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A class of general tests for testing Independence of failure time and cause of failure In a competing risks model

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A class of general tests for testing independence of failure time and cause of failure in a competing risks model

Running Head: Test of independence for competing risks

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Abstract: In this paper we introduce a class of tests for testing independence of failure time and cause of failure for competing risks data. A class of tests using martingale approach is derived. We, then develop a test statistic using likelihood ratio procedure. Asymptotic distributions of the proposed test statistics are derived. The procedures are illustrated using a real life example. A simulation study is carried out to assess the power of the tests.

Key words: Chi-square distribution, competing risks models, likelihood ratio test, tests for independence

1. Introduction

In medical studies or in the analysis of industrial data, failure of individuals or units may be attributable to more than one cause or factor. Competing risks models are usually employed to analyze such situations. Two frame works are often used to deal with competing risks settings in which a non negative failure time variable T and a cause of failure $J \in \{1,2,\ldots,k\}$ can be observed for an individual. One approach is to use cause specific hazard $(\lambda_j(t))$ formulations, where

$$\lambda_j(t) = \lim_{\Delta t \to 0} \frac{P\{T < t + \Delta t, J = j \mid T \geq t\}}{\Delta t} \qquad j = 1,2,\ldots,k. \tag{1}$$

An alternative approach is to compare cumulative incidence functions $(F_j(t))$ where

$$F_j(t) = P(T \leq t, J = j) \qquad j = 1,2,\ldots,k. \tag{2}$$

The analysis of competing risks data using (1) and (2) is extensively discussed in literature. Crowder (2001), Kalbfleisch and Prentice (2002) and Lawless (2003) provide reviews on this topic.

In many applications within the competing risks setting, it is of interest to test equality of cumulative incidence functions (cause specific hazard rates). Aly et.al (1994) and Dykstra et.al (1995) have proposed distribution free tests for testing the equality of cause specific hazard rates against ordered alternatives. Sun and Tiwari (1998) considered the problem of testing the equality of two cumulative incidence functions. Carrirer and Kochar (2000) developed a distribution free test for the problem of testing equality of m distribution functions when the failure times are continuous. Later, Kulathinal and Gasbarra (2002) considered the problem of testing the equality of cause specific hazard rates corresponding to m competing risks in k groups. El- Barami and Kochar (2002) considered the same problem with discrete failure times using likelihood ratio test procedure. El Barami et.al (2006) developed likelihood ratio test for and against ordering of the cumulative incidence functions. For various other tests, one can refer to Kochar (1995), Lam (1998) and Alvarez-Andrade (2007).

The problem of identifiability in modeling the competing risks data in terms of latent failure times is well known (Tsiatis (1978), Crowder (2001)). This problem does not arise if the modeling of the competing risks data is done in terms of cause specific hazard rates or cause specific distribution functions. The nature of dependence between T and J is crucial and useful in such modeling. If T and J are independent, then the bivariate competing risks data reduces to two sets of data on independent variables. This motivated researchers to develop tests for independence between T and J. A class of restricted tests for testing independence between T and J for competing risks data was derived by Dykstra. et.al (1998). Dewan et.al (2004) developed test procedure for testing independence against lack of independence and specific alternatives viz. positive dependence (PQD and RTI) via conditional probabilities. Testing independence between T and J when causes of failure are missing was discussed by Dewan and Kulathinal (2007). All theses tests were based on U-statistics. In this paper, we develop a general class of tests for testing independence between T and J against the alternative that they are not independent. We use two approaches viz. martingale approach and likelihood ratio approach.

The rest of the paper is organized as follows. In Section 2, we develop a test statistic using martingale approach and discuss its limiting distribution. In Section 3, we derive a test statistic using likelihood ratio procedure and work out its asymptotic distribution. The methods are illustrated using a real data due to Hoel (1972) in Section 4. In Section 5, we carry out a simulation study to assess the performance of the tests. Finally, Section 6 provides major conclusions of the study.

2. Martingale Approach

This approach is based on cause specific hazard rates which are fundamental quantities in competing risks data as they can be estimated on the basis of failure time and cause of failure.

Suppose that there are n individuals under study. Let $N_j(t)$ denote the number of transitions from alive to death due to cause j during the time interval $(0,t]$, for $j=1,2,...,k$. We now consider the situation where the failure time T is right censored by the variable Z. In practice one could observe, $Y_i = \min(T_i, Z_i)$ and

$\delta_{ij} = I(Y_i \le Z_i, J_i = j)$ on n individuals, where J_i is observed if $Y_i = T_i$ and $I(.)$ is the usual indicator function for $j = 1,2,...,k$ and $i = 1,2,...,n$. In this situation $N_j(t) = \sum_{i=1}^{n} I(Y_i \le t, \delta_{ij} = 1)$ and $Y(t) = \sum_{i=1}^{n} I(Y_i > t)$, is the number of individuals who have survived beyond t.

The history of the entire process up to time t is represented by $\mathcal{F}_t$, where $\mathcal{F}_t$ represent the σ-field generated by the counting process $\{N_j(t), j = 1,2,...,k\}$. The cause specific hazard process is given by $Y(t)\lambda_j(t)$ for $j = 1,2,...,k$ and $t \ge 0$. The cause specific cumulative hazard rates $\Lambda_j(t)$ are defined as

$$\Lambda_j(t) = \int_0^t \lambda_j(s)\,ds \qquad j = 1,2,...,k.$$

We assume that the k failure causes are mutually exclusive and exhaustive so that an individual can have at most one realized lifetime (identifiable cause). Then the overall cumulative hazard rate $\Lambda(t)$ is obtained as

$$\Lambda(t) = \sum_{j=1}^{k} \Lambda_j(t) \qquad j = 1,2,...,k.$$

Under the assumption of independence between T and J, $\lambda_j(t)$ can be written as $\lambda_j(t) = \pi_j \lambda(t)$ where $\pi_j = P(J = j)$ for $j = 1,2,...,k$ and $\lambda(t) = \sum_1^k \lambda_j(t)$.

Now we test the null hypothesis

$$H_0: \lambda_j(t) = \pi_j \lambda(t) \qquad j = 1,2,...,k \text{ and for all } t \tag{3}$$

against

$$H_1: \lambda_j(t) \ne \pi_j \lambda(t) \quad \text{for at least one } j \text{ and for some } t. \tag{4}$$

The construction of the test statistic is based on the simple idea of comparing the estimates of both sides of (3) The Nelson-Aalen estimator (Anderson et. al. (1993)) of $\Lambda_j(t)$ and $\Lambda(t)$ are given by

$$\hat{\Lambda}_j(t) = \int_0^t \frac{c(u)\, dN_j(u)}{Y(u)}, \tag{5}$$

and

$$\hat{\Lambda}(t) = \int_0^t \frac{c(u)\, dN(u)}{Y(u)}, \tag{6}$$

where $c(u) = I\{Y(u) > 0\}$ and $dN(u) = \sum_{j=1}^{k} dN_j(u)$.

The probabilities π_j can be written as

$$\pi_j = F_j(\tau) = \int_0^{\tau} S(u)\, d\Lambda_j(u), \tag{7}$$

where $S(t) = P(T > t)$ is the survivor function of T and $\tau = \sup\{t : F(t) < 1\}$. Thus the estimator of π_j can be obtained as

$$\hat{\pi}_j = \hat{F}_j(\tau) = \int_0^{\tau} \hat{S}(u)\, d\hat{\Lambda}_j(u), \tag{8}$$

where $\hat{S}(t)$ is the well known Kaplan-Meier estimator of $S(t)$. Note that $\hat{\pi}_j$ is independent of t and in the uncensored set up, $\hat{\pi}_j$ is nothing but the proportion of failures due to the cause j. Consider a measure of departure from the null hypothesis as follows

$$Z_j(t) = \int_0^t w_j(u)\left\{ d\hat{\Lambda}_j(u) - \hat{\pi}_j d\hat{\Lambda}(u) \right\}$$

$$= \int_0^t c(u)\, w_j(u) \left[\frac{dN_j(u)}{Y(u)} - \hat{\pi}_j \frac{dN(u)}{Y(u)} \right], \tag{9}$$

where $w_j(t)$ is a locally bounded predictable weight process. By the Doob-Meyer decomposition, we have

$$M_j(t) = N_j(t) - \int_0^t \lambda_j(u)\, Y(u)\, du \qquad j = 1, 2, \ldots, k, \tag{10}$$

are zero mean martingales with respect to the increasing family of σ-fields $\{\mathcal{F}_t, t \geq 0\}$. Now (9) can be written as

$$Z_j(t) = \int_0^t c(u)\, w_j(u) \left[\frac{dM_j(u)}{Y(u)} - \hat{\pi}_j \frac{dM(u)}{Y(u)} \right], \tag{11}$$

where $M(t) = \sum_{j=1}^{k} M_j(t)$.

Defining

$$c(t)\, w_j(t) = k(t)\, Y(t) \qquad j = 1, 2, \ldots, k, \tag{12}$$

(11) becomes

$$Z_j(t) = \int_0^t k(u) \left(\delta_j - \hat{\pi}_j * dN(u) \right) \quad j = 1, 2, \ldots, k, \tag{13}$$

where $k(t)$ is a locally bounded predictable process, and $\delta_j = I(J = j)$. We note that

$\sum_{j=1}^{k} Z_j(t) = 0$. Further, Z_j's are local square integrable martingales. Under H_0, it can be

shown from Anderson et.al (1993) that $Z(t) = \left[Z_1(t), \ldots, Z_k(t) \right]^T$ is asymptotically distributed as a k variate normal with mean vector 0 and a covariance matrix consistently estimated by $W(t) = \{ W_{jm}(t) \}$, where

$$W_{jm}(t) = \int_0^t k^2(u) \left(\Delta_{jm} - \hat{\pi}_m \right) \hat{\pi}_j dN(u) \tag{14}$$

where Δ_{jm} is the Kronecker delta. Thus a test statistic is given by

$$\chi^2 = Z^T(t)\, \bar{W}(t)\, Z(t) \tag{15}$$

where $\bar{W}(t)$ is the generalized inverse of $W(t)$. If we delete the last row and last column of $W(t)$, to give say, $W_0(t)$, and let $Z_0(t) = Z_1(t),...,Z_{k-1}(t)^{T}$, (15) can be alternatively given as,

$$c^2 = Z_0^T(t)\, W_0(t)^{-1} Z_0(t) \tag{16}$$

where $W_0(t)^{-1}$ is the ordinary inverse of $W(t)$. Under H_0, c^2 is asymptotically distributed as chi-square with $k-1$ degrees of freedom (see Anderson et al. (1993)).

The efficiency of the procedure obviously depends on the choice of the weight function $w_j(t)$. Possible choices for weight function in (11) are

a) $k(t) = S(t)$

b) $k(t) = S^2(t)$

c) $k(t) = Y(t)\,S(t)$

and

d) $k(t) = Y(t)$

When $k(t) = Y(t)$, we get

$$Z_j(\quad) = \int_0 Y(u)\,dN_j(u) - \int_0 Y(u)\,dN(u).$$

Thus, (13) is the generalization of the Wilcoxon and Kruskal-Wallis tests to right censored data due to Gehan (1965) and Breslow (1970).

The efficiency of these tests obviously depends on the choice of weight function. A possible approach to find the value of $k(t)$ that minimizes the mean squared error of the test statistics. However, the exact mean squared error of the test statistic is not available. In practice, we can choose an optimal weight function using bootstrap procedure, which will be discussed in Section 4.

3. Likelihood Ratio Test

Let $N^*(t) = \{(N_1(t),...,N_k(t)) : t \in \mathcal{T}\}$ be a k-variate counting process with intensity process $\lambda^* = (\lambda_1^*,...,\lambda_k^*)'$. Then the likelihood L based on a sample is given by (Anderson et al. (1993))

$$L = \prod_{t \in \mathcal{T}} \prod_{j=1}^{k} \lambda_j^*(t)^{dN_j(t)} \exp\left(-\int_0^t \sum_{j=1}^{k} \lambda_j^*(t)\,dt\right) \tag{17}$$

where $\prod_{t \in \mathcal{T}}$ represents the product integral.

Since $\lambda_j^*(t) = \lambda_j(t)\,Y(t)$, under H_0, the likelihood (17) becomes

$$L^0 = \prod_{t \in \mathcal{T}} \prod_{j=1}^{k} \left(\lambda(t)\,Y(t)\,\pi_j\right)^{dN_j(t)} \exp\left(-\int_0^t \sum_{j=1}^{k} \lambda_j(t)\,Y(t)\,dt\right) \tag{18}$$

Thus, we can obtain the likelihood ratio test statistic for testing (3) as

$$Q = \frac{\prod_{t} \prod_{j=1}^{k} \hat{\lambda}(t)^{dN_j(t)}\,\hat{\pi}_j^{dN_j(t)}}{\prod_{t} \prod_{j=1}^{k} \hat{\lambda}_j(t)^{dN_j(t)}} \tag{19}$$

where $\hat{\lambda}_j(t)$, $\hat{\pi}_j$ and $\hat{\lambda}(t)$ are the maximum likelihood estimates of $\lambda_j(t)$, π_j and $\lambda(t)$.

In the following, we can establish that the asymptotic distribution of $-2\log Q$ is a chi-square distribution with $k-1$ degrees of freedom.

From (19), we can have that,

$$-2\log Q = 2\int_0^t \sum_{j=1}^{k} \left(\log \hat{\lambda}_j(t) - \log\left(\hat{\pi}_j \hat{\lambda}(t)\right)\right) dN_j(t)$$

$$= 2\int_0^t \sum_{j=1}^{k} \left(\log \hat{\lambda}_j(t) - \log\left(\hat{\pi}_j \hat{\lambda}(t)\right)\right) \hat{\lambda}_j(t) Y(t)\,dt \tag{20}$$

By Taylor's series expansion, (20) can be written as

$$-2\log Q = 2\int_0^t \sum_{j=1}^{k} \left(\hat{\lambda}_j(t) - \hat{\pi}_j \hat{\lambda}(t)\right) Y(t)\,dt + \int_0^t \sum_{j=1}^{k} \frac{\left(\hat{\lambda}_j(t) - \hat{\pi}_j \hat{\lambda}(t)\right)^2}{\hat{\lambda}_j(t)} Y(t)\,dt + O\left(\frac{1}{n}\right)$$

(21)

Under H_0, $\sum_{j=1}^{k} \left(\hat{\lambda}_j(t) - \hat{\pi}_j \hat{\lambda}(t)\right) Y(t) = 0$ and thus (21) can be asymptotically written as

$$-2\log Q = \int_0^t \sum_{j=1}^{k} \frac{\left(\hat{\lambda}_j(t) - \hat{\pi}_j \hat{\lambda}(t)\right)^2}{\hat{\pi}_j \hat{\lambda}(t)} Y(t)\,dt \qquad (22)$$

To find the asymptotic distribution of $-2\log Q$, we write $\log L$ in the Taylor's series expansion around $\hat{\lambda}_j(t)$ and $\hat{\pi}_j \hat{\lambda}(t)$, those provide

$$\left(\hat{\lambda}_j(t) - \lambda_j(t)\right) \cong D_1 \frac{\partial \log L}{\partial \lambda_j(t)} \qquad (23)$$

and

$$\left(\hat{\pi}_j \hat{\lambda}(t) - \lambda_j(t)\right) \cong D_2 \frac{\partial \log L}{\partial \lambda_j(t)} \qquad (24)$$

where D_1 and D_2 are respectively, the inverse of $E\left(-\frac{\partial^2 \log L}{\partial \lambda_j(t)^2}\right)$ evaluated at $\lambda_j(t)$ and $\lambda(t)\pi_j$. From (23) and (24) , it follows that

$$\left(\hat{\lambda}_j(t) - \hat{\pi}_j \hat{\lambda}(t)\right) \cong \left(D_1 - D_2\right) \frac{\partial \log L}{\partial \lambda_j(t)} \qquad (25)$$

Since $\frac{\partial \log L}{\partial \lambda_j(t)}$ is asymptotically normally distributed with mean zero and variance

$D = E\left[-\frac{\partial^2 \log L}{\partial \lambda_j(t)^2}\right]$ and $\left(\hat{\Lambda}_j(t) - \hat{\pi}_j \hat{\Lambda}(t)\right) \xrightarrow{d} N\left(0, (D_1 - D_2)^2 D\right)$, where

$$(D_1 - D_2)^2 D = \frac{\left(\pi_j \lambda(t) - \lambda_j(t)\right)^2}{Y(t)\, \pi_j \lambda(t)}.$$

For large n, it follows from (25) that,

$$\frac{\left(\hat{\Lambda}_j(t) - \hat{\pi}_j \hat{\Lambda}(t)\right)}{\left(\hat{D}_1 - \hat{D}_2\right)\sqrt{\hat{D}}} \xrightarrow{d} N(0,1) \tag{26}$$

Now, $\left(\hat{D}_1 - \hat{D}_2\right)\sqrt{\hat{D}} = \frac{\hat{\pi}_j \hat{\Lambda}(t)}{Y(t)}\left[1 - \frac{\hat{\Lambda}_j(t)}{\hat{\pi}_j \hat{\Lambda}(t)}\right]^2$. Under H_0, this is asymptotically equal to

$\frac{\hat{\pi}_j \hat{\Lambda}(t)}{Y(t)}$. Thus, $\sqrt{Y(t)}\,\frac{\left(\hat{\Lambda}_j(t) - \hat{\pi}_j \hat{\Lambda}(t)\right)}{\sqrt{\hat{\pi}_j \hat{\Lambda}(t)}} \xrightarrow{d} N(0,1)$ and hence $Q^* = -2\log Q$ follows chi-square distribution with $k-1$ degrees of freedom.

4. Data Analysis

The two classes of tests are applied to a competing risks data given in Hoel (1972). The data are obtained from a laboratory experiment on RFM strain male mice, which had received a radiation dose of 300 rads at ages of 5 to 6 weeks and were kept in a conventional germ-free environment. There are three causes of death viz. Thymic lymphoma, reticulam cell sarcoma and other causes. The data were studied by different authors including Aly.et.al (1994), Kocher et.al (2002) and Dewan et.al (2004). The estimates of cumulative hazard rate functions due to the three different causes are given in Figure 1.

(“Figure 1 here”)

Figure 1 shows that, the estimates of cause specific cumulative hazard rate functions due to ‘other causes’ yields high values comparing to the same from thymic lymphoma and reticulum cell sarcoma at later ages.

To apply the test procedure (15), we consider different weight functions, (a)-(d) given in Section 2 .Test statistics on theses cases were calculated along with their P-values. Let Q_1, Q_2, Q_3, Q_4 and Q^* represent the test statistics obtained using weight functions (a),(b),(c),(d) and the likelihood ratio test respectively. Table 1 provides the values of the test statistics along with their P-values

Then we consider the ‘other causes’ as censored variables and analyse the same data as a two risks problem. The estimates of the cause specific hazard rate functions for two types of cancer - thymic lymphoma and reticulum cell sarcoma, when considering ‘other causes’ as censored variables are given in Figure 2.

(“Figure 2 here”)

Figure 2 shows that, the estimates of cause specific cumulative hazard rate function due to reticulum cell sarcoma possess high values comparing to the same from thymic lymphoma at later ages. The values of test statistics along with their P-values are given in Table 1. From Table 1, it follows that, the test statistic values for all the four weight functions (a)-(d) are highly significant irrespective of the presence of censoring. Likelihood ratio test statistic is highly significant in both censored and uncensored set up. Thus the failure time and the cause of failure can safely be assumed to be not independent for Hoel’s data set.

(“Table 1 here”)

To find the optimal $k(t)$, we use the bootstrap procedure. We choose optimal $k(t)$, as the value of $k(t)$, which minimizes the bootstrap mean square error estimate for the proposed test statistics. The bootstrap technique for determining optimal $k(t)$ is applied to the real data given above. The bootstrap estimates of the absolute value of the biases and the mean square errors of test statistics Q_1, Q_2, Q_3 and Q_4, computed from the real data based on 250 bootstrap samples of size 181 are given in Table 2.

("Table 2 here")

From Table 2, it is clear that, the estimates of the mean square error based on the bootstrap samples yields the lowest value for the statistic Q_4 in both uncensored and censored set up. Thus we can conclude that the weight function corresponding to Q_4, that is, $k(t) = Y(t)$, is the optimal choice of weight function and provides the best conclusion, if mean squared error is the optimality criterion. Thus, we can conclude that time of failure and causes of failure are not independent for the above failure time data

5. Simulation Study

We carry out a simulation study to assess the performance of test statistics. In martingale approach, the four different weight functions (a)-(d) described in Section 2 are considered. We consider two causes of failure. Lifetimes are generated from exponential distribution with two different parameters. We consider both no censoring and 20% censoring situations. In the censored situation, observations are censored by uniform random variable over $(0, a)$, where a is chosen in such a way that 20% of the observations are censored. We generate random sample of size $n =$ 50,100 and 250 from exponential distributions with hazard rate 2 and hazard rate 8 with proportions 0.2 and 0.8 respectively. Empirical type I error of the test is calculated by generating 1000 such random samples. Empirical power of the test is also calculated in a similar fashion, by generating lifetimes from exponential distributions with hazard rate 2 and 8 proportions 0.8 and 0.2 respectively.

To assess the performance of the likelihood ratio test, random samples of size $n =$ 50,100 and 250 are generated 1000 times. The exponential distribution with same parameters and proportions described above is used for computing empirical type I error and power. Tables 3 and 4 provide the empirical type I error and power, in percent, for different test statistics developed using martingale approach. Table 5 presents the empirical type I error and power of the likelihood ratio test.

("Table 3 here")

From Table 3, it is clear that all the four test statistics have type I error is close to the chosen level   for all values of n. For larger sample size, censoring seems to increase the type I error marginally.

("Table 4 here")

Table 4 shows that all the four test statistics have good power in general. As sample size increases, the power of all the four statistics increases, especially for Q_4. However, Q_2 and Q_3 behaves in similar fashion in most of the simulations, essentially because of the choice of weight functions. The presence of censoring does not provide much difference to the power of the tests.

("Table 5 here")

The results of simulation study using likelihood ratio test show that the test has good power

6. Conclusion

In this paper, we proposed a class of tests for testing independence of failure time and cause of failure in competing risks set up. A class of tests using martingale approach and a test statistic using likelihood ratio method are derived. The nature of dependence between time of failure T and cause of failure J is crucial and useful if the modeling is done in terms of cause specific hazard rates. The performance of the martingale approach obviously depends on the choice of weight function. In practice, one can choose the optimum weight function using bootstrap procedure. The tests are applied to a set of mortality data available in Hoel (1972) and the test statistics suggested that the time of failure and cause of failure are not independent for this data. A simulation study is carried out to assess the empirical type I error and power of the proposed tests. The results of simulation also confirm the adequacy of the proposed techniques in distinguishing between the null and the alternative hypothesis.

The optimal choice of $k(t)$, depends on the underlying distribution, which is difficult to determine analytically. More simulation studies are required to investigate this aspect, which will be presented in a separate paper. We are also looking into the extension of these techniques to the case when some of the causes of failure are unknown.

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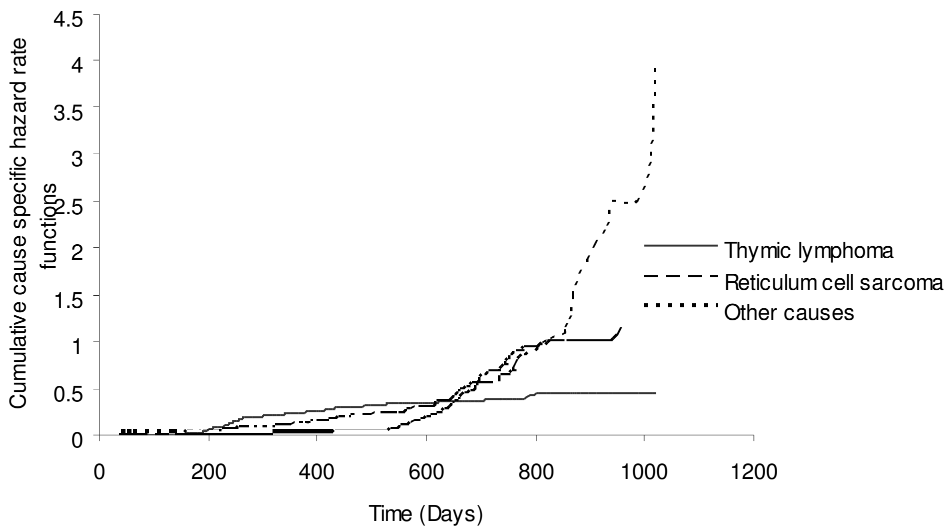

Figure 1 The estimates of the cumulative hazard rate functions of death from thymic lymphoma, reticulum cell sarcoma and other causes.

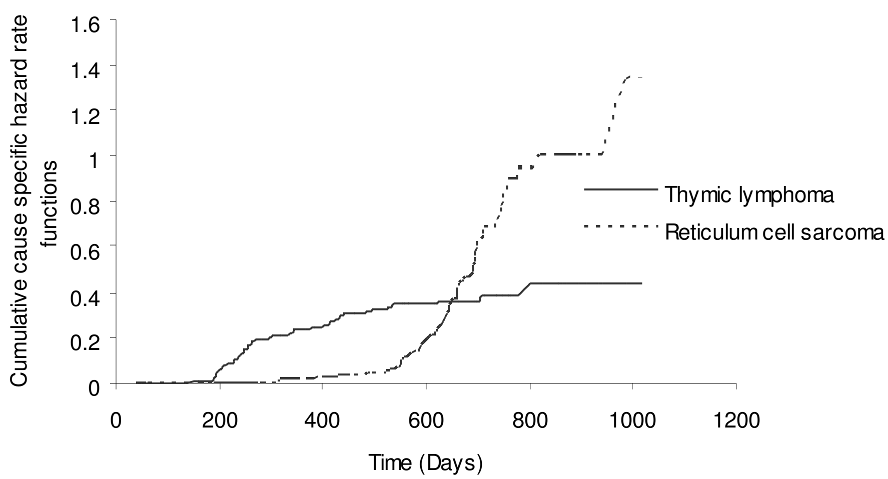

Figure 2 The estimates of the cumulative hazard rate functions due to causes thymic lymphoma and reticulum cell sarcoma in the censored case

Table 1 Test statistic values using different weight functions and Likelihood ratio test statistic

Test statistic	Test statistic value for 3 causes (uncensored data)	P value	Test statistic value for 2 causes (censored data)	P value
Q_1	13.348	<0.01	18.799	< 0.01
Q_2	26.206	<0.01	32.811	< 0.01
Q_3	26.08	<0.01	38.597	< 0.01
Q_4	13.223	<0.01	27.411	<0.01
Q^*	162.597	< 0.01	64.339	<0.01

Table 2 Bootstrap estimates of absolute value of the biases and mean square error of test statistics Q_1, Q_2, Q_3 and Q_4 based on 250 bootstrap samples.

Test statistic	Bias and MSE without censoring		Bias and MSE with censoring	
Q_1	0.50059	0.97267	1.1213	2.3693
Q_2	0.52775	0.908	0.37406	1.2235
Q_3	0.44099	0.84408	1.07	2.4634
Q_4	0.15795	0.70372	0.27757	0.83607

Table 3 Empirical type I error (%) of different test statistics

n	Significance level (%)	Test statistic	No Censoring	20% Censoring
50	5	Q_1	4.4	4.4
50	5	Q_2	5.2	5.1
50	5	Q_3	4.9	4.9
50	5	Q_4	5.1	5.6
50	1	Q_1	1.0	1.2
50	1	Q_2	1.4	1.5
50	1	Q_3	0.9	0.9
50	1	Q_4	1.1	5.7
100	5	Q_1	4.3	4.1
100	5	Q_2	5.2	5.1
100	5	Q_3	4.8	4.2
100	5	Q_4	5.0	5.0
100	1	Q_1	1.1	1.0
100	1	Q_2	1.3	1.3
100	1	Q_3	0.9	0.8
100	1	Q_4	1.4	4.1
250	5	Q_1	3.8	4.1
250	5	Q_2	5.1	5.2
250	5	Q_3	4.8	5.4
250	5	Q_4	4.8	4.9
250	1	Q_1	0.9	1.2
250	1	Q_2	0.8	1.5
250	1	Q_3	1.2	1.6

250	1	Q_4	1.4	1.8

Table 4 Empirical type power (%) of different test statistics

n	Significance level (%)	Test statistic	No Censoring	20% Censoring
50	5	Q_1	50.0	48.9
50	5	Q_2	51.0	51.9
50	5	Q_3	52.8	44.6
50	5	Q_4	23.9	34.4
50	1	Q_1	12.4	10.2
50	1	Q_2	40.1	57.6
50	1	Q_3	44.1	42.8
50	1	Q_4	14.1	18.9
100	5	Q_1	70.5	78.3
100	5	Q_2	76.4	98.2
100	5	Q_3	79.2	81.4
100	5	Q_4	43.0	44.1
100	1	Q_1	16.5	21.1
100	1	Q_2	66.8	93.2
100	1	Q_3	68.1	58.5
100	1	Q_4	28.4	30.9

250	5	Q_1	98.6	95.6
250	5	Q_2	98.1	97.4
250	5	Q_3	98.2	94.3
250	5	Q_4	98.5	94.1
250	1	Q_1	87.1	77.2
250	1	Q_2	88.6	87.8
250	1	Q_3	89.1	90.1
250	1	Q_4	87.2	90.6

Table 5 Empirical type I error and power (%) of the likelihood ratio test statistic

n	Significance level (%)	No Censoring		20% Censoring	
		Empirical type I error	Empirical power	Empirical type I error	Empirical power
50	5	5.5	95.9	5.2	84.8
100	5	4.3	97.1	5.0	96.1
250	5	4.0	99.3	3.9	95.8
50	1	1.0	77.8	1.0	71.2
100	1	0.8	88.5	0.7	60.6

250	1	0.6	98.1	0.6	87.1