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Abstract

We examine a simple model of collusion under a single-object second-price auction. Under the appropriate parameter conditions, in particular as long as collusion is neither too easy, nor too difficult, we find that the optimal policy involves both an effective ceiling, as well as a reserve price set at the lowest bidder valuation.

Key words: Auctions, ceilings, collusion, reserve prices.
JEL Classification Number: C7, D4.

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1 Introduction

In this paper we examine the role of ceilings (i.e. bid-caps) and reserve prices in controlling collusion in auctions. It is well recognized that auctions are prone to collusive behavior among the bidders (see Graham and Marshall (1987)). Given that auctions account for a large volume of economic activity,[1] policies aimed at preventing such collusion are of interest. Further, given that ceilings and reserve prices seem to be rather widely used, in this paper we focus attention on these two policies.

We consider a simple second-price auction with independent private values where the bidders may potentially collude. We examine the case where the seller may impose a ceiling, as well as a reserve price. We characterize the conditions under which the optimal policy involves both an effective ceiling, and a reservation price set at the lowest bidder valuation. Thus interestingly an apparently anti-competitive policy, i.e. the imposition of ceilings, may have a pro-competitive effect.

In the literature the role of bid-caps in collusive auctions is relatively unexplored. It focuses on the case where there is no collusion e.g. Banerjee and Chakroborty (2005), Che and Gale (1998) and Gavious et. al. (2002). There are, however, several papers that examine the role of reserve prices in second-price auctions with collusion, e.g. Graham and Marshall (1987) and Kirkegaard (2005). In contrast to this paper, however, Graham and Marshall (1987) and Kirkegaard (2005) examine reserve prices in isolation, rather than in conjunction with ceilings.

[1] The US government, for example, sells timber rights, offshore oil leases, etc. through auctions. Since 1994, auction theorists have designed spectrum sales in various countries, e.g. electric power auctions in the United States (Milgrom (2004)).

2 The Model

A seller plans to sell an indivisible object to two bidders, 1 and 2 through a second-price auction with a symmetric and random tie-breaking rule. The seller's valuation of the object is zero. Bidder i's valuation of the object, v_i, is private information. v_i equals $\overline{V}$ with probability θ, and $\underline{V}$ with probability $1-\theta$, where $\overline{V} > \underline{V} \geq 0$ and $0 < \theta < 1$.

Bidder i's strategy is to announce a bid $b_i \in [0, M]$, where M denotes a bid-cap. Note that $M \geq \overline{V}$ represents the case where there is no effective bid-cap.

The sequence of actions is as follows.

Stage 1. The seller announces a cap of M, as well as a reserve price of R, such that $R \leq M \leq \overline{V}$.

Stage 2. The bidders sequentially decide on whether to collude, or compete. There is collusion if and only if both the bidders opt to collude.[2]

Stage 3. Bidder i, $i \in \{1, 2\}$, gets to know the realization of v_i.

Stage 4. The bidders simultaneously announce their bids. In case there is no collusion, each bidder i independently announces a bid $b_i \in [0, M]$. Whereas in the case there is collusion, bidder i bids $\min\{v_i, R\}$.

We assume that the bidders can commit to their collusive strategies.[3] Such commitment may be justified on reputational grounds and is more likely if the bidders also interact among themselves in other markets, so that deviations from the collusive strategy may be penalized in these other markets. Further, post-auction, the object cannot be redistributed.[4]

[2]We adopt the tie-breaking rule that in case of indifference the bidders prefer to compete rather than collude.

[3]Our modelling strategy is similar to Eso and Schummer (2004) who also assume that the bidders can commit to their collusive strategies. Alternatively, one can examine collusive strategies that are incentive compatible (e.g. Graham and Marshall (1987)).

[4]Clearly the *ex ante* payoff of the bidders would be greater in case such re-distribution is possible. This, however, may make collusion infeasible as it makes it easier to detect,

Colluding, however, involves a cost of C (> 0) for both the bidders. This might arise because in case there is a suspicion that the bidders are colluding among themselves, they may be blacklisted from future auctions by this, or other sellers.

We then use a backwards induction argument to solve the game. Further, the bidders are assumed to play undominated strategies.[5]

The collusive outcome.

Case 1. In case $R \leq \underline{V}$, recall that both the bidders bid R.[6] Hence the expected payoff of the bidders in stage 2

$$P'(R) = \frac{\theta\overline{V} + (1-\theta)\underline{V} - R}{2} - C, \tag{1}$$

and that of the sellers

$$S'(R) = R. \tag{2}$$

Case 2. In case $R > \underline{V}$, a bidder bids $\underline{V}$ if her valuation is $\underline{V}$, and bids R if her valuation is $\overline{V}$. Thus a bidder's expected payoff in stage 2 is

$$P''(R) = \theta(1-\theta)(\overline{V} - R) + \frac{\theta^2(\overline{V} - R)}{2} - C, \tag{3}$$

and that of the sellers

$$S''(R) = \theta(2-\theta)R. \tag{4}$$

The non-collusive outcome.

Let $B_i(M)$ denote the equilibrium bids of the two bidders in stage 4. It is simple to see that the equilibrium involves $B_i(M) = \min\{v_i, M\}$.

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thus increasing the costs of collusion.

[5]Undominated strategies are used in the stage 4 subgame in case the bidders opt not to collude. In case the set of undominated strategies is not a singleton, and bidding the true valuation is an undominated strategy, then the selection rule is that the bidders bid their true valuations.

[6]The collusion scheme adopted here is related to the one in Eso and Schummer (2004). However, while in our paper collusion is symmetric for $R \leq \underline{V}$, and the object is allocated to the bidders randomly, in Eso and Schummer (2004) only the bribing bidder wins the object.

Case 1. $R \leq \underline{V}$.

1(a). $R \leq M \leq \underline{V}$. Given the equilibrium bidding strategies, the expected payoff of both the bidders in stage 2 is

$$P(M)|_{R\leq M\leq \underline{V}} = \frac{\theta\overline{V} + (1-\theta)\underline{V} - M}{2}, \tag{5}$$

and that of the seller is

$$S(M)|_{R\leq M\leq \underline{V}} = M. \tag{6}$$

1(b). $R \leq \underline{V} < M \leq \overline{V}$. Given the equilibrium bidding strategies, the expected payoff of the bidders in stage 2 is

$$P(M)|_{M>\underline{V}\geq R} = \frac{\theta^2(\overline{V} - M)}{2} + \theta(1-\theta)(\overline{V} - \underline{V}), \tag{7}$$

and that of the seller is

$$S(M)|_{M>\underline{V}\geq R} = \theta^2 M + (1-\theta^2)\underline{V}. \tag{8}$$

Case 2. $\underline{V} < R \leq \overline{V}$. Clearly, in this case the bidders will bid $\min\{\overline{V}, M\}$ if and only if their valuation is $\overline{V}$. Otherwise they bid $\underline{V}$. Thus the expected payoff of the bidders in stage 2 is

$$P(M)|_{R>\underline{V}} = \frac{\theta^2}{2}(\overline{V} - M) + \theta(1-\theta)(\overline{V} - R). \tag{9}$$

and that of the sellers is

$$S(M)|_{R>\underline{V}} = \theta^2 M + 2\theta(1-\theta)R. \tag{10}$$

We then solve for the optimal mechanism $< R, M >$ that maximizes the seller's payoff. We need some notations. Let $M^*(R)$ solve $P(M)|_{R\leq M\leq \underline{V}} = P'(R)$. Hence

$$M^*(R) = R + 2C. \tag{11}$$

Similarly, let $M^{**}(R)$ solve $P(M)|_{M>\underline{V}\geq R} = P'(R)$, so that

$$M^{**}(R) = \frac{R + 2C + \theta(1-\theta)\overline{V} - (1-\theta)(2\theta+1)\underline{V}}{\theta^2}. \tag{12}$$

Note that there are two policies available to the seller for controlling collusion, M and R. Proposition 1 below shows that, under the appropriate parameter conditions (in particular for intermediate values of C), the optimal mechanism involves using *both* ceilings and reserve prices. We proceed as follows. Let us fix R. In the absence of collusion, the payoff of the bidders is decreasing in M for all $M > \underline{V}$ (see (7) and (9)). Thus lowering M may, by increasing $P(M)$, encourage non-collusive behavior by the bidders. Also, from (6), (8) and (10), note that in this case the expected payoff of the seller is increasing in M. Thus, for $M > \underline{V}$,[7] the seller would like to charge the highest possible M that ensures non-collusive behavior by the bidders.

Further, from (1), note that the bidders' payoff under collusion is decreasing in R for $R \leq \underline{V}$. Similarly, from (3), the bidder's payoff under collusion is decreasing in R for $R > \underline{V}$. Next, for the same (R, M), note that the seller's payoff in the collusive case is lower compared to that in the non-collusive case whenever $R \leq \underline{V}$ (from (2), (6) and (8)). A similar argument holds for the case where $R > \underline{V}$ (from (4) and (10)). In fact, for $M > R$, the payoff of the seller is strictly lower in the collusive case. It is thus sufficient to restrict attention to the set of (R, M) such that collusion is prevented. For this set, let us first consider the case where $R \leq \underline{V}$. We identify the vector in this set such that R is maximized. This ensures that the corresponding M is the maximum possible among all (R, M) in this set. We then identify a similar mechanism from the set of (R, M) such that collusion is prevented and $R > \underline{V}$. A comparison of seller payoffs under these two mechanisms yields the optimal mechanism.

Proposition 1. *The optimal mechanism* $< R, M >$ *involves*

(a) Suppose $\underline{V}(1 + \theta) \leq 2\overline{V}\theta$*. Then* $R = M = \overline{V}$*.*

[7]Note that any mechanism that involves $M < \underline{V}$ cannot be optimal as, from the seller's point of view, it is always dominated by a posted price mechanism with a price of $\underline{V}$. Similarly, any mechanism (R, M) such that $M = \underline{V}$, is dominated by another mechanism (R', M') with $R' = \underline{V}$ and $M' = \underline{V} + \epsilon$, where $\epsilon > 0$ is sufficiently small.

(b) Suppose $\underline{V}(1+\theta) > 2\overline{V}\theta$. *Then*

$$\begin{cases} R = M = \overline{V}, & \text{if } C \leq \frac{\theta\overline{V}-\underline{V}(1-\theta+\theta^2)}{2}, \\ R = \underline{V} \text{ and } M = M^{**}(\underline{V}) < \overline{V}, & \text{if } \frac{\theta\overline{V}-\underline{V}(1-\theta+\theta^2)}{2} < C \leq \frac{(\overline{V}-\underline{V})\theta(2\theta-1)}{2}, \\ R = \underline{V} \text{ and } M = \overline{V}, & \text{if } \frac{(\overline{V}-\underline{V})\theta(2\theta-1)}{2} < C. \end{cases}$$

Proof. First consider the case where $R \leq \underline{V}$. To begin with, recall that $M^*(R)$ solves $P(M)|_{M \leq \underline{V}} = P'(R)$. Thus for any $\underline{V} \geq M > M^*(R)$, there will be collusion. Similarly, recall that $M^{**}(R)$ solves $P(M)|_{M > \underline{V}} = P'(R)$. Hence there will be collusion whenever $M > M^{**}(R) \geq \underline{V}$. Hence, for a given R, optimally M is set either at $M^*(R)$, or $M^{**}(R)$.

Observe that the expected payoff of the seller is increasing in M. Since $M^*(R)$ and $M^{**}(R)$ are both increasing in R, it is optimal to set $R = \underline{V}$ and $M = \min\{M^{**}(\underline{V}), \overline{V}\}$.[8]

It is easy to see that $M^{**}(\underline{V}) < \overline{V}$ if and only if

$$2C < (\overline{V} - \underline{V})\theta(2\theta - 1). \tag{13}$$

Thus the expected payoff of the seller under $< R, M > |_{R \leq \underline{V}}$ is

$$2C + \theta(1-\theta)\overline{V} + (1 - \theta + \theta^2)\underline{V}, \tag{14}$$

if (13) holds. Otherwise, it is

$$\theta^2\overline{V} + (1 - \theta^2)\underline{V}. \tag{15}$$

We next consider the case where $\underline{V} < R \leq \overline{V}$. Note that for $R = \overline{V}$ (= M), from equations (3) and (9), the bidders opt not to collude. Moreover, from equations (4) and (10), this maximizes the seller's payoff. Thus, for $\overline{V} \geq R > \underline{V}$, setting $M = R = \overline{V}$ is optimal for the seller. Next, from (10), for $M = R = \overline{V}$, the seller's payoff is

$$\theta(2 - \theta)\overline{V}. \tag{16}$$

Finally we compare the seller's payoff under (14), (15) and (16). The seller's payoff under (14) is at least as much as that under (16) if and only

[8] It is straightforward to check that $M^{**}(\underline{V}) > \underline{V}$.

if $C \geq \frac{\theta\overline{V}-\underline{V}(1-\theta+\theta^2)}{2}$. Similarly, the seller's payoff under (15) is at least as much as that under (16) if and only if $\underline{V}(1+\theta) \geq 2\overline{V}\theta$. Proposition 1 now follows from the observation that the interval $[\frac{\theta\overline{V}-\underline{V}(1-\theta+\theta^2)}{2}, \frac{(\overline{V}-\underline{V})\theta(2\theta-1)}{2}]$ is well defined if and only if $\underline{V}(1+\theta) > 2\overline{V}\theta$. ■

Let us consider Proposition 1(a). In this case $\underline{V}(1+\theta) \leq 2\overline{V}\theta$. Note that this condition ensures that, in the absence of any collusive possibilities, the optimal mechanism involves a posted price mechanism with a price of $\overline{V}$. Since such a mechanism also rules out collusive possibilities, $R = M = \overline{V}$ is optimal even when collusion is possible.

The above argument suggests that for any other possibility to emerge we must have that $\underline{V}(1+\theta) > 2\overline{V}\theta$.[9] This case is considered in Proposition 1(b). We find that the optimal policy involves an effective bid-cap (coupled with a reserve price set at the lowest bidder valuation) whenever C is at an intermediate level. The intuition behind ceilings is as follows. Suppose the bidders are competing among themselves. Then ceilings reduce competition among the bidders. This increases the bidders' payoff from competition, so that the bidders have less incentive to collude. We then consider the use of reserve prices. In the absence of collusion, the seller's payoff is increasing in the ceiling price. Reserve prices reduce the bidders' payoffs from collusion, thus reducing their incentive to collude. This allows the seller to set a higher ceiling price, hence the result.

For other values of C, however, the optimal policy does not involve an effective ceiling. In case C is small, preventing collusion through an effective ceiling leads to a very low ceiling price. Hence the seller's payoff would be rather small, and be dominated by a posted price mechanism with a price of $\overline{V}$. Whereas if C is large, then there is little incentive for collusion. Hence setting $R = \underline{V}$ is sufficient to prevent collusion and a bid-cap is not required. Moreover, this dominates setting $R = \overline{V}$, since the bidders with

[9]This is likely to hold if θ is not too large, and $\overline{V}$ is not too large compared to $\underline{V}$. This is satisfied for example, if $\theta = 0.65$, $\underline{V} = 2$ and $\overline{V} = 2.5$.

low valuations are not ruled out in this case.

3 Conclusion

In a simple second-price auction we examine the role of ceilings and reserve prices in controlling collusion. Under the appropriate parameter conditions, the optimal policy involves an effective ceiling (coupled with a reserve price set at the lowest bidder valuation) as long as collusion is neither too easy, nor too difficult. Thus, even though apparently anti-competitive, ceilings may promote competition by preventing collusion among agents. Further, in this case the optimal policy also involves a reserve price since it allows the seller to set a higher ceiling price and still prevent collusion.

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