. The intuition of this result is as follows. If $(c)$ is true in equilibrium, there will be multiple equilibria in marriage payments since the number of potential brides and grooms are exactly equal. The upper limit of payments, $D_0^1$, is a dowry equal to young women's pre-payments marital surplus and the lower limit is the bride price equal to old men's pre-payments surplus. If old agents are the high-surplus agents who are matched first, then it must be the case that the (absolute value of the) lower limit exceeds the upper limit. This yields an average payment of bride price.[32] The matching rule also implies that $\beta$ has to be sufficiently low or else the two-period marital returns of young agents will outweigh the surpluses of old agents, making the former the high-surplus agents. Since men get married only in the last period of their life - with returns discounted by a low $\beta$[33] - and expect to pay a bride price at that stage, it is not worthwhile for parents to have as many sons per daughter as ensures $\sigma = (1 + \widehat{r})$.The fertility incentives generated by $(c)$ ensures, at best, $\sigma < (1 + \widehat{r})$, which cannot sustain $(c)$ in a steady state general equilibrium.

Suppose $(d)$ were true in steady state equilibrium. This implies $f_1^t = 0$ (because all women are matched when young) and $f_0^t + f_1^t = f_0^t < m_1^t$. Such a demographic structure may be replicated in every period only if the equilibrium growth rate of the population, $(1+\widehat{r})$, is *less* than the sex ratio, $\sigma$. Since $\sigma$ has to be sufficiently large to sustain an equilibrium like $(d)$ there must be an upper limit on $(E_f - E_m)$ in equilibrium, because $\sigma$ varies inversely with it (see Proposition 4, $(F6)$, $(F7.1)$). It may be shown, however, that at the equilibrium marriage payments implied by $(d)$, $(E_f - E_m)$ will be higher than this upper limit and $\sigma$ will be lower than that which can sustain an equilibrium like $(d)$. This is true because at an equilibrium of

old men and young women will be equal in each period if and only if there are more men born in each period than women ($\sigma > 1$) and by the exact magnitude of population growth. Hence $(1 + \widehat{r}) = \sigma$.

[32] Once again assuming that the matchmaker draws the actual payments from a uniform distribution over the feasible range.

[33] Appendix G shows that this result holds even for $\beta = 1$.

the form $(d)$, old men will pay their entire pre-payments marital surplus as bride price to young women. If old men and women are the high surplus agents, then this bride price will be too large for parents to want as many sons per daughter as is required to sustain $(d)$.

Hence $(e)$ is the only demographic configuration possible in equilibrium. Example 3 demonstrates numerically that a steady state equilibrium of the form $(e)$ exists. I show in Appendix G that in an equilibrium of the form $(e)$, $\sigma$ is greater than 1 and the equilibrium marriage payment is a dowry. This is due to the following reasons. First, there are more eligible women than men in the marriage market since young men postpone marriage. Second, young women stand to gain a positive surplus from marriage. This is because they value marriage sufficiently to want to marry young *and* reap this value over two periods. In equilibrium, there are enough men for all the old women (who are matched first), but not for all the young women. Thus within-group competition makes young women bid away their entire surplus which, being positive, is a dowry. Old women engage in between-group competition and match this offer to make men indifferent to the age of their spouse. Hence, they pay dowry too. This structure of payments and matching probabilities ensure that the marriage market returns of men exceed that of women ($E_m > E_f$), so parents choose more sons than daughters in equilibrium ($\sigma > 1$). However, recall that the marriage market returns of men are a single-period return that is discounted by $\beta$ since they choose to delay marriage. Further, if old agents are the high-surplus agents, then $\beta$ must be sufficiently small or else the two-period gains of young agents would exceed that of their older counterparts (see proof in Appendix G). Moreover, since the high-surplus older women do not engage in *within*-group competition for a spouse, the dowry paid in equilibrium is less than the total surplus of old women. Since the entire surplus of old women is not extracted the equilibrium dowry is relatively small. All these factors impose an upper limit of the excess marriage market returns of men, ensuring that the equilibrium $\sigma$ is not high enough to reverse the marriage squeeze against women.

Proposition 5 underlines the importance of adopting a general equilibrium approach to understand the long run relationship between population dynamics and marriage payments in India. As cases (c) and (d) demonstrate, when older men marry younger women in a steady state equilibrium, marriage payments will take the form of bride price when the aggregate male-to-female sex ratio at birth $\sigma$ exceeds the growth factor $(1 + \widehat{r})$. This is because $\sigma$ governs the number of men in the marriage market and $(1 + \widehat{r})$ determines the number of young cohorts, hence the number of young women in the market. When $\sigma$ is *exogenous*, therefore, bride price may prevail at low fertility levels when $1+\widehat{r} \leq \sigma$ - the idea that defines the motivating question of this paper. Example 2 demonstrates the existence of precisely such a case. Proposition 5 shows, however, that when sex ratios are *endogenously* determined based on marriage market incentives, $\sigma$ must necessarily be small compared to the growth factor $(1+\widehat{r})$, so as to render bride price equilibria of the form (c) or (d) impossible. In the only possible steady state general equilibria, $\sigma$ is low compared to $(1+\widehat{r})$ plus the proportion of young women left unmarried in each period.[34] Since this is true regardless of the fertility level, there can only be dowry payments in a steady state general equilibrium. This insight is noteworthy for its remarkable accuracy in describing current conditions in the Indian marriage market, as outlined in the following table.[35]

| Feature | Model | Evidence |
|---|---|---|
| Marriage Payments | Dowry | Dowry |
| Sex Ratio | Masculine | Masculine |
| % Men Married by Age 45-49 | 100 | 97.6 |
| Groom's Age - Bride's Age (at first marriage) | Positive | Positive |

# 5. Extension: Introducing a Cost Parameter in Sex Ratio Choice

With the widespread availability of sex-selective abortion techniques since the 1980s, the cost of biasing the sex ratio is expected to have fallen. Will this be instrumental in skewing the sex ratio sufficiently to reverse the marriage squeeze (and the result of Proposition 5)? This section introduces a cost parameter in sex ratio choice. I show that the result of Proposition 5 continues to hold at low costs of skewing the sex ratio.

Let $\tau$ be a cost parameter in the post-marriage utility function,

$$U^{marr} = c + E_f b_f + E_m b_m - \tau(b_f - b_m)^2, \qquad \tau > 0 \qquad (F0.1a)$$

Note that in the model presented in the previous sections, $\tau = 1$.

[34] In a long run equilibrium, the proportion of young women left unmarried in any period, $(1-p_0)$, governs the number of old women in the marriage market. Therefore, when $\sigma < (1+\widehat{r}) + (1-p_0)$, there is an excess supply of women in the marriage market and hence dowry payments prevail.

[35] The datum on percentage of men married by age 45-49 is from Tertilt (2004).

Proposition 6 outlines the optimal sex-ratio choice of agents when the post-marriage utility function is of the form $(F0.1a)$. It reinstates the results of Proposition 4 (where $\tau = 1$).

**Proposition 6.** [36] *Suppose that the post-marriage utility function of agents is given by $(F0.1a)$. In all non-trivial equilibria, couples choose to have as many offspring as their total fertility allows, ensuring that young mothers have more offspring than old mothers. Maternal-age $(i)$-specific sex ratios $\sigma_i$ (male/female) of offspring are determined as follows:*

*when $|E_f - E_m| < 4\tau\rho_1$,*

$$\sigma_i = \frac{4\tau\rho_i - (E_f - E_m)}{4\tau\rho_i + (E_f - E_m)} \; \epsilon \; (0, \infty) \; for \; i = 0, 1 \qquad (F6a)$$

*when $4\tau\rho_1 < |E_f - E_m| < 4\tau\rho_0$,*

$$\sigma_0 = \frac{4\tau\rho_0 - (E_f - E_m)}{4\tau\rho_0 + (E_f - E_m)} \; \epsilon \; (0, \infty) \qquad (F7.1a)$$

$$\begin{aligned} \sigma_1 &= 0 \; if \; E_m < E_f \\ \sigma_1 &= \infty \; if \; E_m > E_f \end{aligned} \qquad (F7.2a)$$

*Further, there is no non-trivial steady state equilibrium compatible with the condition $|E_f - E_m| > 4\tau\rho_0$.*

The results in Proposition 6 are used to derive Proposition 7, which summarizes the characteristics of a steady state general equilibrium when the post-marriage utility function is of the form $(F0.1a)$.

**Proposition 7.** [37] *Suppose parameter values are such that young men postpone marriage at offered payments, and old women are matched first when men are indifferent to the age of their spouse. Suppose that the post marriage utility function is given by $(F0.1a)$ where $\tau$ $(> 0)$ represents the cost to parents of choosing to skew the sex ratio of offspring. If $\tau < (1 + \beta)$, then the only demographic configuration that is consistent with a steady state general equilibrium is $(e)$ (see $(I1)$) and the equilibrium marriage payment is a dowry. The aggregate male-to-female sex ratio at birth $(\sigma)$ in this equilibrium is greater than 1.*

[36] See Appendix H.1 for proof.
[37] See Appendix H.2 for proof.

Proposition 7 shows that the results of Proposition 5 are reinforced at low $\tau$. The intuition is as follows. Notice in $(F6a)$ and $(F7.1a)$ that $\tau$ has the same effect on sex ratios as fertility $\rho_i$. In other words, a decline in $\tau$ skews the sex ratio in favor of the offspring with the higher expected surplus in the marriage market. This suggests that low $\tau$ may lead to an over-production of boys when dowry is expected, thereby invalidating the result of Proposition 5. This is not true, however, because a low cost of sex ratio choice does not alter the fact that men marry late and hence receive a single-period discounted return from marriage! Thus the upper limits on $(E_m - E_f)$ and $\sigma$ continue to hold and parents do not over-produce boys in the equilibrium $(e)$, allowing dowry to be sustained in the long run. However, low $\tau$ makes a bride price equilibrium even less likely because it makes parents more inclined to over-produce girls when bride price is expected. A high $\tau$ is, therefore, a *necessary* condition for a bride price to be sustained in equilibrium because this precludes agents from 'over-responding' to the associated high excess returns of girls $(E_f - E_m > 0)$. When $\tau$ is low, this over-production of girls serves to reverse the trend of bride price by generating a marriage squeeze against women in the marriage market.

Propositions 5 and 7 demonstrate that at the given parameters and preference structure $E_f$ is more 'sensitive' to marriage market conditions than $E_m$. This is because women are willing to marry young and when they do so, reap the (high) benefits of marriage in both periods of life. Men, on the other hand, marry only when old whereupon their returns are discounted by $\beta$ (which has to be low to justify the matching rule). The excess sensitivity of $E_f$ ensures that a female advantage in the marriage market cannot be sustained in the long run. The limited sensitivity of $E_m$ guarantees that $\sigma$ is not high enough to take away the male advantage in the marriage market in an equilibrium of the form $(e)$.

## 6. Summary and Conclusion

The dowry inflation that has been observed in India since the 1950s has been attributed to a marriage squeeze against women caused by population growth and the resulting excess supply of (younger) women relative to (older) men in the marriage market. Decreasing fertility levels since the 1970s and an increasing (male/female) sex ratio lead to the question of whether the marriage squeeze will reverse and bride price will emerge as the dominant form of marriage payments in future. The answer gains importance in the light of the literature on the positive effects of female decision-making on intra-household allocations.

This paper provides a theoretical framework for understanding the mechanism of a demographic marriage squeeze and its long run effects on marriage market bargaining and sex ratio choice. It incorporates the two-way link between population dynamics and marriage market decisions by analyzing the effect of the former on marriage payments and also on the choice of sex ratio, which in turn determines the strength of the marriage squeeze in future periods.

I show, using an overlapping-generations dynamic general equilibrium framework, that at parameter values consistent with marriage market indicators in India the only possible steady state equilibria are characterized by dowry and a marriage squeeze against women. A persistent pattern of higher age of first marriage for men in India indicates that it may be optimal for young men to delay marriage at the offered payments. At the same time, the universality of female marriage in India indicates that old women are matched before their younger counterparts. When this is true, the total number of eligible women will exceed the total number of eligible men in the only steady state equilibria that may be sustained in the long run. This, coupled with the fact that women ascribe a positive value to marriage (because they gain sufficiently from marriage to enter the market young and also because they enjoy the benefits of marriage for a longer time) imply that a positive price will be paid for scarce grooms in equilibrium. Moreover, although dowry payments make it optimal for parents to beget more boys than girls in equilibrium, there is an upper limit on the masculinity of the sex ratio. This follows from the fact that there is an upper limit on the expected marriage market returns of men, which is a discounted single-period gain because they marry late. Thus, although the male-to-female sex ratio is greater than one in steady state equilibria, it is low enough to sustain the marriage squeeze against women. I have shown that these results are true especially at low costs of skewing the sex ratio, which with the advent of sex-selective abortion techniques in India, is suspected to be the case.

The above result stems from an analysis that focuses purely on the incentives of the marriage market. Labor and capital markets are ignored, as are exogenous sex preferences and differential costs of singlehood, which must stem from gender-specific differences in labor and capital market opportunities. The analysis provides valuable insights on the interplay between marriage decisions and population dynamics and makes a contribution to the literature in an area that has not been traversed before. The significant contribution of this paper lies both in the accuracy of its description of the current state of the marriage market in India as well as the novelty of the approach undertaken to achieve the same. To the extent that modernization and economic development in India reduces inherent sex biases and brings equality of status and opportunity for men and women the main result may even be considered to be a preview of future marriage market equilibrium outcomes.

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# Appendices[38]

## A. Propositions 1 and 2: A Discussion

Here I shall discuss the intuition of Propositions 1 and 2.

Conditions (a) and (b) in Propositions 1 and 2 ensure that there is some group of agents, say $A$, who will not *all* be matched in equilibrium.

$A$ will engage in within-group competition (viz. competition among agents of the same type) for a spouse, thereby paying out their entire surplus as marriage payment in equilibrium. This will be true since paying an amount less than the whole surplus will enable some other agent in this group to offer a slightly higher payment to the potential spouse, still obtain a positive surplus and guarantee himself/herself a match (since the potential spouse will prefer him/her to all other agents of his/her type).

If $A$ is the high surplus group of agents of its sex, then the payment of this surplus cannot be matched by the low surplus agents (say, $B$) of the same sex. In this situation, equilibrium payments will be such that all high-surplus agents ($A$) offer their entire surplus as payment, only some in $A$ get matched in equilibrium, and spouses prefer agents $A$ to $B$ at the marriage payments the latter can afford to offer.[39]

If $A$ is matched with both types of the spouse, they must be indifferent to the type of the latter. Suppose this is not true and that $A$ prefers spouses of type $C$ to those of type $D$ at the equilibrium marriage payments. Notice that for this to be possible, $A$ must have a positive surplus from marrying $C$ (since equilibrium surpluses can only be non-negative and strict preference implies that the surplus from marrying $C$ must be greater than that from marrying $D$). However, this makes it beneficial for some agent in $A$ to offer an amount to $C$ that is slightly higher than what the rest in $A$ are offering, still obtain a positive surplus and guarantee himself/herself a match with a preferred spouse in $C$! This cannot be an equilibrium. By a similar argument, all agents who are matched in equilibrium with both types of the opposite sex, must be indifferent between the two.

If $A$ is the low surplus group of agents, then in equilibrium, the high-surplus agents of the same sex as $A$ (call them $F$) need only offer the amount that makes

[38] Detailed technical appendices are available from the author upon request.

[39] Note that this does not violate Proposition 1 which states that agents must be indifferent to the types of spouses when their type is matched with *both* types of the spouse in equilibrium.

potential spouses indifferent between $A$ and $F$ (since the matching rule ensures that when there is indifference, high-surplus agents are matched first). This is the result of between-group competition among agents in groups $A$ and $F$.

If $A$ are young agents, then their surplus is the expected difference between the utilities from marrying now and postponing marriage to the next period. Hence, a marriage payment that reduces this surplus to zero is one that makes $A$ indifferent between marrying now and postponing marriage to the next period.

Therefore, when conditions (a) and (b) hold, the marriage surplus of the group of agents ($A$) that do not all find matches determines the payment made by this group. All other payments can be uniquely determined by ensuring that all agents who are matched with both types of spouses in equilibrium are indifferent to them. Hence, there is a unique equilibrium in marriage payments when conditions (a) and (b) hold. This also ensures that there is a unique matching rule, viz. one which matches the high-surplus agents of each sex first,[40] since equilibrium payments ensure that agents who may be matched with both types of spouses are indifferent to them (see definition of the matching technology).

When (a) or (b) is violated, it means that at some stage of the matching process, the numbers of potential partners are exactly equal. When this happens, neither of the two groups of agents have a credible threat point against which a single payment may be determined. Hence there are a range of feasible payments that could be settled upon, each of which would leave some agent with an incentive to negotiate further (see Example 2 for such a case). I assume that in such cases, both parties approach the matchmaker with the feasible range of payments and ask her to randomly draw a payment from a uniform distribution over this range. Thus when (a) or (b) is violated, there could be multiple equilibria in marriage payments. Note however, that whatever the actual payments, the *expected* values have to be such that leave agents indifferent to both types of spouses in expected-surplus terms (if they may be matched to either in equilibrium). Hence, the matching rule is still unique and matches high expected-surplus agents of each type before others.

[40] The matching rule is unique up to the type of agents. Identical agents (of the same type) are matched in random order. Also, if parameters are so aligned that both types of agents obtain the same surplus from marriage (viz. there are no high or low surplus agents) then the matching rule matches agents randomly.

# B. Stable Population Equilibrium

## B.1. Derivation of mapping $\phi$

Let $(F_0^t, F_1^t, M_0^t, M_1^t)$ denote the population vector in period $t$, where $F_i^t$ $(M_j^t)$ denotes the number of females of age $i$ (males of age $j$) in the population in time $t$.

Let $(f_0^t, f_1^t, m_0^t, m_1^t)$ denote the number of eligible agents in the marriage market in period $t$, i.e. the number of single and never-married agents in the population

Let $u_{old}^t$ denote the vector of married agents[41] in the population at the *beginning* of period $t$ and let $u^t$ denote the vector of married agents in the population at the *end* of period $t$.

Then,

$$\begin{aligned} u_{old,00}^t &= 0 \\ u_{old,11}^t &= u_{00}^{t-1} \end{aligned} \tag{1}$$

where $u_{old,ij}^t$ is the number of married couples with woman of age $i$ and man of age $j$ at the beginning of period $t$.

By the traditional definition of 'old unions', $u_{old,01}^t = u_{old,10}^t = 0$. However, using this definition in the analysis that is to follow will not eliminate widowed agents from the pool of eligible agents in each period. Since remarriage is not permitted in the model, I redefine the number of 'old unions' in which one spouse is dead to be equal to the number of once-married agents who are alive and who are not included in $u_{old,11}^t$. The significance of this definition will soon be clear. Hence,

$$\begin{aligned} u_{old,01}^t &= u_{10}^{t-1} \\ & (the\ number\ of\ old\ males\ who\ when\ young,\ married\ old\ females) \\ u_{old,10}^t &= u_{01}^{t-1} \\ & (the\ number\ of\ old\ females\ who\ when\ young,\ married\ old\ males) \end{aligned} \tag{2}$$

[41] Anyone who has been married once and is alive in the period of consideration is a married agent, regardless of whether the spouse is dead or living. Remarriage is not permitted in this model.

In each period single agents are determined as follows:

$$\begin{aligned} f_0^t &= F_0^t - u_{old,00}^t \\ m_0^t &= M_0^t - u_{old,00}^t \\ f_1^t &= F_1^t - u_{old,10}^t - u_{old,11}^t \\ m_1^t &= M_1^t - u_{old,01}^t - u_{old,11}^t \end{aligned} \tag{3}$$

Let $u_{new}^t$ denote the vector of new matches made in period $t$ from among the eligible agents $(f_0^t, f_1^t, m_0^t, m_1^t)$. This is determined by the matching rule $\mu$. Thus,

$$u_{new,ij}^t = \mu_{ij}(f_0^t, f_1^t, m_0^t, m_1^t) \tag{4}$$

Hence, the total number of unions at the *end* of period $t$ is

$$\begin{aligned} u_{00}^t &= u_{old,00}^t + u_{new,00}^t \\ u_{11}^t &= u_{old,11}^t + u_{new,11}^t \\ u_{01}^t &= u_{new,01}^t \\ u_{10}^t &= u_{new,10}^t \end{aligned} \tag{5}$$

(1) – (5) define a mapping $\phi^u$:

$$u_{old}^t = \phi^u(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1}) \tag{A}$$

The number of newborns in each period is given by (recall that old unions do not produce children):

$$\begin{aligned} F_0^t &= \sum_i \sum_j b_{ij} u_{new,ij}^{t-1} = \sum_i \sum_j b_{ij} \mu_{ij}(f_0^{t-1}, f_1^{t-1}, m_0^{t-1}, m_1^{t-1}) \\ M_0^t &= \sigma \sum_i \sum_j b_{ij} u_{new,ij}^{t-1} = \sigma \sum_i \sum_j b_{ij} \mu_{ij}(f_0^{t-1}, f_1^{t-1}, m_0^{t-1}, m_1^{t-1}) \end{aligned}$$

Using (3), the above can be written as

$$\begin{aligned} F_0^t &= \sum_i \sum_j b_{ij} \mu_{ij}(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1}) \\ M_0^t &= \sigma \sum_i \sum_j b_{ij} \mu_{ij}(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1}) \end{aligned} \tag{6}$$

Since all young agents live to be old and all old agents die at the end of the period, we have:

$$\begin{aligned} F_1^t &= F_0^{t-1} \\ M_1^t &= M_0^{t-1} \end{aligned} \tag{7}$$

$(6) - (7)$ define a mapping $\phi^1$:

$$(F_0^t, F_1^t, M_0^t, M_1^t) = \phi^1(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1}) \tag{B}$$

$(A)$ and $(B)$ define the mapping $\phi$:

$$(F_0^t, F_1^t, M_0^t, M_1^t, u_{old}^t) = \phi(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1})$$

## B.2. Proof of Proposition 3

**Proof.** Suppose that the total population is in a stable population equilibrium, growing at the rate $(1+\widehat{r})$. Denote $(1+\widehat{r}) = \lambda$.

Then, by Definition 6,

$$\begin{aligned} F_i^t &= \lambda F_i^{t-1} & i &= 0,1 \\ M_j^t &= \lambda M_j^{t-1} & j &= 0,1 \\ u_{old,kl}^t &= \lambda u_{old,kl}^{t-1} & k &= 0,1; l = 0,1 \end{aligned} \tag{8}$$

Using Equation (3) from Appendix B.1, we have

$$\begin{aligned} f_0^{t+1} &= F_0^{t+1} - u_{old,00}^{t+1} = \lambda(F_0^t - u_{old,00}^t) = \lambda f_0^t \\ m_0^{t+1} &= M_0^{t+1} - u_{old,00}^{t+1} = \lambda(M_0^t - u_{old,00}^t) = \lambda m_0^t \\ f_1^{t+1} &= F_1^{t+1} - u_{old,10}^{t+1} - u_{old,11}^{t+1} = \lambda(F_1^t - u_{old,10}^t - u_{old,11}^t) = \lambda f_1^t \\ m_1^{t+1} &= M_1^{t+1} - u_{old,01}^{t+1} - u_{old,11}^{t+1} = \lambda(M_1^t - u_{old,01}^t - u_{old,11}^t) = \lambda m_1^t \end{aligned} \tag{9}$$

Hence, by Definition 7, the eligible population in the marriage market $(f^t, m^t)$ is in a stable population equilibrium and grows at the rate $\lambda = (1+\widehat{r})$. ■

## C. Choice of Optimal Marriage Payments in a Dynamic Setting

In this section, I state the optimization problems that agents solve in order to determine their optimal marriage payments in a dynamic setting. Since the value function is not continuous in the control variables, the problem cannot be solved by differentiating to take first order conditions. Nevertheless, I present the formulation to clarify the decision making process of agents of each sex and age. All the assumptions of Section 3.2 hold.

Let $W_i^g$ denote the value function of an agent of age $i$ and gender $g$. (In reality, parents arrange the matches of offspring based on the utility function from marriage that they ascribe to the latter. However, the outcome of marriage market bargaining will be the same whether it is conducted by parents or by agents using the utility function from marriage that their parents ascribe to them. Therefore I present the bargaining as conducted by agents using their parental utility function.)

Old agents will die at the end of the period. Hence the value functions of old agents in period $t$ are as follows.

Old women:

$$\begin{gathered}
W_1^f(f_0^t, f_1^t, m_0^t, m_1^t) = \underset{D_1^{1t}, D_1^{0t}}{Max}\,[ \\
p_1^{1t}(K + s - 1 + (w - s) - D_1^{1t}) \\
+p_1^{0t}(K + s - 2 + (w - s) - D_1^{0t}) \\
+ (1 - p_1^{1t} - p_1^{0t})(w - s) \\
sub.\ to\ K + s - 1 - D_1^{1t} \geq 0, \\
K + s - 2 - D_1^{0t} \geq 0\ ]
\end{gathered}$$

where $p_i^{jt}$ is the probability that a woman of age $i$ shall match with a man of age $j$ in period $t$.

Old men:

$$W_1^m(f_0^t, f_1^t, m_0^t, m_1^t) = \underset{D_0^{1t}, D_1^{1t}}{Max}[$$
$$q_0^{1t}(K + s + (w - s) + D_0^{1t})$$
$$+ q_1^{1t}(K + s - 1 + (w - s) + D_1^{1t})$$
$$+ (1 - q_0^{1t} - q_1^{1t})(w - s)$$
$$sub.\ to\ K + s + D_0^{1t} \geq 0,$$
$$K + s - 1 + D_1^{1t} \geq 0]$$

where $q_i^{jt}$ is the probability that a man of age $j$ shall match with a woman of age $i$ in period $t$.

Note that $p_i^{jt}$ and $q_i^{jt}$ are functions of $(f_0^t, f_1^t, m_0^t, m_1^t)$ and $\{D_i^{jt}\}_{i,j=0,1}$.

Young agents will live for another period. Hence their value functions include an expectation of future marriage potential if they fail to find a match in the current period. The value functions of young agents in period $t$ are as follows.

Young women:

$$W_0^f(f_0^t, f_1^t, m_0^t, m_1^t) = \underset{D_0^{1t}, D_0^{0t}}{Max}[$$
$$p_0^{1t}\{(K + w)(1 + \beta) - D_0^{1t}\} + p_0^{0t}\{(K - 1 + w)(1 + \beta) - D_0^{0t}\}$$
$$+ (1 - p_0^{1t} - p_0^{0t})\{w + \beta E(W_1^f(f_0^{t+1}, f_1^{t+1}, m_0^{t+1}, m_1^{t+1}/f_0^t, f_1^t, m_0^t, m_1^t)\}$$
$$sub.\ to\ K(1 + \beta) - D_0^{1t} - \beta E(W_1^f(f_0^{t+1}, f_1^{t+1}, m_0^{t+1}, m_1^{t+1}/f_0^t, f_1^t, m_0^t, m_1^t) \geq 0,$$
$$(K - 1)(1 + \beta) - D_0^{0t} - \beta E(W_1^f(f_0^{t+1}, f_1^{t+1}, m_0^{t+1}, m_1^{t+1}/f_0^t, f_1^t, m_0^t, m_1^t) \geq 0]$$

Young men:

$$W_0^m(f_0^t, f_1^t, m_0^t, m_1^t) = \underset{D_0^{0t}, D_1^{0t}}{Max}[$$
$$q_0^{0t}\{(K - 1 + w)(1 + \beta) + D_0^{0t}\} + q_1^{0t}\{(K - 2 + w)(1 + \beta) + D_1^{0t}\}$$
$$+(1 - q_0^{0t} - q_1^{0t})\{w + \beta E(W_1^m(f_0^{t+1}, f_1^{t+1}, m_0^{t+1}, m_1^{t+1}/f_0^t, f_1^t, m_0^t, m_1^t)\}$$
$$subject\ \ to\ (K - 1)(1 + \beta) + D_0^{0t} - \beta E(W_1^m(f_0^{t+1}, f_1^{t+1}, m_0^{t+1}, m_1^{t+1}/f_0^t, f_1^t, m_0^t, m_1^t) \geq 0,$$
$$(K - 2)(1 + \beta) + D_1^{0t} - \beta E(W_1^m(f_0^{t+1}, f_1^{t+1}, m_0^{t+1}, m_1^{t+1}/f_0^t, f_1^t, m_0^t, m_1^t) \geq 0\ ]$$

Recall that $p_i^{jt}$ and $q_i^{jt}$ are functions of $(f_0^t, f_1^t, m_0^t, m_1^t)$ and $\{D_i^{jt}\}_{i,j=0,1}$.

The state variables $(f_0^t, f_1^t, m_0^t, m_1^t)$ evolve according to the following equations:

$$\begin{aligned}
f_0^{t+1} &= \sum_i \sum_j b_{ij} \mu_{ij}^t (f_0^t, f_1^t, m_0^t, m_1^t) \\
m_0^{t+1} &= \sigma \sum_i \sum_j b_{ij} \mu_{ij}^t (f_0^t, f_1^t, m_0^t, m_1^t) \\
f_1^{t+1} &= Max[0,\ f_0^t - \sum_j \mu_{0j}^t (f_0^t, f_1^t, m_0^t, m_1^t)] \\
m_1^{t+1} &= Max[0,\ m_0^t - \sum_i \mu_{i0}^t (f_0^t, f_1^t, m_0^t, m_1^t)]
\end{aligned}$$

where $b_{ij}$ is the number of female children born to couple $(i, j)$, $\sigma$ is the sex ratio (male/female) at birth and $\mu_{ij}^t(f_0^t, f_1^t, m_0^t, m_1^t)$ represents the number of $(i, j)$ matches made from the eligible pool of marriage market participants - $(f_0^t, f_1^t, m_0^t, m_1^t)$ - in period $t$. Note that $i$ stands for the age of the woman and $j$ for the age of the man in a couple $(i, j)$.

## D. Example 2: A Discussion

Let $S_i^j$ denote the total surplus from the marriage of a woman of age $i$ and a man of age $j$.

Let $v_i^j(V_i^j)$ denote the pre-marriage-payments surplus from marriage that goes to a woman of age $i$ (man of age $j$) upon marrying a man of age $j$ (woman of age $i$).

Then, $S_i^i = v_i^j + V_i^j$.

Let $ED_i^j$ denote the expected marriage payment associated with the match of a woman of age $i$ and a man of age $j$.

Younger women's surplus from marriage are as follows:

$$\begin{aligned}
v_0^0 &= (w + K - 1)(1 + \beta) - [w + \beta X^f] \qquad (1') \\
v_0^1 &= (w + K)(1 + \beta) - [w + \beta X^f]
\end{aligned}$$

where $X^f$ denotes young women's expectations of marriage returns in the next period. In the current example,

$$X^f = p(w + K - 1 - ED_1^1) + (1 - p)(w - s) \qquad (1)$$

where $p$ is the probability that the young woman will find a partner in the *next* period.[42]

Younger men's surplus from marriage are

$$\begin{aligned} V_0^0 &= (w+K-1)(1+\beta) - [w+\beta X^m] \\ V_1^0 &= (w+K-2)(1+\beta) - [w+\beta X^m] \end{aligned} \qquad (2')$$

where $X^m$ denotes young men's expectations of marriage returns in the next period. In the current example,

$$X^m = q(w+K-1+ED_1^1) + (1-q)(w-s) \qquad (2)$$

where $q$ is the probability that the young man will find a partner in the *next* period.

The current example has the following parameter values: $\sigma = 1.6$, $b_1 = 1$, $b_0 = 1.6$.

Suppose that older agents are the high surplus agents and are paired first in equilibrium. I shall derive the conditions under which this will be true. Then, elementary arithmetic shows that

$$f_0^t = m_1^t,\ f_1^t = 0,\ m_0^t = 1.6 f_0^t,\ \widehat{r} = 0.6 \qquad (A)$$

defines a stable population equilibrium with these demographic parameters. Since older agents are matched first, $p = q = 1$.

Hence, the surpluses that women receive from marriage (using $(1)' - (2)$ and matching probabilities consistent with $(A)$) are:

$$\begin{aligned} v_0^0 &= (K-1)(1+\beta) - \beta(K-1-ED_1^1) \\ v_0^1 &= K(1+\beta) - \beta(K-1-ED_1^1) \\ v_1^0 &= K+s-2 \\ v_1^1 &= K+s-1 \end{aligned}$$

[42] In equilibrium, older women have to indifferent to the age of men they marry, so it suffices to focus on the overall probability of finding a partner, $p$, (instead of the individual probabilities of finding an old or a young man) since the payoff will be the same regardless of the latter's age. The only case in which older (high-surplus) women may not be indifferent to the type of spouse is when the number of high-surplus men exceeds the number of high-surplus women. In such an event, within-group competition will make high-surplus men pay their entire surplus as bride price - an amount that low-surplus agents cannot match - so older women will prefer the former to the latter. In this example, older men and older women are the high surplus agents, so either way, the representation of expected gains from postponing marriage is correct.

The surpluses that men receive from marriage are:

$$
\begin{aligned}
V_0^0 &= (K-1)(1+\beta) - \beta(K-1+ED_1^1) \\
V_0^1 &= K+s \\
V_1^0 &= (K-2)(1+\beta) - \beta(K-1+ED_1^1) \\
V_1^1 &= K+s-1
\end{aligned}
$$

Denote the equilibrium marriage payments by $D = \{D_0^0, D_0^1, D_1^0, D_1^1\}$.

Let $\widetilde{D}_0^0$ denote the minimum dowry that young men accept in order to marry now, knowing that their expected future probability ($q$) of finding a match is 1. Hence, $\widetilde{D}_0^0 = -V_0^0$.

Let $\widehat{D}_0^0$ denote the maximum dowry that young women are willing to offer to young men, at a future expected matching probability ($p$) of 1. Hence, $\widehat{D}_0^0 = v_0^0$.

Some algebra will show that young men will postpone marriage if:

$$
\begin{aligned}
\widetilde{D}_0^0 &> \widehat{D}_0^0 \\
or,\ K &< 1
\end{aligned}
\tag{3}
$$

Consider the maximum dowry, $\widehat{D}_0^1$ that young women are willing to pay old men:

$$
\begin{aligned}
v_0^1 - \widehat{D}_0^1 &= 0 \\
or,\ \widehat{D}_0^1 &= K + 2\beta + \beta ED_0^1
\end{aligned}
\tag{4}
$$

(Note that $ED_0^1$ may not equal $D_0^1$ since there may be multiple equilibria in the model, as I shall discuss later in the proof.)

Suppose younger men are not willing to marry at the equilibrium marriage payments ($K < 1$). Then (4) gives the maximum dowry that young women are willing to pay to old men. However, since the number of eligible men and women who are willing to marry are exactly equal and since old men have only one period to find a match, the latter could be coerced to pay a bride price upto the point that their marriage surplus is reduced to zero. Thus there may be multiple equilibria in marriage payments (the limits of the range of possible marriage payments are defined by the qauntities that reduce the surplus of each party to 0) due to the fact that neither of the two parties has a credible threat point. Suppose that in such a situation, agents (young women and old men) approach the matchmaker and request her to draw a marriage payment from a uniform distribution over the feasible range of payments.

Let $X$ denote the payment drawn by a matchmaker. Then

$$X \sim U\ [-(K+s),\ K+2\beta+\beta ED_0^1]$$

Hence,

$$E(X) = ED_0^1 = \frac{2\beta - s}{2-\beta} \tag{5}$$

Older men will be indifferent to the age of their spouse when,

$$ED_1^1 = 1 + ED_0^1 = \frac{\beta - s + 2}{2-\beta} \tag{5'}$$

To see if the average payment, $ED_0^1$, is a bride price or dowry, let us look at the condition that must hold to ensure that the matching rule matches older agents first, when payments are given by (5) and (5′).

Older women will be matched first if,

$$\begin{aligned} S_1^1 &> S_0^1 \\ or,\ v_1^1 + V_1^1 &> v_0^1 + V_0^1 \\ or,\ s &> \beta + 2 \end{aligned} \tag{6}$$

Hence, (6) implies

$$ED_0^1 = \frac{2\beta - s}{2-\beta} < 0$$

since $\beta \in (0,1)$. Therefore, the average marriage payment is a bride price.
Denote $ED_0^1 = \frac{2\beta-s}{2-\beta} = x$.

[Older men will be matched first if:

$$\begin{aligned} S_0^1 &> S_0^0 \\ or,\ v_0^1 + V_0^1 &> v_0^0 + V_0^0 \end{aligned}$$

$$or,\ s > \frac{-(2+\beta)}{1-\beta} \tag{7}$$

Note that (7) is true since $s > 0$ and $\beta\ \epsilon\ (0,1)$ ]

Suppose now that $K > 1$.

Then there are some marriage payments that are acceptable to both young men and young women for marriage in the current period. Hence, the outside option of young women contemplating marriage with an old man, is a match with a young man for $\widetilde{D}_0^0$. We obtain the maximum dowry that young women will offer old men to be:

$$\widetilde{D}_0^1 = \widetilde{D}_0^0 + 1 + \beta = \beta ED_0^1 - K + 2 + 2\beta$$

The range of feasible marriage payments $D_0^1$ is therefore $[-(K+s),\ \beta ED_0^1 - K + 2 + 2\beta]$. Assuming that the matchmaker draws a random payment from a uniform distribution over this range, we have

$$ED_0^1 = \frac{2(1-K) + 2\beta - s}{2-\beta} \tag{8}$$

Denote $ED_0^1 = \frac{2+2\beta-2K-s}{2-\beta} = y$.

Recall that in the previous case when young men were not willing to marry young, we computed $ED_0^1 = \frac{2\beta - s}{2-\beta} = x$.

Then the following is true:

(a) $y < 0$ if (6) is true (since $K > 1$, $\beta \epsilon (0,1)$), hence the average marriage payment is a bride price when young men are willing to match and old women are matched first.

(b) $y < x$ (since $y - x = \frac{2(1-K)}{2-\beta} < 0$ when $K > 1$, $\beta \epsilon (0,1)$). So when young men are willing to marry young, the bargaining power of women in the marriage market is increased and the bride price is higher.

## E. Proof of Proposition 4

**Proof.** The optimization problem in $(F1.1) - (F1.2)$ may be solved by the Lagragean method. The Lagrangean function is

$$L = c + E_f b_f + E_m b_m - (b_f - b_m)^2 + \lambda[\rho_i - b_f - b_m] + \pi_f b_f + \pi_m b_m$$

The first order conditions are:

$$L_f = \frac{\partial L}{\partial b_f} = E_f - 2(b_f - b_m) - \lambda + \pi_f = 0 \tag{F2.1}$$

$$L_m = \frac{\partial L}{\partial b_m} = E_m + 2(b_f - b_m) - \lambda + \pi_m = 0 \quad (F2.2)$$

$$\lambda L_\lambda = \lambda(\rho_i - b_f - b_m) = 0; \lambda \geq 0, (\rho_i - b_f - b_m) \geq 0 \quad (F2.3)$$

$$\pi_f L_{\pi_f} = \pi_f b_f = 0; \pi_f \geq 0, \; b_f \geq 0 \quad (F2.4)$$

$$\pi_m L_{\pi_m} = \pi_m b_m = 0; \pi_m \geq 0, \; b_m \geq 0 \quad (F2.5)$$

Suppose $\lambda > 0$ and $\pi_f = \pi_m = 0$ (i.e. $b_f > 0, \; b_m > 0 \; and \; b_f + b_m = \rho_i$).

The interior solution is obtained as

$$\begin{aligned} b^*_{fi} &= \frac{4\rho_i + (E_f - E_m)}{8} \\ b^*_{mi} &= \frac{4\rho_i - (E_f - E_m)}{8} \end{aligned} \quad (F5)$$

Suppose parameters are such that $b^*_{fi} > 0, \; b^*_{mi} > 0$. Let us check that the assumption of $\lambda > 0$ is justified. The marginal utilities of having a boy or a girl at $(b^*_f, b^*_m)$ are given by:

$$\frac{\partial U^{marr}}{\partial b_f} = \frac{\partial U^{marr}}{\partial b_m} = \frac{E_f + E_m}{2} \geq 0$$

It is clearly optimal to conceive as many children as total fertility allows as long as the marginal utility of the additional child is strictly positive. Note that the marginal utility of the additional child will be zero only if $E_f = E_m = 0$. This can occur when both agents have a zero probability of finding a match in their lifetime and may be true only when the population is reduced to zero, viz. a 'trivial' stable population. $E_f$ (or $E_m$) may also be zero if there are so many women (men) in the population that they are forced to pay out their entire surplus as marriage payment in each period. However, when this is the case, men (women) find matches with ease and get paid positive dowries (bride price), hence $E_m$ $(E_f) > 0$. Thus in all meaningful and non-trivial equilibria, the marginal utility of offspring is positive for interior solutions $(F5)$ and couples will produce as many children as the fertility of the mother will allow.

Consider, without loss of generality, a corner solution in which $b_{mi} = 0, \; b_{fi} > 0$. Is the constraint on total fertility $(F2.3)$ binding?

A binding constraint $(F2.3)$ and $b_{mi} = 0$ implies $b_{fi} = \rho_i$. At this $b_{fi}$, $\frac{\partial U^{marr}}{\partial b_f} = E_f - 2\rho_i$. Note also that for $b_{mi} = 0$, we must have $4\rho_i - (E_f - E_m) < 0$ (see $(F5)$). But this implies that $\rho_i < \frac{E_f - E_m}{4} < \frac{E_f}{2} - (\frac{E_f + E_m}{4}) < \frac{E_f}{2}$. Hence, $\frac{\partial U^{marr}}{\partial b_f} = E_f - 2\rho_i > 0$ and the constraint $(F2.3)$ does indeed bind for corner solutions in which offspring of only one sex are desired in equilibrium.

The total fertility constraint is not binding in the case where $b_{fi} = b_{mi} = 0$. However, the only steady state equilibrium that is compatible with $b_{fi} = b_{mi} = 0$ is the trivial equilibrium.

Hence, in all non-trivial population equilibria, couples have as many children as total fertility allows. This implies that younger women have more children than older women, since fertility depends only on the age of the mother and younger mothers are more fertile.

Let us look at the conditions under which constraints $(F2.4)$ and $(F2.5)$ are binding. Since $\rho_1 < \rho_0$, when $|E_f - E_m| > 4\rho_0$, all agents choose $b_{mi} = 0$ if $E_f > E_m$ or $b_{fi} = 0$ if $E_f < E_m$. This cannot be compatible with a steady state equilibrium for the following reason. Suppose, without loss of generality, that $E_f > E_m$, so that $|E_f - E_m| > 4\rho_0$ implies that all agents choose to have baby girls. This will wipe out boys from the population reducing $E_f$ to zero (because the girls cannot find matches) and violating the condition that $E_f > E_m$. When $|E_f - E_m| < 4\rho_1$, both constraints $(F2.4)$ and $(F2.5)$ will fail to bind and an interior solution such as in $(F5)$ is obtained. When $4\rho_1 < |E_f - E_m| < 4\rho_0$ the high fertility couples (with young women) will have offspring of both sexes but the low fertility agents have offspring of one sex only (viz. the one with the higher $E_g$).

This yields the result stated in Proposition 4. ■

## F. Example 3: A Discussion

Assume the following parameter values: $\varphi_0 = 9,\ \varphi_1 = 3,\ K = \beta = 0.5,\ s = 5$

Then a steady state equilibrium exists and has the following characteristics (Claims $(F.1) - (F.5)$):

**Claim F.1.** *Young men are not willing to marry at the equilibrium marriage payments because $K$ is too small.*

**Proof.** Let $\widetilde{D}_0^0$ denote the minimum dowry that young men accept in order to marry now, knowing that their expected future probability of finding a match is one. Then,

$$\begin{aligned}\widetilde{D}_0^0 &= -V_0^0 \\ or,\ \widetilde{D}_0^0 &= \beta ED_0^1 - K + (1+\beta) \\ or,\ \widetilde{D}_0^0 &= 0.5ED_0^1 + 1\end{aligned}$$

Let $\widehat{D}_0^0$ be the highest payment that young women are willing to offer to young men, knowing that their expected future probability of finding a match is one. Then,

$$\begin{aligned}v_0^0 - \widehat{D}_0^0 &= 0 \\ or,\ \widehat{D}_0^0 &= (K+\beta-1) + \beta ED_0^1 \\ or,\ \widehat{D}_0^0 &= 0.5ED_0^1\end{aligned}$$

Clearly, $\widetilde{D}_0^0 > \widehat{D}_0^0$, so young women are unwilling to pay young men the minimum payment they demand in order to marry. Hence young men do not participate in the marriage market. ∎

**Claim F.2.** *The equilibrium matching rule matches old men and women first.*

**Proof.** Assume that equilibrium payments are $D_0^1 = \frac{K+2\beta}{1-\beta} = 3,\ D_1^1 = \frac{K+1+\beta}{1-\beta} = 4.$ (Part 4 proves that these are indeed the equilibrium payments.) Then,

$$\begin{aligned}S_0^1 &= v_0^1 + V_0^1 = K(1+\beta) - \beta(K - 1 - ED_1^1) + K + s \\ or,\ S_0^1 &= 8.5 \\ S_1^1 &= v_1^1 + V_1^1 = 2(K+s-1) \\ or,\ S_1^1 &= 9\end{aligned}$$

$S_1^1 > S_0^1$, so old women are matched before young women when men are indifferent to the age of their spouse.

Old men are matched first because there are no young women in the market at the offered payments. ∎

**Claim F.3.** *In a stable population equilibrium,* $\overline{q}_0 = 0,\ \overline{q}_1 = 1,\ \overline{p}_0 = 0.403,\ \overline{p}_1 = 1$. *That is, in every period, young men refrain from marrying, old men are matched with all old women and some young women. Also, the stable population grows at the rate* $(1+\widehat{r}) = 1.922$, *so* $\widehat{r} = 0.922$.

**Proof.** This is computationally demonstrated by graphing the evolution of the population given the parameter values and the optimal birth rates derived in Claim 5 below. The results are presented in Table (a) at the end of the Appendix H. Table (a) shows graphs of the evolution of the population beginning from some initial vector of marriage market participants. I have presented graphs of $\frac{F_0^{t+1}}{F_0^t}$, $\frac{M_0^{t+1}}{M_0^t}$, $\frac{f_0^{t+1}}{f_0^t}$ and $\frac{m_0^{t+1}}{m_0^t}$ (since the assumed mortality rates imply $F_1^t = F_0^{t-1}$, $M_1^t = M_0^{t-1}$ and the fact that all newborns are single ensure $F_0^t = f_0^t$, $M_0^t = m_0^t$). ■

**Claim F.4.** *The equilibrium marriage payments are* $D_0^1 = \frac{K+2\beta}{1-\beta} = 3$, $D_1^1 = \frac{K+1+\beta}{1-\beta} = 4$. *Hence the equilibrium marriage payments are dowries.*

**Proof.** Assume $\overline{q}_0 = 0$, $\overline{q}_1 = 1$, $\overline{p}_0 = 0.4$, $\overline{p}_1 = 1$. Since all young women do not find a match in equilibrium, within-group competition for spouses must reduce their marriage surplus to zero. So,

$$\begin{aligned} v_0^1 - D_0^1 &= 0 \\ or,\ D_0^1 &= K(1+\beta) - \beta(K - 1 - ED_1^1) \\ or,\ D_0^1 &= \frac{K+2\beta}{1-\beta} = 3 \end{aligned}$$

(since in a steady state general equilibrium, $D_i^j = ED_i^j$ for all $i$, $j$ and $D_0^1 = 1 + D_1^1$ so that old men are indifferent to the age of their spouse). Hence,

$$D_1^1 = D_0^1 + 1 = \frac{K+1+\beta}{1-\beta} = 4$$

The payments are positive, hence dowries. ■

**Claim F.5.** *The optimal maternal-age-specific birth rates and sex ratios are:*
$b_{f0} = 4,\ b_{m0} = 5,\ \sigma_0 = 1.25$
$b_{f1} = 1,\ b_{m1} = 2,\ \sigma_1 = 2$

**Proof.** Given the parameter values and the results derived above, we have (see equation $(F0.2)$ and $(F0.3)$ in the main body of the paper),

$$\begin{aligned} E_f &= \overline{p}_0[K(1+\beta) + \beta s - ED_0^1] + \beta(1-\overline{p}_0)\overline{p}_1[K + s - 1 - ED_1^1] \\ or,\ E_f &= 0.25 \\ E_m &= \overline{q}_0[(K-1)(1+\beta) + \beta s + ED_0^0] + \beta(1-\overline{q}_0)\overline{q}_1[K + s + ED_0^1] \\ or,\ E_m &= 4.25 \end{aligned}$$

Since $|E_f - E_m| < 4\rho_1$, there will be an interior solution to the problem of sex ratio choice (see Proposition 4). Using equation $(F5)$ of Appendix E, this is

$$\begin{aligned} b_{f0} &= \frac{4\rho_0 + (E_f - E_m)}{8} = 4,\ b_{m0} = \frac{4\rho_0 - (E_f - E_m)}{8} = 5,\ \sigma_0 = 1.25 \\ b_{f1} &= \frac{4\rho_1 + (E_f - E_m)}{8} = 1,\ b_{m0} = \frac{4\rho_1 - (E_f - E_m)}{8} = 2,\ \sigma_1 = 2 \end{aligned}$$

■

## G. Proof of Proposition 5

There are five possible demographic configurations that may be obtained in a non-trivial steady state general equilibrium:

$$\begin{aligned} (a)\ f_1^t &> m_1^t > 0 \\ (b)\ f_1^t &= m_1^t < f_1^t + f_0^t \\ (c)\ f_1^t &< f_1^t + f_0^t = m_1^t \\ (d)\ f_1^t &< f_1^t + f_0^t < m_1^t \\ (e)\ f_1^t &< m_1^t < f_1^t + f_0^t \end{aligned} \qquad (I1)$$

Recall that a steady state general equilibrium is one in which the the age-sex structure of the population, marriage payments $\{D_i^j\}_{i,j=0,1}$ and sex ratio decisions $(\ b_{fi}^*,\ b_{mi}^*)_{i=0,1}$ are unchanging over time. This requires that there be a stable population equilibrium as well as equilibrium in the marriage market in each period.

**Proof.** (a) Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $f_1^t > m_1^t > 0$ for all $t$ (case $(a)$).

Since the matching rule matches old women first, none of the young women are matched in equilibrium. Also, since young men choose to postpone marriage and all young agents survive to old age, we have $f_1^t = F_1^t = F_0^{t-1}$ and $m_1^t = M_1^t = M_0^{t-1}$. Hence $f_1^t > m_1^t$ implies $F_0^{t-1} > M_0^{t-1}$, so

$$\sigma^{t-1} = \frac{M_0^{t-1}}{F_0^{t-1}} < 1$$

Since the population is in a stable equilibrium, growing at $\widehat{r}$, the overall sex ratio must be a constant $\sigma$ in all periods. Hence,

$$\sigma < 1 \qquad for\ all\ t \tag{1}$$

Equilibrium marriage payments are

$$D_1^1 = K + s - 1$$

due to within group competition among older women who pay their entire marital surplus as dowry.

Using equations $(F0.2)$ and $(F0.3)$ (in the main body of the paper), I can derive

$$\begin{aligned} E_f &= 0 \\ E_m &= 2\beta(K + s - 1) \\ E_f - E_m &= -2\beta(K + s - 1) \end{aligned} \tag{2}$$

at the equilibrium marriage payments.

Note that for old women to be matched first, $S_1^1 > S_0^1$, must be true in equilibrium. This reduces to the following condition:

$$s > \frac{2 + K\beta}{1 - \beta} \tag{2'}$$

It can easily be shown using $(2)'$, that $(K + s - 1) > 0$, hence

$$E_f - E_m = -2\beta(K + s - 1) < 0 \tag{2''}$$

Since we are interested only in non-trivial equilibria, let $|E_f - E_m| < 4\rho_0$ as outlined in Proposition 4.

Suppose $|E_f - E_m| < 4\rho_1 < 4\rho_0$ (viz. there exists an interior solution to the problem of sex ratio choice).

Then,

$$\begin{aligned} b_{fi} &= b_{fi}^* = \frac{4\rho_i + (E_f - E_m)}{8} \\ b_{mi} &= b_{mi}^* = \frac{4\rho_i - (E_f - E_m)}{8} \end{aligned} \tag{3}$$

$(2)''$ implies that $b_{f_i} < b_{m_i}$ for each $i$, so

$$\sigma = \frac{\sum_i b_{mi}\mu_{i.}^t}{\sum_i b_{f_i}\mu_{i.}^t} > 1$$

where $\mu_{i.}^t$ is the number of matches made in period $t$ involving women of age $i$ (in equilibrium, $\mu_{i.}^t$ also grows at the rate $\widehat{r}$) and $b_{gi}$ is the optimal number of offspring of gender $g$ that a woman of age $i$ mothers.

This contradicts (1).

Suppose $4\rho_1 < |E_f - E_m| < 4\rho_0$.

Then,

$$\begin{aligned} b_{f0} &= \frac{4\rho_0 + (E_f - E_m)}{8} \\ b_{m0} &= \frac{4\rho_0 - (E_f - E_m)}{8} \\ b_{f1} &= 0 \\ b_{m1} &= \rho_1 \end{aligned}$$

Again, $b_{f_i} < b_{m_i}$ for each $i$, so $\sigma = \frac{\sum_i b_{mi}\mu_{i.}}{\sum_i b_{f_i}\mu_{i.}} > 1$.

This contradicts (1).

Hence there cannot be a steady state general equilibrium of the form $(a)$.

(b) Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $f_1^t = m_1^t < f_1^t + f_0^t$ for all $t$.

Since old women are matched first, none of the young women are paired in equilibrium. Also, all young agents live to old age. Hence, $f_1^t = F_1^t = F_0^{t-1}$ and $m_1^t = M_1^t = M_0^{t-1}$. Therefore, $f_1^t = m_1^t$ in equilibrium implies

$$\sigma = 1 \tag{5}$$

Since the numbers of old men and old women are perfectly matched, there will be multiple equilibria in marriage payments. The lower limit of payments $D_1^1$ will be the dowry that makes men indifferent to the age of women. This may be derived as

$$\underline{D}_1^1 = 1 + D_0^1 = 1 + v_0^1$$

$$or,\ \underline{D}_1^1 = 1 + K + \beta + \beta E D_1^1 \tag{7}$$

The upper limit is the dowry that reduces old women's surplus to zero. This is derived as

$$\overline{D}_1^1 = K + s - 1$$

Therefore,

$$D_1^1 \ \epsilon \ [1 + K + \beta + \beta E D_1^1, \ K + s - 1] \tag{8}$$

Assuming that the matchmaker draws the actual marriage payment from a uniform distribution over the above range, we have

$$ED_1^1 = \frac{2K + s + \beta}{2 - \beta}$$

Hence, we can derive

$$\begin{aligned} E_f - E_m &= -2\beta E D_1^1 \\ or, \ E_f - E_m &= -2\beta(\frac{2K + s + \beta}{2 - \beta}) < 0 \end{aligned} \tag{9}$$

As before, $E_f - E_m < 0$ implies that in a non-trivial equilibrium, $b_{f_i} < b_{m_i}$ for each $i$, hence

$$\sigma > 1$$

This contradicts (5).

Hence there cannot be a steady state general equilibrium of the form ($b$).

(c) Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $f_1^t + f_0^t = m_1^t$ for all $t$.

In such an equilibrium, $f_1^t = 0$ since all young women find a match in every period. Therefore the above condition reduces to $f_0^t = m_1^t$. Since $f_0^t = F_0^t = (1 + \widehat{r})F_0^{t-1}$ and $m_1^t = M_0^{t-1}$, this implies

$$(1 + \widehat{r}) = \sigma \tag{10}$$

Also, since $f_1^t = 0,$ the only marriage market participants are young women and old men. Hence, in equilibrium,

$$(F_0^{t+1}, M_0^{t+1}) = [((1 + \widehat{r})F_0^t, (1 + \widehat{r})M_0^t] = [b_{f0}F_0^t, b_{m0}F_0^t]$$

since $F_0^t$ young women are matched in every period $t$.

Hence,

$$1 + \widehat{r} = b_{f0}$$

This implies

$$\begin{aligned} \sigma &= b_{f0} \\ or,\ b_{m0} &= b_{f0}^2 \end{aligned} \tag{11}$$

There will be multiple equilibria in marriage payments since the numbers of marriage market participants are exactly matched. The upper limit of $D_0^1$ is the dowry that reduces young women's marital surplus to zero. This is

$$\overline{D}_0^1 = v_0^1 = K(1+\beta) - \beta(K - 1 - ED_1^1)$$

$$or,\ \overline{D}_0^1 = K(1+\beta) - \beta(K - 2 - ED_0^1) \tag{12}$$

The lower limit is the bride price that reduces old men's surplus to zero. This is

$$\underline{D}_0^1 = -(K+s) \tag{12'}$$

Hence,

$$D_0^1\ \epsilon\ [-(K+s),\ K(1+\beta) - \beta(K - 2 - ED_0^1)]$$

Assuming that the actual marriage payments are drawn by the matchmaker, from a uniform distribution over the above range, we have

$$ED_0^1 = \frac{2\beta - s}{2-\beta} \tag{13}$$

$$ED_1^1 = 1 + ED_0^1 = \frac{2+\beta-s}{2-\beta} \tag{1}$$

For old women to matched first, we must have $S_1^1 > S_0^1$. This reduces to

$$s > (2+\beta) \tag{14}$$

(14) implies $s > 2\beta$ (since $\beta \in (0,1)$), hence

$$ED_0^1 = \frac{(2\beta - s)}{2-\beta} < 0$$

Therefore, the average equilibrium payment, $ED_0^1$, is a bride price.

Using the equilibrium marriage payments, I derive

$$
\begin{aligned}
E_f &= K(1+\beta)+\beta s-\frac{(2\beta-s)}{2-\beta} \qquad (15)\\
E_m &= \beta[K+s+\frac{(2\beta-s)}{2-\beta}]\\
E_f-E_m &= K-\frac{(2\beta-s)}{2-\beta}(1+\beta)>0
\end{aligned}
$$

since $s>2\beta$ (from (14)).

In a non-trivial equilibrium, we have (recall that $f_1^t=0$),

$$
\begin{aligned}
b_{f0} &= \frac{4\rho_0+(E_f-E_m)}{8} \qquad (16)\\
b_{m0} &= \frac{4\rho_0-(E_f-E_m)}{8}
\end{aligned}
$$

Denote $E_f-E_m=E$

Subsituting (16) in (11) yields

$$E^2+(8+8\rho_0)E+(16\rho_0^2-32\rho_0)=0 \qquad (18)$$

The roots are given by

$$E=-(4+4\rho_0)\pm 4(1+4\rho_0)^{0.5} \qquad (19)$$

Denote $E_1=-(4+4\rho_0)+4(1+4\rho_0)^{0.5}$ and $E_2=-(4+4\rho_0)-4(1+4\rho_0)^{0.5}$.

$E_2<0$ since $\rho_0>0$ in a non-trivial equilibrium. (15) shows that $E>0$, so $E_1$ must be the relevant root of (18) for an equilibrium of the form $(c)$.

Note also that the following condition must be true in equilibrium:

$$E_1<Min[4\rho_0,8-4\rho_0] \qquad (20)$$

[ $(E_1<4\rho_0)$ since we are interested in non-trivial equilibria and $(4\rho_0+E_1<8)$, since $b_{f0}=\frac{4\rho_0+E_1}{8}<1$. This is because $E>0$ implies $\sigma<1$ and $b_{f0}=\sigma$ from (11)]

**Claim G.1.** *There do not exist parameter values $K>0$, $\beta\,\epsilon\,(0,1)$ and $s>0$ that justify a matching rule in which older women are matched first [see (14)] and that are consistent with a steady state general equilibrium of the form $(c)$.*

**Proof.** Suppose $E = E_1$ as must be true in an equilibrium. From (15), we have:

$$E_1 = K - \frac{(2\beta - s)}{2 - \beta}(1 + \beta)$$

$$K = E_1 + \frac{(2\beta - s)}{2 - \beta}(1 + \beta) \tag{21}$$

In equilibrium, (21) must be satisfied for some $K > 0$.
Hence, in equilibrium,

$$E_1 + \frac{(2\beta - s)}{2 - \beta}(1 + \beta) > 0 \tag{22}$$

A necessary condition for (22) to hold is that it be true at the maximum value of $E_1$. It is easily shown that $E_1$ is maximized at $\rho_0^{\max} = 0.75$ and $E_1(\rho_0^{\max}) = 1$. (Note that (20) is satisfied at $\rho_0^{\max}$.)
Hence, the necessary condition reduces to

$$2\beta^2 + (1 - s)\beta + 2 - s > 0 \tag{23}$$

The left hand side of (23) is decreasing in $s$. I will show that (23) will not be satisfied even at the lowest value of $s$ (highest value of the left hand side of (23)) permitted by the matching rule (14), hence making an equilibrium impossible.
Let $s = (2 + \beta)$ (see (14))
It is easy to show that for (23) to be true at this $s$, we must have

$$\beta(\beta - 2) > 0$$

Clearly, the above inequality is not satisfied in the range $\beta \in (0, 1)$. ■

I have shown that even when $\rho_0$ is such that $E$ attains the highest possible value consistent with a non-trivial equilibrium, there does not exist an $s$ that justifies the matching rule (14) and is consistent with $K > 0$ and $\beta \in (0, 1)$. This implies that there do not exist a set of parameter values consistent with model assumptions and the matching rule, that allow a steady state general equilibrium of the form ($c$).

(d) Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $m_1^t > f_1^t + f_0^t$ for all $t$.

Here too, $f_1^t = 0$, since all young women find a match in every period. Hence the above condition reduces to $m_1^t > f_0^t$. Since $m_1^t = M_1^t = M_0^{t-1}$ and $f_0^t = F_0^t = (1+\widehat{r})F_0^{t-1}$, this implies

$$\sigma > (1+\widehat{r}) \tag{24}$$

Since $f_1^t = 0$, the only marriage market participants are young women and old men. Hence, in equilibrium,

$$(F_0^{t+1}, M_0^{t+1}) = [((1+\widehat{r})F_0^t, (1+\widehat{r})M_0^t] = [b_{f0}F_0^t, b_{m0}F_0^t]$$

since $F_0^t$ young women are matched in every period $t$.

Hence,

$$1+\widehat{r} = b_{f0}$$

This implies

$$\begin{aligned} \sigma &> b_{f0} \qquad (25) \\ or,\ b_{m0}^2 &> b_{f0} \end{aligned}$$

Equilibrium marriage market payments are

$$D_0^1 = -(K+s)$$

since old men engage in within group competition to secure a spouse.

Hence, I obtain

$$\begin{aligned} E_f &= K(1+\beta) + \beta s + K + s \qquad (26) \\ E_m &= 0 \\ E_f - E_m &= 2K + \beta(K+s) + s > 0 \end{aligned}$$

$S_1^1 > S_0^1$ must be true for old women to be matched first. This reduces to the condition

$$s > 2 - \frac{K\beta}{1+\beta} \tag{27}$$

As in the previous case, $b_{f0}$ and $b_{m0}$ are given by (16) in a non-trivial equilibrium.

(24) and (16) imply

$$E^2 + (8\rho_0 + 8)E + 16\rho_0^2 - 32\rho_0 < 0 \tag{28}$$

Also, as before, the following condition must also be true in equilibrium:

$$0 < E < Min[4\rho_0, 8 - 4\rho_0] \tag{29}$$

Consider the equation:

$$x^2 + (8\rho_0 + 8)x + 16\rho_0^2 - 32\rho_0 = 0 \qquad (see\ (18))$$

The roots are

$$\begin{aligned} x_1 &= -(4 + 4\rho_0) + 4(1 + 4\rho_0)^{0.5} \\ x_2 &= -(4 + 4\rho_0) - 4(1 + 4\rho_0)^{0.5} < 0 \end{aligned}$$

since $\rho_0 > 0$ in a non-trivial equilibrium.

(26) shows that in equilibrium $E > 0$. (28) is satisfied with $E > 0$ when

$$E \ \epsilon \ (0, x_1) \tag{30}$$

Note that $x_1 > 0$ when $\rho_0 < 2$, hence this is a necessary condition for the existence of equilibrium.

Using (26), (30) reduces to the following condition:

$$0 < E = 2K + \beta(K + s) + s < x_1 \tag{31}$$

**Claim G.2.** *There do not exist parameter values $K > 0$, $\beta \ \epsilon \ (0, 1)$ and $s > 0$ that justify a matching rule in which older women are matched first [see (27)] and that are consistent with a steady state general equilibrium of the form $(d)$.*

**Proof.** Since $E$ is increasing in $s$ (see (26)) it is sufficient to show that there do not exist parameter values that satisfy (the right inequality of) (31) at the minimum value of $s$ that the matching rule (27) permits.

Let $s = 2 - \frac{K\beta}{1+\beta}$ (see (27))

At the above $s$,

$$E < x_1 \Leftrightarrow$$

$$2K + 2\beta + 2 < x_1 \tag{32}$$

The left hand side of (32) is greater than 2 when $K > 0$, $\beta > 0$. It is easily shown that $x_1$ attains a maximum of 1 at $\rho_0 = 0.75$ (note that the necessary condition of $\rho_0 < 2$ is satisfied at this value; also (29) is satisfied at this $\rho_0$). Hence, the right hand side of (32) is 1 at its maximum, whereas the left hand side is greater than 2 for the relevant parameter range. Further, (32) has been derived from (31) using the lowest possible $s$ permissible by the matching rule, (27). For larger $s$, the left hand side of (32)) will be even higher, violating the requirement $E < x_1$ in (31). ■

Hence, there do not exist a set of parameter values consistent with model assumptions and the matching rule, that allow a steady state general equilibrium of the form $(d)$

Hence, when parameters are such that young men choose to postpone marriage and older women are matched before young women (when men are indifferent to the age of their spouse), the only possible steady state general equilibrium is of the form $(e)$.

(e) Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $f_1^t < m_1^t < f_1^t + f_0^t$ for all $t$.

Example 3 (see Appendix F for a detailed derivation) illustrates a steady state general equilibrium of this form.

In equilibrium, all old women and all old men find matches. Some but not all young women are matched and hence engage in within group competition to offer dowries that reduce their marital surplus to 0. This yields

$$\begin{aligned} D_0^1 &= \frac{K+2\beta}{1-\beta} > 0 \\ D_1^1 &= \frac{K+1+\beta}{1-\beta} > 0 \end{aligned} \tag{33}$$

Hence the marriage payments corresponding to $(e)$ are dowries.

Therefore,

$$\begin{aligned} E_f &= K(1+\beta) + \beta s - \frac{K+2\beta}{1-\beta} \\ E_m &= \beta[K+s-1+\frac{K+1+\beta}{1-\beta}] \\ E_f - E_m &= -\frac{2\beta(K+\beta+1)}{1-\beta} < 0 \end{aligned} \tag{38}$$

Therefore, in all non-trivial equilibria, $b_{fi} < b_{mi}$, so

$$\sigma > 1$$

(see Proposition 4).

Hence, an equilibrium of the form ($e$) is characterized by dowry and a male–to-female sex ratio at birth that is greater than 1.

Consider the matching rule. If older women are matched first, we must have $S_0^1 < S_1^1$. This yields

$$\begin{aligned} s &> \frac{2+K\beta}{1-\beta} \\ or,\ \beta &< \frac{s-2}{s+K} < 1 \end{aligned}$$

Hence, there is an upper limit on $\beta$ and on $(E_m - E_f)$ that imposes an upper limit on $\sigma$. ■

# H. Propositions 6 and 7

## H.1. Proof of Proposition 6

**Proof.** The problem of sex ratio choice is now:

$$\begin{aligned} &\underset{b_f,\ b_m}{Max}\ c + E_f b_f + E_m b_m - \tau(b_f - b_m)^2 \\ subject\ to\ b_f + b_m &\leq \rho_i,\ b_f \geq 0, \quad b_m \geq 0 \end{aligned}$$

where $i$ is maternal age, $\tau > 0$ represents the cost of skewing the sex ratio.

The problem may be solved by the Lagrangean method as follows:

$$L = c + E_f b_f + E_m b_m - \tau(b_f - b_m)^2 + \lambda[\rho_i - b_f - b_m] + \pi_f b_f + \pi_m b_m$$

At the interior solution, we get:[43]

$$\begin{aligned} b_{fi}^* &= \frac{4\tau\rho_i + (E_f - E_m)}{8\tau} \\ b_{mi}^* &= \frac{4\tau\rho_i - (E_f - E_m)}{8\tau} \end{aligned} \tag{8}$$

An argument identical to the one presented in the Proof of Proposition 4 (see Appendix E) may be applied to prove the results of Proposition 6. ■

[43] It is easy to show that at the interior solution, the maternal-age-specific sex ratio $\sigma_i$ $(= \frac{b_{mi}^*}{b_{fi}^*})$ increases (decreases) with decline in total fertility $\rho_i$ if $E_f - E_m < 0$ $(E_f - E_m > 0)$. In other words, a reduction in fertility skews the sex ratio in favor of offspring with higher marriage market expectations.

### H.2. Proof of Proposition 7

**Proof.** Demographic configurations $(a)$ and $(b)$ (see $(I1)$ in Appendix G) cannot be true in equilibrium for reasons identical to those presented in the proof of Proposition 5 (see Appendix G).

Consider an equilibrium of the form $(c)$: $f_1^t + f_0^t = m_1^t$

Using arguments identical to that presented in the proof of Proposition 5, I obtain that (9) – (11) must be true in an equilibrium of this form:

$$s > (2 + \beta) \tag{9}$$

since old women must be matched first in equilibrium (compare with equation (14) of Appendix G);

$$E_f - E_m = K - \frac{(2\beta - s)}{2 - \beta}(1 + \beta) > 0 \tag{10}$$

(compare with equation (15) of Appendix G);

$$E^2 + (8\tau + 8\rho_0\tau)E + (16\tau^2\rho_0^2 - 32\tau^2\rho_0) = 0 \tag{11}$$

where $E = E_f - E_m$ (compare with equations (18) and (19) of Appendix G):

The roots of (11) are

$$E = -(4\tau + 4\tau\rho_0) \pm 4\tau(1 + 4\rho_0)^{0.5} \tag{12}$$

Denote $E_1 = -(4\tau+4\tau\rho_0)+4\tau(1+4\rho_0)^{0.5}$, $E_2 = -(4\tau+4\tau\rho_0)-4\tau(1+4\rho_0)^{0.5} < 0$

Since $E > 0$, $E_1$ must be the relevant root of (11) for an equilibrium of the form $(c)$.

**Claim H.1.** *When $\tau < 1+\beta$, there do not exist parameter values $K > 0$, $\beta \epsilon (0, 1)$ and $s > 0$ that justify a matching rule in which older women are matched first [see (9)] and that are consistent with a steady state general equilibrium of the form $(c)$.*

**Proof.** Suppose $E = E_1$ as must be true in equilibrium. From (10),

$$\begin{aligned} E_1 &= K - \frac{(2\beta - s)}{2 - \beta}(1 + \beta) \qquad (13) \\ or,\ K &= E_1 + \frac{(2\beta - s)}{2 - \beta}(1 + \beta) \end{aligned}$$

Note that it is a necessary condition for equilibrium that (13) be true for some $K > 0$ at the maximum possible value of $E_1$. It is easily shown that $E_1$ is maximized at $\rho_0^{\max} = 0.75$ and $E_1(\rho_0^{\max}) = \tau$. Hence (13) reduces to

$$K = \tau + \frac{(2\beta - s)}{2 - \beta}(1 + \beta) \tag{14}$$

Now, $K > 0$ implies (using (14))

$$2\beta^2 + (2 - \tau - s)\beta - s + 2\tau > 0 \tag{15}$$

The left hand side of (15) is decreasing in $s$. I shall show that at small values of $\tau$, (15) will not be satisfied even at the lowest value of $s$ (highest value of the left hand side of (15)) permitted by the matching rule (9).

Let $s = (2 + \beta)$ (see (9))

Then, some algebra elementary algebra will demonstrate that for (15) to be true at the above $s$, we must have

$$\tau > 1 + \beta$$

Hence, the necessary conditon for the existence of a steady state of the form ($c$) is *not* satisfied when

$$\tau < 1 + \beta \tag{16}$$

■

Consider a steady state general equilibrium of the form ($d$): $m_1^t > f_1^t + f_0^t$ for all $t$.

Using arguments identical to that in the proof of Proposition 5, I obtain that (17) – (19) must be true in an equilibrium of this form:

$$s > 2 - \frac{K\beta}{1 + \beta} \tag{17}$$

so that old women are matched first in equilibrium (compare with equation (27) of Appendix G);

$$E_f - E_m = 2K + \beta(K + s) + s > 0 \tag{18}$$

(compare with equation (26) of Appendix G);

$$E^2 + (8\tau + 8\rho_0\tau)E + (16\tau^2\rho_0^2 - 32\tau^2\rho_0) < 0 \tag{19}$$

where $E = E_f - E_m$ (compare with equation (28) of Appendix G).

Consider the equation (see (11) above):

$$x^2 + (8\tau\rho_0 + 8\tau)x + (16\tau^2\rho_0^2 - 32\tau^2\rho_0) = 0 \tag{20}$$

The roots are $x_1 = -(4\tau + 4\tau\rho_0) + 4\tau(1 + 4\rho_0)^{0.5}$ and $x_2 = -(4\tau + 4\tau\rho_0) - 4\tau(1 + 4\rho_0)^{0.5} < 0$

(18) shows that at the steady state equilibrium, $E > 0$; (19) is satisfied with $E > 0$ when

$$E \epsilon (0, x_1) \tag{21}$$

The following condition must then be true in an equilibrium of the form $(d)$:

$$0 < E = 2K + \beta(K + s) + s < x_1 \tag{22}$$

**Claim H.2.** *When $\tau < 2$, there do not exist parameter values $K > 0$, $\beta \epsilon (0, 1)$ and $s > 0$ that justify a matching rule in which older women are matched first [see (17)] and that are consistent with a steady state general equilibrium of the form $(d)$.*

**Proof.** Since $E$ is increasing in $s$ (see (18)) it is sufficient to show that at low values of $\tau$, there do not exist parameter values that satisfy (the right inequality of) (22) at the minimum value of $s$ that the matching rule (17) permits.

Let $s = 2 - \frac{K\beta}{1+\beta}$ (see (17))

At this $s$, (22) reduces to

$$E < x_1 \Leftrightarrow$$

$$2K + 2\beta + 2 < x_1 \tag{23}$$

The left hand side is greater than 2 when $K > 0$, $\beta > 0$. It is easily shown that $x_1$ attains a maximum of $\tau$ at $\rho_0 = 0.75$. Hence, the right hand side of (23) is $\tau$ at its maximum, whereas the left hand side is greater than 2 for the relevant parameter range. Hence, there will be no equilibrium of the form $(d)$ if $\tau < 2$.

Since $(1+\beta) < 2$ for $\beta \epsilon (0, 1)$, a sufficient condition that ensures that equilibria of the form $(a) - (d)$ cannot be sustained in a steady state general equilibrium is that $\tau < (1 + \beta)$.

Consider the following parameters: $K = \beta = 0.5$, $\rho_0 = 9$, $\rho_1 = 3$, $\tau = 0.75$. Notice that $\tau < 1+\beta$. Then a steady state equilibrium of the form $(e)$ is obtained and has the following characteristics:

1. Young men are not willing to marry at the equilibrium marriage payments because $K$ is too small ($K < 1$).

2. The equilibrium matching rule matches old men and women first when agents are indifferent to the age of their spouse.

3. In the stable population equilibrium, $\overline{q}_0 = 0$, $\overline{q}_1 = 1$, $\overline{p}_0 = 0.47$, $\overline{p}_1 = 1$. (See Table (b) for graphs of the computational derivation.) That is, in every period, young men refrain from marrying and old men are matched with all old women and some young women. Also, the stable population grows at the rate $(1 + \widehat{r}) = 2.026$, so $\widehat{r} = 1.026$. This is derived at the optimal birth rates and sex ratios derived in (5) below.

4. The equilibrium marriage payments are $D_0^1 = \frac{K+2\beta}{1-\beta} = 3$, $D_1^1 = \frac{K+1+\beta}{1-\beta} = 4$. Hence the equilibrium marriage payments are dowries.

5. The optimal maternal-age-specific birth rates and sex ratios are:

$b_{f0} = 3.83$, $b_{m0} = 5.17$, $\sigma_0 = 1.35$
$b_{f1} = 0.83$, $b_{m1} = 2.17$, $\sigma_1 = 2.61$

The proof is identical to that presented in Appendix F (Example 3), with birth rates being given by

$$
\begin{aligned}
b_{fi} &= \frac{4\tau\rho_i + (E_f - E_m)}{8\tau} \\
b_{mi} &= \frac{4\tau\rho_i - (E_f - E_m)}{8\tau}
\end{aligned}
$$

■

Suppose $(e)$ is obtained in a steady state equilibrium, i.e. $f_1^t < m_1^t < f_1^t + f_0^t$ for all $t$.

As before, I can calculate,

$$D_0^1 = \frac{K+2\beta}{1-\beta} > 0 \tag{24}$$

$$D_1^1 = \frac{K+1+\beta}{1-\beta} > 0 \tag{2}$$

$$E_f - E_m = -\frac{2\beta(K+\beta+1)}{1-\beta} < 0 \tag{3}$$

(24) implies that in all non-trivial equilibria, $b_{f_i} < b_{m_i}$ for each $i$, hence in equilibrium,

$$\sigma > 1$$

(see Proposition 6).

Hence, an equilibrium of the form $(e)$ is characterized by dowry and a male–to-female sex ratio at birth that is greater than 1.

Consider the matching rule. Older women are matched first when $S_0^1 < S_1^1$. This yields (as before)

$$\begin{aligned} s &> \frac{2 + K\beta}{1 - \beta} \\ or,\ \beta &< \frac{s - 2}{s + K} < 1 \end{aligned}$$

The upper limit on $\beta$ imposes an upper limit on $(E_m - E_f)$ and hence, for a given $\tau$, an upper limit on $\sigma$. ■