The Contributions of Paul Milgrom and Robert Wilson to Auction Theory and Practice

Indian Statistical Institute, Delhi

October 22, 2020

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2020 awarded to

Paul R. Milgrom and Robert B. Wilson

"for improvements to auction theory and inventions of new auction formats"

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  Common value model: Wilson (1969)

  General model: Milgrom (1981b), Milgrom and Weber (1982)

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Other details such as minimum reserve price, entry fees, disclosure of bidder identities, bidder deposit etc. lead to many variations in practice

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Second-price auction: Bidders submit sealed bids to the auctioneer. The highest bidder wins and pays the amount of the second-highest bid.

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Multi-object version used by U.S. Treasury to sell government securities; a variation is used in Google search auctions

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Art auctions, eBay

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In IPV model, a second-price auction is equivalent to an English auction

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- Myerson (1981) showed that a second-price auction with reserve prices maximizes expected revenue among all selling mechanisms

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- Wilson introduced the common values model (a.k.a. the mineral rights model) in 1969
- This was the first theoretical analysis of the winner's curse

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Requires more sophisticated analysis than under private values

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- Wilson (1969) was one of the first to use the Harsanyi (1968) formulation of games with incomplete information – Bayesian Nash equilibrium as solution concept
- Wilson (1967), a precursor to Wilson (1969), analyzes a common value auction with extreme asymmetric information
  One bidder is informed about the common value while the other has no information signal about the value – severe winner's curse

Naive estimation and winner's curse

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- Winner's curse is **not** an equilibrium phenomenon

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# of bidders	1	2	5	10
$\mathrm{E}[\max \epsilon_i] = \mathrm{E}[\max(X_i - V)]$	0	0.56σ	1.16σ	1.54σ

Winner's curse in oil lease auctions

Bids on offshore oil tracts ($ millions), 1967-69

	Louisiana	Santa Barbara	Texas	Alaska
Highest bid	32.5	43.5	43.5	10.5
2nd highest bid	17.7	32.1	15.5	5.2
Lowest bid	3.1	6.1	0.4	0.4
Money left on table	14.8	11.4	28	5.3
Highest/Lowest ratio	10	7	109	26

From Capen, Clapp, and Campbell, "Competitive Bidding in High Risk Situations," *Journal of Petroleum Technology,* 1971, 23, 641-653.

Winner's curse in telephone rights auctions in India

Bidding for 9 of 13 licenses ($ millions) 1995

Circle	Highest bidder	Highest bid	Second-highest bid	Ratio of highest to second-highest bid
A.P.	HFCL	4,893	1,124	4.4
Delhi	HFCL	4,804	3,567	1.3
Gujarat	HFCL	4,804	1,003	4.8
Haryana	HFCL	1,293	1,003	1.3
Kerala	HFCL	3,043	401	7.6
Orissa	HFCL	658	82	8
Punjab	HFCL	2,887	1,170	2.5
UP West	HFCL	2,096	859	2.4
Bengal	HFCL	2,887	33	87.5

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- Nowadays, referred to as the **interdependent values model** as each bidder's value depends on her own signal, and on signals of other bidders

Special cases of Milgrom and Weber model

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- Obtained Bayesian Nash equilibrium in common auctions
- Obtained expected revenue ranking of these auctions:

  English auction yields greater exp. revenue than second-price auction

  Second-price auction yields greater exp. revenue than first-price auction

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- Implies that honesty is the best policy for the auctioneer

Multi-object Auctions

Auctions of divisible goods

- Wilson (1979): Divisible goods, common unknown value. After observing a private signal, each bidder submits a demand function.

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- Selling price determined by equating supply with aggregate demand
- Reasons for shading one's bid below estimated value: profit margin, winner's curse, and *price might be determined by one's marginal bid*

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An example

Values of bidders

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Exposure problem for bidder A, who risks ending up with either X or Y but not both

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- Auction ends when no one increases the bid on any object in a round

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- Auction ends when no one increases the bid on any object in a round
- Activity rules force bidders to reveal their interest early

Simultaneous multiple round auction (SMRA)

- If there are, say, 2 objects to be sold then 2 ascending-price auctions are held simultaneously, one for each object
- Bidding occurs in multiple rounds
- In each round, bidders are free to increase bids in any auction
- Auction ends when no one increases the bid on any object in a round
- Activity rules force bidders to reveal their interest early

SMRA was the outcome of two proposals made in 1994 to the F.C.C., one by Milgrom and Wilson and the other by McAfee

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Paul Milgrom's contributions to other areas in economics

- **Reputation Formation:** Milgrom and Roberts (JET 1982), Kreps, Milgrom, Roberts, and Wilson (1982)
- **Game Theory:** Milgrom and Weber (MOR 1985), Milgrom and Roberts (1990, GEB 1991)
- **Industrial Organization:** Milgrom and Roberts (Econometrica 1982, JPE 1986)
- **Organization Economics:** Milgrom and Roberts (AER 1990), Holmstrom and Milgrom (Econometrica 1987, JLEO 1991, AER 1994)
- **Financial Economics:** Glosten and Milgrom (JFE 1985)

Robert Wilson's contributions to other areas in economics

- **Reputation Formation:** Kreps and Wilson (1982a), Kreps, Milgrom, Roberts, and Wilson (1982), Wilson (1985)
- **Game Theory:** Wilson (SIAM 1971), Wilson (Econometrica 1978), Kreps and Wilson (1982a), Govindan and Wilson (2009)
- **Bargaining:** Gul, Sonnenschein, and Wilson (JET 1986), Kennan and Wilson (JEL 1993)
- **Pricing:** Wilson (OUP 1993), Chao and Wilson (AER 1987)

Thank you!