On the relevance of the median voter to resource allocation amongst jurisdictions

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Abstract

This paper examines allocation of local public good over three jurisdictions with individuals with identical tastes and different incomes, in a model with democratic institutions and majority rule. If re-election from a jurisdiction requires 50% or more of the mandate, then the median voter (in income) in each jurisdiction determines the probability of re-election for the incumbent government. The jurisdiction with the median of these median voters is most favored. With identical median voters in jurisdictions, and with re-election not requiring exactly 50% or more of the mandate, the jurisdiction with the median income inequality benefits the most.

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1 Introduction

In the context of demand for the amount of public good to be provided to a jurisdiction, the median voter theorem (M.V.T) states that given single peaked preferences and majority voting, the median demand for the public good is what is going to be supplied. Practical applications of the M.V.T have been severely limited, given that, it is difficult to relate empirical observations on allocation of local public good to the stylized situation posited by M.V.T. This paper presents conditions under which an incumbent government in a democracy, given individuals with identical tastes, but with different incomes in different jurisdictions treats the median voter (median in income) in each jurisdiction as its representative individual. The jurisdiction with the median of these median voters ends up getting the largest share of public resources.

In the model presented here, preference for the median voter emerge from the incentive of the incumbent government to get re-elected from a majority of jurisdictions rather than from a pairwise comparison of votes between alternatives as in the original M.V.T. This model is one of post-election politics where there is apparently no pre-election commitment by candidates to implement certain policies. Analyzing policy outcomes in these circumstances is different from a Downsian one where candidates commit to future policies before elections. Here preferences are essentially over personal consumption and thus the model has more in common with distributive politics framework such as Baron and Ferejohn (1989) and Weingast et. al. (1981)[2].

In the context of redistributive politics, Dixit and Londregan (1996) model situations when voters compromise their political affinities in response to offers by competing parties. They conclude that groups that are likely to have advantage in redistributive politics are (a) those that are indifferent to party ideology relative to private consumption benefits and (b) low income groups whose marginal utility of income is higher, making them more willing to

[2] I thank an anonymous referee for pointing this out

compromise their political preferences for additional private consumption. Dasgupta and Kanbur (2002) consider a model of identical preferences and different incomes of individuals where people make voluntary contributions to finance the public good. For people who do not make voluntary contributions, contributions of others is like a kind transfer rather than a cash transfer of an equivalent amount. If both poor and the middle class are non contributors, the valuation of a given amount of public good, and of an additional unit of public good both increase with their cash income. The authors use this argument to justify the reason, why in public debates it is argued that the middle class benefit more from state expenditure than the poor.

In the empirical work on redistributive politics, LeGrand (1982) finds that much of the expenditures on social services in United Kingdom such as health care, education, housing and transport accrue to people who can be broadly be classified to being in the higher income groups. The middle class are more likely to get opportunities in education than the poor and are more likely to get opportunities in professional jobs. The poor live in areas poorly endowed with social services and have to travel far to avail such services. With data from 24 democracies, Milanovic (2000), find out that when we focus on truly redistributive transfers as unemployment insurance, the middle class gain little from these transfers. According to him the median-voter hypothesis may not be the appropriate collective-decision making mechanism to explain redistribution decisions, and are more appropriate in direct democracies rather than in representative democracies. In this context our theoretical model and its version of the 'median voter' is able to explain LeGrand's observation, circumstances under which, expenditure on public services or public goods accrue only to the top income deciles, with the median voter receiving the largest share of public resources. It also integrates with Milanovic's observation, that although the median voter may not gain in truly redistributive transfers, they do gain in allocation of public goods. It is also a model where the concept of a median voter arises not in the context of a direct representative democracy, so may explain better empirical observation on allocation of public services.

Although it has long been recognized, that majoritarian democracies may be characterized by "tyranny of the majority", where minorities might suffer. It is for this reason, that Buchanan and Tullock (1962) analyzed advantages and disadvantages of unanimity rule and worked out a way of finding the appropriate percent of a mandate required in an ideal democracy. As the mandate required to pass a decision goes up, the present value of external costs imposed on any individual by the action of other individuals goes down, and is zero for a unanimous decision. However, as the proportion required to pass a decision increases, expected decision making costs increase on account of having to convince a larger proportion of individuals. To the authors, the optimal proportion required to carry out a decision should be one that minimizes sum of these two costs. Our work adds to this literature, with the result that if anything else but a 50% rule if followed, will lead to distortions of a discrimination against the jurisdiction with the least or the maximum income inequality.

This paper is organized as follows. The next section describes the basic model. Section 3 discusses resource allocation in a democracy and section 4 analyzes resource allocation when one need not require exactly 50% voter or more to get re-elected from a jurisdiction. Section 5 concludes.

2 The Model

We consider an economy with three[3] jurisdictions with a single individual in each jurisdiction. We assume individuals with identical additively, separable utility function defined over a private and local public good[4]. Individuals differ in their endowments or incomes.

[3]The model can be easily extended to n jurisdictions where n is odd

[4]These are simplifying assumptions which would help us highlight the result better.

A central government decides on a uniform proportional tax rate and the amount of local public good to be supplied to jurisdictions. The voting model incorporates the notion of reservation utility as in Seabright (1996) and Gupta (2001). Individuals are assumed to be immobile across jurisdictions. The central government has to satisfy a majority of jurisdictions (in this case two) in order to get re-elected.

Jurisdictions are represented by i where $i \in \{1, 2, 3\}$. The income of an individual j in jurisdiction i is denoted by y_{ij}. The incomes of individuals in jurisdictions are uniformly distributed in $[y_{il}, y_{ih}]$, where y_{il} is the income of the poorest person and y_{ih} is the income of the richest person in jurisdiction i respectively, i.e., $y_{ih} \geq y_{il}$. The utility function of an individual j in jurisdiction i is given by:

$$W_{ij} = x_{ij} + ln(g_i) \tag{1}$$

where x_{ij} is the amount of private good consumed by the individual j in jurisdiction i and

$$x_{ij} = (1 - t)y_{ij} \tag{2}$$

where t is the uniform proportional tax rate levied by the central government. g_i is the amount of local public good delivered to jurisdiction i by the central government.

The uncertainty regarding an incumbent government's re-election is captured by an electoral uncertainty ϵ, which is a random variable following a uniform distribution over the range $[-q, q]$ and a mean of zero. Let e_{ij} denote the event that the individual is satisfied with the incumbent government and votes in its favor. The event e_{ij} occurs when the welfare of an individual W_{ij} in jurisdiction i, with income y_{ij} net of electoral uncertainty ϵ is greater than a reservation utility V_{ij}, which can be interpreted as the welfare expected from a rival political party. A representative individual in jurisdiction i would be satisfied with the government if

$$W_{ij} + \epsilon \geq V_{ij} \tag{3}$$

Therefore the event e_{ij} occurs when

$$\epsilon \geq V_{ij} - W_{ij} \tag{4}$$

and the probability $p(e_{ij})$ of the individual being satisfied with the incumbent government and voting in its favor is given by

$$p(e_{ij}) = p(\epsilon \geq V_{ij} - W_{ij}) = \frac{q - (V_{ij} - W_{ij})}{2q} \tag{5}$$

It can be seen from the above expression that if the government just manages to provide the reservation utility, it wins with a probability of 0.5, if it provides more it wins with a probability more than 0.5, and the converse holds true. It should be noted that the electoral uncertainty ϵ is common across all individuals and is therefore perfectly correlated across individuals in the jurisdictions. Thus, from any jurisdiction i, if an individual with income y_{ij} votes for the government, all individuals in jurisdiction i, with income level above y_{ij} vote for the government. Therefore for any given level of g_i, and any realized value of ϵ, there exists an income level $y_{i\epsilon}$, above which every individual votes for the incumbent government. Therefore

$$y_{i\epsilon} = \frac{V_{ij} - \epsilon - ln(g_i)}{(1 - t)} \tag{6}$$

Therefore the proportion or the fraction of people voting for the government in any jurisdiction i for any realized value of ϵ will be

$$f_i = \frac{y_{ih} - y_{i\epsilon}}{y_{ih} - y_{il}} \tag{7}$$

In most voting models, the government wins from a jurisdiction if it secures more than 50% of the votes, and let this event be e_i. Therefore, the probability of getting re-elected from a jurisdiction is the probability that it secures more than 50% of the votes (see Appendix 1). Thus

$$p(e_i) = p(f_i \geq \frac{1}{2}) = \frac{1}{2q}[q - V_{ij} + ln(g_i) + (1-t)\frac{y_{ih} + y_{il}}{2}] \tag{8}$$

or

$$p(e_i) = p(f_i \geq \frac{1}{2}) = \frac{1}{2q}[q - V_{ij} + ln(g_i) + (1-t)y_{im}] \tag{9}$$

where

$$y_{im} = \frac{y_{ih} + y_{il}}{2}$$

that is y_{im} is the median voter in jurisdiction i[5]. Let $y_{1m} \geq y_{2m} \geq y_{3m}$. We therefore refer to jurisdiction 1 as the jurisdiction with the richest median voter, jurisdiction 3 as the one with the poorest median voter and jurisdiction 2 as the one with the median, median voter.

The central government has to win from any two of the three jurisdictions. It has to spend the taxes raised from individuals on allocation of local public goods to three jurisdictions. Therefore it is subject to the budget constraint (see Appendix 2):

$$\sum_{i=1}^{3} g_i = t\sum_{i}[\int_{j=y_{il}}^{y_{ih}} y_{ij}p(y_{ij})\,dy_{ij}] = t\sum_{i=1}^{3}\frac{1}{2(y_{ih} - y_{il})}[y_{ih}^2 - y_{il}^2] \tag{10}$$

where $p(y_{ij})$ is the probability that an individual j in jurisdiction i has an income y_{ij}, and $p(y_{ij}) = \frac{1}{y_{ih} - y_{il}}$.

3 Resource Allocation in a Democracy

The central government will set the tax rate and distribute resources for local public good to the jurisdictions in order to maximize the probability of getting re-elected from any two

[5]Since individuals are uniformly distributed in income, the median voter's income lies exactly midway between the poorest and the richest voter in the jurisdiction.

jurisdictions. This would depend not only on the endowment/incomes of the individuals in the jurisdictions, but also on the level of reservation utility of individuals. Given that the electoral uncertainty is perfectly correlated amongst all individuals in all jurisdictions, the probability of getting re-elected from any jurisdiction depends on the gap between the welfare experienced and the reservation utility of the median voter in the jurisdiction. The larger this gap, the more is the probability of getting re-elected from any jurisdiction. Therefore, the central government will always find it *exante* optimal to concentrate on the two jurisdictions with the largest gap and completely ignore a third jurisdiction in the allocation of public good (see appendix 2). As to whom to discriminate against depends on the reservation utility, since this determines the gap. We therefore consider three possible situations (a) equal reservation utilities of all individuals (b) reservation utilities set at the level as that received if the jurisdiction were independent and (c) reservation utilities set as that received from a central Utilitarian social planner. The optimal tax rate and the net gains and losses of each of the jurisdictions is discussed under each of these circumstances.

3.1 Equal Reservation Utilities

If the reservation utility is the same for all individuals, the probability of getting re-elected is dependent on the welfare experienced by the median voters in each of the jurisdictions. Let there be equal allocation of local public goods across the three jurisdictions, i.e. if $g_i = \overline{g}$. Since welfare experienced by any median voter in jurisdiction i is $W_{im} = (1 - t)y_{im} + ln(\overline{g})$, and since $(y_{1m} \geq y_{2m} \geq y_{3m})$, the richest median voter experiences the highest welfare, and the poorest median voter, the least. Therefore, the probability of getting re-elected from jurisdiction 1 is highest, and that from jurisdiction 3 is least. Thus, the probability of re-election can increase if resources for public good are shifted from jurisdiction 3 to jurisdictions 1 and 2. The probability of re-election from the country, is highest, for the highest probability of re-election from jurisdiction 2, i.e. the jurisdiction with the median, median voter. The optimal allocation will be one where probability of re-election from jurisdiction 1 is the same as that from jurisdiction 2 (see Appendix 2). That is

$$p(2) = p(1) \Rightarrow \frac{q - (V - W_{2m})}{2q} = \frac{q - (V - W_{1m})}{2q}$$
$$\Rightarrow (1 - t)y_{2m} + ln(g_2) = (1 - t)y_{1m} + ln(g_1) \tag{11}$$

As seen from (11), equal probabilities of re-election from jurisdictions 1 and 2, imply that the probabilities of the median voters voting for the government in jurisdictions 1 and 2 are the same and with equal reservation utilities it implies that welfare experienced by these two individuals would be the same. Given that $y_{1m} \geq y_{2m}$, (11) would imply that $g_2 \geq g_1$. Therefore, the jurisdiction with the median, median voter gets the largest share of public resources, that with the richest median voter gets some, and the one with the poorest median voter gets none. Therefore, the jurisdiction with the poorest median voter is discriminated against in the allocation of public goods by the central government.

Now the allocation of public goods is decided upon, the central government has to decide upon the uniform tax rate to charge individuals to finance the public good. It will therefore choose a tax rate, and local public good allocations to maximize the probability of getting re-elected from jurisdiction 2. This happens when the probability of getting re-elected from jurisdiction 2 is at least as large as that from jurisdiction 1.

$$\max_{t, g_i} p(e_2)$$
$$such\, that$$
$$p(2) \leq p(1)$$
$$\sum_{i=1}^{3} g_i = t \sum_{i=1}^{3} [\int_{j=y_{il}}^{y_{ih}} y_{ij} p(y_{ij}) dy_{ij}] \tag{12}$$

Solving for (12), we evaluate the optimum tax rate t^* to be $t = \frac{1}{y_{2m}}$ (see Appendix 3). In such a setup the next question that comes up is which jurisdiction gains and which jurisdiction loses in a democracy. For that one has to check on the net contribution of each jurisdiction, which is the tax contribution less its receipts as public good.Let

$$NC_i = ty_{im} - g_i \tag{13}$$

where NC_i, is the net contribution of jurisdiction i. Since jurisdiction 3, the jurisdiction with the poorest median voter only contributes in taxes and receives nothing as public good, it is a loser in a democracy. Jurisdiction 2, the jurisdiction with the median, median voter has a negative net contribution and definitely gains in the process (see Appendix 4 for a formal proof.). As for the jurisdiction 1, that with the richest median voter, its contribution may be negative as well as positive and depends on its income relative of that of the median, median voter. The least income that the richest median voter can have is that equal to the median, median voter. At that level of income, it receives half of the total tax receipts, as public goods and is a net receiver. As the income of the richest median voter goes up, its net receipts decreases (in absolute as well as proportionate terms), and at a certain critical level of income at which its net contribution is zero. Above this level of income, its net contribution is positive, and it is a net loser in a democratic setup. Therefore the results may be summarized as:

Proposition 1 *In a democracy with equal reservation utilities for all individuals, the jurisdiction with the median of the median voters receives the largest allocation of local public good and that with the poorest median voter receives none. The net benefit to the jurisdiction with the richest median voter is decreasing in the income gap between the richest median voter and the median, median voter.*

3.2 Reservation Utilities as in Independent Jurisdictions

It would be interesting to analyze a situation where reservation utility is not the same across individuals. With the same reservation utility for all individuals, it is easier to satisfy the individuals with the higher endowments for the same level of local public good allocation, and hence they are more likely to vote for the government. If individuals with higher endowments are also those with lower reservation utilities, then again it would be easier to satisfy those individuals with higher endowment. The only case where the same need not be true is when people with higher endowments have higher reservation utilities. One can think of rich and poor jurisdictions in a country, the richer jurisdictions contribute more to the central kitty than the poorer ones, but then expect more in terms of infrastructure services.

It would be interesting to analyze whom an incumbent democratic government winning by majority rule, favors or discriminates against in the net, i.e. whether the rich or poor jurisdictions are discriminated against in terms of the amount of tax revenue paid and the amount of public good received. One possible instance of such a situation in this model will arise if the reservation utilities of individuals are at the level of welfare obtained if the jurisdictions were independent, i.e. the local government in the jurisdiction raised resources to finance the local public good. In this situation, one can imagine a local government fixing up the tax rate by maximizing a Utilitarian social welfare function subject to the budget constraint. The problem for the local government would be

$$\max_t \int_{y_{il}}^{y_{ih}} (1 - t)p(y_{ij})y_{ij}dy_{ij} + ln(g_i)$$

$$such\,that$$

$$\int_{y_{il}}^{y_{ih}} \frac{1}{y_{ih} - y_{il}} t y_{ij} dy_{ij} = g_i \tag{14}$$

From (14), $t_i = \frac{1}{y_{im}}$ and $g_i = 1$, emerges as a solution and the welfare experienced by any individual j in jurisdiction i, is $(1 - t_i)y_{ij}$. Thus individuals set their reservation utility at this level i.e. $V_{ij} = (1 - t_i)y_{ij}$. For an equal allocation of public goods to all jurisdictions, the probability of getting re-elected is highest from jurisdiction 3, and least from jurisdiction 1. So in this case, the jurisdiction with the richest median voter that is jurisdiction 1 is the one that is discriminated against in the allocation of public good. The median, median jurisdiction is the most favored in the allocation of public goods (see Appendix 5 for a formal proof). The central government will fix the optimum tax rate at $t = t_2 = \frac{1}{y_{2m}}$, and jurisdictions 2 and 3 are always net beneficiaries while jurisdiction 1 is always a net loser in net allocations from the central government (see Appendix 6 for a formal proof). Therefore, it is slightly different from the equal reservation utilities case where, the jurisdiction receiving the second largest allocation of public goods could be a net gainer or a net loser. Thus the results can be summarized as:

Proposition 2 *In a situation where individuals set their reservation utilities at a level as that they would receive from a social planner if the jurisdiction were independent, the central government in a democracy would favor the jurisdiction with the median, median voter the most and jurisdiction with the richest median voter will be the one that is discriminated against. The jurisdictions with the poorest and the median, median voters are always net gainers in terms of public goods received and taxes paid, while the one with the richest median voter is always a net loser.*

3.3 Reservation Utilities as from a Central Utilitarian Social Planner

In the last section, individuals with higher incomes had higher reservation utilities for the same allocation of public goods, and in such a scenario it was observed that the richest median voter gets discriminated against. To find out if this will be consistent over different scenarios, we now consider a different situation, when reservation utilities are set by a central social planner, one that would maximize the sum of welfare of all individuals in a nation. Since all individuals have the same utility function, a central social planner would give the same allocation of public goods to all jurisdictions. As to which jurisdictions to favor would depend on whether the uniform tax rate in a democracy, which is again $t_d = \frac{1}{y_{2m}}$ is lower or higher than the social planner tax rate $t_s = \frac{1}{\sum_{i=1}^{3} y_{im}}$ (see Appendix 7 for a formal proof). If $t_d \geq t_s$, jurisdictions 1 and 2, i.e. jurisdictions with the poorest and the median, median voters should be favored. If $t_d \leq t_s$, jurisdictions 2 and 3 will be favored. However, $t_d \geq t_s$, implies $y_{2m} \leq \frac{\sum_{i=1}^{3} y_{im}}{3}$, i.e. if the income of the median voter in jurisdiction 2 is lower than the mean income of median voters in all jurisdictions, jurisdiction 3, the one with the richest median voter will be discriminated against. Conversely, $t_d \leq t_s$ implies $y_{2m} \geq \frac{\sum_{i=1}^{3} y_{im}}{3}$, i.e. if the income of the median voter in jurisdiction 2 is higher than the mean of the all median voters, jurisdiction 3, that with the poorest median voter will be discriminated against. Therefore, if the median, median voter is relatively poor, the the poor jurisdictions are favored, if rich, then the rich jurisdictions are favored.

As far as the net contributions of jurisdictions are concerned, that of jurisdiction 2 is always negative, and that of the jurisdiction discriminated against is always positive. When jurisdiction 3, the jurisdiction with the poorest median voter is favored, its net contribution is always negative, so is a net gainer (see Appendix 7 for a formal proof). When jurisdiction 1, the jurisdiction with the richest median voter is favored, its net contribution can be both positive or negative, exactly as in the situation of jurisdiction 1 with equal reservation utilities.

Thus the results can be summarized as:

Proposition 3 *In a situation where individuals set their reservation utilities at a level as that received from a central Utilitarian social planner, the median, median voter would be favored the most. Jurisdiction with the richest median voter is discriminated against if* *the income of the median voter in jurisdiction* 2 *is lower than the mean income of median voters. Jurisdiction with the poorest median voter is discriminated against if income of the median voter in jurisdiction* 2 *is higher than the mean of the all median voters. The jurisdiction with the median, median voter and that with the poorest median voter (when favored) always experience negative net contribution, while the same is not the case for the jurisdiction with the richest median voter*

4 Resource Allocation with not a 50% Majority Rule

We have till now considered allocation when governments get re-elected if they receive 50% or more of the mandate. However constitutional requirements may require that candidates win with more than 50% of the votes in a jurisdiction to get re-elected, in order for the government to be more representative. It might also be the case that candidates may win and get re-elected with even less than 50% of the mandate, in case it is more than a two party contest. In both these cases, the government's objective function gets changed. The event of re-election from any jurisdiction i will be when $f_i = \frac{1}{2} + \eta$, where $0 \geq \eta \geq \frac{1}{2}$. When $\eta \geq 0$, governments need more than 50% of the mandate to get re-elected, when $\eta \leq 0$, governments need less than 50% of the mandate to get re-elected. The governments probability of getting re-elected will now be redefined as

$$\begin{aligned} p(e_i) = p(f_i \geq \frac{1}{2} + \eta) &= \frac{1}{2q}[q - V_{ij} + ln(g_i) + (1-t)y_{im} - \eta(1-t)(y_{ih} - y_{il})] \\ &= \frac{1}{2q}[q - V_{ij} + ln(g_i) + (1-t)y_{im} - \eta(1-t)R(y_i)] \quad (15) \end{aligned}$$

where $R_i = y_{ih} - y_{il}$, i.e. R_i is the range of income distribution in jurisdiction i. Let $y_{im} = y_m$ and $R_1 \geq R_2 \geq R_3$, that is the incomes of the median voters of all jurisdictions are equal, and jurisdiction 1 has the maximum inequality and jurisdiction 3 the least. If $\eta \geq 0$, that is if governments get re-elected only if they get 50% or more of the mandate.

For an equal allocation of public good across jurisdictions, the probability of re-election is highest from jurisdiction 3 and least from jurisdiction 1. So the two jurisdictions with the least inequality (jurisdictions 2 and 3) will be favored in resource allocation. Jurisdiction 2 gets the largest share of public resources and is always a net gainer. As for jurisdiction 3, it receives the same amount of public good as jurisdiction 2 when $R_2 = R_3$ and its net contributions are negative. As R_3 decrease to zero, its net contributions also decreases (see Appendix 8 for a formal proof). Therefore, jurisdiction 3 is always a net receiver.

We now examine the situation when $\eta \leq 0$. In this situation for an equal allocation of public goods, the probability of getting re-elected is highest from jurisdictions 1 and 2. So the two jurisdictions with the highest inequality (jurisdictions 1 and 2) will be favored, with jurisdiction 2 receiving a larger share. The net contribution of jurisdiction 2 is always negative, and that of jurisdiction jurisdiction 1 is negative when $R_2 = R_1$, and it gets equal share of public resources. When R_1 increases, its net contributions decline, and therefore the net contributions of the jurisdiction with the richest median voter is always negative. In this situation, although costs as characterized by Buchanan and Tullock (1962) are absent, the most optimal voting rule is one that requires 50% or more of the mandate. Thus, democracies would function best in the presence of two party contests, and not in the situation of multi-party contests. In the latter, not only does it allow a winner to win with a very small proportion of the votes, it also introduces a distortion in resource allocation.

As far as the optimal tax rate in this situation is concerned, the optimal tax rate will be $t^* = \frac{1}{y_m - \eta R_2}$. With $\eta \geq 0$, the tax rate is least, with $\eta \leq 0$, the tax rate is highest (see Appendix 9 for a formal proof). Therefore taxation and redistribution is highest with re-election requiring less than 50% of the mandate, and it declines as the mandate requirement increases. This situation is different from that observed by Milanovic (2000) where greater income inequality lead to greater income redistribution. Therefore the results can be summarized as:

Proposition 4 *If the median voter in every jurisdiction has the same income, and if more than 50% of the mandate is required to get re-elected from a jurisdiction, then the two jurisdictions with the least income inequality (as captured by range) are favored. If less than 50% of the mandate is required, then the two jurisdictions with the highest income inequality will be favored. The jurisdiction with the median inequality will be favored the most, and jurisdictions being favored will always be net gainers.*

5 Conclusion

This paper presents conditions under which political competition, leads the incumbent government to look at the voter with the median income (the median voter in the jurisdiction) to decide on the allocation of public goods for the jurisdiction. The jurisdiction with the median of the median voters is the one that gets favored the most for resource allocation. Re-election from a jurisdiction depends only on the median voters income as long as exactly 50% or more of the mandate is required. If one requires less or more than 50% of the mandate to get re-elected, then resource allocation also depends on income inequality in a jurisdiction. With not a 50% majority rule, and with identical median voters in jurisdictions, the jurisdiction with the median inequality gets favored the most.

In this situation the government acts out of its own selfish perspective which is divorced from that of its citizens as in Niskanen (1971) as against Besley and Coate (1997) where citizens run for political office and implement their own policy choice. Benefit spillovers occur for individual types not targeted, but living in favored jurisdictions. This model gives a theoretical illustration as to why and how the middle class in democratic societies gain the most out of government policies. A limitation in this model is the expectations or reservation utilities of citizens are exogenous. Future work needs to concentrate on deriving reservation utilities out of political considerations and then working out who gains in the process.

Appendix 1: Probability of getting re-elected from a jurisdiction

Re-election from a jurisdiction e_i happens when 50% or more of the population vote for the government.

$$f_i \geq \frac{1}{2} \Rightarrow \frac{y_{ih} - y_{i\epsilon}}{y_{ih} - y_{il}} \geq \frac{1}{2} \tag{16}$$

Substituting for $y_{i\epsilon} = \frac{V_{ij} - \epsilon - ln(g_i)}{(1-t)}$ in (16), we get

$$\frac{y_{ih}}{y_{ih} - y_{il}} - \frac{V_{ij} - \epsilon - ln(g_i)}{(1-t)(y_{ih} - y_{il})} \geq \frac{1}{2} \tag{17}$$

$$\Rightarrow \epsilon \geq V_{ij} - ln(g_i) - (1-t)y_{ih} + \frac{1}{2}(1-t)(y_{ih} - y_{il}) \tag{18}$$

$$p(e_i) = \frac{1}{2q}[q - V_{ij} + ln(g_i) + (1-t)y_{ih} - \frac{1}{2}(1-t)(y_{ih} - y_{il})] \tag{19}$$

$$p(e_i) = \frac{1}{2q}[q - V_{ij} + ln(g_i) + (1-t)\frac{y_{ih} + y_{il}}{2}] \tag{20}$$

$$p(e_i) = \frac{1}{2q}[q - V_{ij} + ln(g_i) + (1-t)y_{im}] \tag{21}$$

Appendix 2: Local public good allocation with equal reservation utilities

We assume that the reservation utility is the same for all individuals at $V_{ij} = V$. The central government has to decide on the allocation of local public good to jurisdictions for any given tax rate t. The total resources at the disposal of the central government will be $\sum_{i=1}^{3} g_i = t\sum_i[\int_{j=y_{il}}^{y_{ih}} y_{ij}p(y_{ij})dy_{ij}]$, let us go for equal allocation of local public good across jurisdictions. Therefore the amount of local public good being given to a jurisdiction i, $i \in \{1, 2, 3\}$ is $\overline{g} = \frac{1}{3}t\sum_{i=1}^{3}[\int_{j=y_{il}}^{y_{ih}} y_{ij}p(y_{ij})dy_{ij}]$. Then

Then

$$p(e_i) \;=\; \frac{1}{2q}[q \;-\; V \;+\; ln\overline{g} \;+\; (1-t)y_{im}] \tag{22}$$

Let $y_{1m} \;>\; y_{2m} \;>\; y_{3m}$. Therefore $p(e_1) \;>\; p(e_2) \;>\; p(e_3)$

The central government has to win from two of the three jurisdictions, so will maximize the probability of re-election from any two of the three jurisdictions, the objective function given by

$$Z \;=\; p(e_1 \cap e_2 \cap -e_3) \;+\; p(e_1 \cap -e_2 \cap e_3) \;+\; p(-e_1 \cap e_2 \cap e_3) \;+\; p(e_1 \cap e_2 \cap e_3) \tag{23}$$

where $-e_i$ is the event of not satisfying jurisdiction i. The central government will maximize the above objective function subject to the budget constraint $\sum_{i=1}^{3} g_i \;=\; t\sum_{i=1}^{3}\frac{1}{2(y_{ih}-y_{il})}[y_{ih}^2 - y_{il}^2]$, to get the optimal resource allocation.

Given common electoral shock, the event e_1, e_2, or e_3 will occur, when

$$\epsilon \geq \underline{V} \;-\; (1-t)y_{im} \;-\; \overline{g} \tag{24}$$

and $i \;\in\; \{1, 2, 3\}$. Therefore when e_3 occurs, e_1 and e_2, necessarily occur, since $y_{1m} > y_{2m} > y_{3m}$. By similarly reasoning, when e_2 occurs, e_1 will definitely occur, which implies $p(e_1 \mid e_2) \;=\; 1$. Therefore

$$p(-e_1 \cap e_2 \cap e_3) \;=\; p(e_1 \cap -e_2 \cap e_3) \;=\; 0 \tag{25}$$

and the objective function reduces to

$$Z \;=\; p(e_1 \cap e_2 \cap -e_3) \;+\; p(e_1 \cap e_2 \cap e_3) \;=\; p(e_1 \cap e_2) \;=\; p(e_2).p(e_1 \mid e_2) \;=\; p(e_2) \tag{26}$$

Therefore, with equal allocation of local public goods across jurisdictions, the probability of getting re-elected is the probability of getting re-elected from jurisdiction 2, i.e. the jurisdiction with the median voter, whose income is the median of the median voters in the three jurisdictions. One should also note that with equal allocation of local public goods, $p(1) \geq p(2) \geq p(3)$. Therefore, one can do better, i.e. increase the probability of getting re-elected, by redistributing local public good allocation of jurisdiction 3 to jurisdictions 1 and 2. So the optimal allocation would be a g_1^*, g_2^* and $g_3^* = 0$ at which the government budget constraint is satisfied and one where $p(e_1) = p(e_2)$. Therefore the jurisdiction with the poorest median voter gets no allocation of local public good, and is discriminated against.

Appendix 3: Optimal tax rate with equal reservation utilities

Therefore for any amount of revenue raised, we now know the optimal allocation. The government also has to decide on the optimal tax rate at which the probability of winning from the jurisdiction with the median median voter is maximized. The government's problem is as follows:

$$\max_{t,g_i} p(e_2)$$
$$such\, that$$
$$p(2) \leq p(1)$$
$$\sum_{i=1}^{3} g_i = t\sum_{i=1}^{3}[\int_{j=y_{il}}^{y_{ih}} y_{ij}p(y_{ij})\, dy_{ij}] \tag{27}$$

Therefore, at the optimum

$$p(2) = p(1) \Rightarrow W_{2m} = W_{1m}$$
$$\Rightarrow (1 - t)y_{2m} + ln(g_2) = (1 - t)y_{1m} + ln(g_1) \tag{28}$$

In this situation since reservation utilities are the same for all individuals, equal probabilities of winning as in (28) imply equal welfare for the median voters in the jurisdictions.

For any given tax rate t, there exists a λ_i, $0 \geq \lambda_i \geq 1$ and $\sum_1^3 \lambda_i = 1$ such that

$$g_i = \lambda_i t \sum_{i=1}^{3} y_{im} \tag{29}$$

In this situation $\lambda_3 = 0$ and $\lambda_2 = (1 - \lambda_1)$. Therefore

$$(1 - t)y_{2m} + ln(1 - \lambda_1)t \sum_{i=1}^{3} y_{im} = (1 - t)y_{1m} + ln\lambda_1 t \sum_{i=1}^{3} y_{im} \tag{30}$$

(29) ensures balancing of the government budget and (30) ensures that the second condition for optimal welfare, the equal welfare of the jurisdictions being favored is ensured. Therefore, the optimal tax rate, which maximizes the median voter's utility is given by

$$\max_t Z_1 = (1 - t)y_{2m} + ln(1 - \lambda_1)t \sum_{i=1}^{3} y_{im} \tag{31}$$

As first order condition the following equation is obtained:

$$\frac{\partial Z}{\partial t} = -y_{2m} + \frac{1}{t} = 0 \tag{32}$$

Therefore from 32, we get the optimal tax rate t^* to be $\frac{1}{y_{2m}}$, and we notice that the optimal tax rate is independent of λ_i or y_{1m}.

Appendix 4: Net contributions of jurisdictions with equal reservation utilities

To analyze the net contributions of jurisdictions in a democracy, we first analyze the effect of an increase in the income of the richest median voter on λ_1, the share of tax revenues going to jurisdiction 1. Let (30) be re-written as

$$(1 - t)(y_{1m} - y_{2m}) + ln(\frac{\lambda_1}{1 - \lambda_1}) = 0 \tag{33}$$

Differentiating partially (33) with respect to y_{1m} and rearranging the terms we get

$$(1-t)+\frac{(1-\lambda_1)}{\lambda_1}\frac{\partial}{\partial y_{1m}}ln(\frac{\lambda_1}{1-\lambda_1})=0 \tag{34}$$

or

$$(1-t)+\frac{1}{\lambda_1(1-\lambda_1)}[(1-\lambda_1)\frac{\partial\lambda_1}{\partial y_{1m}}-\lambda_1(-\frac{d\lambda_1}{dy_{1m}})]=0 \tag{35}$$

or

$$(1-t)+\frac{1}{\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial y_{1m}}=0 \tag{36}$$

or

$$\frac{\partial\lambda_1}{\partial y_{1m}}=-(1-t)\lambda_1(1-\lambda_1)<0 \tag{37}$$

Therefore the share of tax revenues going to the richest jurisdiction declines as the income of the richest median voter increases. From (33), one notes that the minimum value of y_{1m} is y_{2m}, in which case the maximum value of λ_1 is $\frac{1}{2}$. Therefore the minimum value of $(1-\lambda_1)$, that is the share of tax revenues going to jurisdiction 2 is $\frac{1}{2}$.

The net contribution of any jurisdiction i is given by:

$$NC_i=ty_{im}-g_i \tag{38}$$

The net contribution of jurisdiction 2 is given by

$$NC_2=ty_{2m}-t(1-\lambda_1)\sum_{i=1}^{3}y_{im} \tag{39}$$

Since the minimum value of $(1-\lambda_1)$ is $\frac{1}{2}$, therefore, for any t, NC_2 reaches its maximum value at $(1-\lambda_1)=\frac{1}{2}$. NC_2 evaluated at $(1-\lambda_1)=\frac{1}{2}$ is

$$NC_2=t[y_{2m}-\frac{1}{2}(y_{1m}+y_{2m}+y_{3m})] \tag{40}$$

Since $y_{1m}\geq y_{2m}$, $\frac{1}{2}(y_{1m}+y_{2m}+y_{3m})\geq y_{2m}$. Therefore, NC_2 is negative at $(1-\lambda_1)=\frac{1}{2}$, and since it is the maximum value of NC_2, the median, median voter is always a net receiver in a democracy.

To analyze the net contributions of jurisdiction 1, the jurisdiction with the richest median voter, let us start from the initial situation where $y_{1m} = y_{2m}$. In this case jurisdiction 1 will receive exactly the same level of public good as jurisdiction 2, that is receive half the share of tax revenues as public goods and $\lambda_1 = \frac{1}{2}$. In this situation, it is a net receiver, just like the median, median voter at $(1 - \lambda_1) = \frac{1}{2}$.

The net contribution of jurisdiction 1 is

$$NC_1 = ty_{1m} - \lambda t \sum_{i=1}^{3} y_{im} \tag{41}$$

The effect on NC_1 from a rise in the income of the richest median voter will therefore be

$$\begin{aligned} \frac{\partial NC_1}{\partial y_{1m}} &= t - \lambda_1 t - t \sum_{i=1}^{3} y_{im} \frac{\partial \lambda_1}{\partial y_{1m}} \\ &= (1 - \lambda_1)t + t \sum_{i=1}^{3} y_{im} \lambda_1 (1 - \lambda_1)(1 - t) > 0 \end{aligned} \tag{42}$$

Therefore, with an increase in the income of the richest median voter, the net contribution (in absolute terms) of the jurisdiction with the richest median voter goes up. It should be noted whether the jurisdiction with the richest median voter is a net contributor if NC_1 is positive and a net receiver if NC_1 is negative. As the income of the richest median voter increases, the net receipts of this jurisdiction declines as given by (42), and for a particular income of the richest median voter, the net contribution by the jurisdiction with the richest median voter is zero. This happens when:

$$ty_{1m} = g_1 \Rightarrow ty_{1m} = \lambda_1 \sum_{i=1}^{3} y_{im} \Rightarrow \lambda_1^* = \frac{y_{1m}}{\sum_{i=1}^{3} y_{im}} \tag{43}$$

Therefore for a particular value of y_{1m}, given y_{2m} there will be a $\lambda_1 = \frac{y_{1m}}{\sum_{i=1}^{3} y_{im}}$ at which net contribution for this jurisdiction is zero. Above this value of y_{1m}, NC_1 is positive, and jurisdiction 1 is a net contributor. Since λ_1 is decreasing in y_{1m}, and $\lambda_{1max} = \frac{1}{2}$ at which net contribution of jurisdiction 1 is negative, there exists a value of $\lambda_1 = \lambda_1^*$, at which net contributions of jurisdiction 1 is zero.

Appendix 5: Resource allocation with reservation utilities as in independent jurisdictions

We first evaluate the level of taxes rate, which will determine the level of public good supplied in any jurisdiction, and therefore the level of welfare experienced by individuals at which they set their reservation utilities. The local government's problem as in (14) can be re-written as

$$\begin{aligned} &\max_t \int_{y_{il}}^{y_{ih}} \frac{1}{y_{ih} - y_{il}}(1 - t)y_{ij}dy_{ij} + ln(g_i) \\ &\qquad\qquad\qquad\qquad such\, that \\ &\qquad\qquad\qquad\qquad ty_{im} = g_i \end{aligned} \tag{44}$$

or

$$\max_t (1 - t)y_{im} + ln(ty_{im}) \tag{45}$$

Solving for (45) gives us the optimal tax rate in jurisdiction i t_i to be $t_i = \frac{1}{y_{mi}}$. Therefore the amount of local public good received by any jurisdiction i, is $g_i = 1$, therefore the welfare experienced by any individual in jurisdiction i with income y_{ij} is $W_{ij} = (1 - t_i)y_{ij} + ln1 = (1 - t)y_{ij}$. This is also the level of welfare at which individuals set their reservation utility. Thus, the probability of getting re-elected from any jurisdiction i, for a uniform tax rate t, by the central government and an equal allocation of public good of $g_i = \overline{g}$ will be given by

$$p(e_i) = \frac{1}{2q}[q + ln\overline{g} - (t - t_i)y_{mi}] \tag{46}$$

Therefore, the probability of getting re-elected from any jurisdiction depends on $(t - t_i)y_{mi}$. Since $t_i y_{mi} = 1$, for any uniform tax rate t, the probability of getting re-elected is least from jurisdiction 1, the jurisdiction with the richest median voter, and highest from jurisdiction 3, the jurisdiction with the poorest median voter. Thus in this case, resources for public good will be shifted from jurisdiction 3 to jurisdictions 1 and 2. The probability of getting re-elected for the central government is again the probability of getting re-elected from jurisdiction 2, the jurisdiction with the median, median voter (proof along the same line as in Appendix 2). In this case, one will again maximize the probability of re-election from jurisdiction 2, the jurisdiction with the median, median voter. The government's optimization problem will be

$$\max_{t,g_i} p(e_2)$$

$$such\, that$$

$$p(2) \leq p(3)$$

$$\sum_{i=1}^{3} g_i = t\sum_{i=1}^{3}[\int_{j=y_{il}}^{y_{ih}} y_{ij}p(y_{ij})dy_{ij}] \tag{47}$$

Therefore, at the optimum the following conditions should be satisfied

$$p(e_2) = p(e_3) \tag{48}$$

and

$$t\sum_{i=1}^{3} y_{im} = g_2 + g_3 \tag{49}$$

(48 would imply

$$\frac{1}{2q}[q + ln(g_2) - (t - t_2)y_{2m}] = \frac{1}{2q}[q + ln(g_3) - (t - t_3)y_{3m}] \tag{50}$$

Since $t_2 y_2 = t_3 y_3 = 1$, (50) would imply

$$ln(g_2) - ty_{2m} = ln(g_3) - ty_{3m} \tag{51}$$

Since $y_{2m} \geq y_{3m}$, (51) would imply that $g_2 \geq g_3$. Therefore, the jurisdiction with the median, median voter gets the largest share of public resources, that with the poorest median voter gets some, while the jurisdiction with the richest median voter gets none. The optimum uniform central tax rate in this case is again $t^* = \frac{1}{y_{2m}}$ (proof along the same line as Appendix 3).

Appendix 6: Net contributions of jurisdictions with with reservation utilities as in independent jurisdictions

The net contribution of jurisdiction 1, that with the richest median voter is positive, since it pays taxes and receives no public good and is always a loser in this setup. The net contribution of jurisdiction 2, the one with the median, median voter is also always positive. In this situation $\lambda_1 = 0$, and $\lambda_2 = (1 - \lambda_3)$.

Thus (51) can be re-written as

$$t(y_{3m} - y_{2m}) = ln\frac{\lambda_3}{1 - \lambda_3} \tag{52}$$

Differentiating partially with respect to y_{3m}, we get

$$t = \frac{(1 - \lambda_3)}{\lambda_3}.\frac{1}{(1 - \lambda_3)^2}[(1 - \lambda_3)\frac{\partial\lambda_3}{\partial y_{3m}} + \lambda_3\frac{\partial\lambda_3}{\partial y_{3m}}] \tag{53}$$

or

$$\frac{\partial\lambda_3}{\partial y_{3m}} = \lambda_3(1 - \lambda_3)t \tag{54}$$

Therefore, with an increase in income of the poorest median voter, the share of tax revenues going to jurisdiction 3, the one with the poorest median voter increases. Since $y_{2m} \geq y_{3m} \geq 0$, when $y_{3m} = y_{2m}$, its situation is exactly like the median, median voter and $\lambda_3 = \frac{1}{2}$ and it is a net beneficiary from the central government. Net contribution of jurisdiction 3 equal to zero would imply

$$\begin{aligned} NC_3 &= ty_{3m} - t\lambda_3\sum_{i=1}^{3} y_{im} = 0 \\ &= t[y_{3m} - \lambda_3\sum_{i=1}^{3} y_{im}] = 0 \end{aligned} \tag{55}$$

If $NC_3 = 0$, it implies that $[y_{3m} - \lambda_3 \sum_{i=1}^{3} y_{im}] = 0$, which in turn implies that if $\lambda_3 = \frac{y_{3m}}{\sum_{i=1}^{3} y_{im}}$. Above this value of λ_3, net contribution of jurisdiction 3 is negative, and vice versa.

Differentiating (55), partially with respect to y_{3m}, we get

$$\begin{aligned}
\frac{\partial NC_3}{y_{3m}} &= t - \lambda_3 t - t \sum_{i=1}^{3} y_{im} \frac{\partial \lambda_3}{y_{3m}} \\
&= (1 - \lambda_3)t - t \sum_{i=1}^{3} y_{im} \lambda_3 (1 - \lambda_3) t \\
&= (1 - \lambda_3) t [1 - t \lambda_3 \sum_{i=1}^{3} y_{im}] \qquad (56)
\end{aligned}$$

and

$$\frac{\partial^2 NC_3}{\partial y_{3m}^2} = -[1 - t\lambda_3 \sum_{i=1}^{3} y_{im}] t \frac{\partial \lambda_3}{\partial y_{3m}} + (1 - \lambda_3) t [-t\lambda_3 - t \sum_{i=1}^{3} y_{im} \frac{\partial \lambda_3}{\partial y_{3m}}] \qquad (57)$$

It should be noted that for

$$[1 - t\lambda_3 \sum_{i=1}^{3} y_{im}] = 0 \qquad (58)$$

$\frac{\partial NC_3}{y_{3m}} = 0$ and $\frac{\partial^2 NC_3}{\partial y_{3m}^2} \leq 0$. $\lambda_3 = \frac{1}{t \sum_{i=1}^{3} y_{im}}$ satisfies (58), so the net contribution of jurisdiction 3 is maximum at this value of λ_3. Since optimal tax rate is always $t = \frac{1}{y_{2m}}$, at the maximum contribution of jurisdiction 3, $\lambda_3 = \frac{y_{2m}}{\sum_{i=1}^{3} y_{im}}$, which is greater that $\frac{y_{3m}}{\sum_{i=1}^{3} y_{im}}$, at which value of λ_3, the net contribution of jurisdiction 3 is zero. So jurisdiction with the poorest median voter is always negative and it is always a net gainer in this situation.

Appendix 7: Resource allocation with reservation utilities as with a central central Utilitarian Social Planner

A central Utilitarian social planner will maximize the sum of welfare of all individuals. Its problem is thus:

$$\begin{aligned}
\max_{t, g_i} W = \sum_{i=1}^{3} \int_{y_{il}}^{y_{ih}} W_{ij} p(y_{ij}) dy_{ij} &= \sum_{i=1}^{3} [\int_{y_{il}}^{y_{ih}} \{(1-t) y_{ij} + ln(g_i)\} p(y_{ij}) dy_{ij}] \\
&\textit{subject to} \\
\sum_{i=1}^{3} g_i &= t \sum_{i=1}^{3} y_{im} \qquad (59)
\end{aligned}$$

The policy parameters for the social planner are the uniform tax rate t and the amount of local public good g_i to be given to jurisdictions 1, 2 and 3.

The lagrangian function for this optimization model is

$$Z_s = \sum_{i=1}^{3}[\int_{y_{il}}^{y_{ih}}\{(1-t)y_{ij} + ln(g_i)\}p(y_{ij})dy_{ij}] + \mu[t\sum_{i=1}^{3} y_{im} - \sum_{i=1}^{3} g_i] \tag{60}$$

As first order conditions for optimization we get

$$\frac{\partial Z_s}{\partial \mu} = t\sum_i y_{im} - \sum_i g_i = 0 \tag{61}$$

$$\frac{\partial Z_s}{\partial t} = -\sum_i y_{im} + \mu\sum_i y_{im} = 0 \tag{62}$$

$$\frac{\partial Z_s}{\partial g_i} = \frac{1}{g_i} - \mu = 0 \tag{63}$$

From the first order conditions we get the optimal values of μ, g_i, t as $\mu^* = 1$, $g_i^* = 1$ and $t^* = \frac{3}{\sum_i y_{im}}$.

The welfare obtained by any individual in a central Utilitarian Social Planner allocation may thus be given as

$$W_{ij}^* = (1-t^*)y_{ij} + ln(g^*) = (1-t^*)y_i \quad for\, i \in \{1, 2, 3\} \tag{64}$$

Let us start from the central Utilitarian Social Planner allocation $g_i^* = 1$ and $t^* = \frac{3}{\sum_i y_i}$, henceforth referred to as $g^s = 1$ and t^s. The welfare obtained by an individual in jurisdiction i, with a social planner be referred to as

$$W_{ij}^s = (1-t_s)y_i + ln(g_s) = (1-t_s)y_i \quad for\, i \in \{1, 2, 3\}$$

Therefore, when reservation utilities are set at the level provided by a Utilitarian Social Planner, i.e. $V_{ij} = W_{ij}^s$, the probability of getting re-elected in any jurisdiction i is given by

$$p(i) = \frac{q - (W_{ij}^s - W_i)}{2q} \tag{65}$$

If the democratic planner starts with the central Utilitarian social planner allocation, then $W_i = W_i^s$, and the probability of getting elected from any jurisdiction is 0.5. However, the government can do better than this by re-distributing equally the one unit of local public good from any jurisdiction to the other two jurisdictions, and the government is indifferent which jurisdiction it favors at the social planner tax rate t_s. However, this may not be true for any other tax rate t, one actually might gain by diverting local public goods to either the richer or to the poorer jurisdictions. For any tax rate t, let us start with an equal allocation of local public good $\frac{\overline{g}=t\sum_i y_i}{3}$. Therefore the probability of getting re-elected from any jurisdiction i, $i \in, \{1, 2, 3\}$ will be a function of $(W_i^s - W_i)$, the lower is this value, the higher is the probability of getting elected from any jurisdiction. However,

$$W_i^s - W_i = (1 - t^s)y_{im} + ln(1) - (1 - t)y_{im} - ln(\overline{g}) = (t - t_s)y_{im} - ln(\overline{g}) \tag{66}$$

From (66), it is clear that for an equal allocation of local public good amongst jurisdictions, the probability of winning from any jurisdiction would depend on $(t - t_s)y_i$. The lower is this value, the higher is the probability of winning from the jurisdiction. If $t \geq t_s$, then $p(e_1) \geq p(e_2) \geq p(e_3)$, then jurisdictions 1 and 2 should be favored, if $t \leq t_s$, then $p(e_1) \leq p(e_2) \leq p(e_3)$, then jurisdictions 2 and 3 should be favored. In either of these situations, it is the probability of re-election from jurisdiction 2 that has to be maximized and in the optimal allocation, the probability of re-election from jurisdiction 2 should be equal to the probability of re-election from jurisdiction 1 or 3.

To analyze which direction the optimal tax rate will move in this case, let us evaluate the effect of a change in tax rate, on the probability of re-election from jurisdiction 2. The probability of re-election from jurisdiction 2 is given by:

$$\frac{1}{2q}[q - (t - t_s)y_{im} + ln(\lambda_2 t \sum_{i=1}^{3} y_{im})] \qquad (67)$$

Differentiating (67) with respect to t gives

$$\frac{\partial p(e_i)}{\partial t} = \frac{1}{2q}[-y_{2m} + \frac{1}{t\lambda_2 \sum_{i=1}^{3} y_{im}}(\lambda_2 \sum_{i=1}^{3} y_{im})] = \frac{1}{2q}[-y_{im} + \frac{1}{t}] \qquad (68)$$

At $t = t_s = \frac{3}{\sum_{i=1}^{3} y_{im}}$, the probability of re-election will increase with an increase in tax rate if

$$[-y_{2m} + \frac{1}{t_s}] \geq 0 \Rightarrow [-y_{2m} + \frac{\sum_{i=1}^{3} y_{im}}{3}] \geq 0 \Rightarrow y_{2m} \leq \frac{\sum_{i=1}^{3} y_{im}}{3} \qquad (69)$$

Thus, from (69) and (68), we find that $\frac{\partial p(e_i)}{\partial t} \geq 0$, if $y_{2m} \leq \frac{\sum_{i=1}^{3} y_{im}}{3}$ and so $t_d \geq t_s$, and jurisdictions 1 and 2 will be favored. $\frac{\partial p(e_i)}{\partial t} \leq 0$, if $y_{2m} \geq \frac{\sum_{i=1}^{3} y_{im}}{3}$ and so $t_d \leq t_s$, and jurisdictions 2 and 3 will be favored. In both cases, the median voter will get the largest share of public resources.

As far as the net contributions of jurisdictions are concerned, that of jurisdiction 2 is always negative, and that of the jurisdiction 1 (the one with the richest median voter), may be positive or negative when favored (proof along the same lines as in Appendix 4). As for jurisdiction 3, the one with the poorest median voter, its net contribution is zero when $\lambda_3 = \frac{y_{3m}}{\sum_{i=1}^{3} y_{im}}$. If it receives a share above it, its net contributions are negative and vice versa (proof along the same line as in Appendix 6). The highest net contribution of jurisdiction 3, when favored is $\frac{1}{(t - t_s)\sum_{i=1}^{3} y_{im}}$ (proof along the same lines as in Appendix 6). So if $\frac{1}{(t - t_s)} \geq y_{3m}$, then the net contribution of jurisdiction 3 is always negative.

$$\begin{aligned}\frac{1}{t - t_s} &= \frac{1}{\frac{1}{y_{2m}} - \frac{3}{\sum_{i=1}^{3} y_{im}}} \\ &= \frac{1}{\frac{\sum_{i=1}^{3} y_{im} - 3y_{2m}}{y_{2m}\sum_{i=1}^{3} y_{im}}}\end{aligned}$$

$$
\begin{aligned}
&= \frac{y_{2m}\sum_{i=1}^{3} y_{im}}{\sum_{i=1}^{3} y_{im} - 3y_{2m}} \\
&= \frac{y_{2m}}{1 - \frac{3y_{2m}}{\sum_{i=1}^{3} y_{im}}} \\
&= \frac{y_{2m}}{1 - \frac{y_{2m}}{\sum_{i=1}^{3} y_{im}/3}}
\end{aligned} \tag{70}
$$

When jurisdiction 3 is favored, $y_{2m} \leq \sum_{i=1}^{3} y_{im}/3$, the denominator is less than one, and the numerator is greater than y_{3m}, so the net contribution of jurisdiction 3 when favored is always negative.

Appendix 8: Resource allocation with not a 50% majority rule

Re-election from a jurisdiction with not a 50% majority rule will be when

$$p(e_i) = f_i \geq \frac{1}{2} + \eta \Rightarrow \frac{y_{ih} - y_{i\epsilon}}{y_{ih} - y_{il}} \geq \frac{1}{2} + \eta \tag{71}$$

Substituting for the value of $y_{i\epsilon} = \frac{V_{ij} - \epsilon - ln(g_i)}{(1-t)}$ in (71), we get

$$\frac{y_{ih}}{y_{ih} - y_{il}} - \frac{V_{ij} - \epsilon - ln(g_i)}{(1-t)(y_{ih} - y_{il})} \geq \frac{1}{2} + \eta \tag{72}$$

Therefore, for re-election from any jurisdiction i, one would require

$$\epsilon \geq V_{ij} - ln(g_i) - (1-t)y_{ih} + \frac{1}{2}(1-t)(y_{ih} - y_{il}) + \eta(1-t)(y_{ih} - y_{il}) \tag{73}$$

Thus

$$
\begin{aligned}
p(e_i) &= \frac{1}{2q}[q - V_{ij} + ln(g_i) + (1-t)y_{im} - \eta(1-t)(y_{ih} - y_{il})] \\
p(e_i) &= \frac{1}{2q}[q - V_{ij} + ln(g_i) + (1-t)y_{im} - \eta(1-t)R_i
\end{aligned} \tag{74}
$$

where $R_i = (y_{ih} - y_{il})$ in (74). Let $R_1 \geq R_2 \geq R_3$. At the optimum, probability of winning from jurisdiction 2 and jurisdiction k, where $k \in \{1, 3\}$ will be equal (proof along the same lines as in Appendix 2) That is

$$\frac{1}{2q}[q + ln(g_2) + (1-t)y_{2m} - \eta(1-t)R_2] = \frac{1}{2q}[q + ln(g_3) + (1-t)y_{km} - \eta(1-t)R_k] \quad (75)$$

or

$$(1 - t)(y_{km} - y_{2m}) + \eta(1 - t)(R_2 - R_k) + ln(\frac{\lambda_k}{1 - \lambda_k}) = 0 \quad (76)$$

Differentiating (76) with respect to R_k and rearranging the terms we get

$$\frac{\partial \lambda_k}{\partial R_k} = -\eta(1 - t)\lambda_k(1 - \lambda_k) \quad (77)$$

and

$$\frac{\partial NC_k}{\partial R_k} = -\sum_{i=1}^{3} y_{im}\frac{\partial \lambda_k}{\partial R_k} \quad (78)$$

If $\eta \geq 0$, $k = 3$, $\frac{\partial \lambda_k}{\partial R_k} \leq 0$ and $\frac{\partial NC_k}{\partial R_k} \geq 0$. Since $R_2 \geq R_3 \geq 0$, at $R_3 = R_2$, $\lambda_3 = \frac{1}{2}$ and its net contributions are negative. As R_3 declines from this value, its net contributions decline further, and therefore the net contribution of the least unequal jurisdiction is always negative when it is favored.

If $\eta \leq 0$, $k = 1$, $\frac{\partial \lambda_k}{\partial R_k} \geq 0$ and $\frac{\partial NC_k}{\partial R_k} \leq 0$. Since $R_1 \geq R_2$, at $R_1 = R_2$, $\lambda_1 = \frac{1}{2}$ and its net contributions are negative. As R_1 increases from this value, its net contributions decline further, and therefore the net contribution of the most unequal jurisdiction is always negative when it is favored. Therefore, net contributions of jurisdictions being favored in this situation are always negative.

As for the optimal tax rate, it will be decided by the tax rate at which the probability of re-election from jurisdiction 2 is highest.

$$p(e_2) = \frac{1}{2q}[q - V + ln(\lambda_2 t\sum_{i=1}^{3} y_m) + (1 - t)y_m - \eta(1 - t)R_2] \quad (79)$$

Differentiating (79) partially with respect to t, we get

$$\frac{\partial p(e_2)}{\partial t} = \frac{1}{2q}[\frac{1}{t} - \{y_m - \eta R_2\}] \tag{80}$$

and

$$\frac{\partial^2 p(e_2)}{\partial t^2} = -\frac{1}{2q}(\frac{1}{t^2}) \tag{81}$$

At $t = \frac{1}{y_m - \eta R_2}$, the probability of getting re-elected is maximum, since $\frac{\partial p(e_2)}{\partial t} = 0$ and $\frac{\partial^2 p(e_2)}{\partial t^2} \leq 0$. So this is the optimal tax rate in this situation.

References

Baron D. P., Ferejohn J. 1989. Bargaining in legislatures. American Political Science Review 83, 1181-1206.

Besley T., Coate S. 1997. An economic model of representative democracy. Quarterly Journal of Economics 112(1), 85-114.

Buchanan J., Tullock G. 1962. The Calculus of Consent: Logical Foundations of Constitutional Democracy. The University of Michigan Press.

Dasgupta I., Kanbur R. 2002. Class, Community, Inequality. Credit Research Paper No. 02/04.

Le Grand J. 1982. The Strategy of Equality: Redistribution and the social services. Allen and Unwin, London.

Dixit A., Londregan J. 1996. The Determinants of Success of Special Interests in Redistributive Politics. Journal of Politics 58(4), 1132-55.

Gupta S. 2001 Political Accountability and Fiscal Federalism. International Tax and Public Finance 8(3) 263-279.

Milanovic B. 2000 The median-voter hypothesis, income inequality, and income redistribution: an empirical test with required data. European Journal of Political Economy 16, 367-410.

Niskanen W., 1971. Bureaucracy and Representative Government. New York: Aldine Atherton.

Seabright P. 1996. Accountability and decentralisation in government: An incomplete contracts model. European Economic Review 40, 61-89.

Weingast B., Shepsle K., and Johnson G. 1981. The political economy of benefits and costs: A neo-classical approach to distributive politics. Journal of Political Economy 89, 42-664.