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Abstract

Consider a Bertrand-Edgeworth duopoly with linear cost functions. If the firms produce to stock then no Nash equilibrium in pure strategies exists. If, however, the firms produce to order then all subgame perfect Nash equilibria involve the firms charging a price equal to marginal cost.

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1 Introduction

Consider a Bertrand duopoly where the firms have identical and linear cost functions and must supply the whole of the demand coming to them. It is well known that there is a unique Nash equilibrium where both the firms charge a price equal to marginal cost. In the literature this result is known as the Bertrand (1833) paradox, since it suggests that competition among only two firms may be sufficient to yield the perfectly competitive outcome (see, for example, Tirole (1988), pp. 209-211).

Efforts at resolving the Bertrand paradox have involved relaxing the various assumptions underlying the model, e.g. that the cost functions are linear, or that the product is homogeneous, etc. (see Tirole (1988), Chapter 5). In the process the Bertrand paradox has played an important role in the development of the literature.

Here we focus on another critical assumption behind the Bertrand paradox, that firms must supply all demand. It is often implicitly assumed that the result goes through even if this assumption is relaxed. In this paper, however, we argue that the Bertrand paradox is fundamentally altered if the firms are free to supply less than the quantity demanded (this assumption is due to Edgeworth (1897)).

Given the Edgeworth (1897) assumption there are two ways of modelling a game of price competition. Under the production to stock (or PTS) framework, the firms simultaneously decide on both their price and output levels.[1] Under the production to order (or PTO) framework, however, the

[1] The PTS game can be interpreted as one with advance production, so that firms must decide on their output levels before trading starts. Thus they make their price and output decisions without knowing the price and output decisions of the other firms. Retail markets are often characterized by such production conditions (see Mestelman et al. (1987)).

firms play a two stage game where they first simultaneously decide on their prices, and then on their output levels.

We find that under PTS competition no pure strategy Nash equilibrium exists, i.e. in this case we are faced with the Edgeworth (1897), rather than the Bertrand (1833) paradox.[2]

Under PTO competition, however, all subgame perfect Nash equilibria in pure strategies involve both the firms charging a price equal to the marginal cost. The equilibria, however, are non-unique in terms of output and may involve an aggregate supply that is less than demand. Thus in this case the Bertrand (1833) paradox can be said to hold, but only partially.

2 The Model

There are 2 identical firms, both producing the same homogeneous good. The market demand function is $q = d(p)$ and the cost function of both the firms is cq.

Assumption 1. $\forall p > 0$, $d(p)$ is well defined and once differentiable, with $d'(p) < 0$ and bounded.

[2]Edgeworth (1897) used a Bertrand duopoly model with linear, but capacity constrained cost functions to argue that in such models equilibria in pure strategies may not exist. This is the well known Edgeworth paradox (though Edgeworth (1897) himself thought of this as a case of indeterminate equilibrium, with prices cycling within a certain range). For a formal analysis of the Edgeworth (1897) paradox see, among others, Levitan and Shubik (1972), Maskin and Tirole (1988) and Tasnádi (1999a).

Given the Edgeworth paradox, one strand of the literature looks for existence in mixed strategies. One can mention, among others, Maskin (1986) who uses the fixed point theorems for discontinuous games developed by Dasgupta and Maskin (1986) to show existence of mixed strategy equilibria for both PTS and PTO games.

Let $R_i(p_1, p_2, q_j)$, $j \neq i$, denote the residual demand facing firm i, where

$$R_i(p_1, p_2, q_j) = \begin{cases} \max[0, d(p_i) - q_j\{\lambda + (1-\lambda)\frac{d(p_i)}{d(p_j)}\}], & \text{if } p_i > p_j, \\ \max[\frac{d(p_i)}{2}, d(p_i) - q_j], & \text{if } p_i = p_j, \\ d(p_i), & \text{if } p_i < p_j, \end{cases} \tag{1}$$

where $\lambda \in [0, 1]$.

The first line of equation 1 (i.e. the rationing rule) draws heavily on the combined rationing rule introduced by Tasnádi (1999b). Clearly, for $\lambda = 1$ we have the efficient rationing rule, whereas for $\lambda = 0$ we have the proportional rationing rule (see Tirole (1988) and Vives (1999) for a discussion of these two rationing rules). For intermediate values of λ other rationing rules emerge. Thus this formulation allows for a large class of rationing rules, including the two most well known one, the efficient and the proportional, as special cases.

The second line of equation 1 (i.e. the tie-breaking rule) follows Davidson and Deneckere (1986) and Kreps and Scheinkman (1983). One nice feature of this formulation is that it allows for the spill-over of unmet residual demand from one firm to another. However we later argue, in Remarks 1 and 2, that our results go through for other tie-breaking rules also.

We can now define the profit function of the i-th firm.

$$\pi_i(p_1, p_2, q_1, q_2) = p_i \min\{q_i, R_i(p_1, p_2, q_j)\} - cq_i, \quad i = 1, 2. \tag{2}$$

2.1 Production to Stock Framework

We first examine a simultaneous move game where the i-th firm's strategy consists of choosing both a price $p_i \in [0, \infty)$ and an output $q_i \in [0, \infty)$.

We solve for the pure strategy Nash equilibrium of this game.

Lemma 1 below is useful. The proof, which is standard, has been relegated to the appendix.

Lemma 1. *Any Nash equilibrium must involve both the firms charging a price equal to c.*

Proposition 1 below shows that this game has no Nash equilibrium in pure strategies.

Proposition 1. *The production to stock game with linear cost functions has no Nash equilibrium in pure strategies.*

Proof. Given Lemma 1, it is sufficient to argue that any outcome, $(\tilde{p}_1, \tilde{p}_2, \tilde{q}_1, \tilde{q}_2)$, where $\tilde{p}_1 = \tilde{p}_2 = c$, cannot be a Nash equilibrium.

Consider some outcome $(\tilde{p}_1, \tilde{p}_2, \tilde{q}_1, \tilde{q}_2)$, where $\tilde{p}_1 = \tilde{p}_2 = c$. Suppose to the contrary that this outcome is Nash. Since both the firms are charging c, and the outcome is Nash, it must be that $\tilde{q}_1 + \tilde{q}_2 \leq d(c)$. Thus there exists $\tilde{q}_j$ such that $d(c) - \tilde{q}_j \geq \frac{d(c)}{2} > 0$. Without loss of generality let $j = 2$. Since $d(c) - \tilde{q}_2 \geq \frac{d(c)}{2}$, it follows that $R_1(p_1, c, \tilde{q}_2)$ is right continuous at $p_1 = c$.

Next suppose that firm 1 deviates by charging a price p_1 greater than c. Note that, for $p_1 \geq c$, it is optimal for firm 1 to supply $R_1(p_1, c, \tilde{q}_2)$. Thus for $p_1 \geq c$, the profit of firm 1, assuming that its output level is optimal, is

$$\pi_1(p_1, c, R_1(p_1, c, \tilde{q}_2), \tilde{q}_2) = (p_1 - c)R_1(p_1, c, \tilde{q}_2). \tag{3}$$

Hence, for $p_1 \geq c$,

$$\frac{\partial \pi_1(p_1, c, R_1(p_1, c, \tilde{q}_2), \tilde{q}_2)}{\partial p_1} = R_1(p_1, c, \tilde{q}_2) + (p_1 - c)\frac{\partial R_1(p_1, c, \tilde{q}_2)}{\partial p_1}. \tag{4}$$

Consequently,

$$\frac{\partial \pi_1(p_1, c, R_1(p_1, c, \tilde{q}_2), \tilde{q}_2)}{\partial p_1}|_{p_1=c} = R_1(c, c, \tilde{q}_2) = d(c) - \tilde{q}_2 > 0. \tag{5}$$

Thus firm 1 can increase its price slightly and gain. ■

Remark 1. Note that Proposition 1 goes through even if the tie-breaking rule is of the form $\frac{d(p_i)}{2}$, or $d(p_i)\frac{q_i}{q_1+q_2}$ (if $q_1 = q_2 = 0$, then the second tie-breaking rule takes the form $\frac{d(p_i)}{2}$). Recall that these are the two examples of tie-breaking rules provided in Maskin (1986). In both the cases it is sufficient to observe that for the outcome, $(\tilde{p}_1, \tilde{p}_2, \tilde{q}_1, \tilde{q}_2)$, where $\tilde{p}_1 = \tilde{p}_2 = c$, it must be the case that $\tilde{q}_1 + \tilde{q}_2 \leq d(c)$,[3] so that $d(c) - \tilde{q}_j \geq \frac{d(c)}{2} > 0$, for some j. Thus the argument in Proposition 1 goes through.

2.2 Production to Order Framework

We next examine a two stage game where, in stage 1, the firms simultaneously decide on their price levels, and in stage 2, they simultaneously decide on their quantity levels.

We then solve for the subgame perfect Nash equilibria of this game in pure strategies.

Proposition 2. *If the firms produce to order and cost functions are linear then, any outcome (p'_1, p'_2, q'_1, q'_2), where $p'_1 = p'_2 = c$ and $q'_1 + q'_2 \leq d(c)$, can be supported as a subgame perfect Nash equilibrium. Moreover, no other subgame perfect Nash equilibrium exists.*

Proof. It is clear that Lemma 1 applies in this case as well.

Next let us consider some outcome (p'_1, p'_2, q'_1, q'_2), where $p'_1 = p'_2 = c$ and $q'_1 + q'_2 \leq d(c)$. It is sufficient to see that the following strategies sustain this outcome as a subgame perfect Nash equilibrium:

Stage 1. Both the firms charge a price equal to c.

Stage 2. In case both the firms charge c in stage 1, then, in stage 2, firm 1 supplies q'_1 and firm 2 supplies q'_2. If, in stage 1, one of the

[3]If, to the contrary, $\tilde{q}_1 + \tilde{q}_2 > d(c)$, then one of the firms must be incurring losses.

firms charges c, while the other firm charges a strictly higher price, then in stage 2 the firm charging c supplies $d(c)$, while the other firm supplies nothing. ■

Interestingly, while the equilibrium price equals the perfectly competitive level, the aggregate supply, $q_1' + q_2'$, may be less than the demand $d(c)$. Thus in this case the Bertrand paradox applies only partially. Of course, the result, that in equilibrium supply can be less than demand, is paradoxical in itself.

Remark 2. It is clear that Proposition 2 goes through even if the tie-breaking rule is of the form $d(p_i)\frac{q_i}{q_1+q_2}$. Whereas if the tie-breaking rule is of the form $\frac{d(p_i)}{2}$, then any outcome (p_1', p_2', q_1', q_2'), where $p_1' = p_2' = c$ and $q_1', q_2' \leq \frac{d(c)}{2}$, constitutes a subgame perfect Nash equilibrium. Moreover, no other subgame perfect Nash equilibrium exists.[4]

3 Conclusion

In this paper we examine a model of Bertrand-Edgeworth duopoly where the firms are free to supply less than the quantity demanded and cost functions are linear. If the competition is of the production to stock type, then no Nash equilibrium in pure strategy exists. If, however, the competition is of the production to order type, then all subgame perfect Nash equilibria in pure strategies involve both the firms charging a price equal to the marginal cost. The aggregate supply, however, may be less than demand.

[4] It is easy to see that in both the cases the strategies outlined in the proof of Proposition 2 will work.

4 Appendix

Proof of Lemma 1. Consider an outcome $(\hat{p}_1, \hat{p}_2, \hat{q}_1, \hat{q}_2)$. We argue that for this outcome to be a Nash equilibrium it is necessary that $\hat{p}_1 = \hat{p}_2 = c$.

Case 1. Suppose $\hat{p}_i > \hat{p}_j > c$. Then firm i can deviate by undercutting firm j slightly and gain. Hence such a price configuration cannot be a part of a Nash equilibrium.

Case 2. Suppose $\hat{p}_i > c \geq \hat{p}_j$. Then firm j can charge some p_j'', such that $\hat{p}_i > p_j'' > c$, and gain.

Case 3. Suppose that $\hat{p}_i < \hat{p}_j \leq c$. Then firm j can charge a price slightly higher than c and gain.

Case 4. Suppose $\hat{p}_1 = \hat{p}_2 > c$. Then firm 1 can undercut slightly and gain.

Case 5. Suppose $\hat{p}_1 = \hat{p}_2 < c$. Then firm 1 can charge a price slightly higher than c and gain. ∎

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